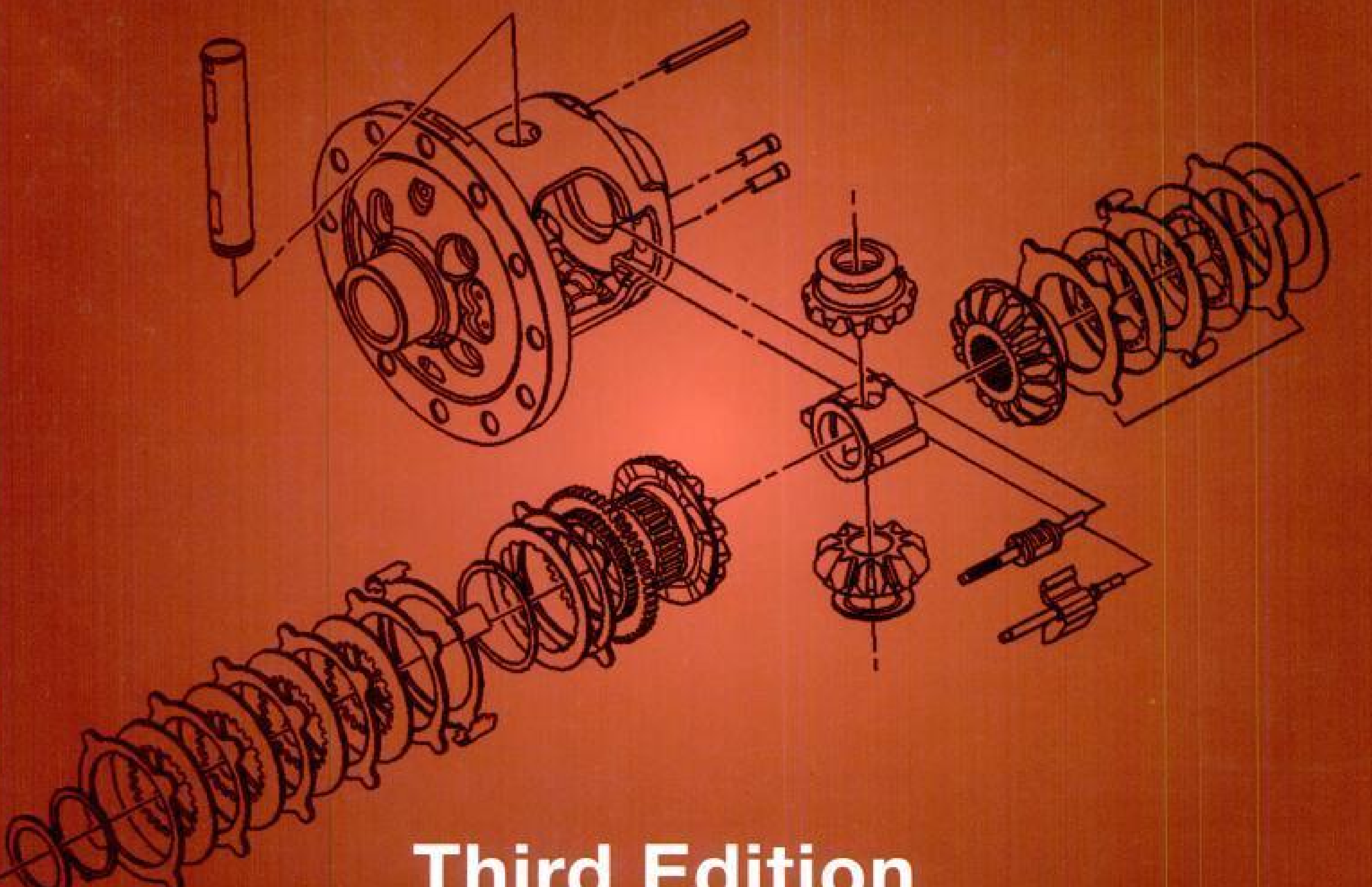

Third Edition

THEORY OF MACHINES

S S RATTAN

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PREFACE

Mechanisms and machines have considerable fascination for most students of mechanical engineering since the theoretical principles involved have immediate applications to practical problems. The main objective of writing this book has been to give a clear understanding of the concepts underlying engineering design. A sincere effort has been made to maintain the physical perceptions in the various derivations and to give the shortest comprehending solution to a variety of problems. The parameters kept in mind while writing the book are the coverage of contents, prerequisite knowledge of students, lucidity of writing, clarity of diagrams and the variety of solved and unsolved numerical problems.

The book is meant to be useful to the degree-level students of mechanical engineering as well as those preparing for AMIE and various other competitive examinations. However, diploma-level students will also find the book to be highly useful. The book will also benefit postgraduate students to some extent as it also contains advanced topics like curvature theory, analysis of rigid and elastic cam systems, complex number and vector methods, force balancing of linkages and field balancing. The salient features of the book are

- Concise and compact covering all major topics
- Presentation of concepts in a logical, innovative and lucid manner
- Evolving the basic theory from simple and readily understood principles
- A balanced presentation of the graphical and analytical approaches
- Computer programs in user-friendly C-language
- Large number of solved examples
- Summary, review questions as well as a number of unsolved problems at the end of each chapter
- An appendix containing objective-type questions
- Another appendix containing important relations and results

It is expected that the students using this book might have completed a course in applied mechanics. The book is divided broadly into two sections, kinematics and dynamics of machines. Kinematics involves study from the geometric point of view to know the displacement, velocity and acceleration of various components of mechanisms, whereas dynamics is the study of the effects of the applied and inertia forces. Chapters 1 to 11 are devoted to the study of the kinematics and the rest to that of dynamics. **Chapter 1** introduces the concepts of mechanisms and machines. **Chapters 2 and 3** describe graphical methods of velocity and acceleration analysis whereas the analytical approach is discussed in **Chapter 4**. Synthesis or designing of mechanisms is important to have the desirable motion of various components of machinery—the detail procedures for the same, both graphical and analytical, are given in **Chapter 5**. Various types of mechanisms with higher number of links are discussed in **Chapter 6**. Friction in various components of machines is very important as it affects their efficiency and is described in **Chapter 8**. Cams, belts, gears, gear trains are meant to transmit power from one shaft to another and are discussed in **chapters 7, 9, 10 and 11** respectively.

Forces are mainly of static and dynamic nature. **Chapters 12 and 13** are devoted to their effects on the components of the mechanisms. **Chapter 13** also includes the topic of flywheels which are essential components for rotary machines to regulate speeds. Speed regulation is also affected by governors which are described in **Chapter 16**. Unbalanced forces and vibrations in various components of rotating machines are mostly undesirable since the efficiency is reduced. A detailed study of these is undertaken in **chapters 14 and 18**. Brakes are essential for any moving components of machinery and are discussed in **Chapter 15**.

Moving bodies like aeroplanes, ships, two- and four-wheelers, etc., experience gyroscopic effect while taking turns. It is described in **Chapter 17**. Automatic control of machinery is very much desirable these days and an introduction of the same is given in **Chapter 19**.

The first edition of the book aimed at providing the fundamentals of the subject in a simple manner for easy comprehension by students. Simple mathematical methods were preferred instead of more elegant but less obvious methods so that those with limited mathematical skills could easily understand the expositions. However, to make the book more purposeful and acceptable to a wider section of users, the second edition also consisted of methods involving vector and complex numbers usually preferred by those who excel in mathematical skills. Such methods frequently lead to computer-aided solutions of the problems. The computer programs were rewritten in the more user-friendly C language. A Summary of each chapter was added at the end and theoretical questions were added to the exercises. One appendix containing objective-type questions was also included. All the previous figures were redrawn.

The present edition is aimed at making the book more exhaustive. Many more worked examples as well as unsolved problems have been added. Many new sections have been added in most of the chapters apart from rewriting some previous sections. Another appendix containing important relations and results has also been added. Effort has been made to remove all sorts of errors and misprints as far as possible. In spite of addition of a large amount of material, care has been taken to let the book remain concise and compact. Hints to most of the numerical problems at the end of each chapter have been provided at the publisher's website of the book for the benefit of average and weak students. Full solutions of the same are available to the faculty members at the same site. The facility can be availed by logging on to http://www.mhhe.com/rattan/tom3e.

I am grateful to all those teachers and students who pointed out errors and mistakes of the previous editions and also gave many valuable suggestions. I acknowledge the efforts of the editorial staff of Tata McGraw Hill Education Private Limited for bringing out the new edition in an excellent format.

Finally, I make an affectionate acknowledgement to my wife, Neena, and my children, Ravneet and Jasmeet, for their patience, support and putting up with it all so cheerfully. But for their sacrifice, I would not have been able to complete this work in the most satisfying way.

For further improvement of the book, readers are requested to post their comments and suggestions at ss_rattan@hotmail.com.

S S Rattan

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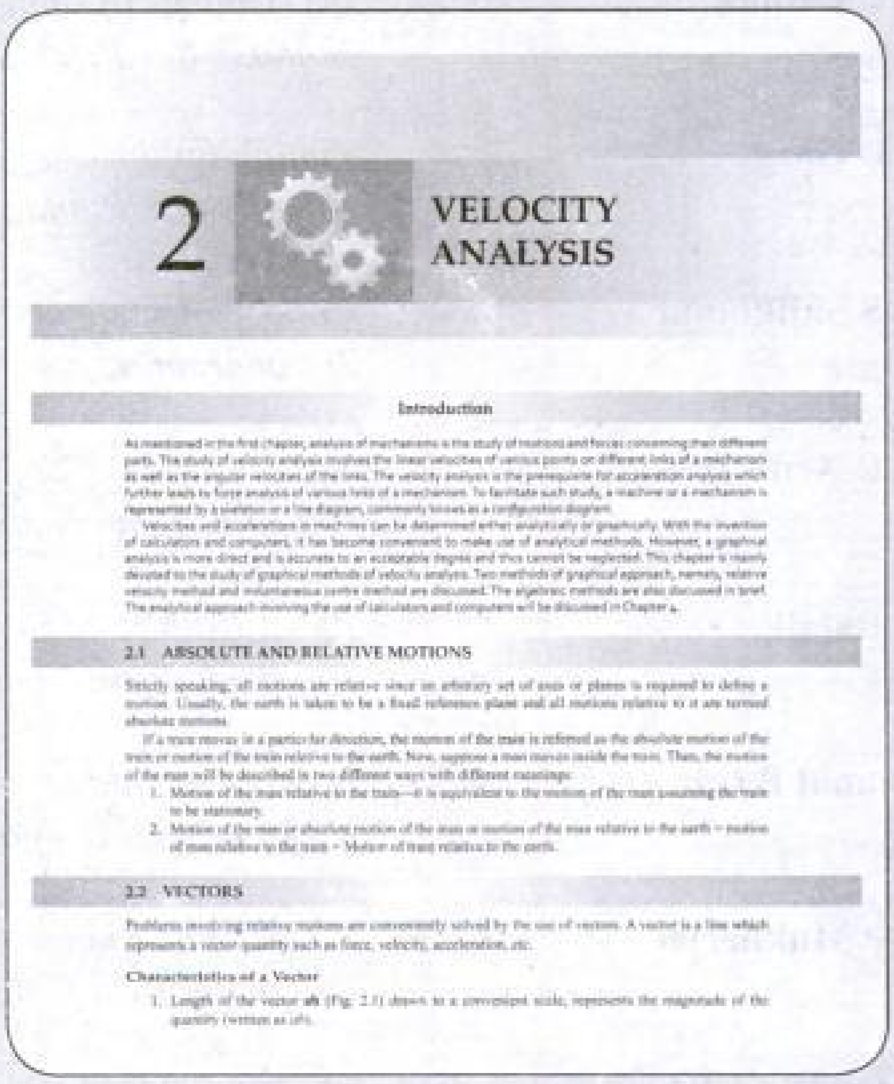
VISUAL WALKTHROUGH

Introduction at the beginning of each chapter sums up the aim and contents of the chapter.



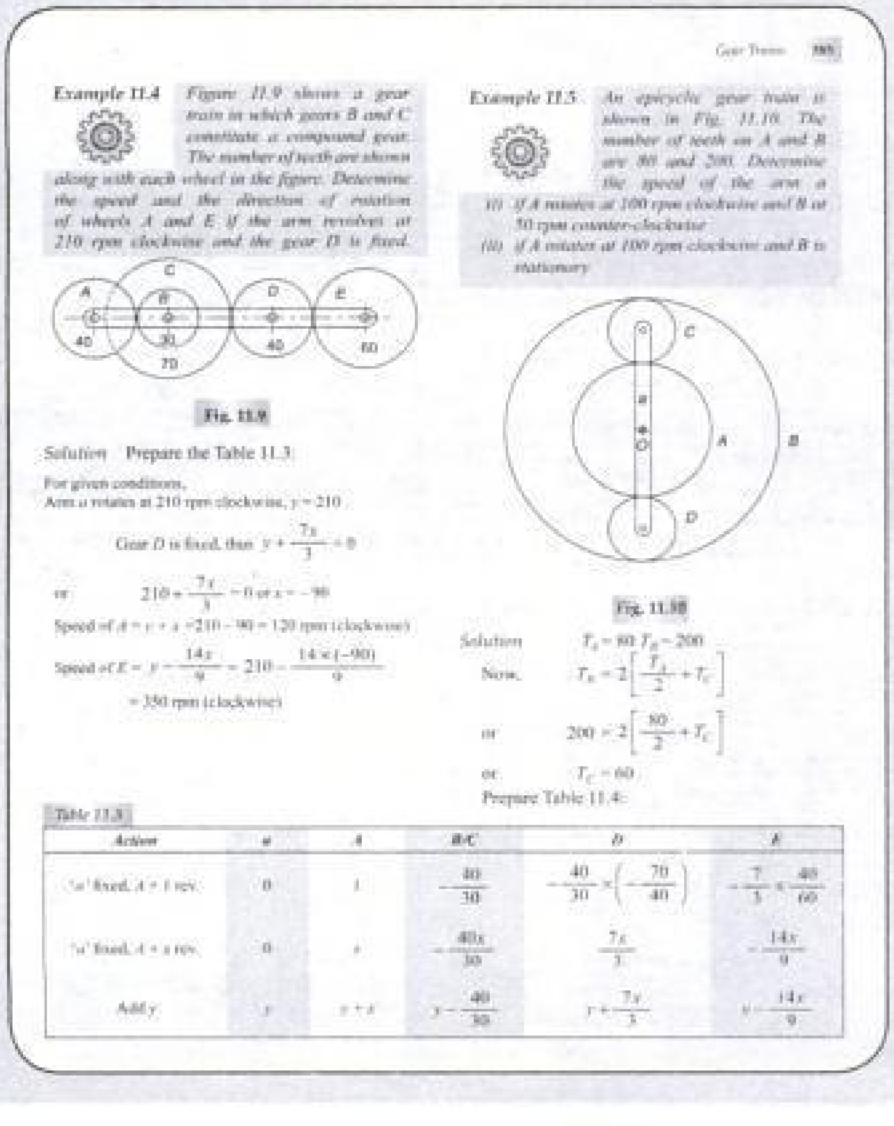
2 VELOCITY ANALYSIS

Introduction

2.1 ABSOLUTE AND RELATIVE MOTIONS

2.2 VECTORS

Characteristics of a Vector



Example 11.4

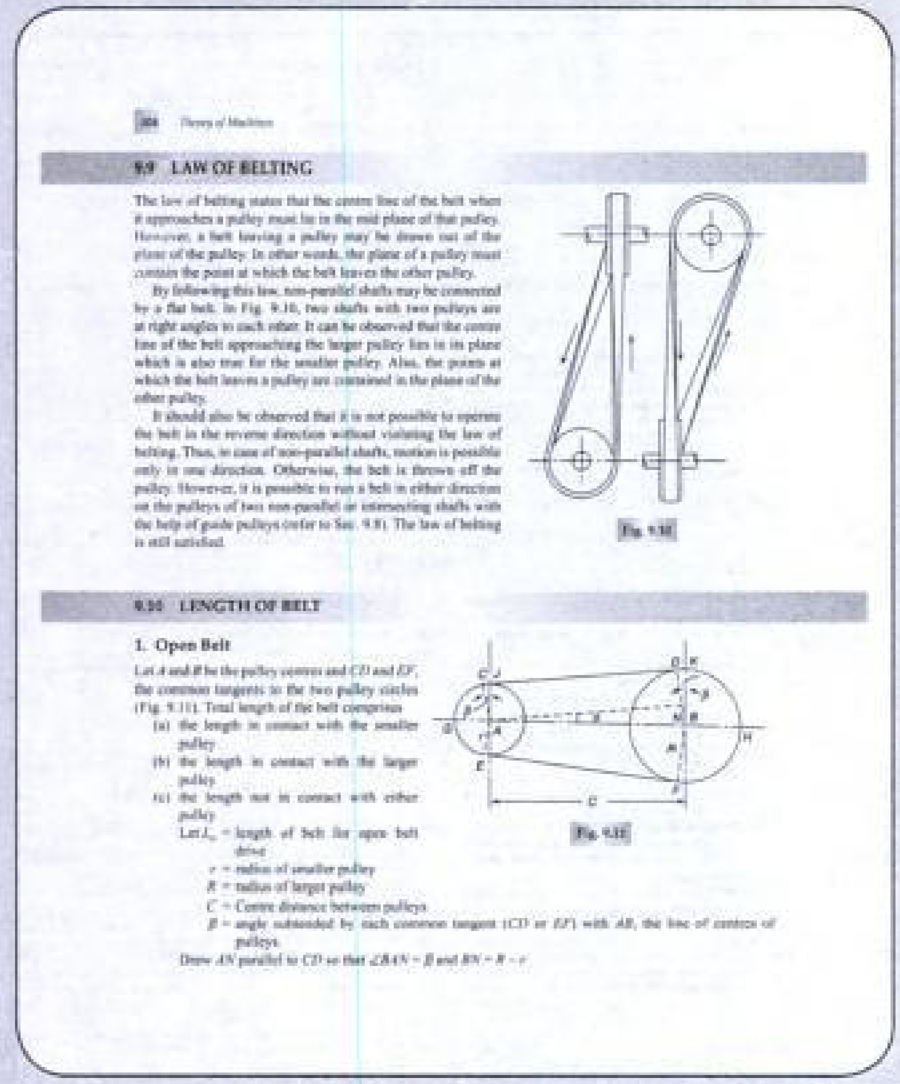
Example 11.5

Fig. 11.9

Fig. 11.10

Table 11.3

A variety of solved examples are given to reinforce the concepts.



9.9 LAW OF BELTING

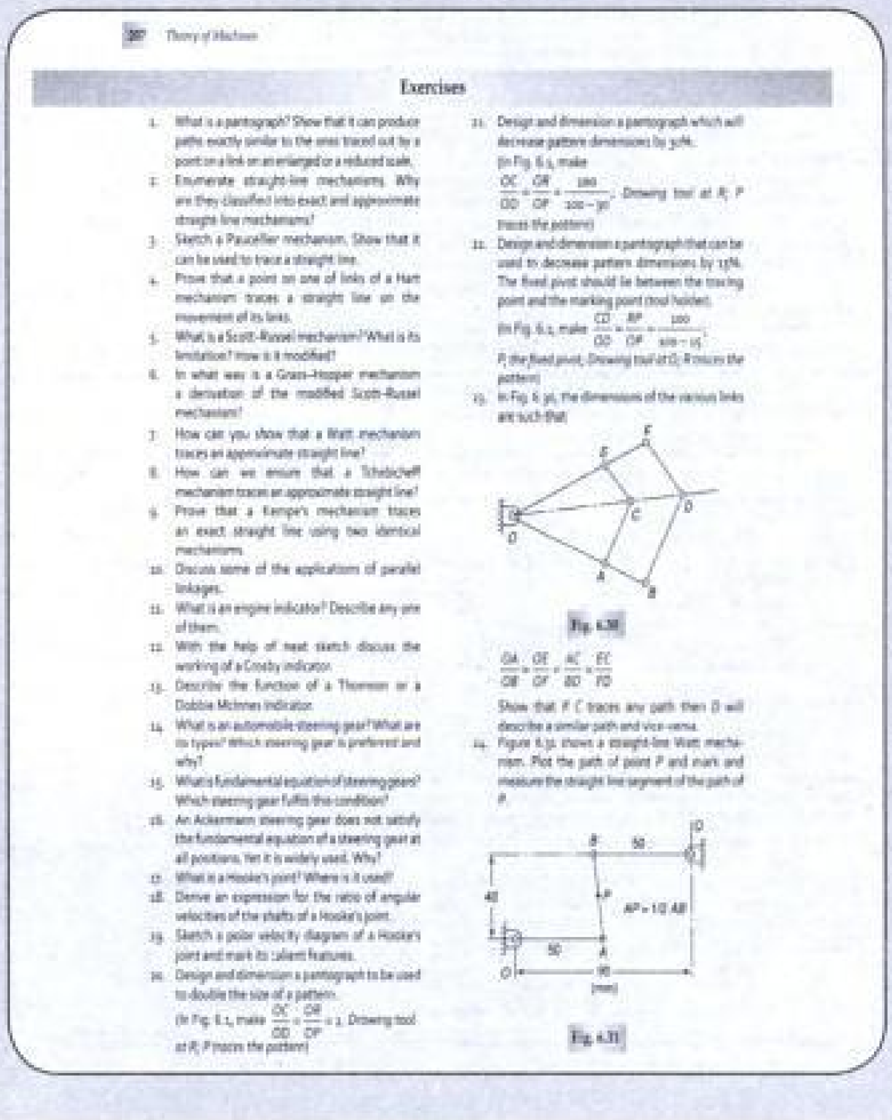
Fig. 9.10

9.10 LENGTH OF BELT

1. Open Belt

Fig. 9.11

Each chapter has a concise and comprehensive treatment of topics with emphasis on fundamental concepts.



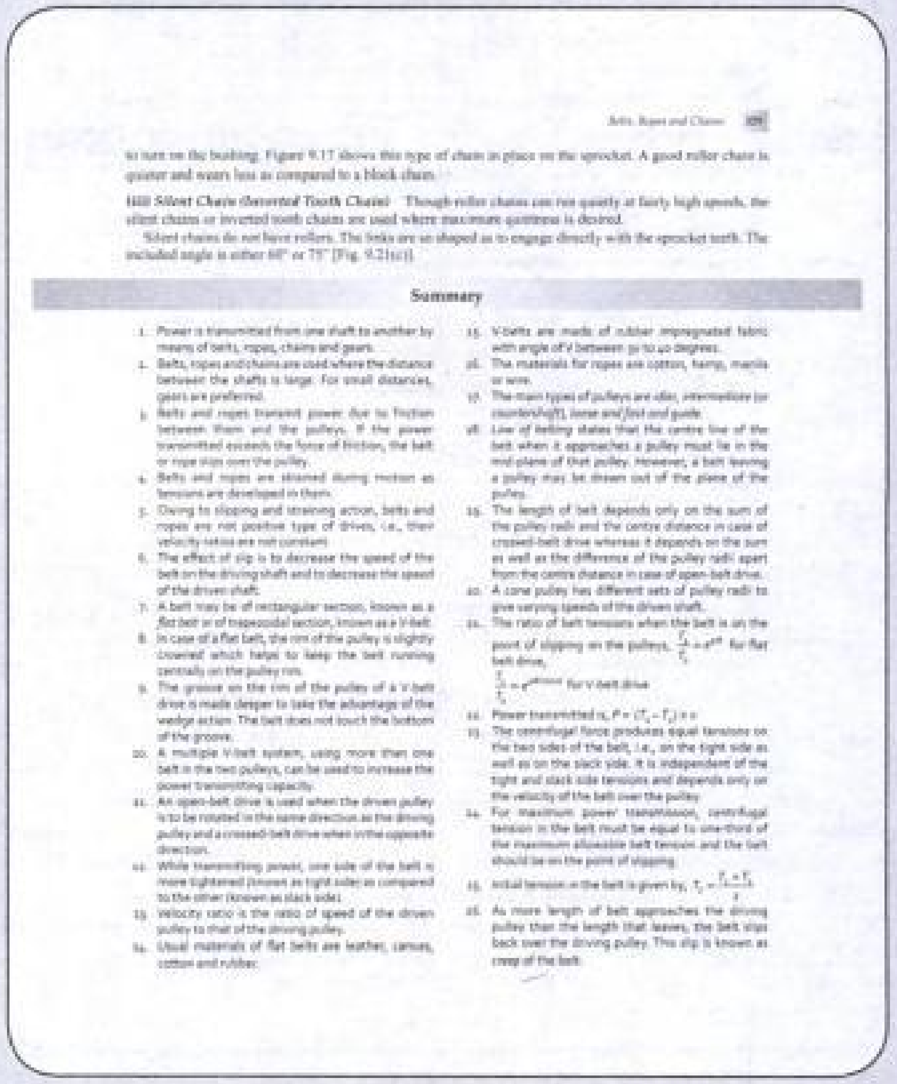
Exercises

Fig. 6.30

Fig. 6.31

A number of theoretical questions and unsolved exercises are given for practice to widen the horizon of comprehension of the topic.

A Summary at the end of each chapter recapitulates the inferences for quick revision.



Belts, Ropes and Chains 419

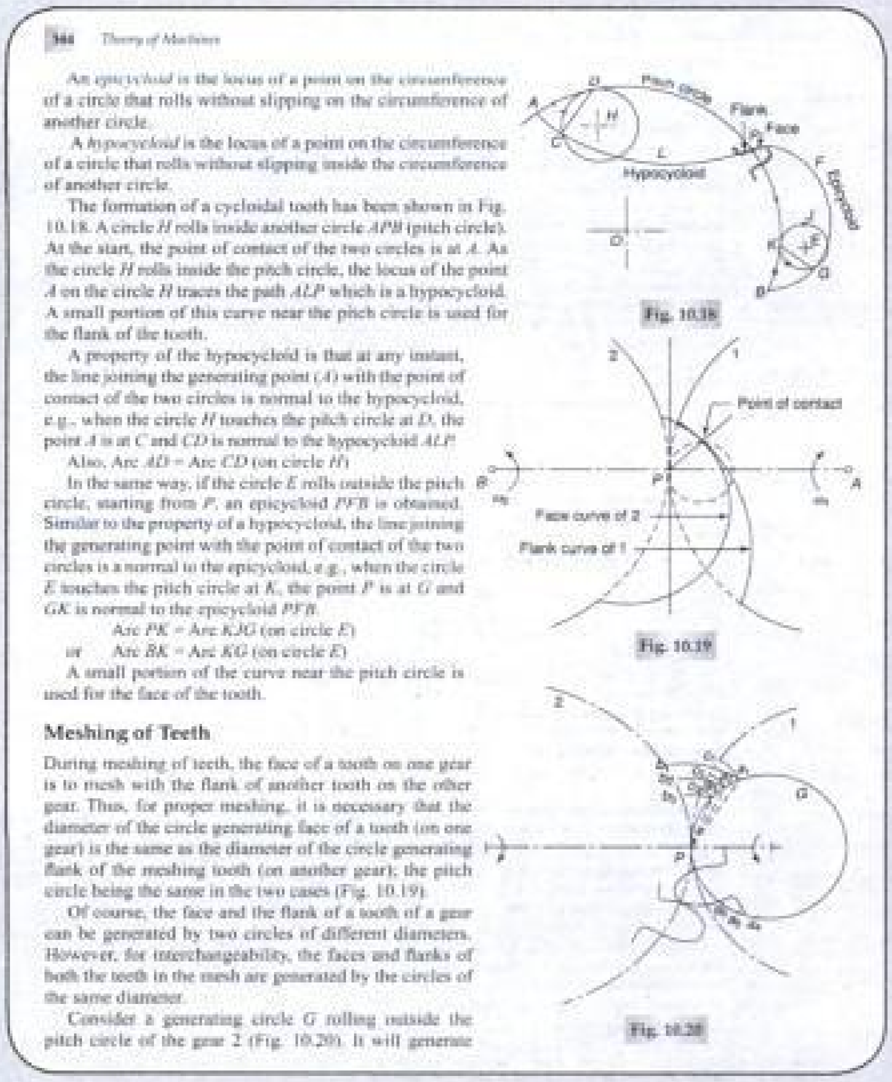
to turn on the bushing. Figure 9.17 shows this type of chain in place on the sprocket. A good roller chain is quieter and wears less as compared to a block chain.

(iii) Silent Chain (Inverted Tooth Chain) Though roller chains can run quietly at fairly high speeds, the silent chains or inverted tooth chains are used where maximum quietness is desired.

Silent chains do not have rollers. The links are so shaped as to engage directly with the sprocket teeth. The included angle is either 60° or 75° [Fig. 9.21(c)].

Summary

1. Power is transmitted from one shaft to another by means of belts, ropes, chains and gears.
2. Belts, ropes and chains are used where the distance between the shafts is large. For small distances, gears are preferred.
3. Belts and ropes transmit power due to friction between them and the pulleys. If the power transmitted exceeds the force of friction, the belt or rope slips over the pulley.
4. Belts and ropes are strained during motion as tensions are developed in them.
5. Owing to slipping and straining action, belts and ropes are not positive type of drives, i.e., their velocity ratios are not constant.
6. The effect of slip is to decrease the speed of the belt on the driving shaft and to decrease the speed of the driven shaft.
7. A belt may be of rectangular section, known as a *flat belt* or of trapezoidal section, known as a *V-belt*.
8. In case of a flat belt, the rim of the pulley is slightly *crowned* which helps to keep the belt running centrally on the pulley rim.
9. The groove on the rim of the pulley of a V-belt drive is made deeper to take the advantage of the wedge action. The belt does not touch the bottom of the groove.
10. A multiple V-belt system, using more than one belt in the two pulleys, can be used to increase the power transmitting capacity.
11. An open-belt drive is used when the driven pulley is to be rotated in the same direction as the driving pulley and a crossed-belt drive when in the opposite direction.
12. While transmitting power, one side of the belt is more tightened (known as tight side) as compared to the other (known as slack side).
13. Velocity ratio is the ratio of speed of the driven pulley to that of the driving pulley.
14. Usual materials of flat belts are leather, canvas, cotton and rubber.
15. V-belts are made of rubber impregnated fabric with angle of V between 30 to 40 degrees.
16. The materials for ropes are cotton, hemp, manila or wire.
17. The main types of pulleys are idler, intermediate (or countershaft), loose and fast and guide.
18. *Law of belting* states that the centre line of the belt when it approaches a pulley must lie in the mid-plane of that pulley. However, a belt leaving a pulley may be drawn out of the plane of the pulley.
19. The length of belt depends only on the sum of the pulley radii and the centre distance in case of crossed-belt drive whereas it depends on the sum as well as the difference of the pulley radii apart from the centre distance in case of open-belt drive.
20. A cone pulley has different sets of pulley radii to give varying speeds of the driven shaft.
21. The ratio of belt tensions when the belt is on the point of slipping on the pulleys, $\frac{T_1}{T_2} = e^{\mu\theta}$ for flat belt drive,
$\frac{T_1}{T_2} = e^{\mu\theta \operatorname{cosec} \alpha}$ for V-belt drive
22. Power transmitted is, $P = (T_1 - T_2) \times v$
23. The centrifugal force produces equal tensions on the two sides of the belt, i.e., on the tight side as well as on the slack side. It is independent of the tight and slack side tensions and depends only on the velocity of the belt over the pulley.
24. For maximum power transmission, centrifugal tension in the belt must be equal to one-third of the maximum allowable belt tension and the belt should be on the point of slipping.
25. Initial tension in the belt is given by, $T_i = \frac{T_1 + T_2}{2}$
26. As more length of belt approaches the driving pulley than the length that leaves, the belt slips back over the driving pulley. This slip is known as creep of the belt.



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An *epicycloid* is the locus of a point on the circumference of a circle that rolls without slipping on the circumference of another circle.

A *hypocycloid* is the locus of a point on the circumference of a circle that rolls without slipping inside the circumference of another circle.

The formation of a cycloidal tooth has been shown in Fig. 10.18. A circle *H* rolls inside another circle *APB* (pitch circle). At the start, the point of contact of the two circles is at *A*. As the circle *H* rolls inside the pitch circle, the locus of the point *A* on the circle *H* traces the path *ALP* which is a hypocycloid. A small portion of this curve near the pitch circle is used for the flank of the tooth.

A property of the hypocycloid is that at any instant, the line joining the generating point (*A*) with the point of contact of the two circles is normal to the hypocycloid, e.g., when the circle *H* touches the pitch circle at *D*, the point *A* is at *C* and *CD* is normal to the hypocycloid *ALP*.

Also, Arc *AD* = Arc *CD* (on circle *H*)

In the same way, if the circle *E* rolls outside the pitch circle, starting from *P*, an epicycloid *PFB* is obtained. Similar to the property of a hypocycloid, the line joining the generating point with the point of contact of the two circles is a normal to the epicycloid, e.g., when the circle *E* touches the pitch circle at *K*, the point *P* is at *G* and *GK* is normal to the epicycloid *PFB*.

Arc *PK* = Arc *KJG* (on circle *E*)

or Arc *BK* = Arc *KG* (on circle *E*)

A small portion of the curve near the pitch circle is used for the face of the tooth.

Meshing of Teeth

During meshing of teeth, the face of a tooth on one gear is to mesh with the flank of another tooth on the other gear. Thus, for proper meshing, it is necessary that the diameter of the circle generating face of a tooth (on one gear) is the same as the diameter of the circle generating flank of the meshing tooth (on another gear); the pitch circle being the same in the two cases (Fig. 10.19).

Of course, the face and the flank of a tooth of a gear can be generated by two circles of different diameters. However, for interchangeability, the faces and flanks of both the teeth in the mesh are generated by the circles of the same diameter.

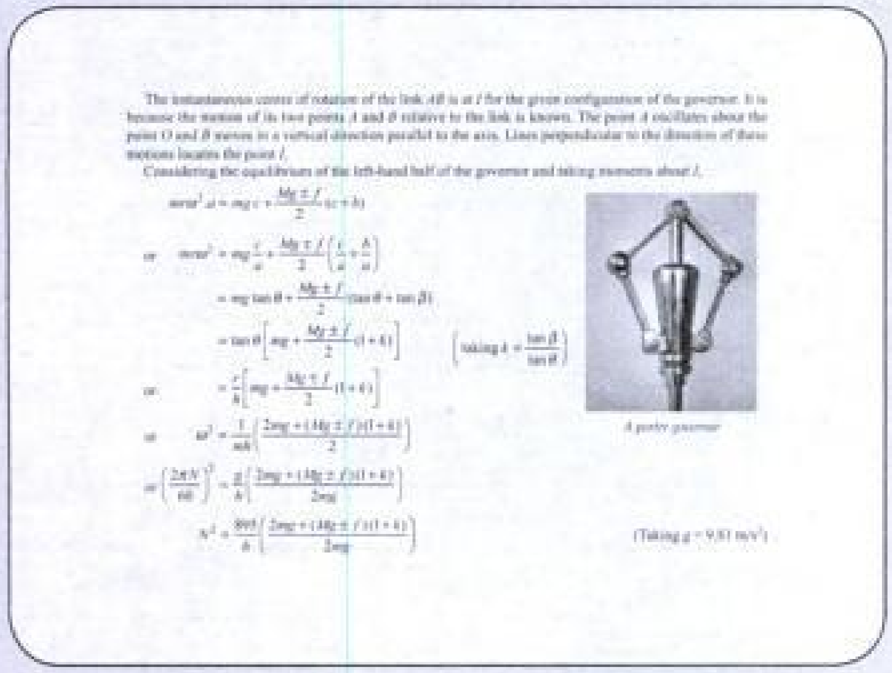
Consider a generating circle *G* rolling outside the pitch circle of the gear 2 (Fig. 10.20). It will generate

Fig. 10.18

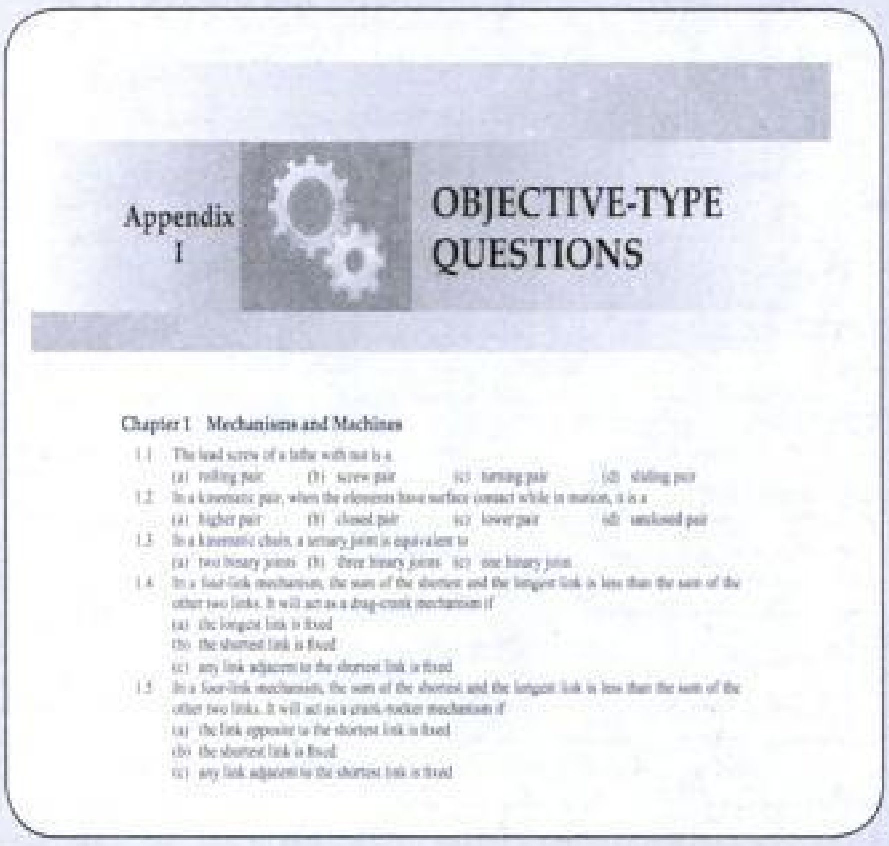
Fig. 10.19

Fig. 10.20

Simple diagrams are given for easy visualization of the explanations.



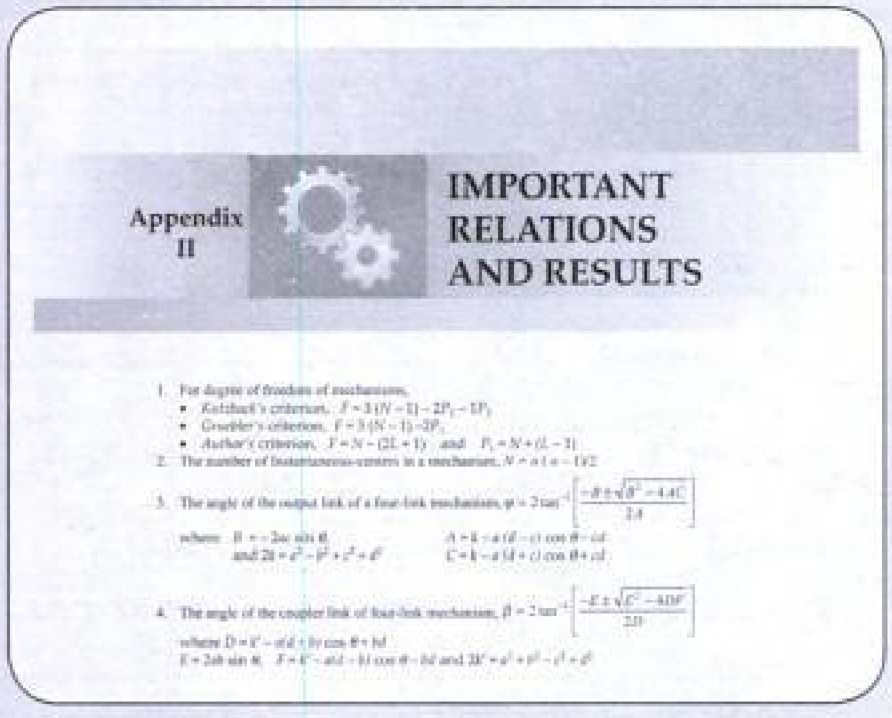
A number of photographs are given to emphasize the factual shape of various components.



Appendix I

OBJECTIVE-TYPE QUESTIONS

An Appendix containing multiple choice questions is given at the end to help students prepare for competitive examinations.



Appendix II

IMPORTANT RELATIONS AND RESULTS

An Appendix containing important relations is given for ready reference.

1 MECHANISMS AND MACHINES

Introduction

If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a *mechanism*. A mechanism transmits and modifies a motion. A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work. Thus, a mechanism is a fundamental unit and one has to start with its study.

The study of a mechanism involves its analysis as well as synthesis. *Analysis* is the study of motions and forces concerning different parts of an existing mechanism, whereas *synthesis* involves the design of its different parts. In a mechanism, the various parts are so proportioned and related that the motion of one imparts requisite motions to the others and the parts are able to withstand the forces impressed upon them. However, the study of the relative motions of the parts does not depend on the strength and the actual shapes of the parts.

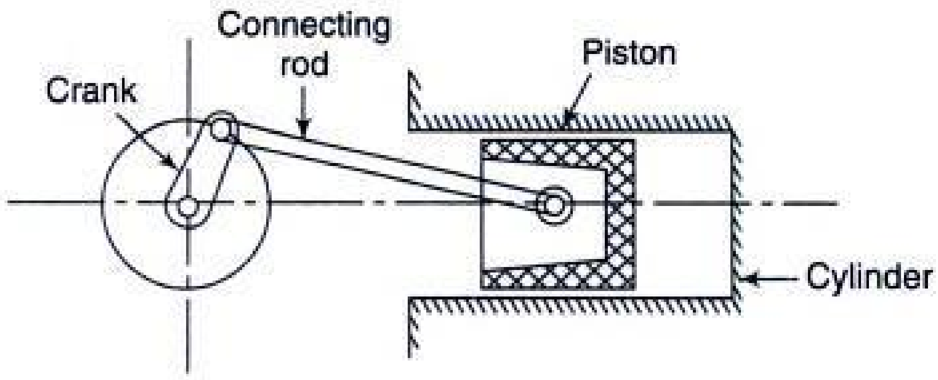


Fig. 1.1

In a reciprocating engine, the displacement of the piston depends upon the lengths of the connecting rod and the crank (Fig. 1.1). It is independent of the bearing strength of the parts or whether they are able to withstand the forces or not. Thus for the study of motions, it is immaterial if a machine part is made of mild steel, cast iron or wood. Also, it is not necessary to know the actual shape and area of the cross section of the part. Thus, for the study of motions of different parts of a mechanism, the study of forces is not necessary and can be neglected. The study of mechanisms, therefore, can be divided into the following disciplines:

Kinematics It deals with the relative motions of different parts of a mechanism without taking into consideration the forces producing the motions. Thus, it is the study, from a geometric point of view, to know the displacement, velocity and acceleration of a part of a mechanism.

Dynamics It involves the calculations of forces impressed upon different parts of a mechanism. The forces can be either static or dynamic. Dynamics is further subdivided into *kinetics* and *statics*. Kinetics is the study of forces when the body is in motion whereas statics deals with forces when the body is stationary.

1.1 MECHANISM AND MACHINE

As mentioned earlier, a combination of a number of bodies (usually rigid) assembled in such a way that the motion of one causes constrained and predictable motion to the others is known as a *mechanism*. Thus, the function of a mechanism is to transmit and modify a motion.

A *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work. It is neither a source of energy nor a producer of work but helps in proper utilization of the same. The motive power has to be derived from external sources.

A slider-crank mechanism (Fig. 1.2) converts the reciprocating motion of a slider into rotary motion of the crank or vice-versa. However, when it is used as an automobile engine by adding valve mechanism, etc., it becomes a machine which converts the available energy (force on the piston) into the desired energy (torque of the crank-shaft). The torque is used to move a vehicle. Reciprocating pumps, reciprocating compressors and steam engines are other examples of machines derived from the slider-crank mechanism.

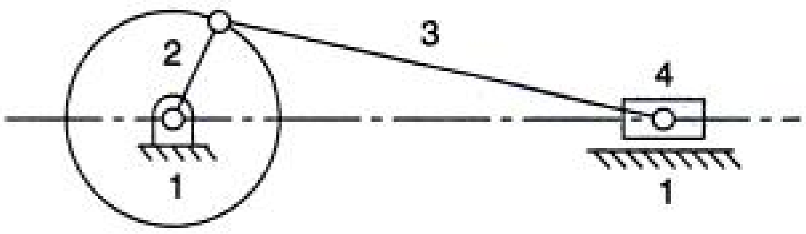


Fig. 1.2

Some other examples of mechanisms are typewriters, clocks, watches, spring toys, etc. In each of these, the force or energy provided is not more than what is required to overcome the friction of the parts and which is utilized just to get the desired motion of the mechanism and not to obtain any useful work.

1.2 TYPES OF CONSTRAINED MOTION

There are three types of constrained motion:

(i) ***Completely constrained motion*** When the motion between two elements of a pair is in a definite direction irrespective of the direction of the force applied, it is known as completely constrained motion. The constrained motion may be linear or rotary. The sliding pair of Fig. 1.3(a) and the turning pair of Fig. 1.3(b) are the examples of the completely constrained motion. In sliding pair, the inner prism can only slide inside the hollow prism. In case of a turning pair, the inner shaft can have only rotary motion due to collars at the ends. In each case the force has to be applied in a particular direction for the required motion.

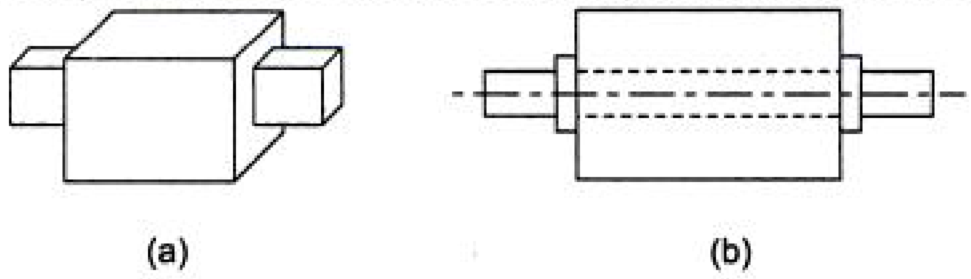


Fig. 1.3

(ii) ***Incompletely constrained motion*** When the motion between two elements of a pair is possible in more than one direction and depends upon the direction of the force applied, it is known as incompletely constrained motion. For example, if the turning pair of Fig. 1.4 does not have collars, the inner shaft may have sliding or rotary motion depending upon the direction of the force applied. Each motion is independent of the other.

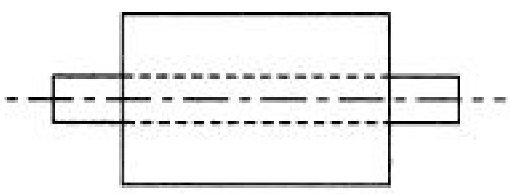
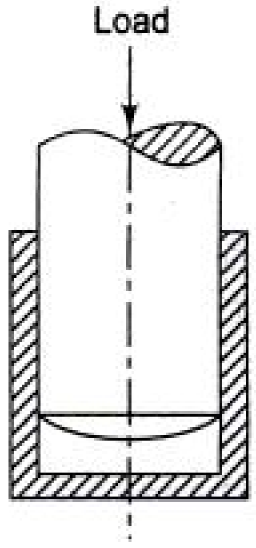


Fig. 1.4

(iii) ***Successfully constrained motion*** When the motion between two elements of a pair is possible in more than one direction but is made to have motion only in one direction by using some external means, it is a successfully constrained motion. For example, a shaft in a footstep bearing may have vertical motion apart from rotary motion (Fig. 1.5). But due to load applied on the shaft it is constrained to move in



Footstep bearing

Fig. 1.5

that direction and thus is a successfully constrained motion. Similarly, a piston in a cylinder of an internal combustion engine is made to have only reciprocating motion and no rotary motion due to constrain of the piston pin. Also, the valve of an IC engine is kept on the seat by the force of a spring and thus has successfully constrained motion.

1.3 RIGID AND RESISTANT BODIES

A body is said to be *rigid* if under the action of forces, it does not suffer any distortion or the distance between any two points on it remains constant.

Resistant bodies are those which are rigid for the purposes they have to serve. Apart from rigid bodies, there are some semi-rigid bodies which are normally flexible, but under certain loading conditions act as rigid bodies for the limited purpose and thus are resistant bodies. A belt is rigid when subjected to tensile forces. Therefore, the belt-drive acts as a resistant body. Similarly, fluids can also act as resistant bodies when compressed as in case of a hydraulic press. For some purposes, springs are also resistant bodies.

These days, resistant bodies are usually referred as rigid bodies.

1.4 LINK

A mechanism is made of a number of resistant bodies out of which some may have motions relative to the others. A resistant body or a group of resistant bodies with rigid connections preventing their relative movement is known as a *link*. A link may also be defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them. Thus, a link may consist of one or more resistant bodies. A slider-crank mechanism consists of four links: frame and guides, crank, connecting-rod and slider. However, the frame may consist of bearings for the crankshaft. The crank link may have a crankshaft and flywheel also, forming one link having no relative motion of these.

A link is also known as *kinematic link* or *element*.

Links can be classified into *binary, ternary* and *quaternary* depending upon their ends on which revolute or turning pairs (Sec. 1.5) can be placed. The links shown in Fig. 1.6 are rigid links and there is no relative motion between the joints within the link.

Fig. 1.6

1.5 KINEMATIC PAIR

A kinematic pair or simply a pair is a joint of two links having relative motion between them. In a slider-crank mechanism (Fig. 1.2), the link 2 rotates relative to the link 1 and constitutes a revolute or turning pair. Similarly, links 2, 3 and 3, 4 constitute turning pairs. Link 4 (slider) reciprocates relative to the link 1 and is a sliding pair.

Types of Kinematic Pairs Kinematic pairs can be classified according to

- nature of contact
- nature of mechanical constraint
- nature of relative motion

Kinematic Pairs according to Nature of Contact

(a) Lower Pair A pair of links having surface or area contact between the members is known as a lower pair. The contact surfaces of the two links are similar.

Examples Nut turning on a screw, shaft rotating in a bearing, all pairs of a slider-crank mechanism, universal joint, etc.

(b) Higher Pair When a pair has a point or line contact between the links, it is known as a higher pair. The contact surfaces of the two links are dissimilar.

Examples Wheel rolling on a surface, cam and follower pair, tooth gears, ball and roller bearings, etc.

Kinematic Pairs according to Nature of Mechanical Constraint

(a) Closed Pair When the elements of a pair are held together mechanically, it is known as a closed pair. The two elements are geometrically identical; one is solid and full and the other is hollow or open. The latter not only envelops the former but also encloses it. The contact between the two can be broken only by destruction of at least one of the members.

All the lower pairs and some of the higher pairs are closed pairs. A cam and follower pair (higher pair) shown in Fig. 1.7(a) and a screw pair (lower pair) belong to the closed pair category.

(b) Unclosed Pair When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this, the links are not held together mechanically, e.g., cam and follower pair of Fig. 1.7(b).

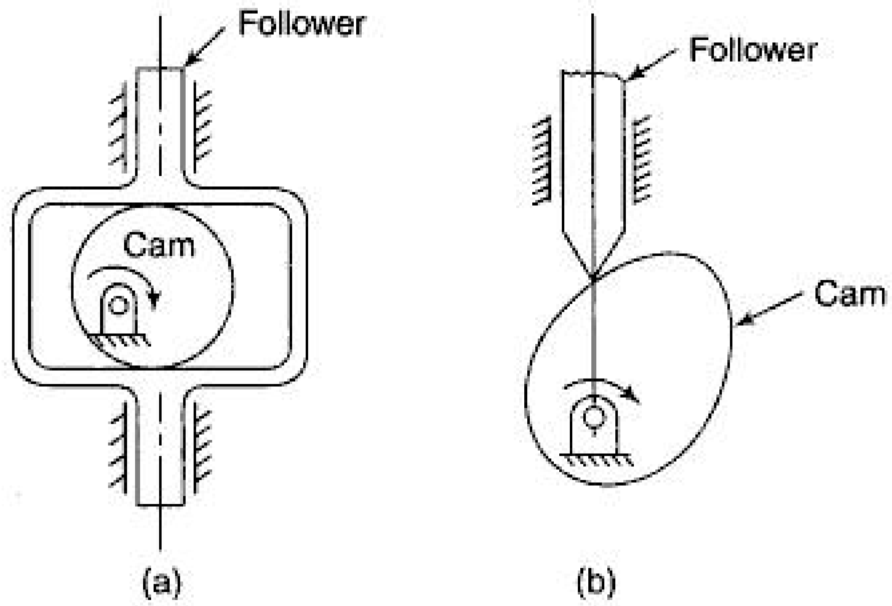


Fig. 1.7

Kinematic Pairs according to Nature of Relative Motion

(a) Sliding Pair If two links have a sliding motion relative to each other, they form a sliding pair.

A rectangular rod in a rectangular hole in a prism is a sliding pair [Fig. 1.8(a)].

(b) Turning Pair When one link has a turning or revolving motion relative to the other, they constitute a turning or revolving pair [Fig. 1.8(b)].

In a slider-crank mechanism, all pairs except the slider and guide pair are turning pairs. A circular shaft revolving inside a bearing is a turning pair.

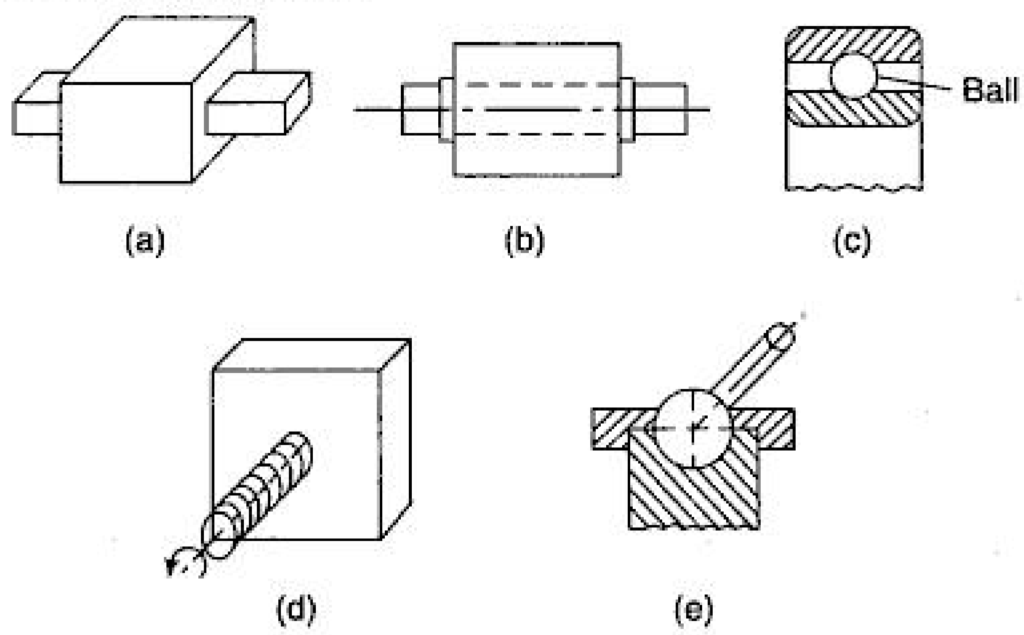


Fig. 1.8

(c) Rolling Pair When the links of a pair have a rolling motion relative to each other, they form a rolling pair, e.g., a rolling wheel on a flat surface, ball and roller bearings, etc. In a ball bearing [Fig. 1.8(c)], the ball and the shaft constitute one rolling pair whereas the ball and the bearing is the second rolling pair.

(d) Screw Pair (Helical Pair) If two mating links have a turning as well as sliding motion between them, they from a screw pair. This is achieved by cutting matching threads on the two links.

The lead screw and the nut of a lathe is a screw pair [Fig. 1.8(d)].

(e) Spherical Pair When one link in the form of a sphere turns inside a fixed link, it is a spherical pair.

The ball and socket joint is a spherical pair [Fig. 1.8(e)].

1.6 TYPES OF JOINTS

The usual types of joints in a chain are

- Binary joint
- Ternary joint
- Quaternary joint

Binary Joint If two links are joined at the same connection, it is called a binary joint. For example, Fig. 1.9 shows a chain with two binary joints named *B*.

Ternary Joint If three links are joined at a connection, it is known as a ternary joint. It is considered equivalent to two binary joints since fixing of any one link constitutes two binary joints with each of the other two links. In Fig. 1.9 ternary links are mentioned as *T*.

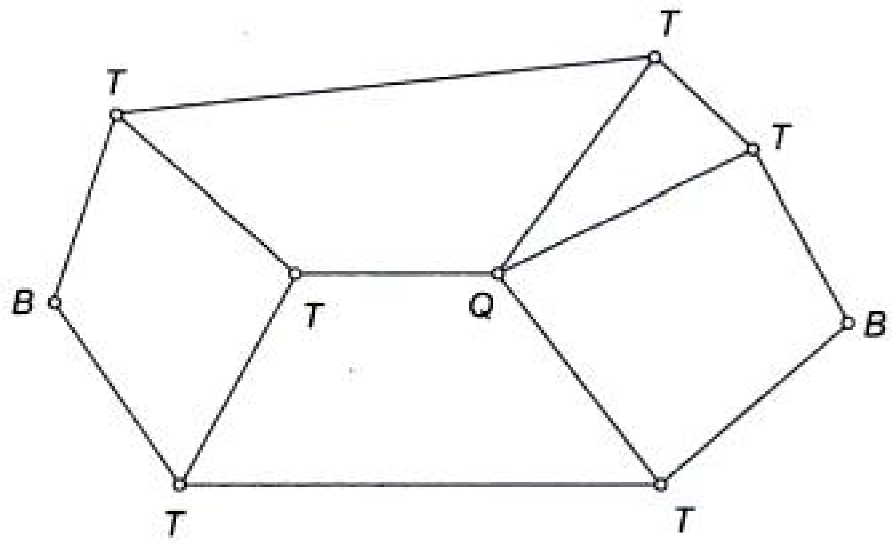


Fig. 1.9

Quaternary Joint If four links are joined at a connection, it is known as a quaternary joint. It is considered equivalent to three binary joints since fixing of any one link constitutes three binary joints. Figure 1.9 shows one quaternary joint.

In general, if n number of links are connected at a joint, it is equivalent to $(n - 1)$ binary joints.

1.7 DEGREES OF FREEDOM

An unconstrained rigid body moving in space can describe the following independent motions (Fig. 1.10):

1. Translational motions along any three mutually perpendicular axes x, y and z
2. Rotational motions about these axes

Thus, a rigid body possesses six degrees of freedom. The connection of a link with another imposes certain constraints on their relative motion. The number of restraints can never be zero (joint is disconnected) or six (joint becomes solid).

Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.

Degrees of freedom = 6 – Number of restraints

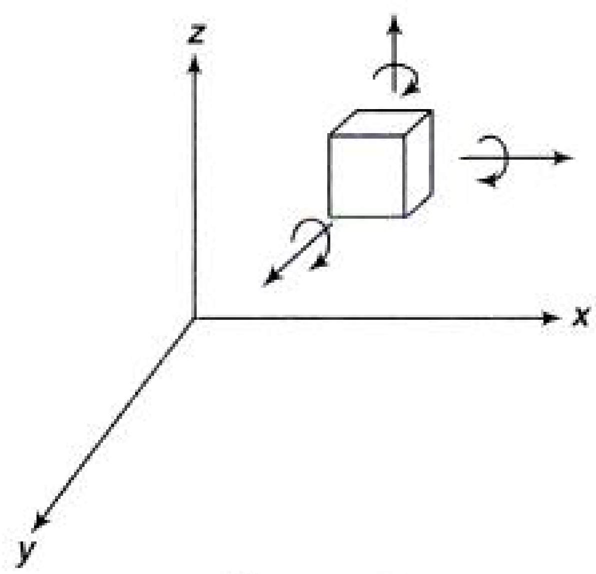


Fig. 1.10

1.8 CLASSIFICATION OF KINEMATIC PAIRS

Depending upon the number of restraints imposed on the relative motion of the two links connected together, a pair can be classified as given in Table 1.1 which gives the possible form of each class.

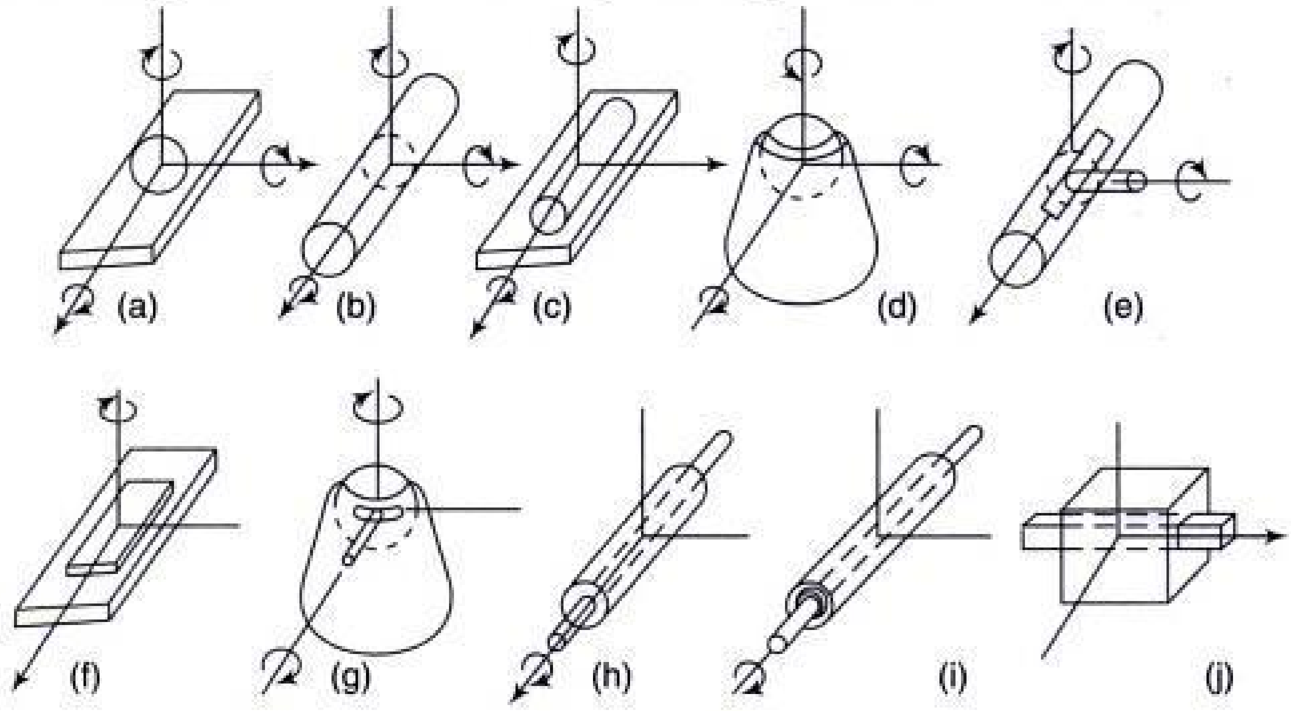


Fig. 1.11

Different forms of each class have also been shown in Fig. 1.11. Remember that a particular relative motion between two links of a pair must be independent of the other relative motions that the pair can have. A screw and nut pair permits translational and rotational motions. However, as the two motions cannot be accomplished independently, a screw and nut pair is a kinematic pair of the fifth class and not of the fourth class.

1.9 KINEMATIC CHAIN

A *kinematic chain* is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the other is definite [Fig.1.12 (a), (b), and (c)].

Table 1.1

Class	Number of Restraints	Form	Restraints on Translatory motion	Restraints on Rotary motion	Kinematic pair	Fig. 1.11
I	1	1st	1	0	Sphere-plane	a
II	2	1st	2	0	Sphere-cylinder	b
		2nd	1	1	Cylinder-plane	c
III	3	1st	3	0	Spheric	d
		2nd	2	1	Sphere-slotted cylinder	e
		3rd	1	2	Prism-plane	f
IV	4	1st	3	1	Slotted-spheric	g
		2nd	2	2	Cylinder	h
V	5	1st	3	2	Cylinder (collared)	i
		2nd	2	3	Prismatic	j

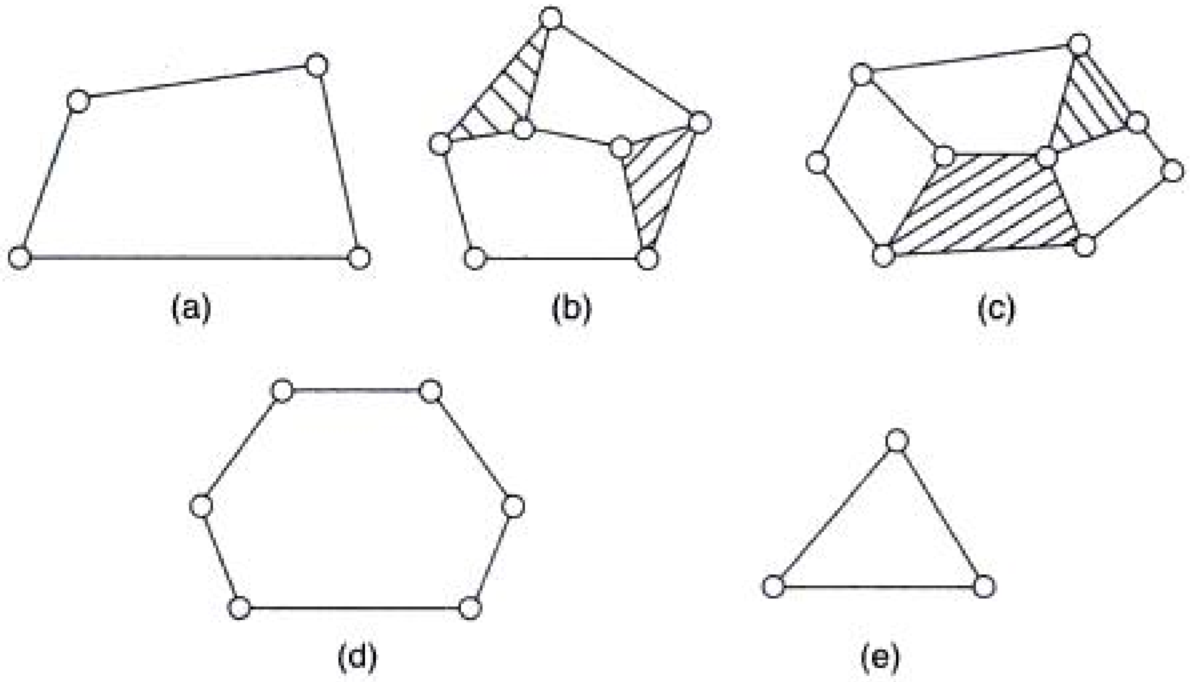


Fig. 1.12

In case the motion of a link results in indefinite motions of other links, it is a *non-kinematic chain* [Fig. 1.12(d)]. However, some authors prefer to call all chains having relative motions of the links as kinematic chains.

A *redundant chain* does not allow any motion of a link relative to the other [Fig. 1.12(e)].

1.10 LINKAGE, MECHANISM AND STRUCTURE

A *linkage* is obtained if one of the links of a kinematic chain is fixed to the ground. If motion of any of the moveable links results in definite motions of the others, the linkage is known as a *mechanism*. However, this distinction between a mechanism and a linkage is hardly followed and each can be referred in place of the other.

If one of the links of a redundant chain is fixed, it is known as a *structure* or *a locked system*. To obtain constrained or definite motions of some of the links of a linkage (or mechanism), it is necessary to know how many inputs are needed. In some mechanisms, only one input is necessary that determines the motions of other links and it is said to have one degree of freedom. In other mechanisms, two inputs may be necessary to get constrained motions of the other links and they are said to have two degrees of freedom, and so on.

The degree of freedom of a structure or a locked system is zero. A structure with negative degree of freedom is known as a *superstructure*.

1.11 MOBILITY OF MECHANISMS

A mechanism may consist of a number of pairs belonging to different classes having different number of restraints. It is also possible that some of the restraints imposed on the individual links are common or general to all the links of the mechanism. According to the number of these general or common restraints, a mechanism may be classified into a different order. A zero-order mechanism will have no such general restrain. Of course, some of the pairs may have individual restraints. A first-order mechanism has one general restraint; a second-order mechanism has two general restraints, and so on, up to the fifth order. A sixth-order mechanism cannot exist since all the links become stationary and no movement is possible.

Expressing the number of degrees of freedom of a linkage in terms of the number of links and the number of pair connections of different types is known as *number synthesis*. *Degrees of freedom* of a mechanism in space can be determined as follows:

Let

N = total number of links in a mechanism

F = degrees of freedom

P_1 = number of pairs having one degree of freedom

P_2 = number of pairs having two degrees of freedom, and so on

In a mechanism, one link is fixed.

Therefore,

Number of movable links = $N - 1$

Number of degrees of freedom of $(N - 1)$ movable links = $6(N - 1)$

Each pair having one degree of freedom imposes 5 restraints on the mechanism, reducing its degrees of freedom by $5P_1$.

Each pair having two degrees of freedom will impose 4 restraints, reducing the degrees of freedom of the mechanism by $4P_2$.

Similarly, other pairs having 3, 4 and 5 degrees of freedom reduce the degrees of freedom of the mechanism. Thus,

$$F = 6(N - 1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5 \tag{1.1}$$

The above criterion is hardly necessary to find the degrees of freedom, as space mechanisms, especially of the zero order are not practical. Most of the mechanisms are two-dimensional such as a four-link or a slider-crank mechanism in which displacement is possible along two axes (one restraint) and rotation about only one axis (two restraints). Thus, there are three general restraints.

Therefore, for plane mechanisms, the following relation may be used to find the degrees of freedom

$$F = 3(N - 1) - 2P_1 - 1P_2 \tag{1.2}$$

This is known as *Gruebler's criterion* for degrees of freedom of plane mechanisms in which each movable link possesses three degrees of freedom. Each pair with one degree of freedom imposes two further restraints on the mechanisms, thus reducing its degrees of freedom. Similarly, each pair with two degrees of freedom reduces the degrees of freedom of the mechanism at the rate of one restraint each.

Some authors mention the above relation as *Kutzback's criterion* and a simplified relation $[F = 3(N-1) - 2P_1]$ which is applicable to linkages with a single degree of freedom only as Gruebler's criterion. However, many authors make no distinction between Kutzback's criterion and Gruebler's criterion.

Thus, for linkages with a single degree of freedom only, $P_2 = 0$

$$F = 3(N - 1) - 2P_1 \tag{1.3}$$

Most of the linkages are expected to have one degree of freedom so that with one input to any of the links, a constrained motion of the others is obtained.

Then,

$$1 = 3(N - 1) - 2P_1$$

or

$$2P_1 = 3N - 4 \tag{1.4}$$

As P_1 and N are to be whole numbers, the relation can be satisfied only if N is even. For possible linkages made of binary links only,

$N = 4$,	$P_1 = 4$	No excess turning pair
$N = 6$,	$P_1 = 7$	One excess turning pair
$N = 8$,	$P_1 = 10$	Two excess turning pairs

and so on.

Thus, with the increase in the number of links, the number of excess turning pairs goes on increasing. Getting the required number of turning pairs from the required number of binary links is not possible. Therefore, the excess or the additional pairs or joints can be obtained only from the links having more than two joining points, i.e., ternary or quaternary links, etc.

For a six-link chain, some of the possible types are Watts six-bar chain, in which the ternary links are directly connected [Fig. 1.13(a)] and Stephenson's six-bar chain, in which ternary links are not directly connected [Fig. 1.13(b)]. Another possibility is also shown in Fig. 1.13(c). However, this chain is not a six-link chain but a four-link chain as links 1, 2 and 3 are, in fact, one link only with no relative motion of these links.

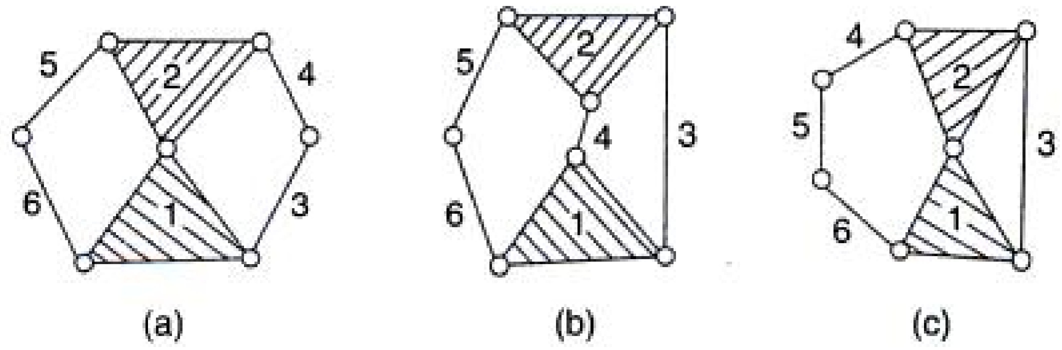


Fig. 1.13

Two excess turning pairs required for an eight-link chain can be obtained by using (apart from binary links):

four ternary links [Figs 1.14(a) and (b)]
two quaternary links [Fig.1.14(c)]
one quaternary and two ternary links [Fig. 1.14(d)].

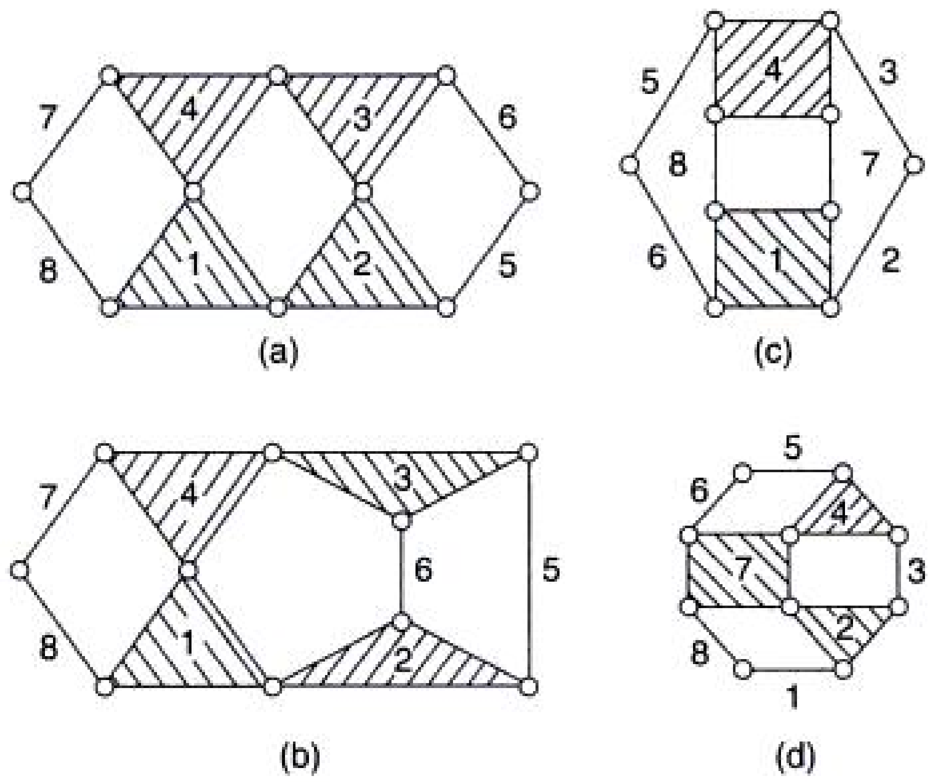


Fig. 1.14

Now, consider the kinematic chain shown in Fig. 1.15. It has 8 links, but only three ternary links. However, the links 6, 7 and 8 constitute a double pair so that the total number of pairs is again 10. The degree of freedom of such a linkage will be

$$F = 3\,(8 - 1) - 2 \times 10$$
$$= 1$$

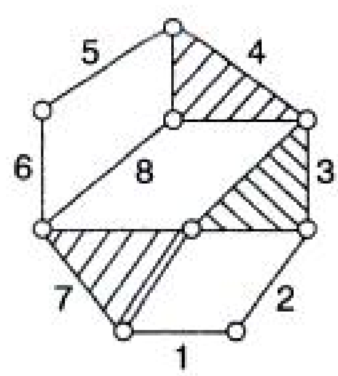


Fig. 1.15

This shows that the number of ternary or quaternary links in a chain can be reduced by providing double joints also.

The following empirical relations formulated by the author provide the degree of freedom and the number of joints in a linkage when the number of links and the number of loops in a kinematic chain are known. These relations are valid for linkages with turning pairs,

$$F = N - (2L + 1) \quad (1.5)$$

$$P_1 = N + (L - 1) \quad (1.6)$$

where

L = number of loops in a linkage.

Thus, for different number of loops in a linkage, the degrees of freedom and the number of pairs are as shown in Table 1.2.

Table 1.2

L	F	P_1
1	$N-3$	N
2	$N-5$	$N+1$
3	$N-7$	$N+2$
4	$N-9$	$N+3$
5	$N-11$	$N+4$
and so on		

For example, if in a linkage, there are 4 loops and 11 links, its degree of freedom will be 2 and the number of joints, 14. Similarly, if a linkage has 3 loops, it will require 8 links to have one degree of freedom, 9 links to have 2 degrees of freedom, 7 links to have –1 degree of freedom, etc.

Sometimes, all the above empirical relations can give incorrect results, e.g., Fig.1.16(a) has 5 links, 6 turning pairs and 2 loops. Thus, it is a structure with zero degree of freedom. However, if the links are arranged in such a way as shown in Fig. 1.16(b), a *double parallelogram linkage* with one degree of freedom is obtained. This is due to the reason that the lengths of the links or other dimensional properties are not considered in these empirical relations. So, exceptions are bound to come with equal lengths or parallel links.

Sometimes, a system may have one or more links which do not introduce any extra constraint. Such links are known as *redundant links* and should not be counted to find the degree of freedom. For example, the mechanism of Fig. 1.16(b) has 5 links, but the function of the mechanism is not affected even if any one of the links 2, 4 or 5 are removed. Thus, the effective number of links in this case is 4 with 4 turning pairs, and thus has one degree of freedom.

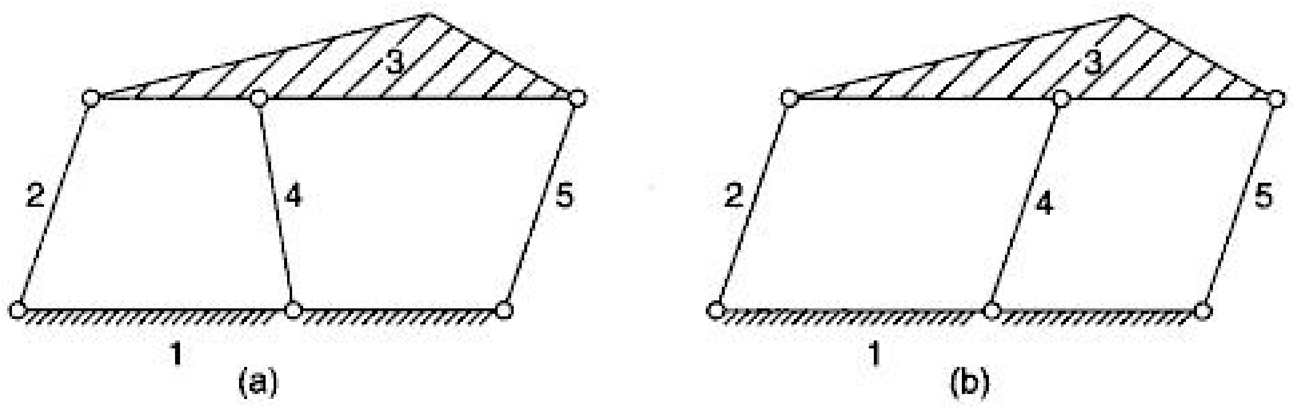


Fig. 1.16

Sometimes, one or more links of a mechanism can be moved without causing any motion to the rest of the links of the mechanism. Such a link is said to have a *redundant degree of freedom*. Thus in a mechanism, it is necessary to recognize such links prior to investigate the degree of freedom of the whole mechanism. For example, in the mechanism shown in Fig. 1.17, roller 3 can rotate about its axis without causing any movement to the rest of the mechanism. Thus, the mechanism represents a redundant degree of freedom.

In case of a mechanism possessing some redundant degree of freedom, the effective degree of freedom is given by

$$F = 3(N-1) - 2P_1 - 1P_2 - F_r$$

where F_r is the number of redundant degrees of freedom. Now, as the above mechanism has a cam pair, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 4

Number of pairs with 1 degree of freedom = 3

Number of pairs with 2 degrees of freedom = 1

$$F = 3(N-1) - 2P_1 - 1P_2 - F_r$$
$$= 3(4-1) - 2 \times 3 - 1 \times 1 - 1$$
$$= 1$$

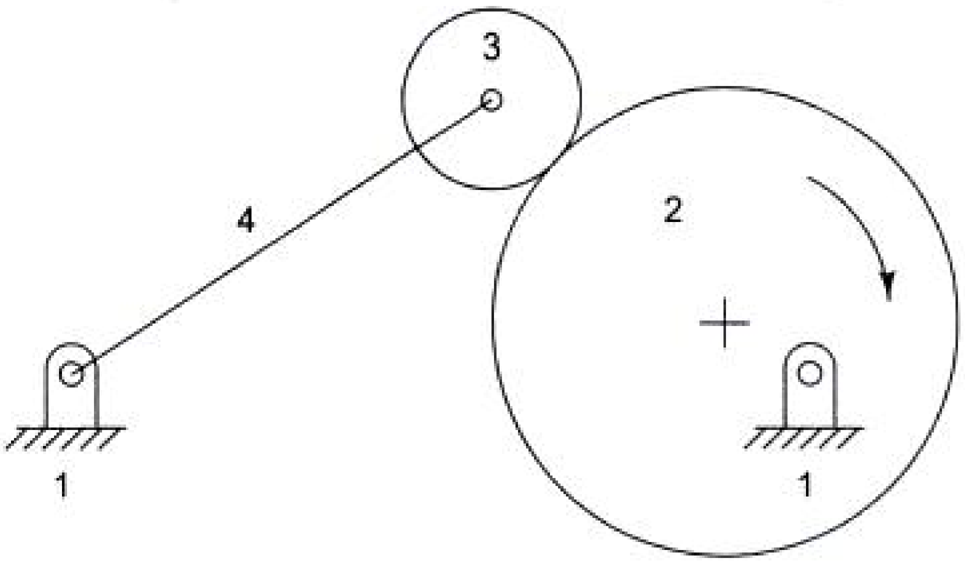


Fig. 1.17

Example 1.1 *For the kinematic linkages shown in Fig. 1.18, calculate the following:*

- *the number of binary links* (N_b)
- *the number of ternary links* (N_t)
- *the number of other (quaternary, etc.) links* (N_o)
- *the number of total links* (N)
- *the number of loops* (L)
- *the number of joints or pairs* (P_1)
- *the number of degrees of freedom* (F)

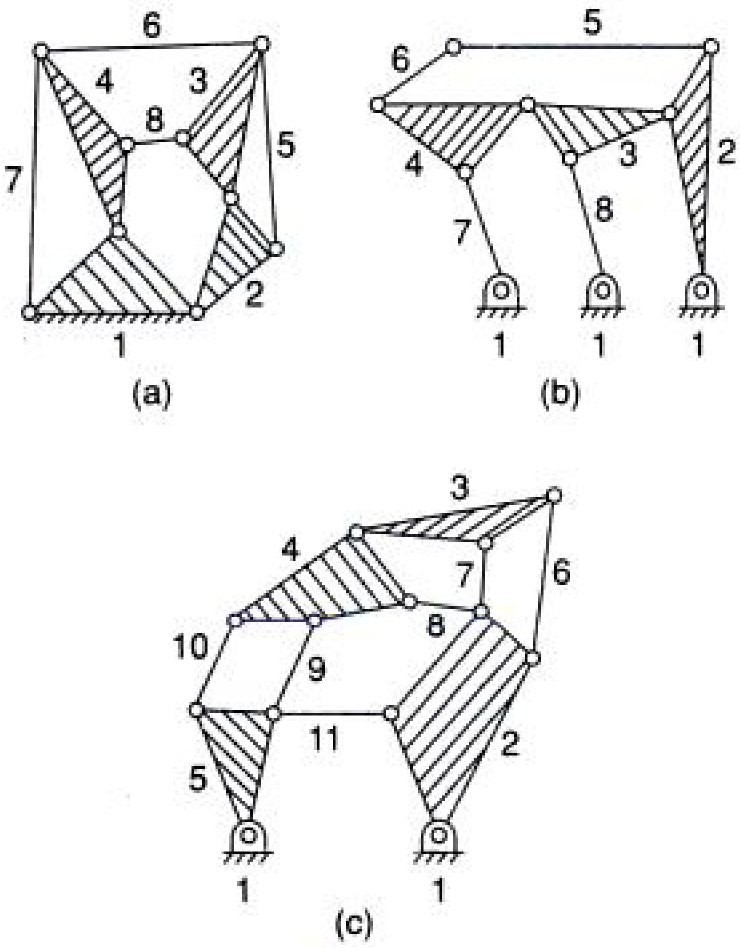


Fig. 1.18

Solution

(a) $N_b = 4$; $N_t = 4$; $N_o = 0$; $N = 8$; $L = 4$

$P_1 = 11$ by counting

or $P_1 = (N + L - 1) = 11$

$F = 3(N-1) - 2P_1$

$= 3(8-1) - 2 \times 11 = -1$

or $F = N - (2L + 1)$

$= 8 - (2 \times 4 + 1) = -1$

The linkage has negative degree of freedom and thus is a superstructure.

(b) $N_b = 4$; $N_t = 4$; $N_o = 0$; $N = 8$; $L = 3$

$P_1 = 10$ (by counting)

or $P_1 = (N + L - 1) = 10$

$F = N - (2L + 1) = 8 - (2 \times 3 + 1) = 1$

or $F = 3(N-1) - 2P_1$

$= 3(8-1) - 2 \times 10 = 1$

i.e., the linkage has a constrained motion when one of the seven moving links is driven by an external source.

(c) $N_b = 7$; $N_t = 2$; $N_o = 2$; $N = 11$

$L = 5$; $P_1 = 15$

$F = N - (2L + 1) = 11 - (2 \times 5 + 1) = 0$

Therefore, the linkage is a structure.

Example 1.2 *State whether the linkages shown in Fig. 1.19 are mechanisms with one degree of freedom. If not, make suitable changes. The number of links should not be varied by more than 1.*

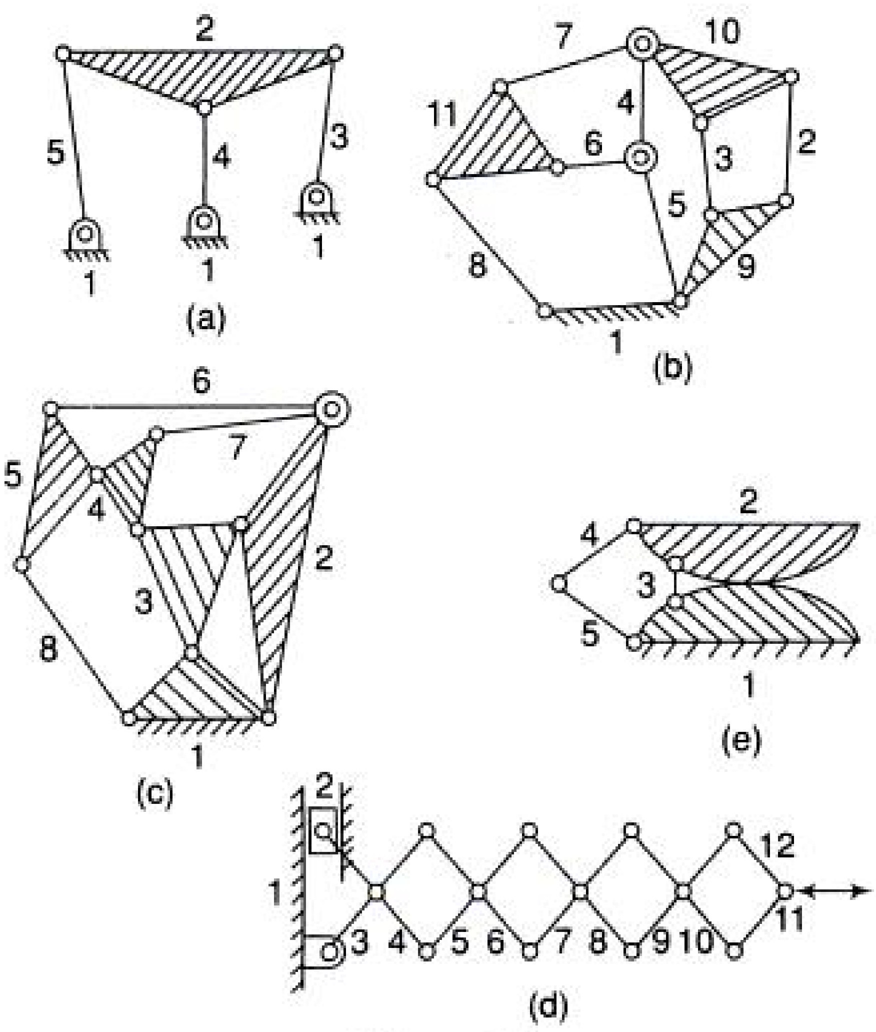


Fig. 1.19

Solution (a) The linkage has 2 loops and 5 links.

$F = N - (2L + 1) = 5 - (2 \times 2 + 1) = 0$

Thus, it is a structure. Referring Table 1.2, for a 2-loop mechanism, n should be six to have one degree of freedom. Thus, one more link should be added to the linkage to make it a mechanism of $F = 1$. One of the possible solutions has been shown in Fig. 1.20(a).

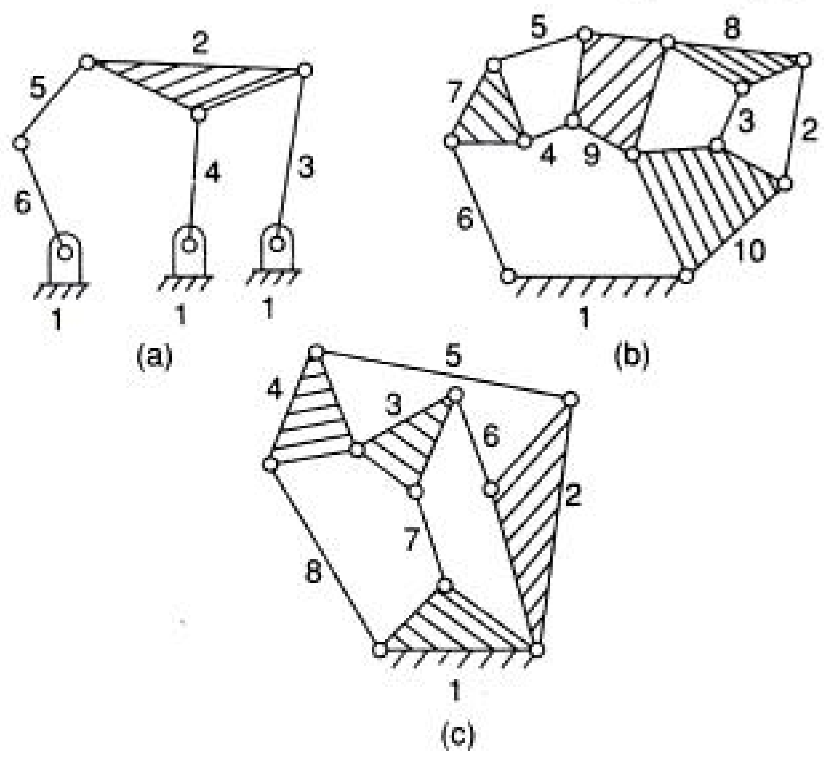


Fig. 1.20

(b) The linkage has 4 loops and 11 links. Referring Table 1.2, it has 2 degrees of freedom. With 4 loops and 1 degree of freedom, the number of links should be 10 and the number of joints 13. Three excess joints can be formed by
 6 ternary links or
 4 ternary links and 1 quaternary link or
 2 ternary links, and 2 quaternary links, or
 3 quaternary links, or
 a combination of ternary and quaternary links with double joints.

Figure 1.20(b) shows one of the possible solutions.

(c) There are 4 loops and 8 links.

$F = N - (2L +) = 8 - (4 \times 2 + 1) = -1$

It is a superstructure. With 4 loops, the number of links must be 10 to obtain one degree of freedom. As the number of links is not to be increased by more than one, the number of loops has to be decreased. With 3 loops, 8 links and 10 joints, the required linkage can be designed. One of the many solutions is shown in Fig. 1.20(c).

(d) It has 5 loops and 12 links. Referring Table 1.2, it has 1 degree of freedom and thus is a mechanism.

(e) The mechanism has a cam pair, therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 5

Number of pairs with 1 degree of freedom = 5

Number of pairs with 2 degrees of freedom = 1

$F = 3(N - 1) - 2P_1 - P_2$

$= 3(5 - 1) - 2 \times 5 - 1 = 1$

Thus, it is a mechanism with one degree of freedom.

Example 1.3 *Determine the degree of freedom of the mechanisms shown in Fig. 1.21.*

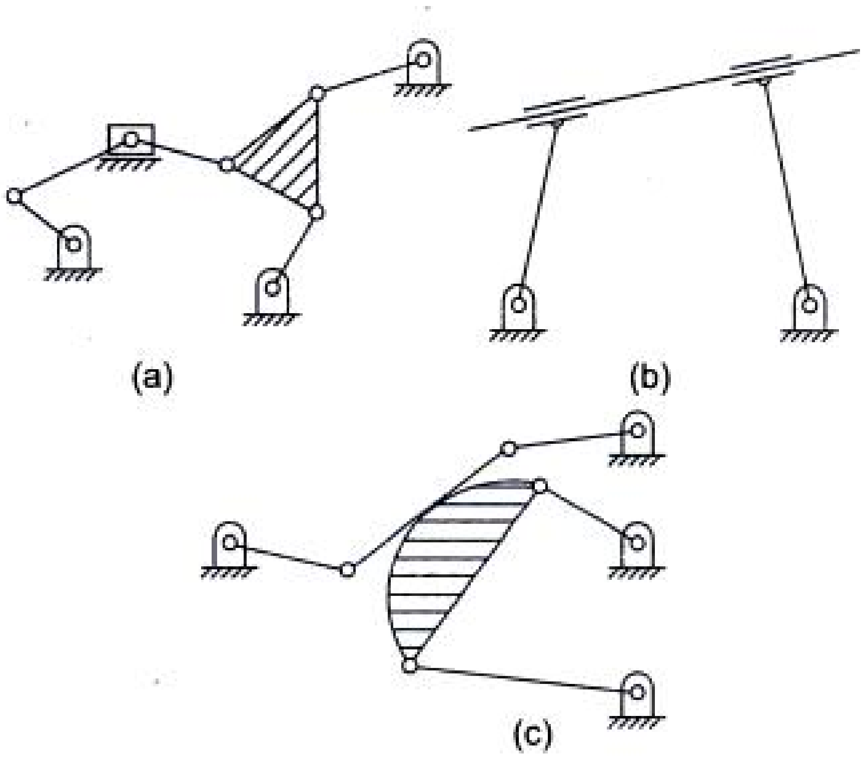


Fig. 1.21

Solution

(a) The mechanism has a sliding pair. Therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 8 (Fig. 1.22)

Number of pairs with 1 degree of freedom = 10

(At the slider, one sliding pair and two turning pairs)

$F = 3\,(N - 1) - 2P_1 - P_2$

$= 3(8 - 1) - 2 \times 10 - 0 = 1$

Thus, it is a mechanism with a single degree of freedom.

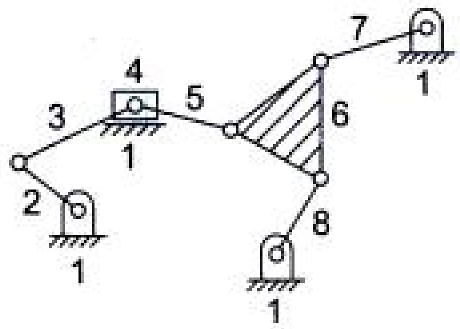


Fig. 1.22

(b) The system has a redundant degree of freedom as the rod of the mechanism can slide without causing any movement in the rest of the mechanism.

∴ effective degree of freedom

$= 3\,(N - 1) - 2P_1 - P_2 - F_r$

$= 3(4 - 1) - 2 \times 4 - 0 - 1 = 0$

As the effective degree of freedom is zero, it is a locked system.

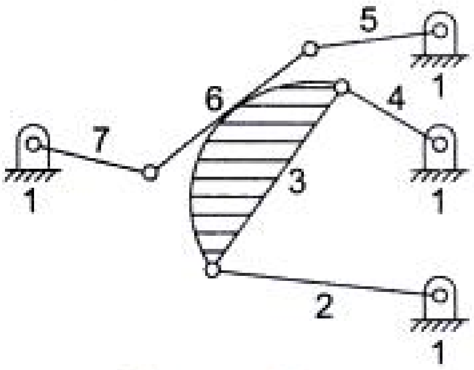


Fig. 1.23

(c) The mechanism has a cam pair. Therefore, its degree of freedom must be found from Gruebler's criterion.

Total number of links = 7 (Fig. 1.23)

Number of pairs with 1 degree of freedom = 8

Number of pairs with 2 degrees of freedom = 1

$F = 3\,(N - 1) - 2P_1 - P_2$

$= 3(7 - 1) - 2 \times 8 - 1 = 1$

Thus, it is a mechanism with one degree of freedom.

Example 1.4 *How many unique mechanisms can be obtained from the 8-link kinematic chain shown in Fig. 1.24?*

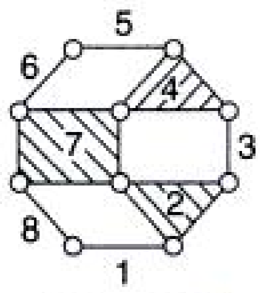


Fig. 1.24

Solution The kinematic chain has 8 links in all. A unique mechanism is obtained by fixing one of the links to the ground each time and retaining only one out of the symmetric mechanisms thus obtained.

The given kinematic chain is symmetric about links 3 or 7. Thus, identical inversions (mechanisms) are obtained if the links 2, 1, 8 or 4, 5, 6 are fixed. In addition, two more unique mechanisms can be obtained from the 8-link kinematic chain as shown in Fig. 1.25.

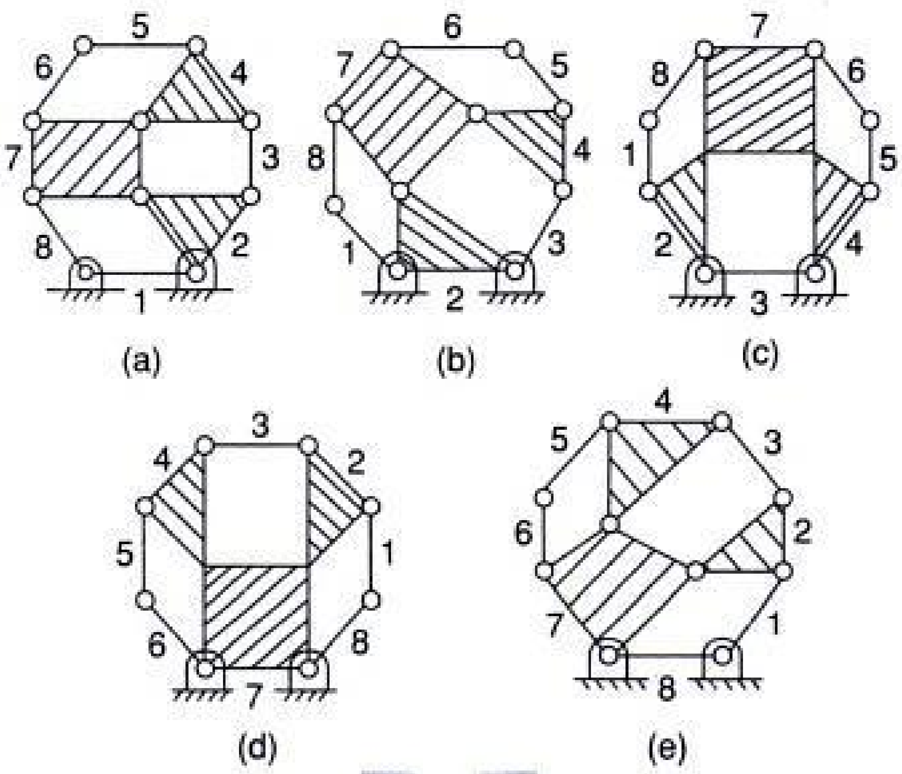


Fig. 1.25

Example 1.5 *A linkage has 11 links and 4 loops. Calculate its degree of freedom and the number of ternary and quaternary links it will have if it has only single turning pairs.*

Solution $F = N - (2L + 1) = 11 - (2 \times 4 + 1) = 2$

$P_1 = N + (L - 1) = 11 + (4 - 1) = 14$

The linkage has 3 excess joints and if all the joints are single turning pairs, the excess joints can be provided either by

- 6 ternary links or
- 4 ternary links and one quaternary link or
- 2 ternary links and two quaternary links or
- 3 quaternary links

1.12 EQUIVALENT MECHANISMS

It is possible to replace turning pairs of plane mechanisms by other types of pairs having one or two degrees of freedom, such as sliding pairs or cam pairs. This can be done according to some set rules so that the new mechanisms also have the same degrees of freedom and are kinematically similar.

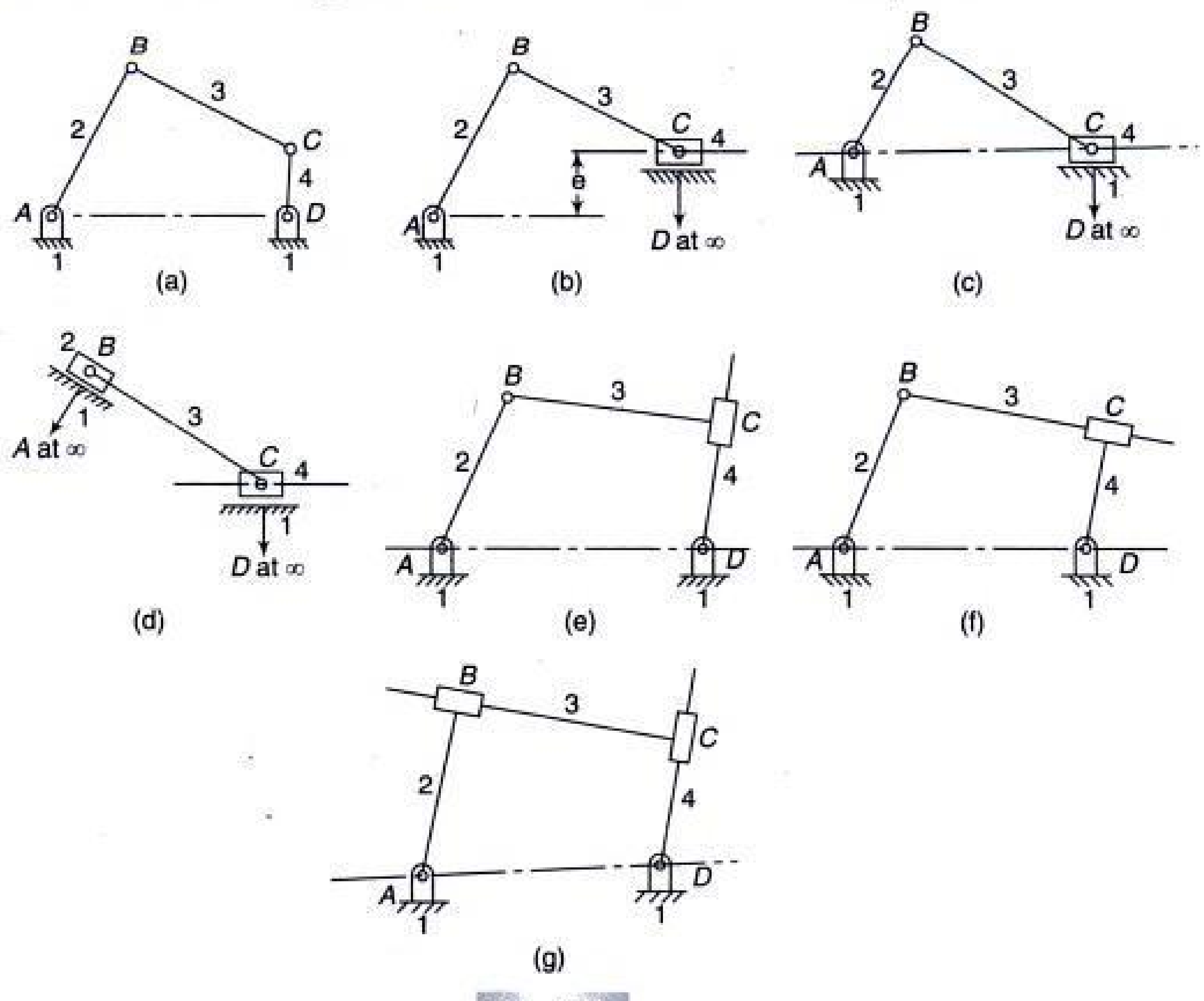


Fig. 1.26

1. Sliding Pairs in Place of Turning Pairs

Figure 1.26(a) shows a four-link mechanism. Let the length of the link 4 be increased to infinity so that *D* lies at infinity. Now, with the rotation of the link 2, *C* will have a linear motion perpendicular to the axis of the link 4. The same motion of *C* can be obtained if the link 4 is replaced by a slider, and guides are provided for its motion as shown in Fig. 1.26(b). In this case, the axis of the slider does not pass through *A* and there is an eccentricity. Figure 1.26(c) shows a slider-crank mechanism with no eccentricity. In this way, a binary link is replaced by a slider pair.

Note that the axis of the sliding pair must be in the plane of the linkage or parallel to it.

Similarly, the turning pair at *A* can also be replaced by a sliding pair by providing a slider with guides at *B* [Fig. 1.26(d)].

In case the axes of the two sliding pairs are in one line or parallel, the two sliders along with the link 3 act as one link with no relative motion among these links. Then the arrangement ceases to be a linkage. Thus, in order to replace two turning pairs in a linkage with sliding pairs, the axes of the sliding pairs must intersect.

In the same way, the turning pairs at *B* and *C* can be replaced by sliding pairs by fixing a slider to any of the two links forming the pair [Figs 1.26(e) and (f)]. Figure 1.26(g) shows both of the turning pairs at *B* and *C* replaced by sliding pairs.

2. Spring in Place of Turning Pairs

The action of a spring is to elongate or to shorten as it becomes in tension or in compression. A similar variation in length is accomplished by two binary links joined by a turning pair. In Fig. 1.27(a), the length *AB* varies as *OB* is moved away or towards point *A*. Figure 1.27(b) shows a 6-link mechanism in which links 4 and 5 have been shown replaced by a spring.

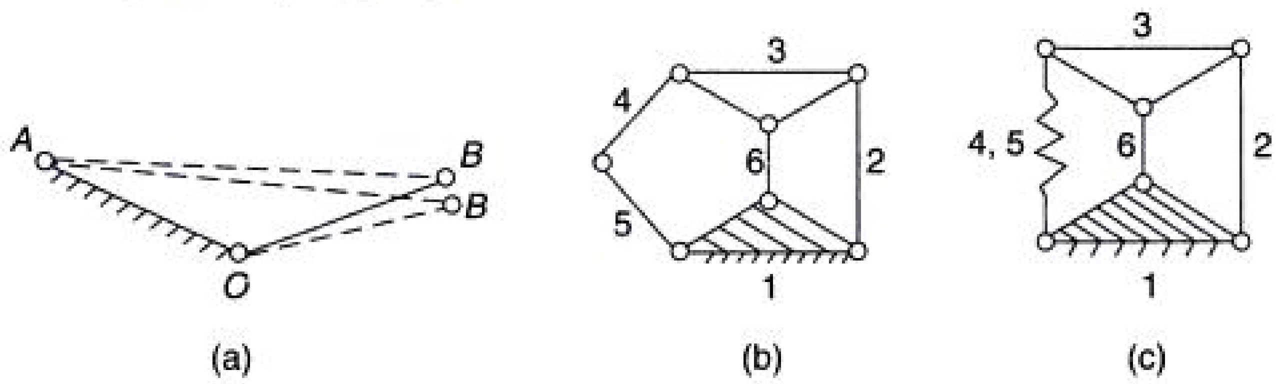


Fig. 1.27

Remember that the spring is not a rigid link but is simulating the action of two binary links joined by a turning pair. Therefore, to find the degree of freedom of such a mechanism, the spring has to be replaced by the binary links.

3. Cam Pair in Place of Turning Pair

A cam pair has two degrees of freedom. For linkages with one degree of freedom, application of Gruebler's equation yields,

$$F = 3(N-1) - 2P_1 - 1P_2$$

or $$1 = 3N - 3 - 2P_1 - 1 \times 1$$

or $$P_1 = \frac{3N-5}{2}$$

This shows that to have one cam pair in a mechanism with one degree of freedom, the number of links and turning pairs should be as below:

$N = 3, \quad P_1 = 2$
$N = 5, \quad P_1 = 5$
$N = 7, \quad P_1 = 8$
$N = 9, \quad P_1 = 11$ and so on.

A comparison of this with linkages having turning pairs only (Table 1.2) indicates that a cam pair can be replaced by one binary link with two turning pairs at each end.

Figure 1.28(a) shows link CD (of a four-link mechanism) with two turning pairs at its ends replaced by a cam pair. The centres of curvatures at the point of contact X of the two cams lie at D and C. Figures 1.28(b) and (c) show the link BC with turning pairs at B and C replaced by a cam pair. The centres of curvature at the point of contact X lie at B and C respectively. Figure 1.28(d) shows equivalent mechanism for a disc cam with reciprocating curved-face follower. The centres of curvature of the cam and the follower at the instant lie at A and B respectively.

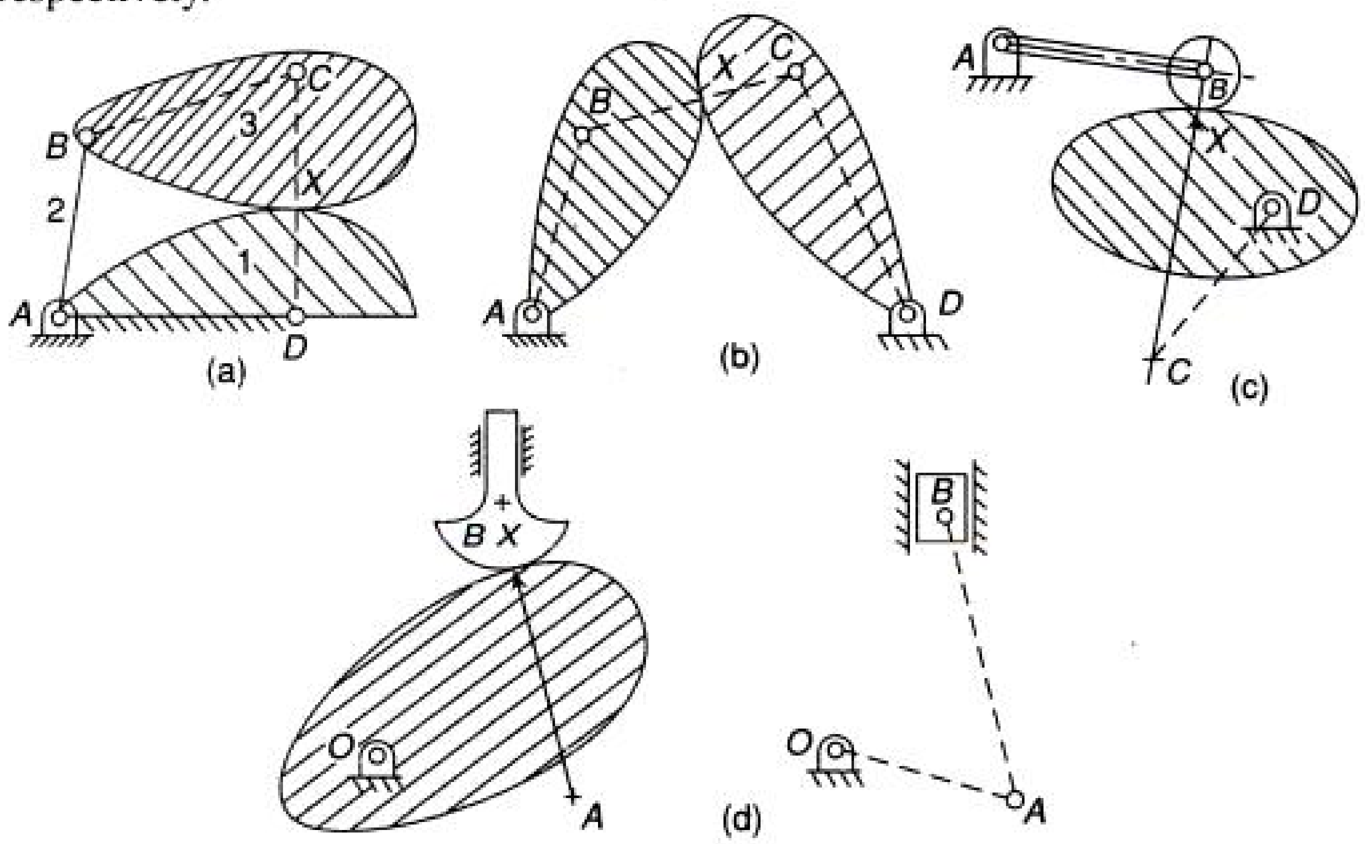


Fig. 1.28

Example 1.6 *Sketch a few slider-crank mechanisms derived from Stephenson's and Watt's six-bar chains.*

Solution Figure 1.29(a) shows a Stephenson's chain in which the ternary links are not directly connected. Thus, any of the binary links 3 or 6 can be replaced by a slider to obtain a slider-crank mechanism as shown in Fig. 1.29(b) and (c).

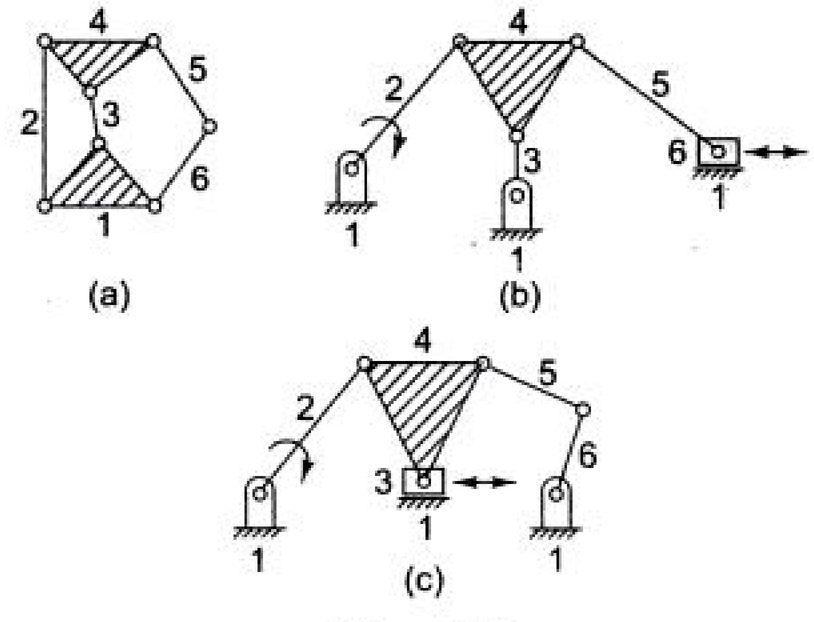


Fig. 1.29

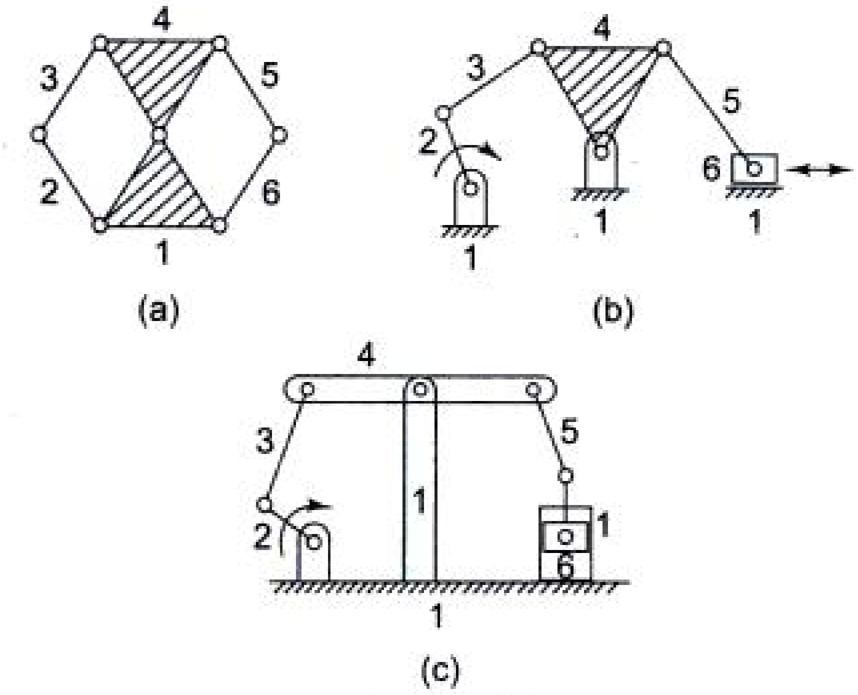


Fig. 1.30

Figure 1.30(a) shows a Watt's chain in which the ternary links are directly connected. Thus, any of the binary links 2 or 6 can be replaced by a slider to obtain a slider-crank mechanism. Figure 1.30 (b) and (c) show two variations of the slider obtained by replacing the binary link 6. The slider-crank mechanism of Fig. 1.30(c) is known as *beam engine*.

Example 1.7 *Sketch the equivalent kinematic chains with turning pairs for the chains shown in Fig. 1.31.*

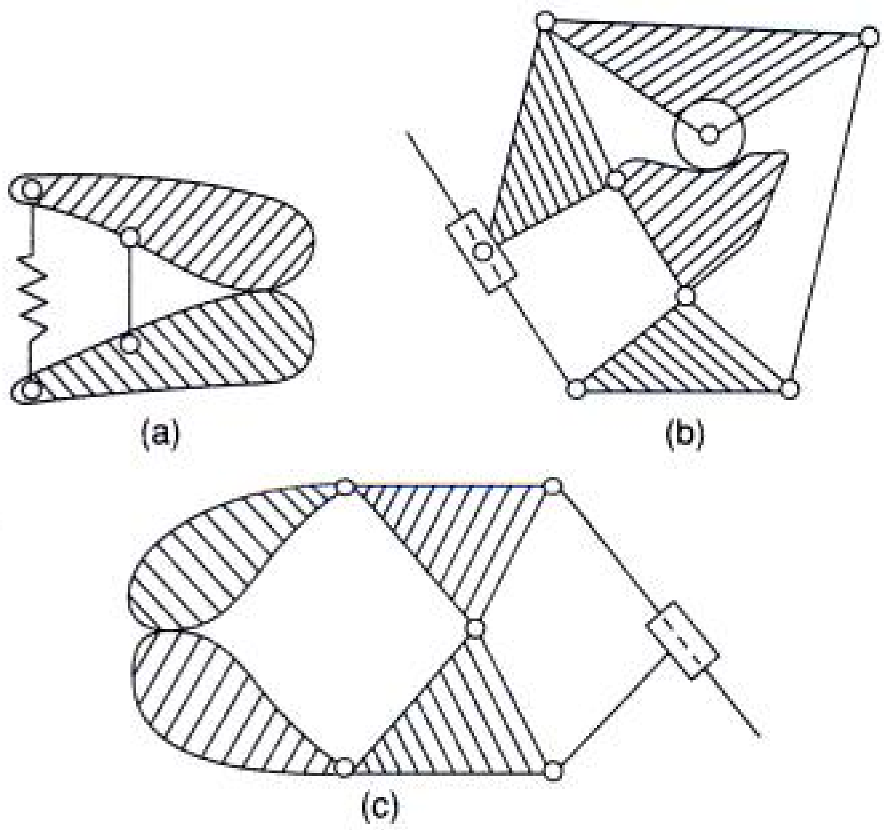


Fig. 1.31

Solution

(a) A spring is equivalent to two binary links connected by a turning pair. A cam pair is equivalent of one binary link with turning pairs at each end. The equivalent chain with turning pairs is shown in Fig. 1.32(a).

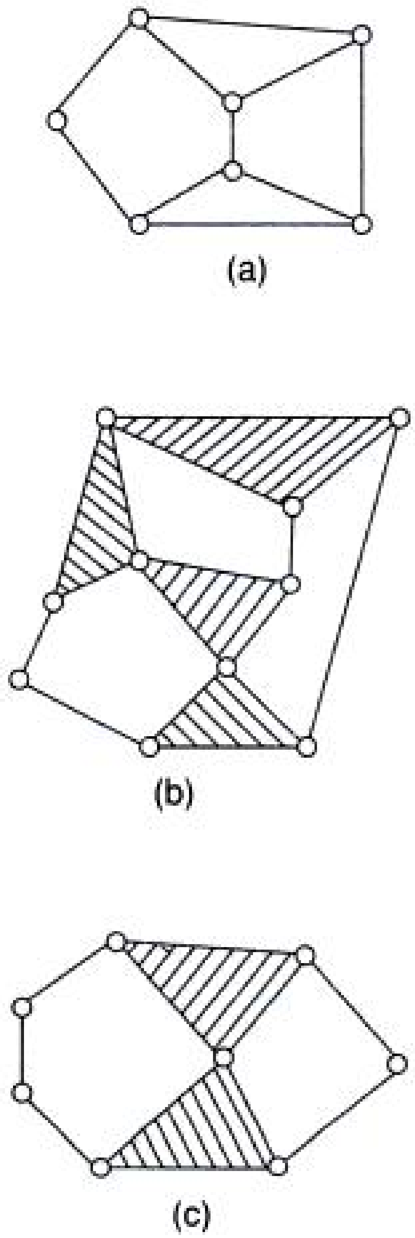


Fig. 1.32

(b) A slider pair can be replaced by one link with a turning pair at the other end. A cam pair with a roller follower can be replaced by a binary link with turning pairs at each end similar to the case of a curved-face follower of Fig. 1.28(d). the equivalent chain is shown in Fig. 1.32(b).

(c) The equivalent chain has been shown in Fig. 1.32(c).

1.13 THE FOUR-BAR CHAIN

A four-bar chain is the most fundamental of the plane kinematic chains. It is a much preferred mechanical device for the mechanisation and control of motion due to its simplicity and versatility. Basically, it consists of four rigid links which are connected in the form of a quadrilateral by four pin-joints. When one of the links is fixed, it is known as a *linkage* or *mechanism*. A link that makes complete revolution is called the *crank*, the link opposite to the fixed link is called the *coupler*, and the fourth link is called a *lever* or *rocker* if it oscillates or another crank, if it rotates.

Note that it is impossible to have a four-bar linkage if the length of one of the links is greater than the sum of the other three. This has been shown in Fig. 1.33 in which the length of link d is more than the sum of lengths of a, b and c, and therefore, this linkage cannot exist.

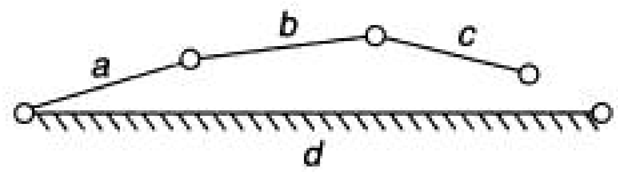


Fig. 1.33

Consider a four-link mechanism shown in Fig. 1.34(a) in which the length a of the link AB is more than d, the length of the fixed link AD. The linkage has been shown in various positions. It can be observed from these configurations that if the link a is to rotate through a full revolution, i.e., if it is to be a crank, then the following conditions must be met:

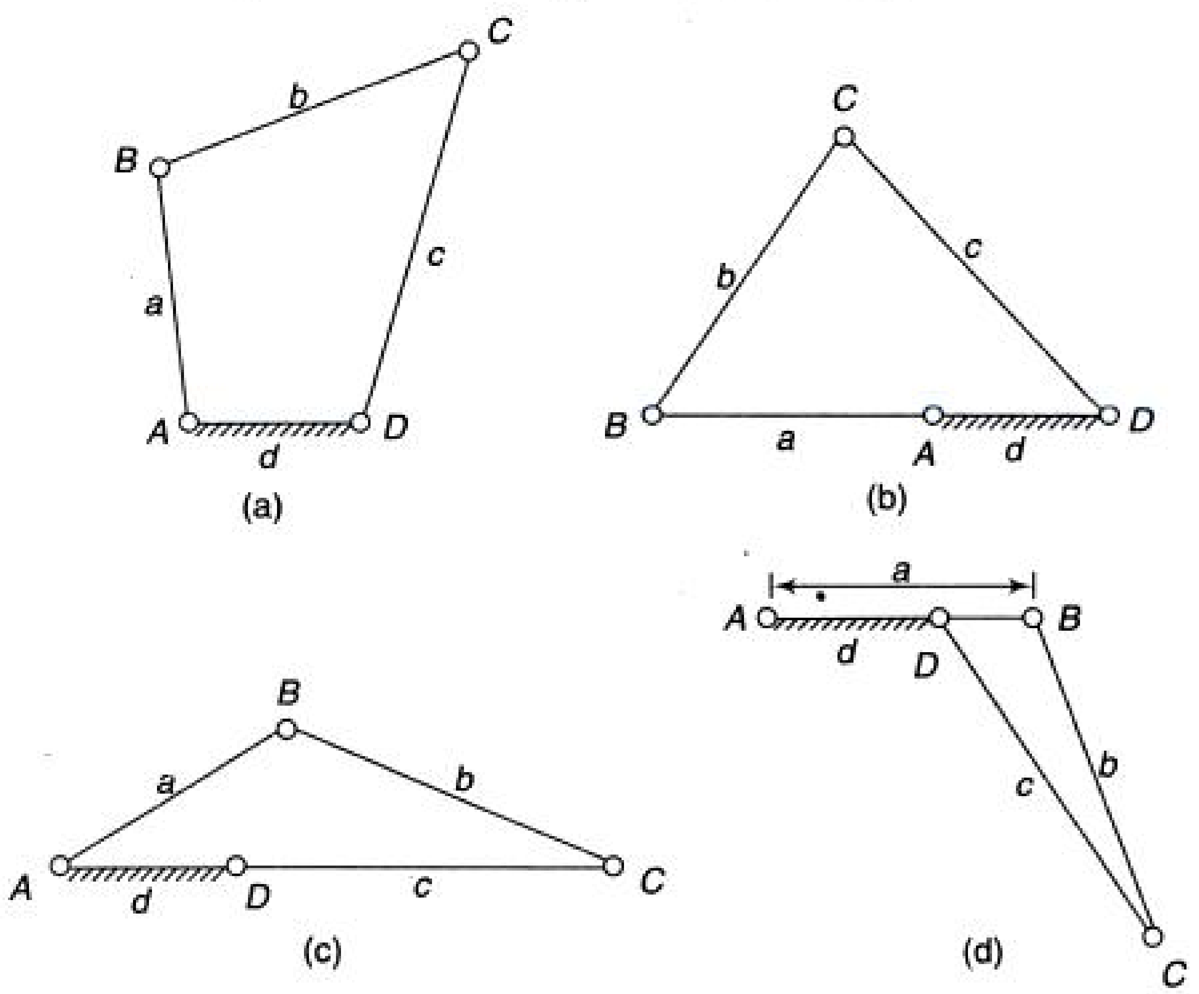


Fig. 1.34

From Fig. 1.34(b), $d + a < b + c$ (i)

From Fig. 1.34(c), $d + c < a + b$ (ii)

From Fig. 1.34(d), $b < c + (a - d)$ or $d + b < c + a$ (iii)

Adding (i) and (ii), $2d + a + c < 2b + a + c$

or $d < b$

Similarly, adding (ii) and (iii), and (iii) and (i) we get

$d < a$

and $d < c$

Thus, d is less than a, b and c, i.e., it is the shortest link if a is to rotate a full circle or act as a crank. The above inequalities also suggest that out of a, b and c, whichever is the longest, the sum of that with d, the shortest link will be less than the sum of the remaining two links. Thus, the necessary conditions for the link a to be a crank is

- the shortest link is fixed, and
- the sum of the shortest and the longest links is less than the sum of the other two links.

In a similar way, it can be shown that if the link c is to rotate through a full circle, i.e., if it is to be a crank then the conditions to be realised are the same as above. Also, it can be shown that if both the links a and c rotate through full circles, the link b also makes one complete revolution relative to the fixed link d.

The mechanism thus obtained is known as *crank-crank* or *double-crank* or *drag-crank mechanism* or *rotary-rotary converter.* Figure 1.35 shows all the three links a, b and c rotating through one complete revolution.

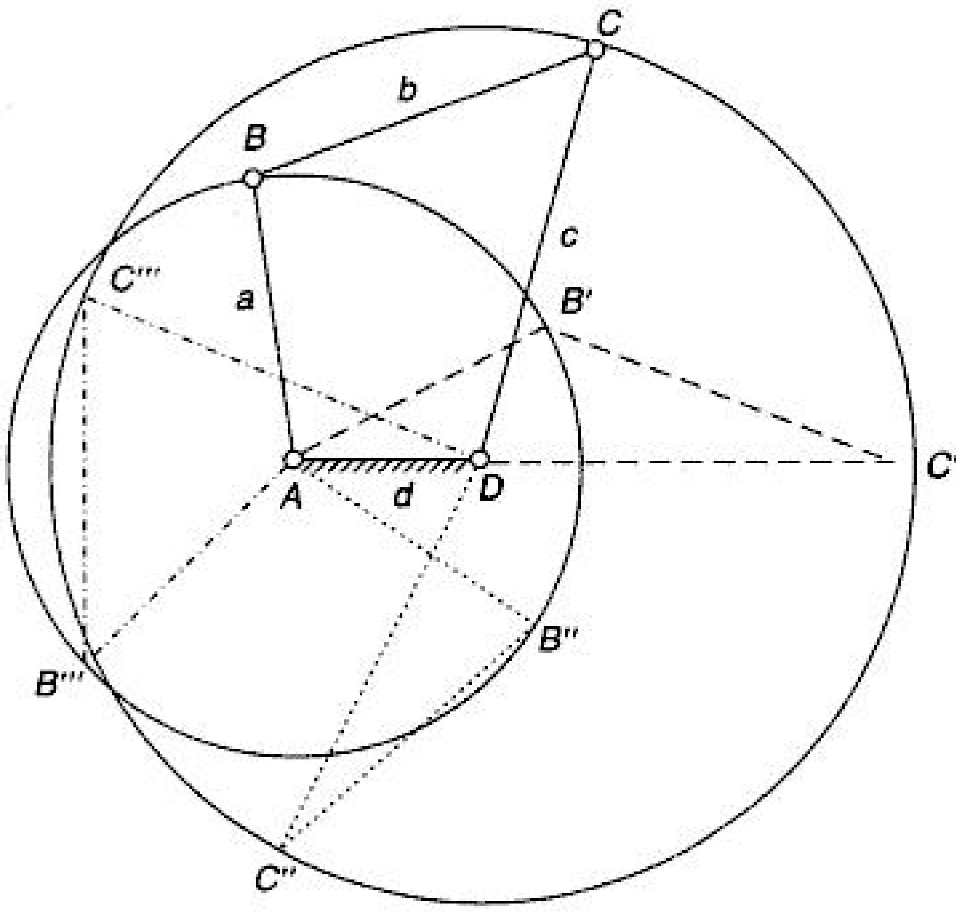


Fig. 1.35

In the above consideration, the rotation of the links is observed relative to the fixed link d. Now, consider the movement of b relative to either a or c. The complete rotation of b relative to a is possible if the angle $\angle ABC$ can be more than 180° and relative to c if the angle $\angle DCB$ more than 180°. From the positions of the links in Fig. 1.35(b) and (c), it is clear that these angles cannot become more than 180° for the above stated conditions.

Now, as the relative motion between two adjacent links remains the same irrespective of which link is fixed to the frame, different mechanisms (known as *inversions*) obtained by fixing different links of this kind of chain will be as follows:

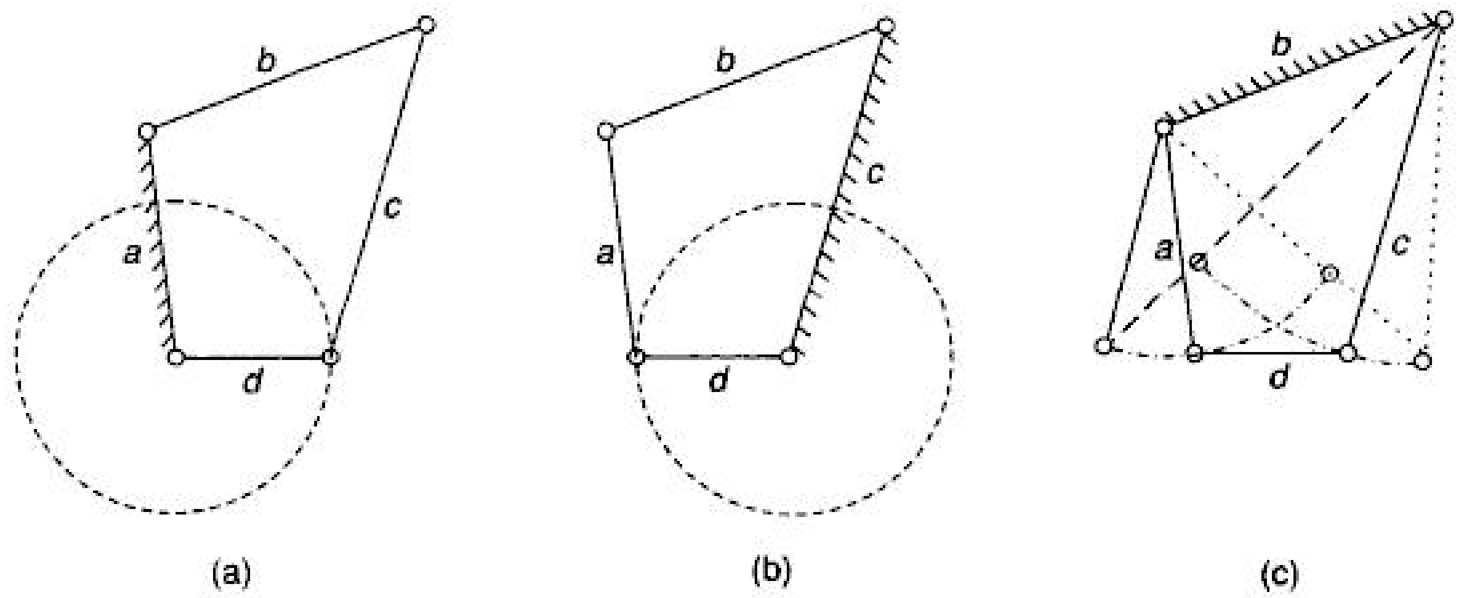


Fig. 1.36

1. If any of the adjacent links of link *d*, i.e., *a* or *c* is fixed, *d* can have a full revolution (crank) and the link opposite to it oscillates (rocks). In Fig. 1.36(a), *a* is fixed, *d* is the crank and *b* oscillates whereas in Fig. 1.36(b), *c* is fixed, *d* is the crank and *b* oscillates. The mechanism is known as *crank-rocker* or *crank-lever mechanism* or *rotary-oscillating converter.*
2. If the link opposite to the shortest link, i.e., link *b* is fixed and the shortest link *d* is made a coupler, the other two links *a* and *c* would oscillate [Fig. 1.36(c)]. The mechanism is known as a *rocker-rocker* or *double-rocker* or *double-lever mechanism* or *oscillating-oscillating converter.*

A linkage in which the sum of the lengths of the longest and the shortest links is less than the sum of the lengths of the other two links, is known as a *class-I,* four-bar linkage.

When the sum of the lengths of the largest and the shorted links is more than the sum of the lengths of the other two links, the linkage is known as a *class-II,* four-bar linkage. In such a linkage, fixing of any of the links always results in a *rocker-rocker* mechanism. In other words, the mechanism and its inversions give the same type of motion (of a *double-rocker* mechanism).

The above observations are summarised in *Grashof's law* which states *that a four-bar mechanism has at least one revolving link if the sum of the lengths of the largest and the shortest links is less than the sum of lengths of the other two links.*

Further, if the *shortest link is fixed,* the chain will act as a double-crank mechanism in which links adjacent to the fixed link will have complete revolutions. If the *link opposite to the shortest link is fixed,* the chain will act as double-rocker mechanism in which links adjacent to the fixed link will oscillate. If the *link adjacent to the shortest link is fixed,* the chain will act as crank-rocker mechanism in which the shortest link will revolve and the link adjacent to the fixed link will oscillate.

If the sum of the lengths of the largest and the shorted links is equal to the sum of the lengths of the other two links, i.e., when equalities exist, the four inversions, in general, result in mechanisms similar to those as given by Grashof's law, except that sometimes the links may become collinear and may have to be guided in the proper direction. Usually, the purpose is served by the inertia of the links. A few special cases may arise when equalities exist. For example, *parallel-crank four-bar linkage* and *deltoid* linkage.

Parallel-Crank Four-Bar Linkage If in a four-bar linkage, two opposite links are parallel and equal in length, then any of the links can be made fixed. The two links adjacent to the fixed link will always act as two cranks. The four links form a parallelogram in all the positions of the cranks, provided the cranks rotate in the same sense as shown in Fig. 1.37.

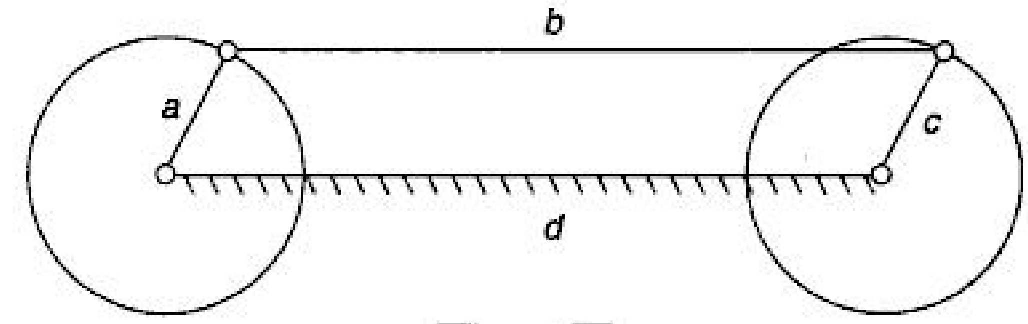


Fig. 1.37

The use of such a mechanism is made in the coupled wheels of a locomotive in which the rotary motion of one wheel is transmitted to the other wheel. For kinematic analysis, link *d* is treated as fixed and the relative motions of the other links are found. However, in fact, *d* has a translatory motion parallel to the rails.

Deltoid Linkage In a deltoid linkage (Fig. 1.38), the equal links are adjacent to each other. When any of the shorter links is fixed, a double-crank mechanism is obtained in which one revolution of the longer link causes two revolutions of the other shorter link. As shown in Fig. 1.38 (a), when the link *c* rotates through half a revolution and assumes the position *DC'*, the link *a* has completed a full revolution.

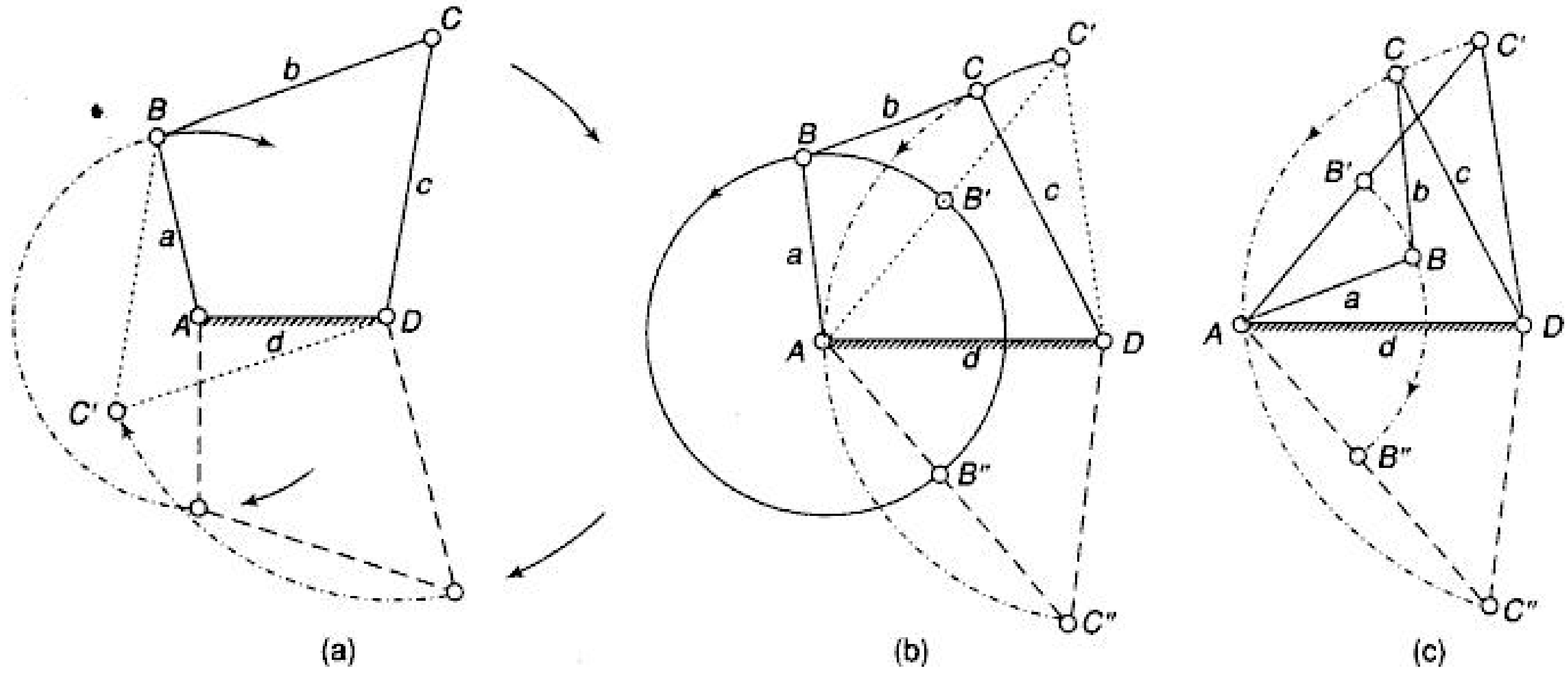


Fig. 1.38

When any of the longer links is fixed, two crank-rocker mechanisms are obtained [Fig. 1.38(b) and (c)]

Example 1.8 *Find all the inversion of the chain given in Fig. 1.39.*

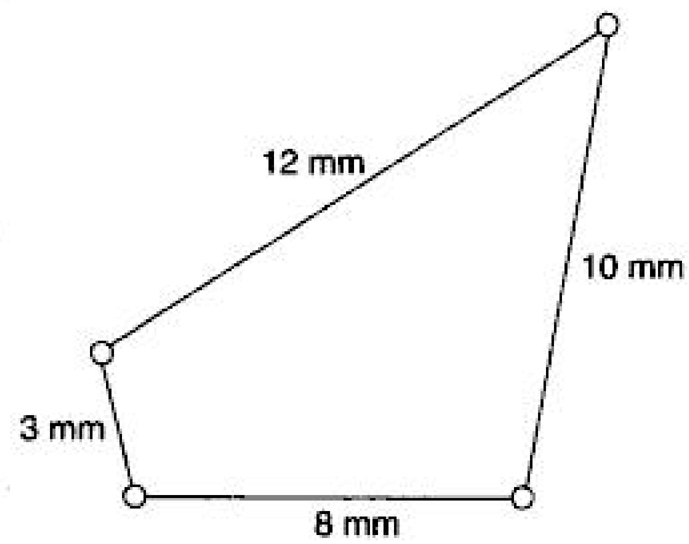


Fig. 1.39

Solution

(a) Length of the longest link = 12 mm
Length of the shortest link = 3 mm
Length of other links = 10 mm and 8 mm
Since 12 + 3 < 10 + 8, it belongs to the class-I mechanism and according to Grashoff's law, three distinct inversions are possible.

Shortest link fixed, i.e., when the link with 3-mm length is fixed, the chain will act as double-crank mechanism in which links with lengths of 12 mm and 8 mm will have complete revolutions.

Link opposite to the shortest link fixed, i.e., when the link with 10-mm length is fixed, the chain will act as double-rocker mechanism in which links with lengths of 12 mm and 8 mm will oscillate.

Link adjacent to the shortest link fixed, i.e., when any of the links adjacent to the shortest link, i.e., link with a length of 12-mm or 8 mm is fixed, the chain will act as crank-rocker mechanism in which the shortest link of 3-mm length will revolve and the link with 10-mm length will oscillate.

Example 1.9 *Figure 1.40 shows some four-link mechanisms in which the figures indicate the dimensions in standard units of length. Indicate the type of each mechanism whether crank-rocker or double-crank or double-rocker.*

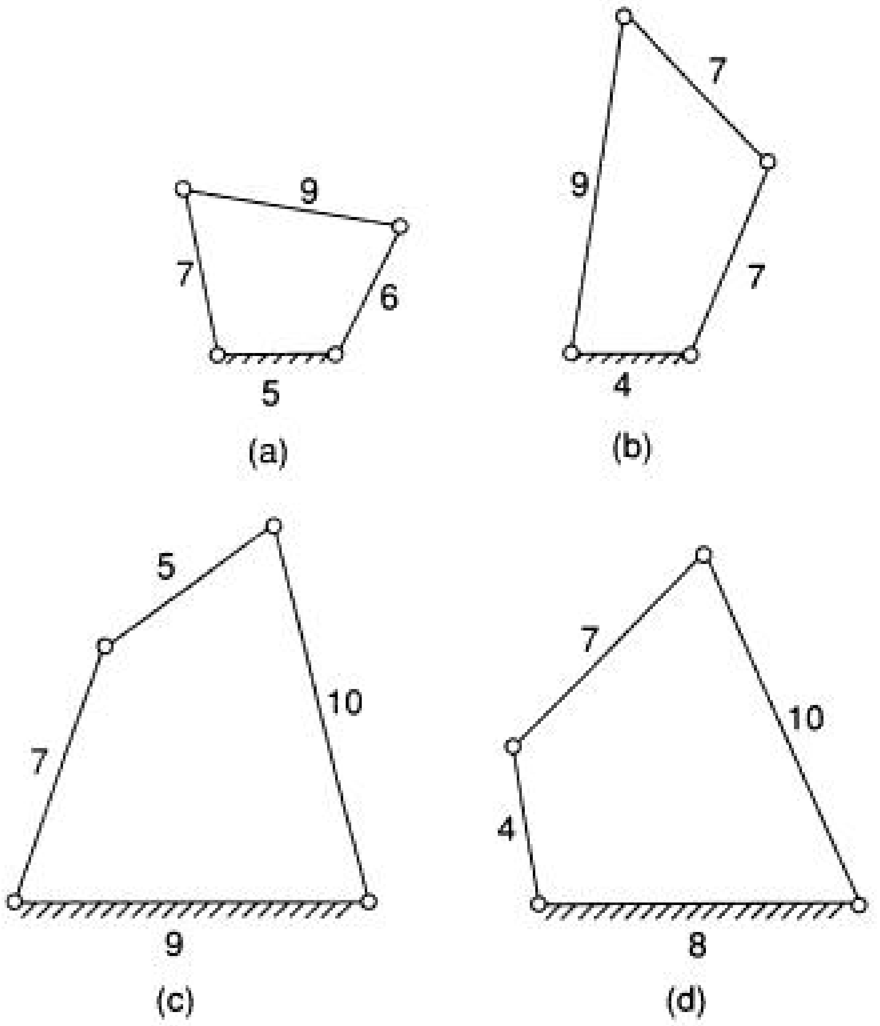


Fig. 1.40

Solution

(a) Length of the longest link = 9
Length of the shortest link = 5
Length of other links = 7 and 6
Since 9 + 5 > 7 + 6, it does not belong to the class-I mechanism. Therefore, it is a double-rocker mechanism.

(b) Length of the longest link = 9
Length of the shortest link = 4
Length of other links = 7 and 7
Since 9 + 4 < 7 + 7, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism.

(c) Length of the longest link = 10
Length of the shortest link = 5
Length of other links = 9 and 7
Since 10 + 5 < 9 + 7, it belongs to the class-I mechanism. In this case as the link opposite to the shortest link is fixed, it is a double-rocker mechanism.

(d) Length of the longest link = 10
Length of the shortest link = 4
Length of other links = 8 and 7
Since 10 + 4 < 8 + 7, it belongs to the class-I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

Example 1.10 *Figure 1.41 shows a plane mechanism in which the figures indicate the dimensions in standard units of length. The slider C is the driver. Will the link AG revolve or oscillate?*

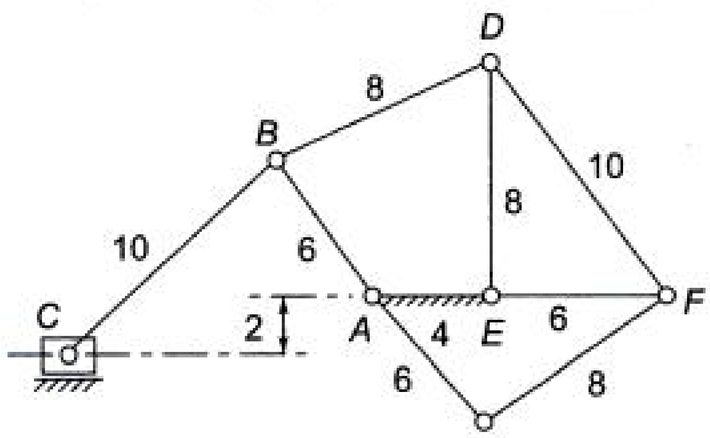


Fig. 1.41

Solution The mechanism has three sub-chains:

(i) *ABC*, a slider-crank chain
(ii) *ABDE*, a four-bar chain
(iii) *AEFG*, a four-bar chain

DEF is a locked chain as it has only three links.

- As the length *BC* is more than the length *AB* plus the offset of 2 units, *AB* acts as a crank and can revolve about *A*.
- In the chain *ABDE*,
  Length of the longest link = 8
  Length of the shortest link = 4
  Length of other links = 8 and 6

Since 8 + 4 < 8 + 6, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism and thus *AB* and *ED* can revolve fully.

- In the chain *AEFG*,
  Length of the longest link = 8
  Length of the shortest link = 4
  Length of other links = 6 and 6

Since 8 + 4 = 6 + 6, it belongs to the class-I mechanism. As the shortest link is fixed, it is a double-crank mechanism and thus *EF* and *AG* can revolve fully.

As *DEF* is a locked chain with three links, the link *EF* revolves with the revolving of *ED*. With the revolving of *ED*, *AG* also revolves.

1.14 MECHANICAL ADVANTAGE

The *mechanical advantage* (MA) of a mechanism is the ratio of the output force or torque to the input force or torque at any instant. Thus for the linkage of Fig. 1.42, if friction and inertia forces are ignored and the input torque T_2 is applied to the link 2 to drive the output link 4 with a resisting torque T_4 then

Power input = Power output

$$T_2\,\omega_2 = T_4\,\omega_4$$

or $$\text{MA} = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}$$

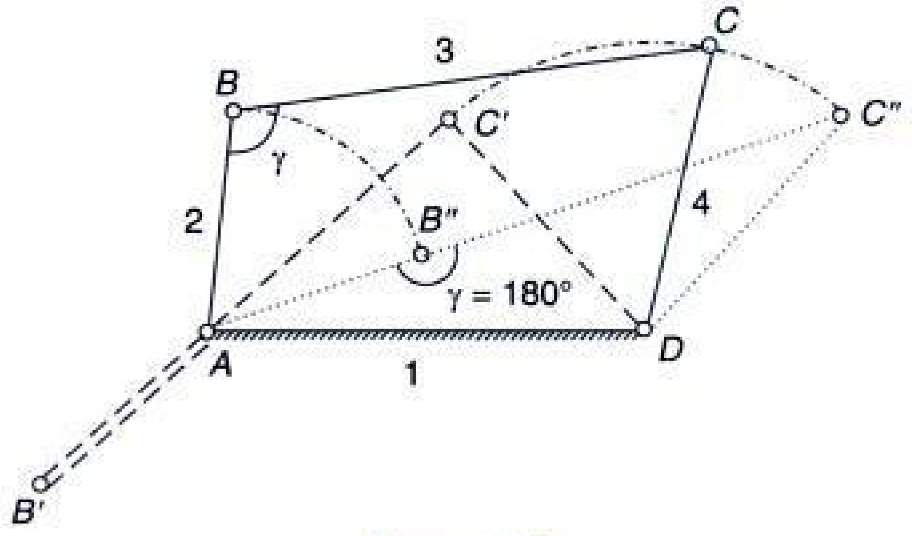


Fig. 1.42

Thus, it is the reciprocal of the velocity ratio. In case of crank-rocker mechanisms, the velocity ω_4 of the output link *DC* (rocker) becomes zero at the extreme positions (*AB′C′D* and *AB″C″D*), i.e., when the input link *AB* is in line with the coupler *BC* and the angle γ between them is either zero or 180°, it makes the mechanical advantage to be infinite at such positions. Only a small input torque can overcome a large output torque load. The extreme positions of the linkage are known as *toggle* positions.

1.15 TRANSMISSION ANGLE

The angle μ between the output link and the coupler is known as *transmission angle*. In Fig. 1.43, if the link *AB* is the input link, the force applied to the output link *DC* is transmitted through the coupler *BC*. For a particular value of force in the coupler rod, the torque transmitted to the output link (about the point *D*) is maximum when the transmission angle μ is 90°. If links *BC* and *DC* become coincident, the *transmission angle* is zero and the mechanism would lock or jam. If μ deviates significantly from 90°, the torque on the output link decreases. Sometimes, it may not be sufficient to overcome the friction in the system and the mechanism may be locked or jammed. Hence μ is usually kept more than 45°. The best mechanisms, therefore, have a transmission angle that does not deviate much from 90°.

Applying cosine law to triangles *ABD* and *BCD* (Fig. 1.43),

$$a^2 + d^2 - 2ad\cos\theta = k^2 \quad \text{(i)}$$

and $$b^2 + c^2 - 2bc\cos\mu = k^2 \quad \text{(ii)}$$

From (i) and (ii),

$$a^2 + d^2 - 2ad\cos\theta = b^2 + c^2 - 2bc\cos\mu$$

or $$a^2 + d^2 - b^2 - c^2 - 2ad\cos\theta + 2bc\cos\mu = 0$$

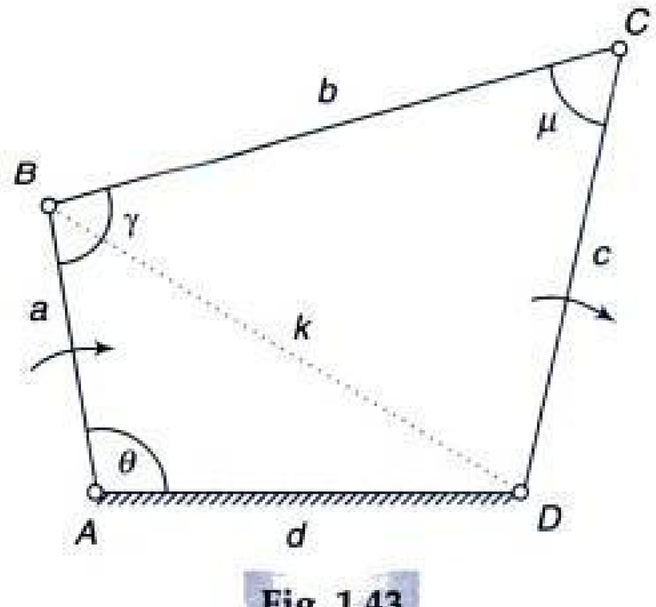


Fig. 1.43

The maximum or minimum values of the transmission angle can be found by putting $d\mu/d\theta$ equal to zero.

Differentiating the above equation with respect to θ,

$$2\,ad \sin\theta - 2bc \sin\mu.\frac{d\mu}{d\theta} = 0$$

or $$\frac{d\mu}{d\theta} = \frac{ad \sin\theta}{bc \sin\mu}$$

Thus, if $d\mu/d\theta$ is to be zero, the term $ad \sin\theta$ has to be zero which means θ is either 0° or 180°. It can be seen that μ is maximum when θ is 180° and minimum when θ is 0°. However, this would be applicable to the mechanisms in which the link a is able to assume these angles, i.e., in double-crank or crank-rocker mechanisms. Figures 1.44(a) and (b) show a crank-rocker mechanism indicating the positions of the maximum and the minimum transmission angles. Figures 1.45(a) and (b) show the maximum and the minimum transmission angles for a double-rocker mechanism.

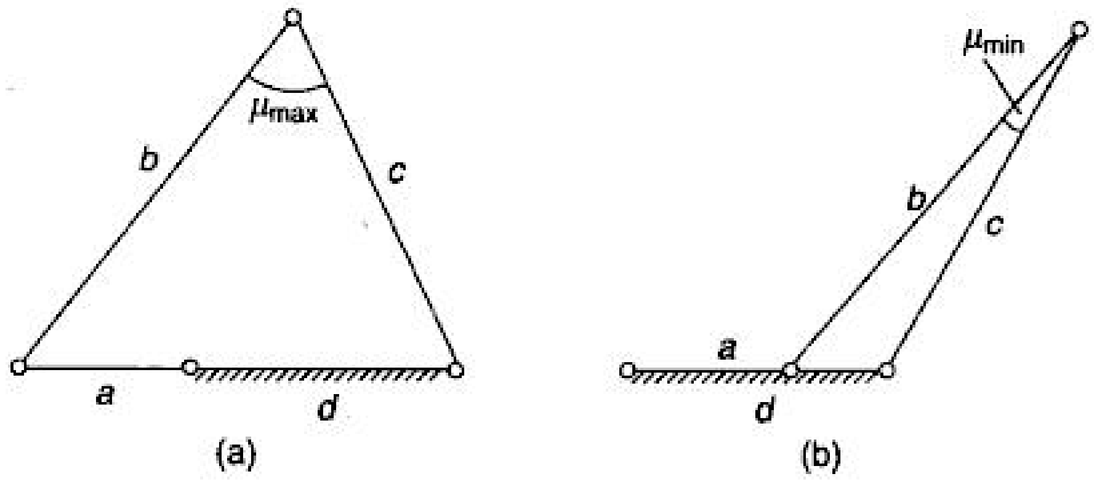


Fig. 1.44

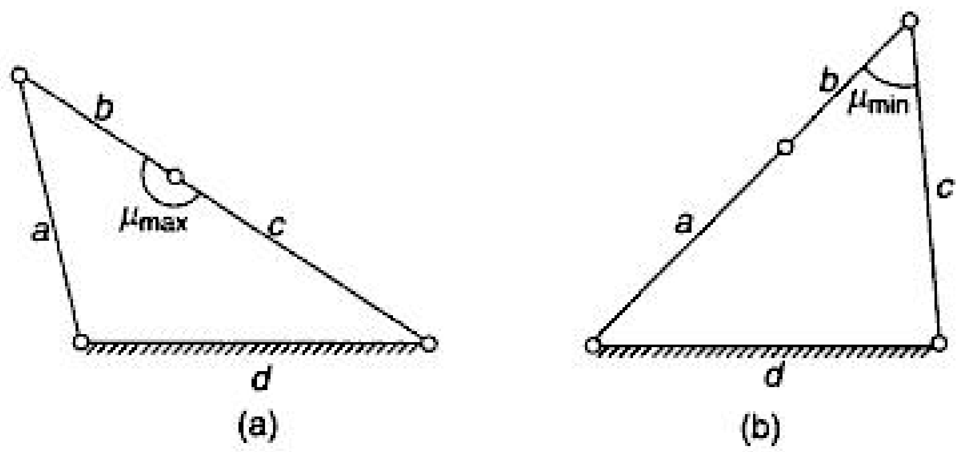


Fig. 1.45

Example 1.11 *Find the maximum and minimum transmission angles for the mechanisms shown in Fig. 1.46. The figures indicate the dimensions in standard units of length.*

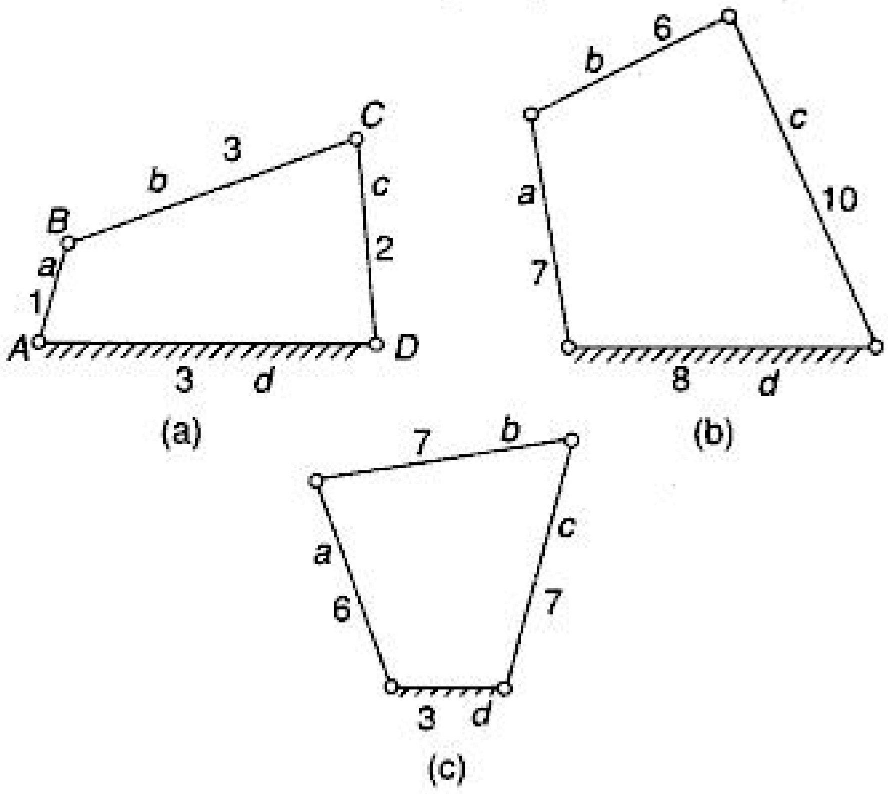


Fig. 1.46

Solution

(a) In this mechanism,
Length of the longest link = 3
Length of the shortest link = 1
Length of other links = 3 and 2

Since 3 + 1 < 3 + 2, it belongs to the class I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

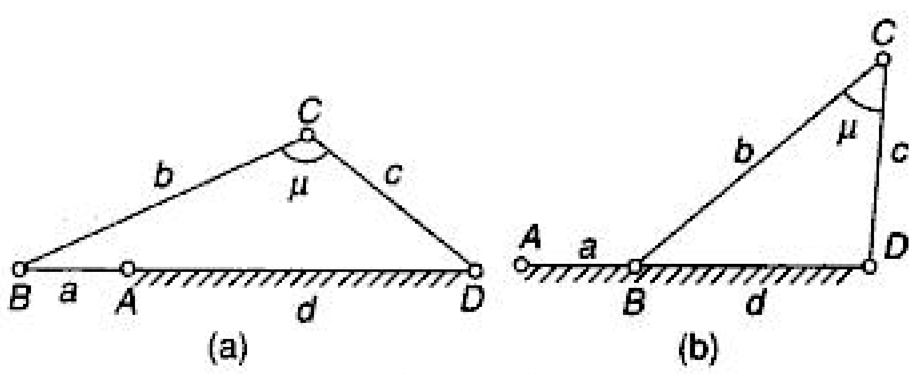


Fig. 1.47

Maximum transmission angle is when θ is 180° [Fig. 1.47(a)],

Thus $(a + d)^2 = b^2 + c^2 - 2bc \cos\mu$

$(1 + 3)^2 = 3^2 + 2^2 - 2 \times 3 \times 2 \cos\mu$

$16 = 9 + 4 - 12 \cos\mu$

$$\cos\mu = -\frac{3}{12} = -0.25$$

$$\mu = 104.5^\circ$$

Minimum transmission angle is when θ is 0° [Fig. 1.47(b)],

Thus $(d - a)^2 = b^2 + c^2 - 2bc\cos\mu$

$$(3 - 1)^2 = 3^2 + 2^2 - 2 \times 3 \times 2\cos\mu$$

$$4 = 9 + 4 - 12\cos\mu$$

$$\cos\mu = \frac{3}{4} = 0.75$$

$$\mu = 41.4^\circ$$

(b) In this mechanism,

Length of the longest link = 10

Length of the shortest link = 6

Length of other links = 8 and 7

Since $10 + 6 > 8 + 7$, it belongs to the class-II mechanism and thus is a double-rocker mechanism.

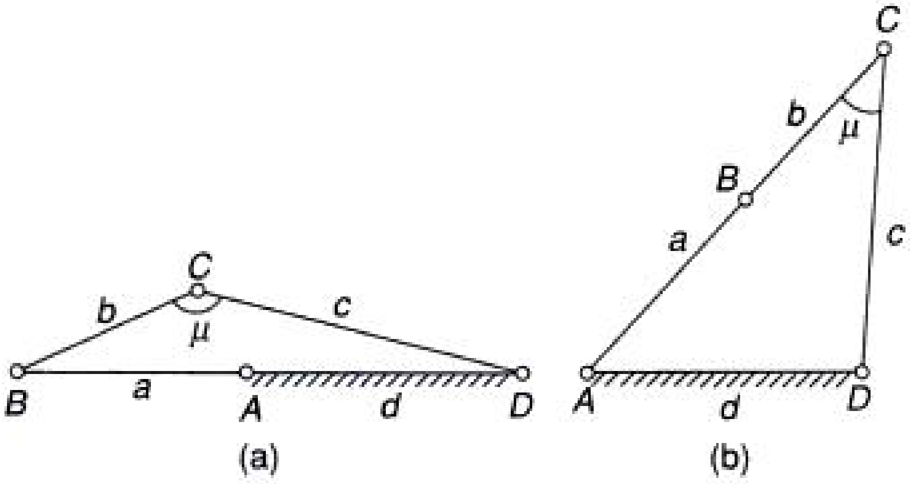


Fig. 1.48

Maximum transmission angle is when θ is 180° [Fig. 1.48(a)],

Thus, $(a + d)^2 = b^2 + c^2 - 2bc\cos\mu$

$$(7 + 8)^2 = 6^2 + 10^2 - 2 \times 6 \times 10\cos\mu$$

$$225 = 36 + 100 - 120\cos\mu$$

$$\cos\mu = -\frac{89}{120} = -0.742$$

$$\mu = 137.9^\circ$$

Minimum transmission angle is when the angle at B is 180° [Fig. 1.48(b)],

Thus, $d^2 = (a + b)^2 + c^2 - 2(a + b)c\cos\mu$

$$8^2 = (7 + 6)^2 + 10^2 - 2(7 + 6) \times 10 \times \cos\mu$$

$$64 = 169 + 100 - 260\cos\mu$$

$$\cos\mu = \frac{205}{260} = 0.788$$

$$\mu = 38^\circ$$

(c) In this mechanism,

Length of the longest link = 7

Length of the shortest link = 3

Length of other links = 6 and 6

Since $7 + 3 < 6 + 6$, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank or drag-link mechanism.

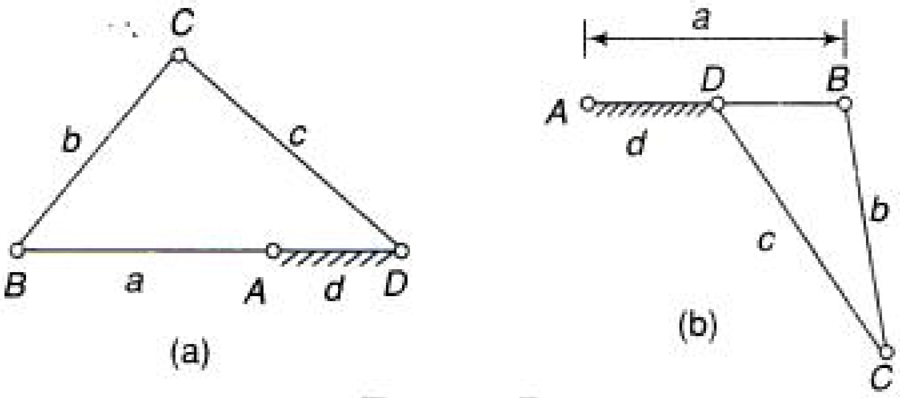


Fig. 1.49

Maximum transmission angle is when θ is 180° [Fig. 1.49(a)],

Thus $(a + d)^2 = b^2 + c^2 - 2bc\cos\mu$

$$(6 + 3)^2 = 6^2 + 7^2 - 2 \times 6 \times 7\cos\mu$$

$$81 = 36 + 49 - 84\cos\mu$$

$$\cos\mu = \frac{4}{84} = 0.476$$

$$\mu = 87.27^\circ$$

Minimum transmission angle is when θ is 0° [Fig. 1.49(b)],

Thus $(a - d)^2 = b^2 + c^2 - 2bc\cos\mu$

$$(6 - 3)^2 = 6^2 + 7^2 - 2 \times 6 \times 7\cos\mu$$

$$9 = 36 + 49 - 84\cos\mu$$

$$\cos\mu = \frac{76}{84} = 0.9048$$

$$\mu = 25.2^\circ$$

Example 1.12 *A crank-rocker mechanism has a 70-mm fixed link, a 20-mm crank, a 50-mm coupler, and a 70-mm rocker. Draw the mechanism and determine the maximum and minimum values of the transmission angle. Locate the two toggle positions and find the corresponding crank angles and the transmission angles.*

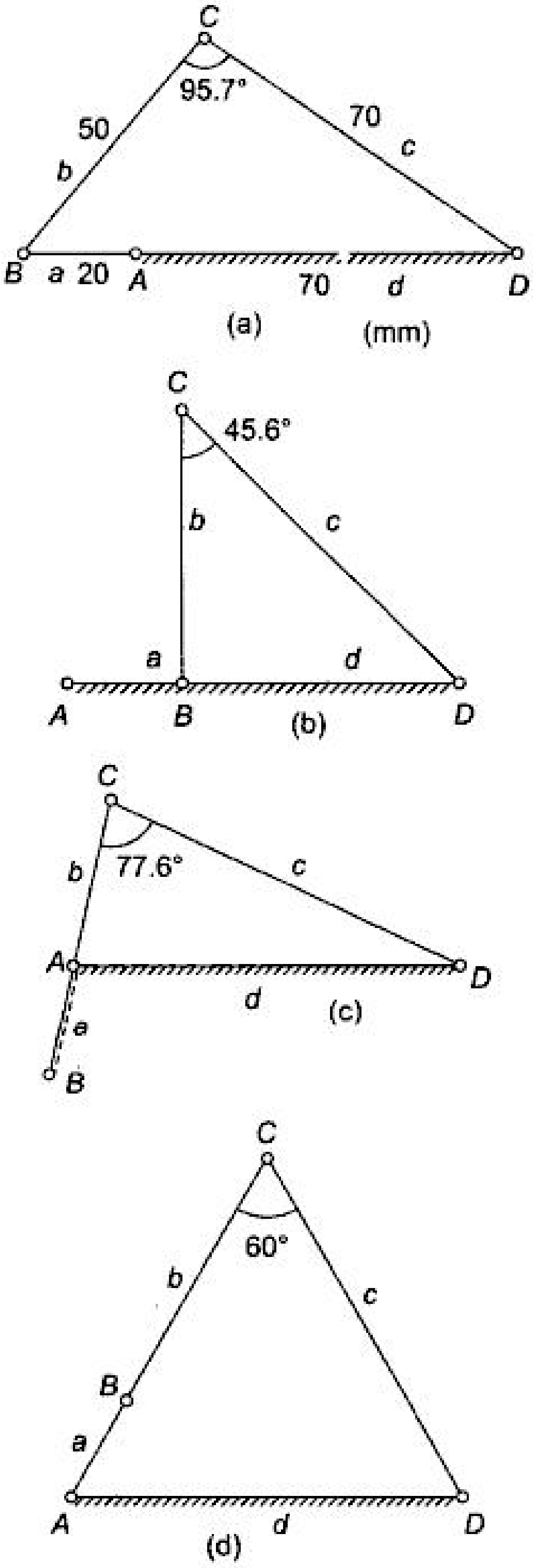


Fig. 1.50

Solution In this mechanism,

Length of the longest link = 70 mm

Length of the shortest link = 20 mm

Length of other links = 70 and 50 mm

Since 70 + 20 < 70 + 50, it belongs to the class-I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

Maximum transmission angle is when θ is 180° [Fig. 1.50(a)],

Thus $(a + d)^2 = b^2 + c^2 - 2bc \cos \mu$

$$(20 + 70)^2 = 50^2 + 70^2 - 2 \times 50 \times 70 \cos \mu$$

$$8100 = 2500 + 4900 - 7000 \cos \mu$$

$$\cos \mu = -0.1$$

$$\mu = 95.7^\circ$$

Minimum transmission angle is when θ is 0° [Fig. 1.50(b)],

Thus $(70 - 20)^2 = 50^2 + 70^2 - 2 \times 50 \times 70 \cos \mu$

$$2500 = 2500 + 4900 - 7000 \cos \mu$$

$$\cos \mu = 0.7$$

$$\mu = 45.6^\circ$$

The two toggle positions are shown in Figs 1.50(c) and (d).

Transmission angle for first position,

$$d^2 = (b - a)^2 + c^2 - 2(b - a)c \cos \mu$$

$$70^2 = 30^2 + 70^2 - 2 \times 30 \times 70 \cos \mu$$

$$4900 = 900 + 4900 - 4200 \cos \mu$$

$$\cos \mu = 0.214$$

$$\mu = 77.6^\circ$$

As c and d are of equal length [Fig. 1.50(c)], it is an isosceles triangle and thus input angle $\theta = (77.6^\circ + 180^\circ) = 257.6^\circ$

Transmission angle for second position Fig. 1.50(d),

$$d^2 = (b + a)^2 + c^2 - 2(b + a)c \cos \mu$$

$$70^2 = 70^2 + 70^2 - 2 \times 70 \times 70 \cos \mu$$

$$4900 = 4900 + 4900 - 9800 \cos \mu$$

$$\cos \mu = 0.5$$

$$\mu = 60^\circ$$

(or as all the sides of the triangle of Fig. 1.50(d) are of equal length, it is an equilateral triangle and thus transmission angle is equal to 60°)

And the input angle, $\theta = 60^\circ$

- The above results can also be obtained graphically by drawing the figures to scale and measuring the angles.

1.16 THE SLIDER-CRANK CHAIN

When one of the turning pairs of a four-bar chain is replaced by a sliding pair, it becomes a *single slider-crank chain* or simply a *slider-crank chain*. It is also possible to replace two sliding pairs of a four-bar chain to get a *double slider-crank chain* (Sec. 1.17). Further, in a slider-crank chain, the straight line path of the slider may be passing through the fixed pivot O or may be displaced. The distance e between the fixed pivot O and the straight line path of the slider is called the *offset* and the chain so formed an *offset slider-crank chain*.

Different mechanisms obtained by fixing different links of a kinematic chain are known as its *inversions*. A slider-crank chain has the following inversions:

First Inversion

This inversion is obtained when link 1 is fixed and links 2 and 4 are made the crank and the slider respectively [Fig. 1.51(a)].

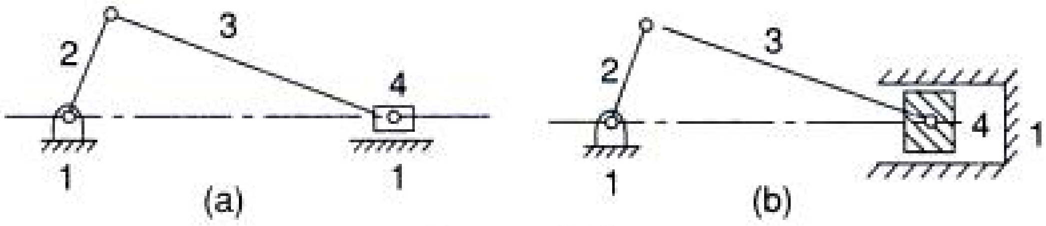


Fig. 1.51

Applications

1. Reciprocating engine
2. Reciprocating compressor

As shown in Fig. 1.51(b), if it is a reciprocating engine, 4 (piston) is the driver and if it is a compressor, 2 (crank) is the driver.

Second Inversion

Fixing of the link 2 of a slider-crank chain results in the second inversion.

The slider-crank mechanism of Fig. 1.51(a) can also be drawn as shown in Fig. 11.52(a). Further, when its link 2 is fixed instead of the link 1, the link 3 along with the slider at its end B becomes a crank. This makes the link 1 to rotate about O along with the slider which also reciprocates on it [Fig. 1.52(b)].

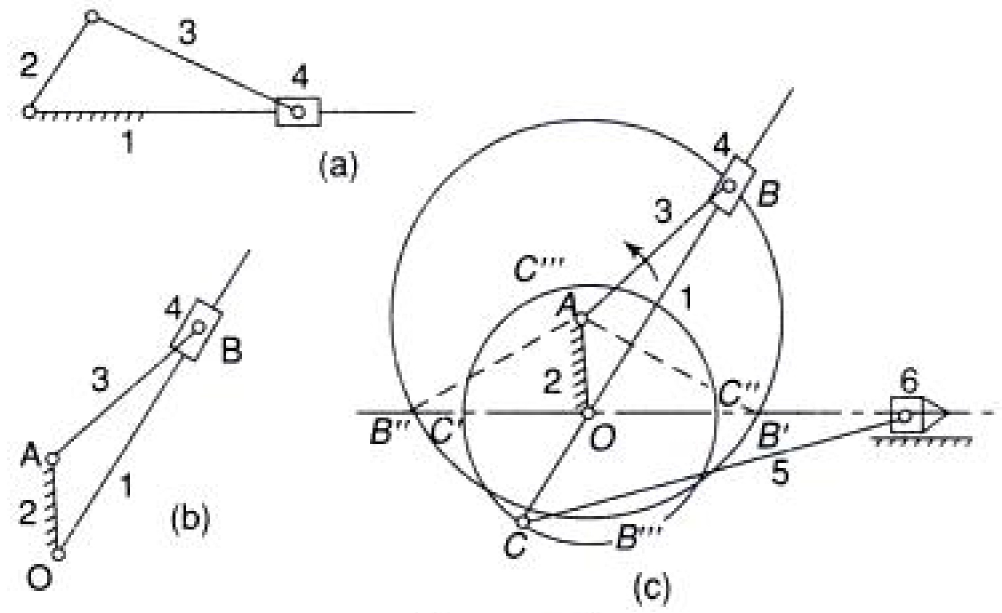


Fig. 1.52

Applications

1. Whitworth quick-return mechanism
2. Rotary engine

Whitworth Quick-Return Mechanism It is a mechanism used in workshops to cut metals. The forward stroke takes a little longer and cuts the metal whereas the return stroke is idle and takes a shorter period.

Slider 4 rotates in a circle about A and slides on the link 1 [Fig. 1.52(c)]. C is a point on the link 1 extended backwards where the link 5 is pivoted. The other end of the link 5 is pivoted to the tool, the forward stroke of which cuts the metal. The axis of motion of the slider 6 (tool) passes through O and is perpendicular to OA, the fixed link. The crank 3 rotates in the counter-clockwise direction.

Initially, let the slider 4 be at B' so that C be at C'. Cutting tool 6 will be in the extreme left position. With the movement of the crank, the slider traverses the path $B'BB''$ whereas the point C moves through $C'CC''$. Cutting tool 6 will have the forward stroke. Finally, the slider B assumes the position B'' and the cutting tool 6 is in the extreme right position. The time taken for the forward stroke of the slider 6 is proportional to the obtuse angle $B''AB'$ at A.

Similarly, the slider 4 completes the rest of the circle through the path $B''B'''B'$ and C passes through $C''C'''C'$. There is backward stroke of the tool 6. The time taken in this is proportional to the acute angle $B''AB'$ at A.

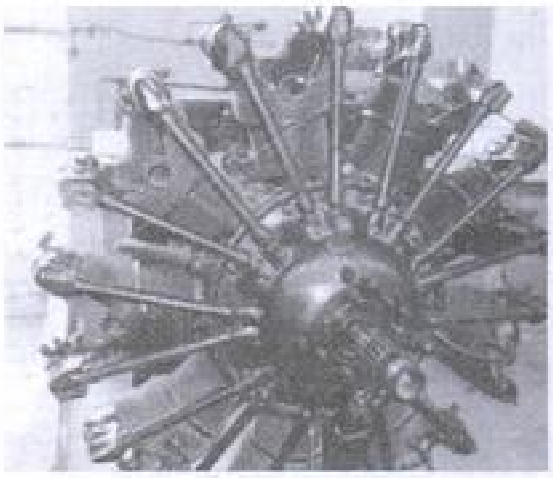
Let

θ = obtuse angle $B'AB''$ at A

β = acute angle $B'AB''$ at A

Then,

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\theta}{\beta}$$



A nine-cylinder rotary engine

Rotary Engine Referring Fig. 1.52(b), it can be observed that with the rotation of the link 3, the link 1 rotates about O and the slider 4 reciprocates on it. This also implies that if the slider is made to reciprocate on the link 1, the crank 3 will rotate about A and the link 1 about O.

In a rotary engine, the slider is replaced by a piston and the link 1 by a cylinder pivoted at O. Moreover, instead of one cylinder, seven or nine cylinders symmetrically placed at regular intervals in the same plane or in parallel planes, are used. All the cylinders rotate about the same fixed centre and form a balanced system. The fixed link 2 is also common to all cylinders (Fig. 1.53).

Thus, in a rotary engine, the crank 2 is fixed and the body 1 rotates whereas in a reciprocating engine (1st inversion), the body 1 is fixed and the crank 2 rotates.

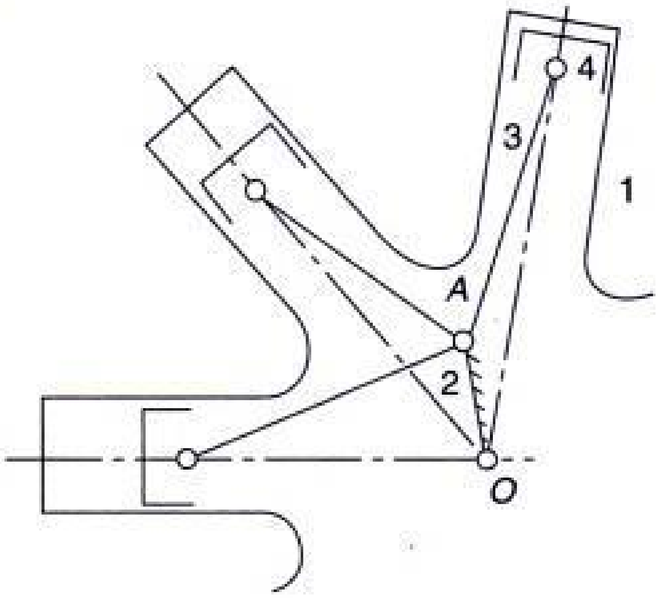


Fig. 1.53

Third Inversion

By fixing the link 3 of the slider-crank mechanism, the third inversion is obtained [Fig. 1.54(a)]. Now the link 2 again acts as a crank and the link 4 oscillates.

Applications

1. Oscillating cylinder engine
2. Crank and slotted-lever mechanism

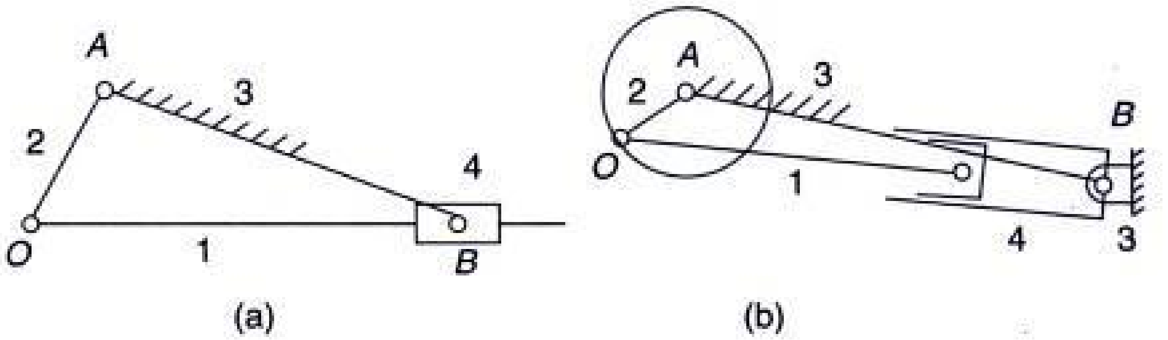


Fig. 1.54

Oscillating Cylinder Engine As shown in Fig. 1.54(b), the link 4 is made in the form of a cylinder and a piston is fixed to the end of the link 1. The piston reciprocates inside the cylinder pivoted to the fixed link 3. The arrangement is known as oscillating cylinder engine, in which as the piston reciprocates in the oscillating cylinder, the crank rotates.

Crank and Slotted-Lever Mechanism If the cylinder of an oscillating cylinder engine is made in the form of a guide and the piston in the form of a slider, the arrangement as shown in Fig. 1.55(a) is obtained. As the crank rotates about A, the guide 4 oscillates about B. At a point C on the guide, the link 5 is pivoted, the other end of which is connected to the cutting tool through a pivoted joint.

Figure 1.55(b) shows the extreme positions of the oscillating guide 4. The time of the forward stroke is proportional to the angle θ whereas for the return stroke, it is proportional to angle β, provided the crank rotates clockwise.

Comparing a crank and slotted-lever quick-return mechanism with a Whitworth quick-return mechanism, the following observations are made:

1. Crank 3 of the Whitworth mechanism is longer than its fixed link 2 whereas the crank 2 of the slotted-lever mechanism is shorter than its fixed link 3.
2. Coupler link 1 of the Whitworth mechanism makes complete rotations about its pivoted joint *O* with the fixed link. However, the coupler link 4 of the slotted-lever mechanism oscillates about its pivot *B*.
3. The coupler link holding the tool can be pivoted to the main coupler link at any convenient point *C* in both cases. However, for the same displacement of the tool, it is more convenient if the point *C* is taken on the extension of the main coupler link (towards the pivot with the fixed link) in case of the Whitworth mechanism and beyond the extreme position of the slider in the slotted-lever mechanism.

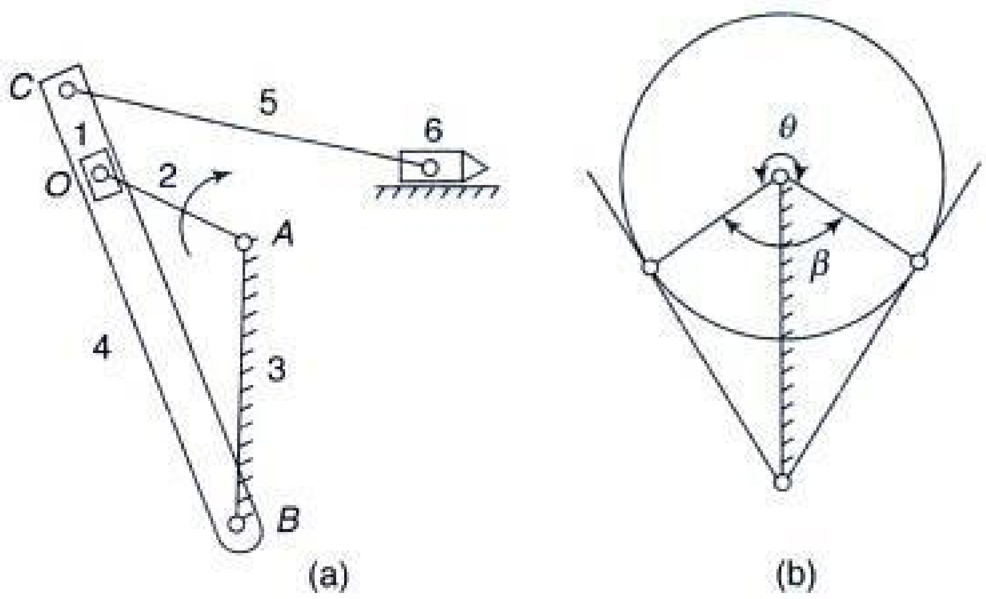


Fig. 1.55

A shaping machine. Shaping machines are fitted with quick-return mechanisms.

Fourth Inversion

If the link 4 of the slider-crank mechanism is fixed, the fourth inversion is obtained [Fig. 1.56(a)]. Link 3 can oscillate about the fixed pivot *B* on the link 4. This makes the end *A* of the link 2 to oscillate about *B* and the end *O* to reciprocate along the axis of the fixed link 4.

Application Hand-pump

Figure 1.56(b) shows a hand-pump. Link 4 is made in the form of a cylinder and a plunger fixed to the link 1 reciprocates in it.

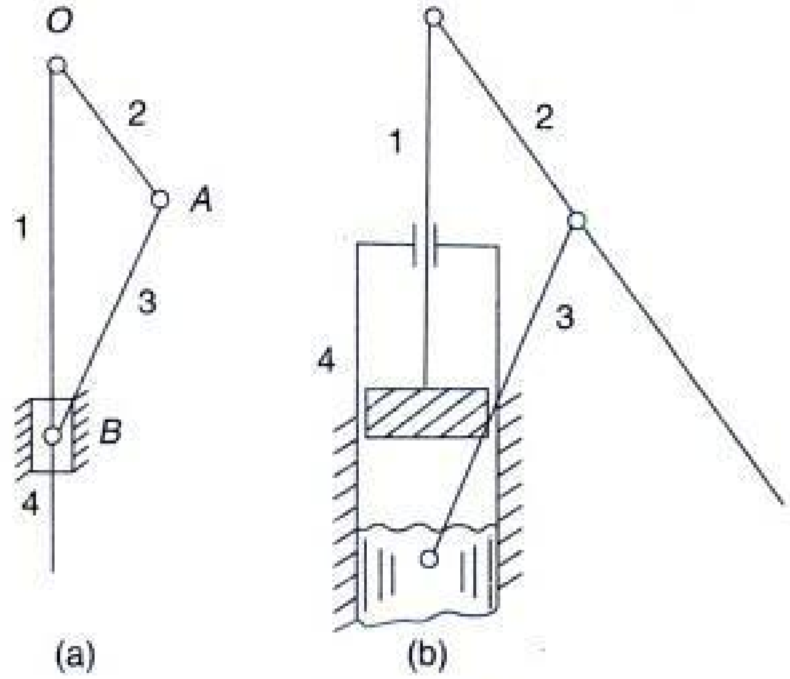


Fig. 1.56

Example 1.13 *The length of the fixed link of a crank and slotted-lever mechanism is 250 mm and that of the crank is 100 mm. Determine the*

(i) inclination of the slotted lever with the vertical in the extreme position,

(ii) ratio of the time of cutting stroke to the time of return stroke, and

(iii) length of the stroke, if the length of the slotted lever is 450 mm and the line of stroke passes through the extreme positions of the free end of the lever.

Solution Refer Fig. 1.57.

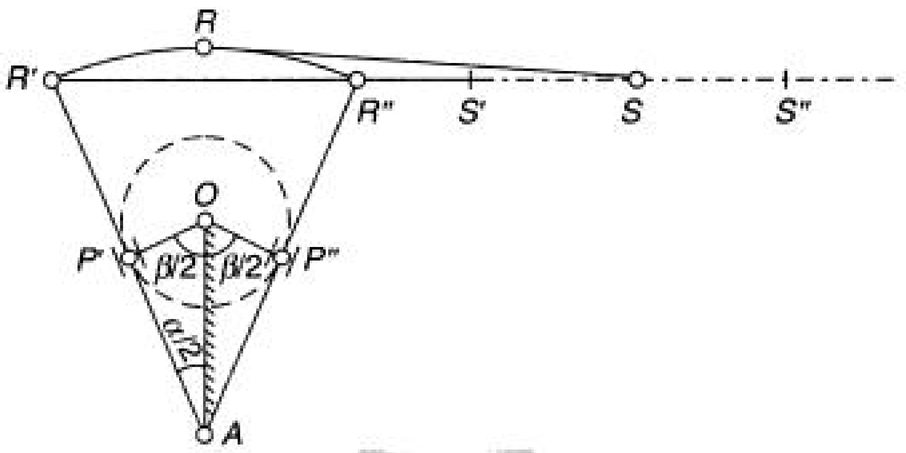


Fig. 1.57

$OA = 250$ mm $\quad OP' = OP'' = 100$ mm

$AR' = AR'' = AR = 450$ mm

$$\cos\frac{\beta}{2} = \frac{OP'}{OA} = \frac{100}{250} = 0.4$$

or $\frac{\beta}{2} = 66.4°$ or $\beta = 132.8°$

(i) Angle of the slotted lever with the vertical $\alpha/2 = 90° - 66.4° = 23.6°$

(ii) $\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}}$

$$= \frac{360° - \beta}{\beta} = \frac{360° - 132.8°}{132.8°} = 1.71$$

(iii) Length of stroke $= S'S'' = R'R''$

$= 2\,AR'.\sin(\alpha/2)$

$= 2 \times 450 \sin 23.6°$

$= 360.3$ mm

1.17 DOUBLE SLIDER-CRANK CHAIN

A four-bar chain having two turning and two sliding pairs such that two pairs of the same kind are adjacent is known as a double-slider-crank chain [Fig. 1.58(a)]. The following are its inversions.

First Inversion

This inversion is obtained when the link 1 is fixed and the two adjacent pairs 23 and 34 are turning pairs and the other two pairs 12 and 41 sliding pairs.

Application Elliptical trammel

Elliptical Trammel Figure 1.58(b) shows an elliptical trammel in which the fixed link 1 is in the form of guides for sliders 2 and 4. With the movement of the sliders, any point C on the link 3, except the midpoint of AB will trace an ellipse on a fixed plate. The midpoint of AB will trace a circle.

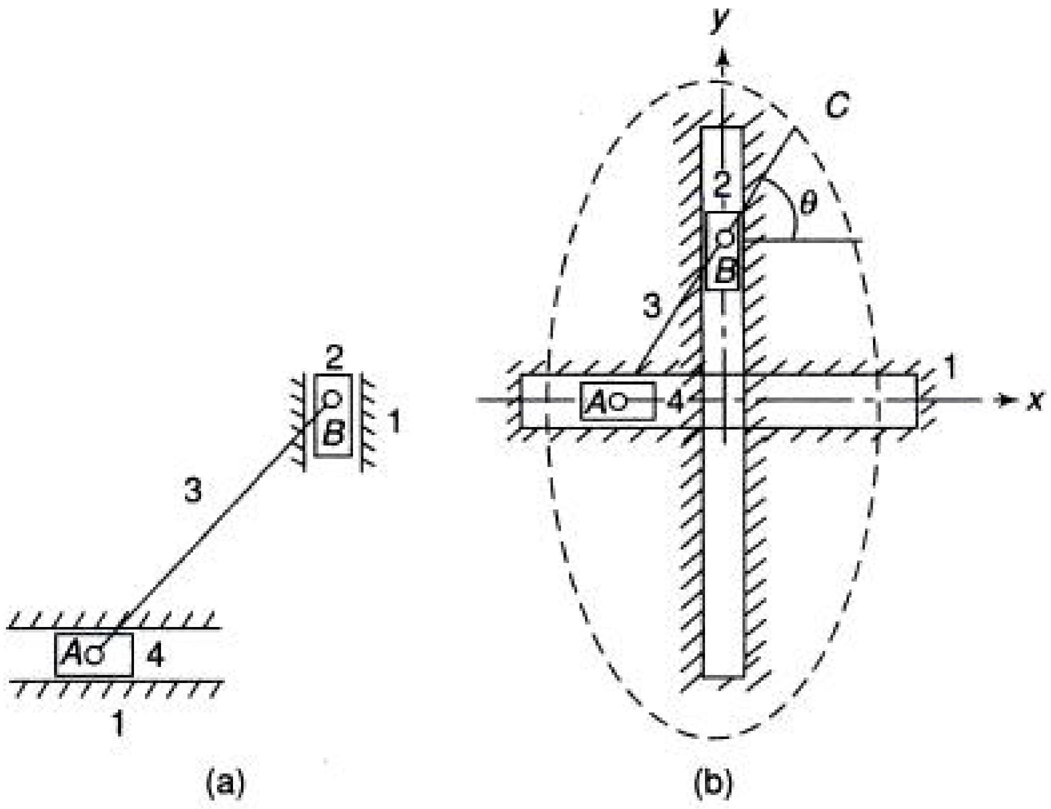


Fig. 1.58

Let at any instant, the link 3 make angle θ with the X-axis. Considering the displacements of the sliders from the centre of the trammel,

$$x = BC\cos\theta \text{ and } y = AC\sin\theta$$

$$\therefore \quad \frac{x}{BC} = \cos\theta \text{ and } \frac{y}{AC} = \sin\theta$$

Squaring and adding,

$$\frac{x^2}{(BC)^2}+\frac{y^2}{(AC)^2}=\cos^2\theta+\sin^2\theta=1$$

This is the equation of an ellipse. Therefore, the path traced by C is an ellipse with the semi-major and semi-minor axes being equal to AC and BC respectively.

When C is the midpoint of AB; $AC = BC$,

and

$$\frac{x^2}{(BC)^2}+\frac{y^2}{(AC)^2}=1 \quad \text{or} \quad x^2+y^2=(AC)^2$$

which is the equation of a circle with AC (=BC) as the radius of the circle.

Second Inversion

If any of the slide-blocks of the first inversion is fixed, the second inversion of the double-slider-crank chain is obtained. When the link 4 is fixed, the end B of the crank 3 rotates about A and the link 1 reciprocates in the horizontal direction.

Application Scotch yoke

Scotch Yoke A scotch-yoke mechanism (Fig. 1.59) is used to convert the rotary motion into a sliding motion. As the crank 3 rotates, the horizontal portion of the link 1 slides or reciprocates in the fixed link 4.

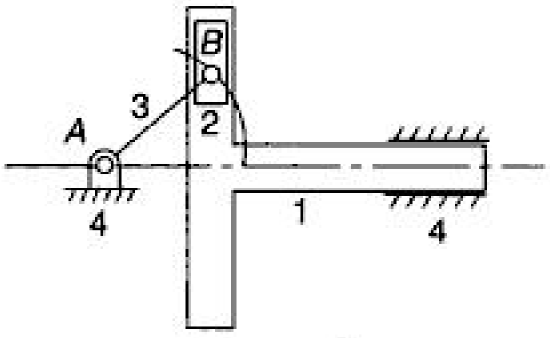


Fig. 1.59

Third Inversion

This inversion is obtained when the link 3 of the first inversion is fixed and the link 1 is free to move.

The rotation of the link 1 has been shown in Fig. 1.60 in which the full lines show the initial position. With rotation of the link 4 through 45° in the clockwise direction, the links 1 and 2 rotate through the same angle whereas the midpoint of the link 1 rotates through 90° in a circle with the length of link 3 as diameter. Thus, the angular velocity of the midpoint of the link 1 is twice that of links 2 and 4.

The sliding velocity of the link 1 relative to the link 4 will be maximum when the midpoint of the link 1 is at the axis of the link 4. In this position, the sliding velocity is equal to the tangential velocity of the midpoint of the link 1.

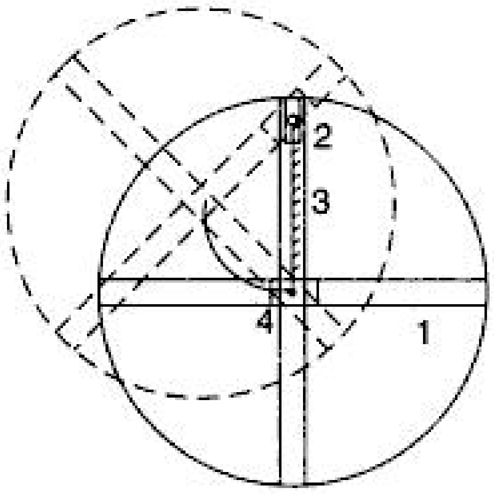


Fig. 1.60

Maximum sliding velocity = tangential velocity of midpoint of the link 1
= angular velocity of midpoint of the link 1 × radius
= (2 × angular velocity of the link 4) × (distance between axes of links 2 and 4)/2
= angular velocity of link 4 × distance between axes of links 2 and 4

The sliding velocity of the link 1 relative to the link 4 is zero when the midpoint of 1 is on the axis of the link 2.

Application Oldham's coupling

Oldham's Coupling If the rotating links 2 and 4 of the mechanism are replaced by two shafts, one can act as the driver and the other as the driven shaft with their axes at the pivots of links 2 and 4.

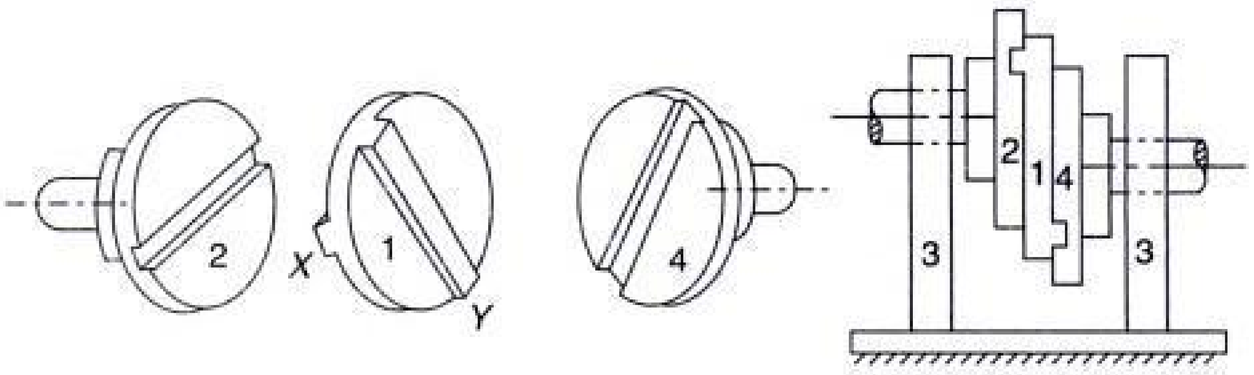


Fig. 1.61

Figure 1.61 shows an actual Oldham's coupling which is used to connect two parallel shafts when the distance between their axes is small. The two shafts have flanges at the ends and are supported in the fixed bearings representing the link 3. In the flange 2, a slot is cut in which the tongue X of the link 1 is fitted and has a sliding motion. Link 1 is made circular and has another tongue Y at right angles to the first and which fits in the recess of the flange of the shaft 4. Thus, the intermediate link 1 slides in the two slots in the two flanges while having the rotary motion.

As mentioned earlier, the midpoint of the intermediate piece describes a circle with distance between the axes of the shafts as diameter. The maximum sliding velocity of each tongue in the slot will be the peripheral velocity of the midpoint of the intermediate disc along the circular path.

Maximum sliding velocity = peripheral velocity along the circular path
= angular velocity of shaft × distance between shafts

Example 1.14 *The distance between two parallel shafts is 18 mm and they are connected by an Oldham's coupling. The driving shaft revolves at 160 rpm. What will be the maximum speed of sliding of the tongue of the intermediate piece along its groove?*

Solution $\omega = \dfrac{2\pi \times 160}{60} = 16.75 \text{ rad/s}$

Maximum velocity of sliding = $\omega \times d$
$= 16.75 \times 0.018$
$= 0.302$ m/s

1.18 MISCELLANEOUS MECHANISMS

Snap-Action Mechanisms

The mechanisms used to overcome a large resistance of a member with a small driving force are known as *snap action* or *toggle* mechanisms. They find their use in a variety of machines such as stone crushers, embossing presses, switches, etc. Figure 1.62(a) shows such a type of mechanism in which links of equal lengths 4 and 5 are connected by a pivoted joint at B. Link 4 is free to oscillate about the pivot C and the link 5 is connected to a sliding link 6. Link 3 joins links 4 and 5. When force is applied at the point B through the link 3, the angle α decreases and links 4 and 5 tend to become collinear. At this instant, the force is greatly multiplied at B, i.e., a very small force is required to overcome a great resistance R at the slider. This is because a large movement at B produces a relatively slight displacement of the slider at D. As the angle α approaches zero, reaction at the pivot becomes equal to R and for force balance in the link BC or BD,

$$\frac{F}{2\sin\alpha} = \frac{R}{\cos\alpha}$$

or $$2\tan\alpha = \frac{F}{R}$$

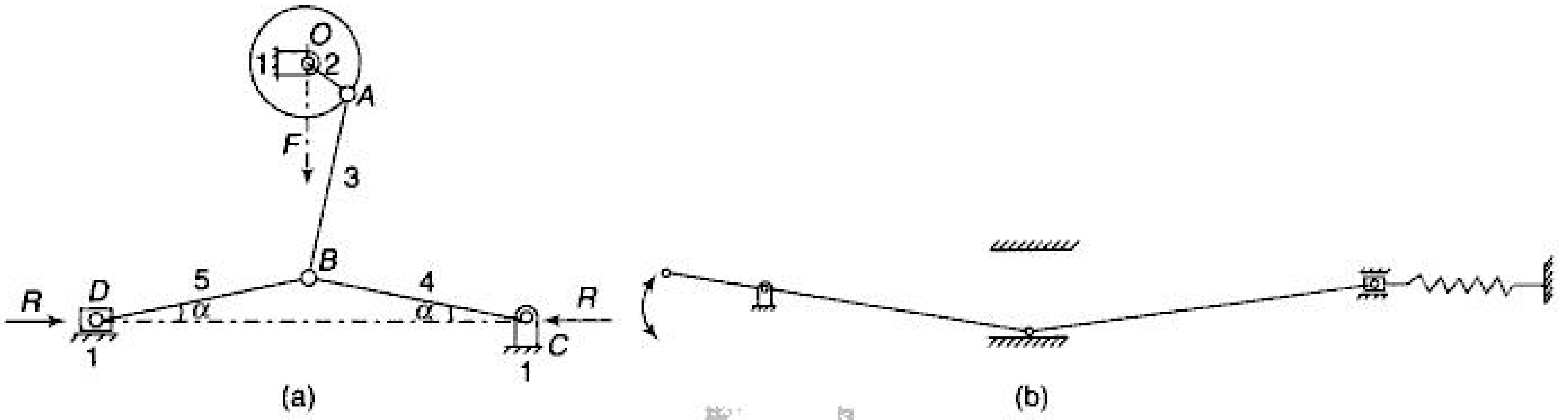


Fig. 1.62

As $\alpha \to 0$, $\tan\alpha \to 0$. Thus for a small value of the force *F*, *R* approaches infinity. In a stone crusher, a large resistance at *D* is overcome with a small force *F* in this way. Figure 1.62(b) shows another such mechanism.

Indexing Mechanisms

An *indexing mechanism* serves the purpose of dividing the periphery of a circular piece into a number of equal parts. Indexing is generally done on gear cutting or milling machines.

An indexing mechanism consists of an index head in which a spindle is carried in a headstock [Fig. 1.63(a)]. The work to be indexed is held either between centres or in a chuck attached to the spindle. A 40-tooth worm wheel driven by a single-threaded right-hand worm is also fitted to the spindle. At the end of the worm shaft an adjustable index crank with a handle and a plunger pin is also fitted. The plunger pin can be made to fit into any hole in the index plate which has a number of circles of equally spaced holes as shown in Fig. 1.63(b). An index head is usually provided with a number of interchangeable index plates to cover a wide range of work. However, the figure shows only the circle of 17 holes for sake of clarity.

As the worm wheel has 40 teeth, the number of revolutions of the index crank required to make one revolution of the work is also 40. The number of revolutions of the crank, needed for a proper division of the work into the desired number of divisions, can be calculated as follows:

- If a work is to be divided into 40 divisions, the crank should be given one complete revolution; if 20 divisions, two revolutions for each division, and so on.
- If the work is to be divided into 160 divisions, obviously the crank should be rotated through one-fourth of a rotation. For such cases, an index plate with a number of holes divisible by 4 such as with 16 or 20 holes can be chosen.
- If the work is to be divided into 136 parts, the use of the index plate will be essential since the rotation of the crank for each division will be 40/136 or five-seventeenth of a turn. Thus, a plate with 17 holes is selected in this case. To obviate the necessity of counting the holes at each partial turn of the crank, an index sector with two arms which can be set and clamped together at any angle is also available. In this case, this can be set to measure off 5 spaces. Starting with the crankpin in the hole *a*, a cut would be made in the work. The crank is rotated and the pin is made to enter into the hole *b*, 5 divisions apart and a second cut is made in the work. In a similar way, a third cut is made by rotating the crank again through five divisions with the help of an index sector, and so on. Usually, index tables are provided to ascertain the number of turns of the crank and the number of holes for the given case.

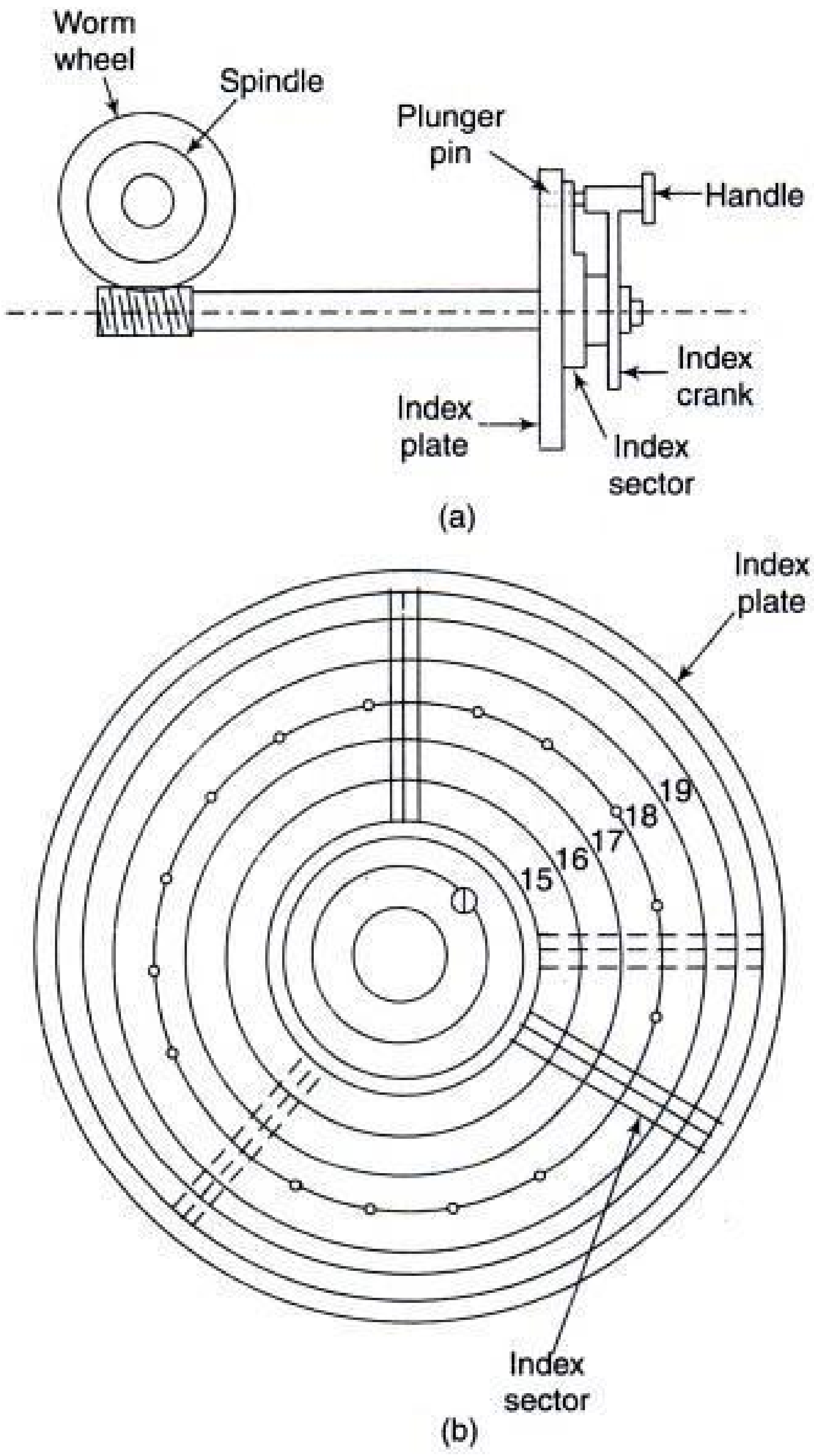
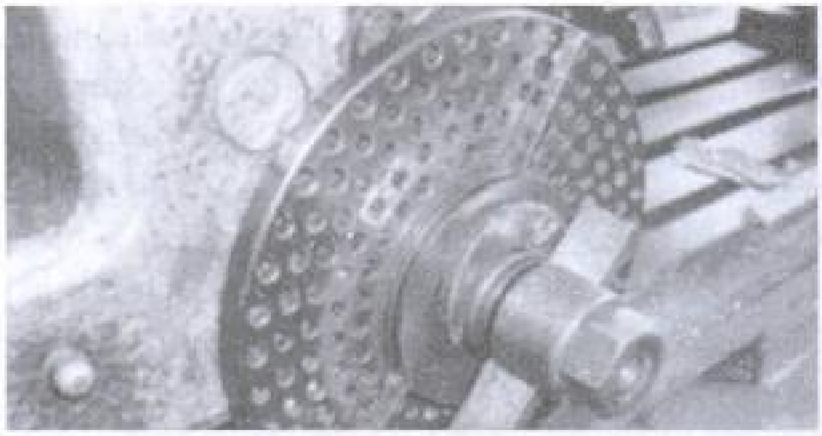


Fig. 1.63



Index plate of an indexing mechanism

Summary

1. *Kinematics* deals with the relative motions of different parts of a mechanism without taking into consideration the forces producing the motions whereas *dynamics* involves the calculation of forces impressed upon different parts of a mechanism.
2. *Mechanism* is a combination of a number of rigid bodies assembled in such a way that the motion of one causes constrained and predictable motion of the others whereas a *machine* is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of useful work.
3. There are three types of constrained motion: *completely constrained, incompletely constrained* and *successfully constrained.*
4. A *link* is a resistant body or a group of resistant bodies with rigid connections preventing their

relative movement. A link may also be defined as a member or a combination of members of a mechanism, connecting other members and having motion relative to them.

5. A *kinematic pair* or simply a pair is a joint of two links having relative motion between them.
6. A pair of links having surface or area contact between the members is known as a *lower pair* and a pair having a point or line contact between the links, a *higher pair*.
7. When the elements of a pair are held together mechanically, it is known as a *closed pair*. The two elements are geometrically identical. When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an *unclosed pair*.
8. Usual types of joints in a chain are binary joint, ternary joint and quaternary joint
9. *Degree of freedom of a pair* is defined as the number of independent relative motions, both translational and rotational, a pair can have.
10. A *kinematic chain* is an assembly of links in which the relative motions of the links is possible and the motion of each relative to the other is definite.
11. A *redundant chain* does not allow any motion of a link relative to the other.
12. A *linkage* or *mechanism* is obtained if one of the links of a kinematic chain is fixed to the ground.
13. *Degree of freedom of a mechanism* indicates how many inputs are needed to have a constrained motion of the other links.
14. *Kutzback's criterion* for the degree of freedom of plane mechanisms is

    $F = 3(N-1) - 2P_1 - 1P_2$

15. *Gruebler's criterion* for degree of freedom of plane mechanisms with single-degree of freedom joints only is

    $F = 3(N-1) - 2P_1$

16. *Author's criterion* for degree of freedom and the number of joints of plane mechanisms with turning pairs is

    $F = N - (2L + 1)$

    $P_1 = N + (L - 1)$

17. In a four-link mechanism, a link that makes a complete revolution is known as a *crank*, the link opposite to the fixed link is called the *coupler* and the fourth link is called a *lever* or *rocker* if it oscillates or another crank, if it rotates.
18. In a Watts six-bar chain, the ternary links are direct connected whereas in a Stephenson's six-bar chain, they are not direct connected.
19. If a system has one or more links which do not introduce any extra constraint, it is known as *redundant link* and is not counted to find the degree of freedom.
20. If a link of a mechanism can be moved without causing any motion to the rest of the links of the mechanism, it is said to have a *redundant degree of freedom*.
21. The *mechanical advantage* (MA) of a mechanism is the ratio of the output force or torque to the input force or torque at any instant.
22. The angle μ between the output link and the coupler is known as *transmission angle*.
23. Different mechanisms obtained by fixing different links of a kinematic chain are known as its *inversions*.
24. The mechanisms used to overcome a large resistance of a member with a small driving force are known as *snap action* or *toggle* mechanisms.
25. An *indexing mechanism* serves the purpose of dividing the periphery of a circular piece into a number of equal parts.

Exercises

1. Distinguish between
   (i) mechanism and machine
   (ii) analysis and synthesis of mechanisms
   (iii) kinematics and dynamics
2. Define: kinematic link, kinematic pair, kinematic chain.
3. What are rigid and resistant bodies? Elaborate.
4. How are the kinematic pairs classified? Explain with examples.
5. Differentiate giving examples:
   (i) lower and higher pairs
   (ii) closed and unclosed pairs
   (iii) turning and rolling pairs
6. What do you mean by degree of freedom of a

kinematic pair? How are pairs classified? Give examples.

7. Discuss various types of constrained motion.
8. What is a redundant link in a mechanism?
9. How do a Watt's six-bar chain and Stephenson's six-bar chain differ?
10. What is redundant degree of freedom of a mechanism?
11. What are usual types of joints in a mechanism?
12. What is the degree of freedom of a mechanism? How is it determined?
13. What is Kutzback's criterion for degree of freedom of plane mechanisms? In what way is Gruebler's criterion different from it?
14. How are the degree of freedom and the number of joints in a linkage can be found when the number of links and the number of loops in a kinematic chain are known?
15. What is meant by equivalent mechanisms?
16. Define Grashof's law. State how is it helpful in classifying the four-link mechanisms into different types.
17. Why are parallel-crank four-bar linkage and deltoid linkage considered special cases of four-link mechanisms?
18. Define mechanical advantage and transmission angle of a mechanism.
19. Describe various inversions of a slider-crank mechanism giving examples.
20. What are quick-return mechanisms? Where are they used? Discuss the functioning of any one of them.
21. How are the Whitworth quick-return mechanism and crank and slotted-lever mechanism different from each other?
22. Enumerate the inversions of a double-slider-crank chain. Give examples.
23. Describe briefly the functions of elliptical trammel and scotch yoke.
24. In what way is Oldham's coupling useful in connecting two parallel shafts when the distance between their axes is small?
25. What are snap-action mechanisms? Give examples.
26. What is an indexing mechanism? Describe how it is used to divide the periphery of a circular piece into a number of equal parts.
27. For the kinematic linkages shown in Fig. 1.64, find the degree of freedom (F).

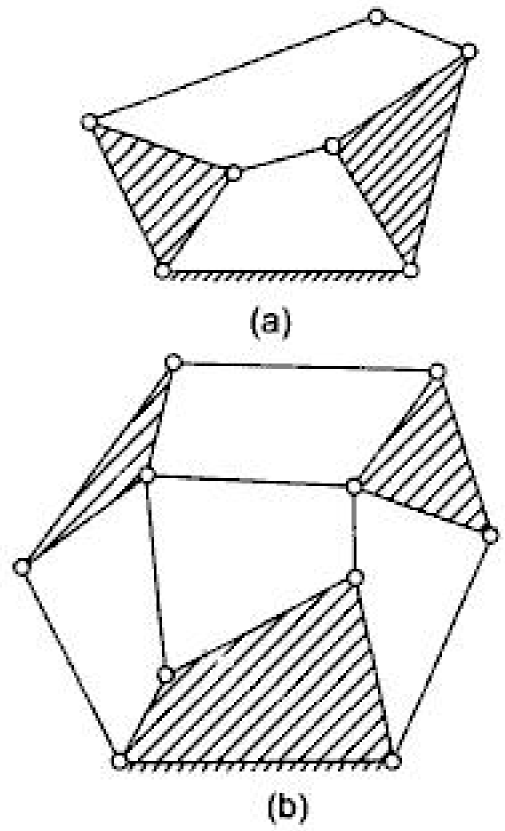


Fig. 1.64

[(a) 1 (b) 0]

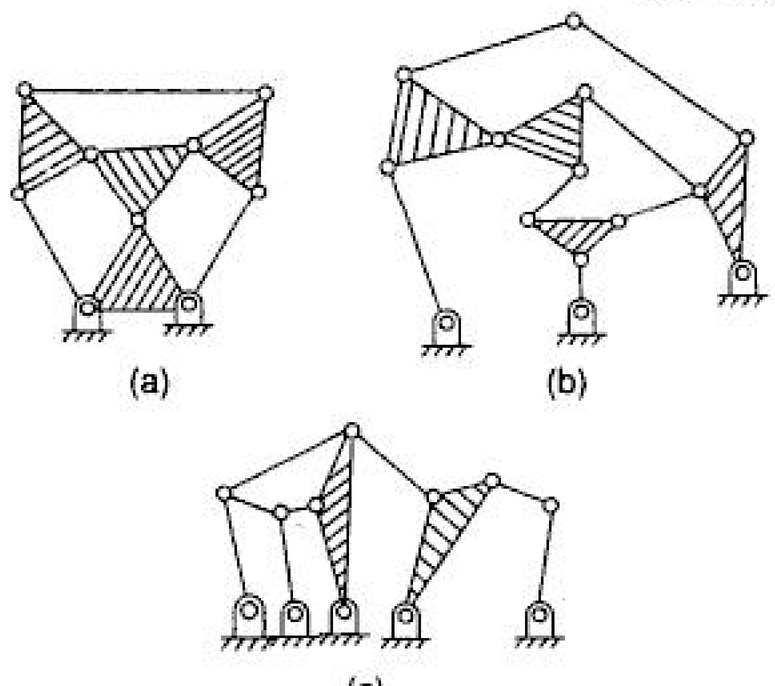


Fig. 1.65

28. For the kinematic linkages shown in Fig 1.65, find the number of binary links (N_b), ternary links (N_t), other links (N_o), total links N, loops L, joints or pairs (P_1), and degree of freedom (F).

[(a) $N_b = 3$; $N_t = 4$; $N_o = 0$; $N = 7$; $L = 3$; $P_1 = 9$; $F = 0$

(b) $N_b = 7$; $N_t = 5$; $N_o = 0$; $N = 12$; $L = 4$; $P_1 = 15$; $F = 3$

(c) $N_b = 8$; $N_t = 2$; $N_o = 1$; $N = 11$; $L = 5$; $P_1 = 15$; $F = 0$]

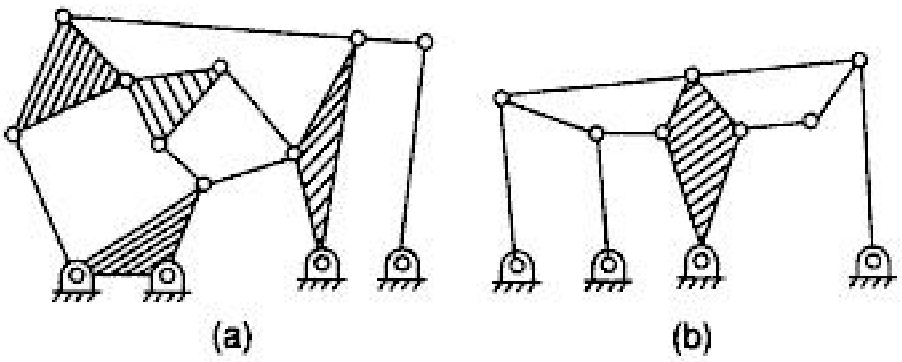


Fig. 1.66

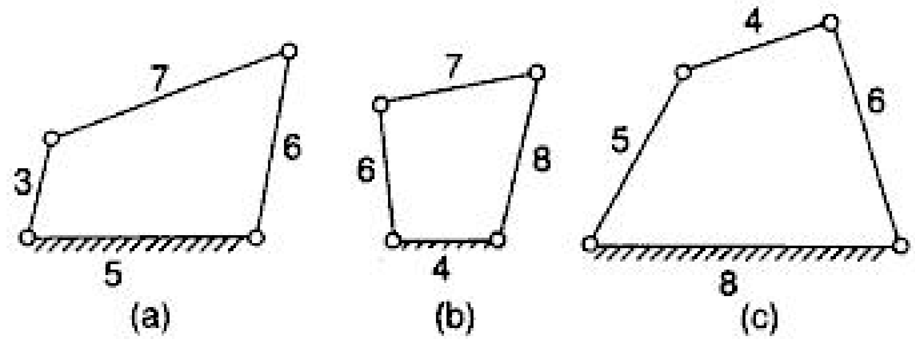


Fig. 1.67

29. Show that the linkages shown in Fig. 1.66 are structures. Suggest some changes to make them mechanisms having one degree of freedom. The number of links should not be changed by more than ±1.
30. A linkage has 14 links and the number of loops is 5. Calculate its
    (i) degrees of freedom
    (ii) number of joints
    (iii) maximum number of ternary links that can be had.

    Assume that all the pairs are turning pairs.

    (3; 18; 8)
31. Figure 1.67 shows some four-link mechanisms in which the figures indicate the dimensions in standard units of length. Indicate the type of each mechanism, whether it is crank-rocker or double-crank or double-rocker.

    [(a) crank-rocker (b) double-crank (c) double-rocker]
32. A crank-rocker mechanism *ABCD* has the dimensions *AB* = 30 mm, *BC* = 90 mm, *CD* = 75 mm and *AD* (fixed link) = 100 mm. Determine the maximum and the minimum values of the transmission angle. Locate the toggle positions and indicate the corresponding crank angles and the transmission angles.

    (103°, 49°, θ = 228°, μ = 92°, θ =38.5°, μ = 56°)

2 VELOCITY ANALYSIS

Introduction

As mentioned in the first chapter, analysis of mechanisms is the study of motions and forces concerning their different parts. The study of velocity analysis involves the linear velocities of various points on different links of a mechanism as well as the angular velocities of the links. The velocity analysis is the prerequisite for acceleration analysis which further leads to force analysis of various links of a mechanism. To facilitate such study, a machine or a mechanism is represented by a skeleton or a line diagram, commonly knows as a *configuration diagram*.

Velocities and accelerations in machines can be determined either analytically or graphically. With the invention of calculators and computers, it has become convenient to make use of analytical methods. However, a graphical analysis is more direct and is accurate to an acceptable degree and thus cannot be neglected. This chapter is mainly devoted to the study of graphical methods of velocity analysis. Two methods of graphical approach, namely, relative velocity method and instantaneous centre method are discussed. The algebraic methods are also discussed in brief. The analytical approach involving the use of calculators and computers will be discussed in Chapter 4.

2.1 ABSOLUTE AND RELATIVE MOTIONS

Strictly speaking, all motions are relative since an arbitrary set of axes or planes is required to define a motion. Usually, the earth is taken to be a fixed reference plane and all motions relative to it are termed absolute motions.

If a train moves in a particular direction, the motion of the train is referred as the absolute motion of the train or motion of the train relative to the earth. Now, suppose a man moves inside the train. Then, the motion of the man will be described in two different ways with different meanings:

1. Motion of the man relative to the train—it is equivalent to the motion of the man assuming the train to be stationary.
2. Motion of the man or absolute motion of the man or motion of the man relative to the earth = motion of man relative to the train + Motion of train relative to the earth.

2.2 VECTORS

Problems involving relative motions are conveniently solved by the use of vectors. A vector is a line which represents a vector quantity such as force, velocity, acceleration, etc.

Characteristics of a Vector

1. Length of the vector **ab** (Fig. 2.1) drawn to a convenient scale, represents the magnitude of the quantity (written as *ab*).

Direction of the line is parallel to the direction in which the quantity acts.

The initial end **a** of the line is the tail and the final end **b**, the head. An arrowhead on the line indicates the direction-sense of the quantity which is always from the tail to the head, i.e., **a** to **b**.

If the sense is as shown in Fig. 2.1(a), the vector is read as **ab** and if the sense is opposite [Fig. 2.1 (b)], the vector is read as **ba**. This implies that **ab** = – **ba**

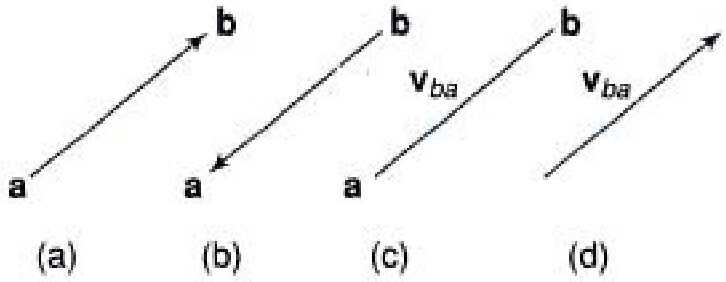


Fig. 2.1

2. Vector **ab** may also represent a vector quantity of a body *B* relative to a body *A* such as velocity of *B* relative to *A*.

   If the body *A* is fixed, **ab** represents the absolute velocity of *B*. If both the bodies *A* and *B* are in motion, the velocity of *B* relative to *A* means the velocity of *B* assuming the body *A* to be fixed for the moment.

   The vector **ab** can also be shown as $\mathbf{v}_{ba}$ [Fig. 2.1(c)], meaning the velocity of *B* relative to *A* provided *a* and *b* are indicated at the ends or an arrowhead is put on the vector [Fig. 2.1(d)].

3. Vector **ab** may also represent a vector quantity of a point *B* relative to a point *A* in the same body.

   If a vector $\mathbf{v}_{ba}$ or **ab** represents the velocity of *B* relative to *A*, the same vector in the opposite sense represents the velocity of *A* relative to *B* and will be read as $\mathbf{v}_{ab}$ or **ba**.

2.3 ADDITION AND SUBTRACTION OF VECTORS

Let

$\mathbf{v}_{ao}$ = velocity of *A* relative to *O*

$\mathbf{v}_{ba}$ = velocity of *B* relative to *A*

$\mathbf{v}_{bo}$ = velocity of *B* relative to *O*

The law of vector addition states that the velocity of *B* relative to *O* is equal to the vectorial sum of the velocity of *B* relative to *A* and the velocity of *A* relative to *O*.

Velocity of *B* relative to *O* = velocity of *B* relative to *A* + velocity of *A* relative to *O* (2.1)

i.e.

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$
$$= \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

or

$$\mathbf{ob} = \mathbf{oa} + \mathbf{ab}$$

Take the vector **oa** and place the vector **ab** at the end of the vector **oa**. Then **ob** is given by the closing side of the two vectors (Fig. 2.2).

Note that the arrows of the two vectors to be added are in the same order and that of the resultant is in the opposite order.

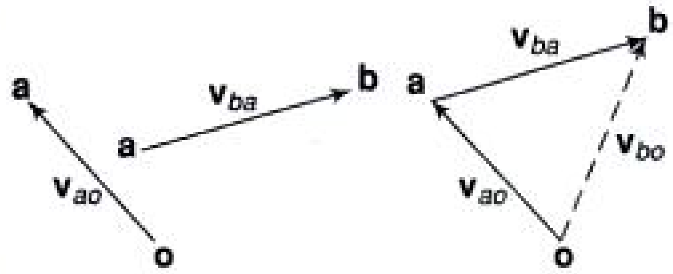


Fig. 2.2

Any number of vectors can be added as follows:

1. Take the first vector.
2. At the end of the first vector, place the beginning of the second vector.

3. At the end of the second vector, place the beginning of the third vector, and so on.
4. Joining of the beginning of the first vector and the end of the last vector represents the sum of the vectors. Figure 2.3 shows the addition of four vectors.

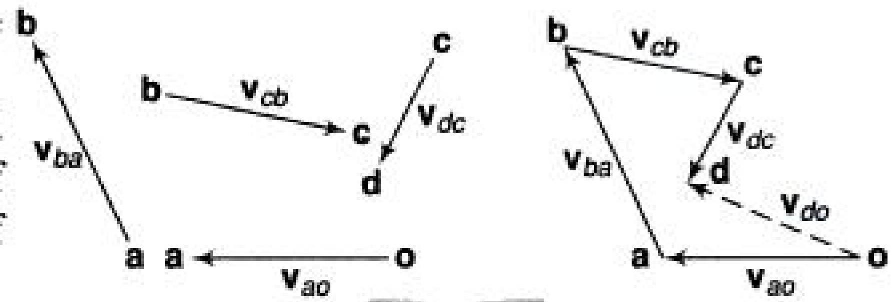


Fig. 2.3

$$\mathbf{v}_{do} = \mathbf{v}_{dc} + \mathbf{v}_{cb} + \mathbf{v}_{ba} + \mathbf{v}_{ao} \tag{2.2}$$
$$= \mathbf{v}_{ao} + \mathbf{v}_{ba} + \mathbf{v}_{cb} + \mathbf{v}_{dc}$$
$$\mathbf{od} = \mathbf{oa} + \mathbf{ab} + \mathbf{bc} + \mathbf{cd}$$

Equation 2.1 may be written as,

Vel. of B rel. to A = Vel. of B rel. to O – Vel. of A rel. to O (2.3)

$$\mathbf{v}_{ba} = \mathbf{v}_{bo} - \mathbf{v}_{ao}$$

or $$\mathbf{ab} = \mathbf{ob} - \mathbf{oa}$$

This shows that in Fig. 2.2, **ab** also represents the subtraction of **oa** from **ob** [Fig. 2.4(a)]

Also $$\mathbf{v}_{ab} = -\mathbf{v}_{ba} = \mathbf{v}_{ao} - \mathbf{v}_{bo}$$

or $$\mathbf{ba} = \mathbf{oa} - \mathbf{ob}$$

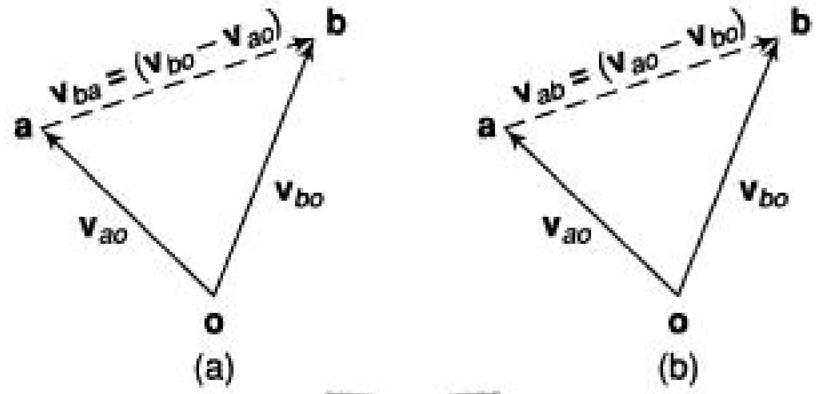


Fig. 2.4

This has been shown in Fig. 2.4 (b).

Thus, the difference of two vectors is given by the closing side of a triangle, the other two sides of which are formed by placing the two vectors tail to tail, the sense being towards the vector quantity from which the other is subtracted.

2.4 MOTION OF A LINK

Let a rigid link OA, of length r, rotate about a fixed point O with a uniform angular velocity ω rad/s in the counter-clockwise direction [Fig.2.5 (a)]. OA turns through a small angle $\delta\theta$ in a small interval of time δt. Then A will travel along the arc AA' as shown in [Fig.2.5(b)].

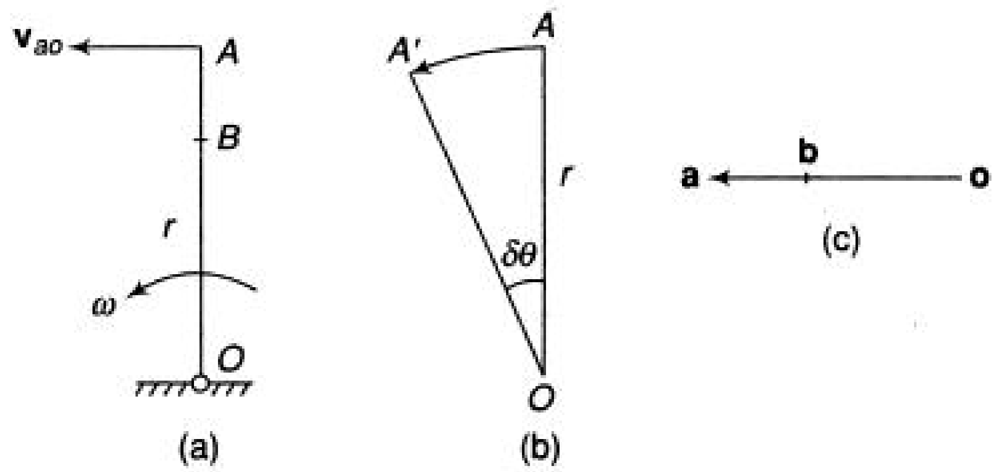


Fig. 2.5

Velocity of A relative to $O = \dfrac{\text{Arc}AA'}{\delta t}$

or

$$\mathbf{v}_{ao} = \frac{r\delta\theta}{\delta t}$$

In the limits, when $\delta t \to 0$

$$\mathbf{v}_{ao} = r\frac{d\theta}{dt} \tag{2.4}$$
$$= r\omega$$

The direction of v_{ao} is along the displacement of A. Also, as δt approaches zero ($\delta t \to 0$), AA' will be perpendicular to OA. Thus, velocity of A is ωr and is perpendicular to OA. This can be represented by a vector **oa** (Fig. 2.5 c). The fact that the direction of the velocity vector is perpendicular to the link also emerges from the fact that A can neither approach nor recede from O and thus, the only possible motion of A relative to O is in a direction perpendicular to OA.

Consider a point B on the link OA.

Velocity of $B = \omega \cdot OB$ perpendicular to OB.

If **ob** represents the velocity of B, it can be observed that

$$\frac{\mathbf{ob}}{\mathbf{oa}} = \frac{\omega OB}{\omega OA} = \frac{OB}{OA} \tag{2.5}$$

i.e., **b** divides the velocity vector in the same ratio as B divides the link.

Remember, the velocity vector v_{ao} [Fig. 2.5(c)] represents the velocity of A at a particular instant. At other instants, when the link OA assumes another position, the velocity vectors will have their directions changed accordingly.

Also, the magnitude of the instantaneous linear velocity of a point on a rotating body is proportional to its distance from the axis of rotation.

2.5 FOUR-LINK MECHANISM

Figure 2.6(a) shows a four-link mechanism (quadric-cycle mechanism) $ABCD$ in which AD is the fixed link and BC is the coupler. AB is the driver rotating at an angular speed of ω rad/s in the clockwise direction if it is a crank or moving at this angular velocity at this instant if it is a rocker. It is required to find the absolute velocity of C (or velocity of C relative to A).

Writing the velocity vector equation,

Vel. of C rel. to A = Vel. of C rel. to B + vel. of B rel. to A

$$\mathbf{v}_{ca} = \mathbf{v}_{cb} + \mathbf{v}_{ba} \tag{2.6}$$

The velocity of any point relative to any other point on a fixed link is always zero. Thus, all the points on a fixed link are represented by one point in the velocity diagram. In Fig. 2.6(a), the points A and D, both lie on the fixed link AD. Therefore, the velocity of C relative to A is the same as velocity of C relative to D.

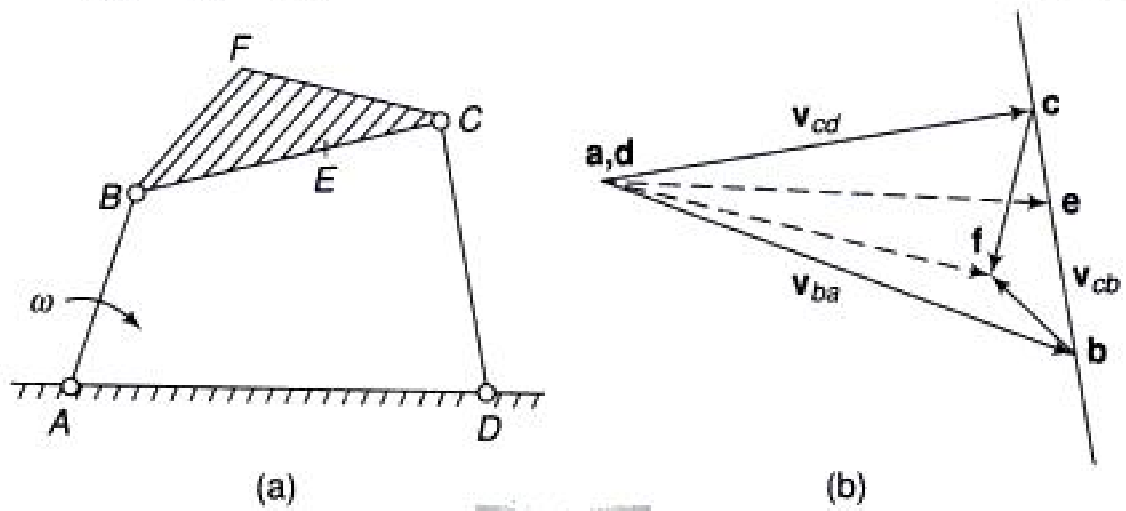


Fig. 2.6

Equation (2.6) may be written as,

$$\mathbf{v}_{cd} = \mathbf{v}_{ba} + \mathbf{v}_{cb}$$

or

$$\mathbf{dc} = \mathbf{ab} + \mathbf{bc}$$

where $\mathbf{v}_{ba}$ or **ab** $= \omega AB$; $\perp$ to AB

$\mathbf{v}_{cb}$ or **bc** is unknown in magnitude ; $\perp$ to BC

$\mathbf{v}_{cd}$ or **dc** is unknown in magnitude ; ⊥ to DC

The velocity diagram is constructed as follows:

1. Take the first vector **ab** as it is completely known.
2. To add vector **bc** to **ab**, draw a line ⊥ BC through **b,** of any length. Since the direction-sense of **bc** is unknown, it can lie on either side of **b.** A convenient length of the line can be taken on both sides of **b.**
3. Through **d,** draw a line ⊥ DC to locate the vector **dc.** The intersection of this line with the line of vector **bc** locates the point **c**.
4. Mark arrowheads on the vectors **bc** and **dc** to give the proper sense. Then **dc** is the magnitude and also represents the direction of the velocity of C relative to A (or D). It is also the absolute velocity of the point C (A and D being fixed points).
5. Remember that the arrowheads on vector **bc** can be put in any direction because both ends of the link BC are movable. If the arrowhead is put from **c** to **b**, then the vector is read as **cb**. The above equation is modified as

$$\mathbf{dc} = \mathbf{ab} - \mathbf{cb} \qquad (\mathbf{bc} = -\,\mathbf{cb})$$

or

$$\mathbf{dc} + \mathbf{cb} = \mathbf{ab}$$

Intermediate Point

The velocity of an intermediate point on any of the links can be found easily by dividing the corresponding velocity vector in the same ratio as the point divides the link. For point E on the link BC,

$$\frac{\mathbf{be}}{\mathbf{bc}} = \frac{BE}{BC}$$

ae represents the absolute velocity of E.

Offset Point

Write the vector equation for point F,

$$\mathbf{v}_{fb} + \mathbf{v}_{ba} = \mathbf{v}_{fc} + \mathbf{v}_{cd}$$

or

$$\mathbf{v}_{ba} + \mathbf{v}_{fb} = \mathbf{v}_{cd} + \mathbf{v}_{fc}$$

or

$$\mathbf{ab} + \mathbf{bf} = \mathbf{dc} + \mathbf{cf}$$

The vectors $\mathbf{v}_{ba}$ and $\mathbf{v}_{cd}$ are already there on the velocity diagram.

$\mathbf{v}_{fb}$ is ⊥ BF, draw a line ⊥ BF through **b;**

$\mathbf{v}_{fc}$ is ⊥ CF, draw a line ⊥ CF through **c;**

The intersection of the two lines locates the point **f.**

af or **df** indicates the velocity of F relative to A (or D) or the absolute velocity of F.

2.6 VELOCITY IMAGES

Note that in Fig. 2.6, the triangle **bfc** is similar to the triangle BFC in which all the three sides **bc**, **cf** and **fb** are perpendicular to BC, CF and FB respectively. The triangles such as **bcf** are known as velocity images and are found to be very helpful devices in the velocity analysis of complicated shapes of the linkages. Thus, any offset point on a link in the configuration diagram can easily be located in the velocity diagram by drawing the velocity image. While drawing the velocity images, the following points should be kept in mind:

1. The velocity image of a link is a scaled reproduction of the shape of the link in the velocity diagram from the configuration diagram, rotated bodily through 90° in the direction of the angular velocity.
2. The order of the letters in the velocity image is the same as in the configuration diagram.
3. In general, the ratio of the sizes of different images to the sizes of their respective links is different in the same mechanism.

2.7 ANGULAR VELOCITY OF LINKS

1. Angular Velocity of BC

(a) Velocity of C relative to *B*, v_{cb} = **bc** (Fig. 2.6)

Point *C* relative to *B* moves in the direction-sense given by $\mathbf{v}_{cb}$ (upwards). Thus, *C* moves in the counter-clockwise direction about *B*.

$$\mathbf{v}_{cb} = \omega_{cb} \times BC = \omega_{cb} \times CB$$

$$\omega_{cb} = \frac{v_{cb}}{CB}$$

(b) Velocity of *B* relative to *C*, $\mathbf{v}_{bc}$ = **cb**

B relative to *C* moves in a direction-sense given by v_{bc} (downwards, opposite to **bc**), i.e., *B* moves in the counter-clockwise direction about *C* with magnitude ω_{bc} given by

$$\frac{v_{bc}}{BC}$$

It can be seen that the magnitude of $\omega_{cb} = \omega_{bc}$ as $v_{cb} = v_{bc}$ and the direction of rotation is the same. Therefore, angular velocity of a link about one extremity is the same as the angular velocity about the other. In general, the angular velocity of link *BC* is ω_{bc} (= ω_{cb}) in the counter-clockwise direction.

2. Angular Velocity of CD

Velocity of *C* relative to *D*,

$$v_{cd} = \mathbf{dc}$$

It is seen that *C* relative to *D* moves in a direction-sense given by v_{cd} or *C* moves in the clockwise direction about *D*.

$$\omega_{cd} = \frac{v_{cd}}{CD}$$

2.8 VELOCITY OF RUBBING

Figure 2.7 shows two ends of the two links of a turning pair. A pin is fixed to one of the links whereas a hole is provided in the other to fit the pin. When joined, the surface of the hole of one link will rub on the surface of the pin of the other link. The velocity of rubbing of the two surfaces will depend upon the angular velocity of a link relative to the other.

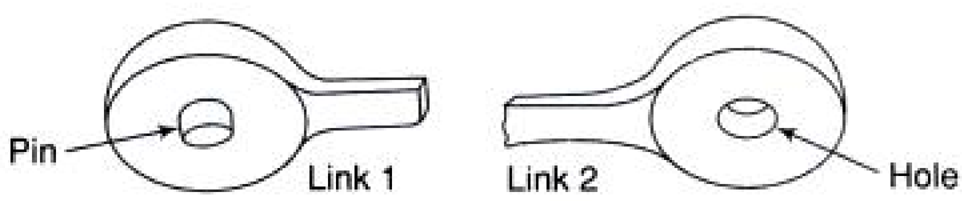


Fig. 2.7

Pin at *A* (Fig. 2.6a)

The pin at *A* joins links *AD* and *AB*. *AD* being fixed, the velocity of rubbing will depend upon the angular velocity of *AB* only.

Let r_a = radius of the pin at *A*

Then velocity of rubbing = $r_a.\omega$

Pin at *D*

Let r_d = radius of the pin at *D*

Velocity of rubbing = $r_d.\omega_{cd}$

Pin at *B*

$\omega_{ba} = \omega_{ab} = \omega$ clockwise

$\omega_{bc} = \omega_{cb} = \dfrac{v_{bc}}{BC}$ counter-clockwise

Since the directions of the two angular velocities of links *AB* and *BC* are in the opposite directions, the angular velocity of one link relative to the other is the sum of the two velocities.

Let r_b = radius of the pin at *B*

Velocity of rubbing = $r_b(\omega_{ab} + \omega_{bc})$

Pin at *C*

$\omega_{bc} = \omega_{cb}$ counter-clockwise

$\omega_{dc} = \omega_{cd}$ clockwise

Let r_c = radius of the pin at *C*

Velocity of rubbing = $r_c(\omega_{bc} + \omega_{dc})$

In case it is found that the angular velocities of the two links joined together are in the same direction, the velocity of rubbing will be the difference of the angular velocities multiplied by the radius of the pin.

2.9 SLIDER-CRANK MECHANISM

Figure 2.8(a) shows a slider-crank mechanism in which *OA* is the crank moving with uniform angular velocity ω rad/s in the clockwise direction. At point *B*, a slider moves on the fixed guide *G*. *AB* is the coupler joining *A* and *B*. It is required to find the velocity of the slider at *B*.

Writing the velocity vector equation,

Vel. of *B* rel. to *O* = Vel. of *B* rel. to *A* + Vel. of *A* rel. to *O*

$$v_{bo} = v_{ba} + v_{ao}$$

$$v_{bg} = v_{ao} + v_{ba}$$

or **gb = oa + ab**

$\mathbf{v}_{bo}$ is replaced by v_{bg} as *O* and *G* are two points on a fixed link

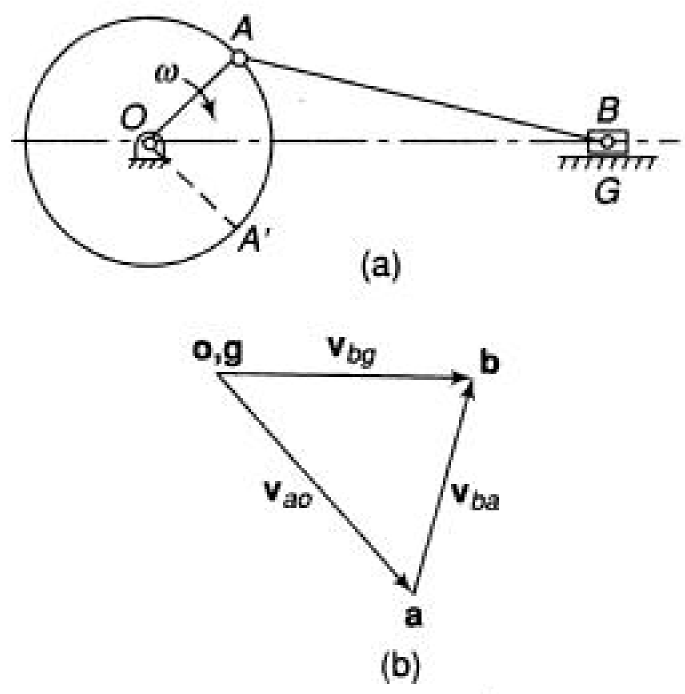


Fig. 2.8

with zero relative velocity between them.

Take the vector $\mathbf{v}_{ao}$ which is completely known.

$\mathbf{v}_{ao} = \omega . OA$; ⊥ to OA

$\mathbf{v}_{ba}$ is ⊥ AB, draw a line ⊥ AB through **a**;

Through **g** (or **a**), draw a line parallel to the motion of B (to locate the vector $\mathbf{v}_{bg}$).

The intersection of the two lines locates the point **b**.

gb (or **ob**) indicates the velocity of the slider B relative to the guide G. This is also the absolute velocity of the slider (G is fixed). The slider moves towards the right as indicated by **gb.** When the crank assumes the position OA' while rotating, it will be found that the vector **gb** lies on the left of **g** indicating that B moves towards left.

For the given configuration, the coupler AB has angular velocity in the counter-clockwise direction, the magnitude being $\dfrac{v_{ba}}{BA(\text{or } AB)}$

Example 2.1 *In a four-link mechanism, the dimensions of the links are as under:*

$AB = 50$ mm, $BC = 66$ mm, $CD = 56$ mm and $AD = 100$ mm

At the instant when $\angle DAB = 60°$, the link AB has an angular velocity of 10.5 rad/s in the counter-clockwise direction. Determine the

(i) velocity of the point C

(ii) velocity of the point E on the link BC when BE = 40 mm

(iii) angular velocities of the links BC and CD

(iv) velocity of an offset point F on the link BC if BF = 45 mm, CF = 30 mm and BCF is read clockwise

(v) velocity of an offset point G on the link CD if CG = 24 mm, DG = 44 mm and DCG is read clockwise

(vi) velocity of rubbing at pins A, B, C and D when the radii of the pins are 30, 40, 25 and 35 mm respectively.

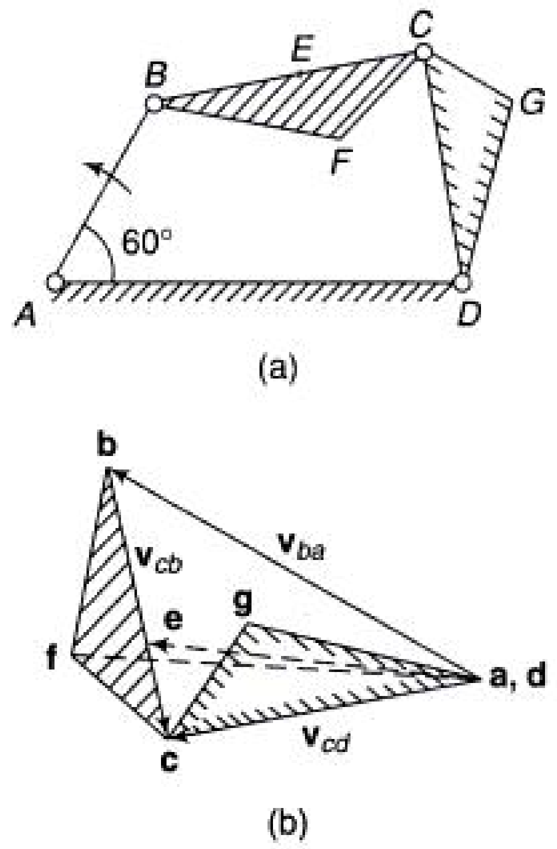


Fig. 2.9

Solution The configuration diagram has been shown in Fig. 2.9(a) to a convenient scale.

Writing the vector equation,

Vel. of C rel. to A = Vel. of C rel. to B + Vel. of B rel. to A

$$\mathbf{v}_{ca} = \mathbf{v}_{cb} + \mathbf{v}_{ba}$$

or $$\mathbf{v}_{cd} = \mathbf{v}_{ba} + \mathbf{v}_{cb}$$

or $$\mathbf{dc} = \mathbf{ab} + \mathbf{bc}$$

We have,

$$\mathbf{v}_{ba} = \omega_{ba} \times BA = 10.5 \times 0.05 = 0.525 \text{ m/s}$$

Take the vector $\mathbf{v}_{ba}$ to a convenient scale in the proper direction and sense [Fig. 2.9(b)].

$\mathbf{v}_{cb}$ is ⊥ BC, draw a line ⊥ BC through **b;**

$\mathbf{v}_{cd}$ is ⊥ DC, draw a line ⊥ DC through **d;**

The intersection of the two lines locates the point **c.**

Note In the velocity diagram shown in Fig. 2.9(b), arrowhead has been put on the line joining points b and c in such a way that it represents the vector for velocity of C relative to B. This satisfies the above equation. As the same equation

$$\mathbf{v}_{cd} = \mathbf{v}_{ba} + \mathbf{v}_{cb}$$

can also be put as

$$\mathbf{v}_{cd} + \mathbf{v}_{bc} = \mathbf{v}_{ba}$$

$$\mathbf{dc} + \mathbf{cb} = \mathbf{ab}$$

This shows that on the same line joining **b** and **c**, the arrowhead should be marked in the other direction so that it represents the vector of velocity of *B* relative to *C* to satisfy the latter equation.

Thus, it implies that in case both the ends of a link are in motion, the arrowhead may be put in either direction or no arrowhead is put at all. This is because every time it is not necessary to write the velocity equation. The velocity equation is helpful only at the initial stage for better comprehension.]

(i) v_c = **ac** (or **dc**) = 0.39 m/s

(ii) Locate the point **e** on **bc** such tha $\frac{be}{bc} = \frac{BE}{BC}$

bc = 0.34 m/s from the velocity diagram.

$$\mathbf{be} = 0.34 \times \frac{0.040}{0.066} = 0.206 \text{ m/s}$$

Therefore, v_e = **ae** (or **de**) = 0.415 m/s

(iii) $\omega_{cb} = \frac{v_{cb}}{CB} = \frac{0.340}{0.066} = 5.15$ rad/s clockwise

$\omega_{cd} = \frac{v_{cd}}{CD} = \frac{0.390}{0.056} = 6.96$ rad/s counter-clockwise

(iv) $\mathbf{v}_{fb}$ is $\perp BF$, draw a line $\perp BF$ through **b;**
$\mathbf{v}_{fc}$ is $\perp CF$, draw a line $\perp CF$ through **c;**
The intersection locates the point **f.**
v_f (i.e., v_{fa} or v_{fd}) = **af** = 0.495 m/s

The point **f** can also be located by drawing the velocity image **bcf** of the triangle *BCF* as discussed earlier.

(v) $\mathbf{v}_{gd}$ is $\perp DG$, draw **dg** $\perp DG$ through **d;**
$\mathbf{v}_{gc}$ is $\perp CG$, draw **cg** $\perp CG$ through **c.**
The intersection locates the point **g.**
v_g = **dg** = 0.305 m/s

However, the velocity of *G* could be found directly since *G* is a point on the link *DC* which rotates about a fixed point *D* and the velocity of *C* is already known.

$$\frac{v_g}{v_c} = \frac{DG}{DC}$$

or

$$v_g = 0.390 \times \frac{0.044}{0.056} = 0.306 \text{ m/s}$$

The point **g** can also be located by drawing the velocity image **dcg** of the triangle *dCG*.

(vi) (a) ω_{ba} (or ω_{ab}) is counter-clockwise and ω_{cb} (or ω_{bc}) is clockwise,

Velocity of rubbing at pin $B = (\omega_{ab} + \omega_{cb})r_b$
$= (10.5 + 5.15) \times 0.040$
$= 0.626$ m/s

(b) ω_{dc} is counter-clockwise and ω_{bc} is clockwise,

Velocity of rubbing at the pin *C*
$= (\omega_{dc} + \omega_{bc})r_c$
$= (6.96 + 5.15) \times 0.025$
$= 0.303$ m/s

(c) Velocity of rubbing at the pin *A*
$= \omega_{ba} r_a = 10.5 \times 0.03 = 0.315$ m/s

(d) Velocity of rubbing at the pin *D*
$= \omega_{cd} r_d = 6.96 \times 0.035 = 0.244$ m/s

Example 2.2 *In a slider-crank mechanism, the crank is 480 mm long and rotates at 20 rad/s in the counter-clockwise direction The length of the connecting rod is 1.6 m. When the crank turns 60° from the inner-dead centre, determine the*

(i) velocity of the slider

(ii) velocity of a point E located at a distance 450 mm on the connecting rod extended

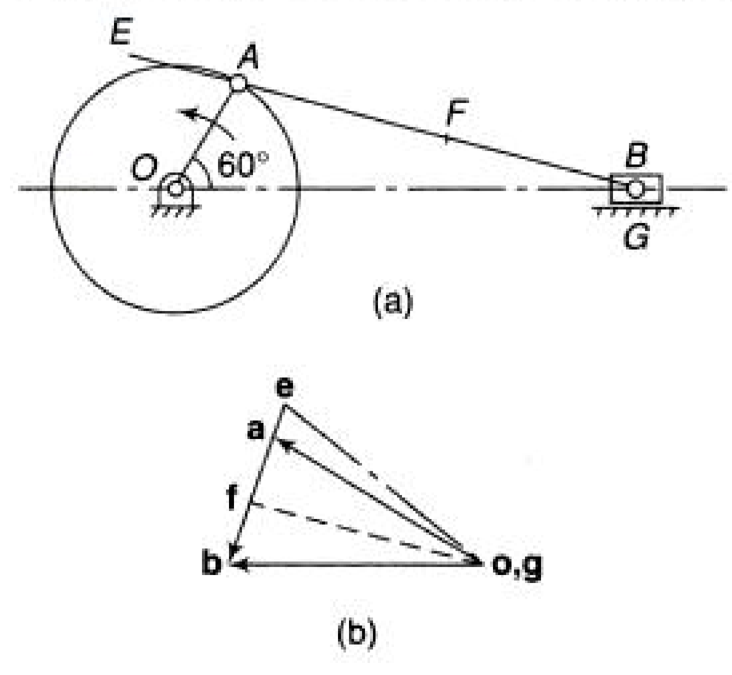


Fig. 2.10

(iii) *position and velocity of a point F on the connecting rod having the least absolute velocity*

(iv) *angular velocity of the connecting rod*

(v) *velocities of rubbing at the pins of the crankshaft, crank and the cross-head having diameters 80, 60 and 100 mm respectively.*

Solution Figure 2.10(a) shows the configuration diagram to a convenient scale.

$$v_{ao} = \omega_{ao} \times OA = 20 \times 0.48 = 9.6 \text{ m/s}$$

The vector equation is $\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$

or

$$\mathbf{v}_{bg} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$

or

$$\mathbf{v}_{bg} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

or

$$\mathbf{gb} = \mathbf{oa} + \mathbf{ab}$$

Take the vector $\mathbf{v}_{ao}$ to a convenient scale in the proper direction and sense [Fig. 2.10 (b)].

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line $\perp AB$ through **a;**

The slider B has a linear motion relative to the guide G. Draw a line parallel to the direction of motion of the slider through **g** (or **o**). Thus, the point **b** is located.

(i) Velocity of the slider, v_b = **ob** = 9.7 m/s

(ii) Locate the point **e** on **ba** extended such that

$$\frac{\mathbf{ae}}{\mathbf{ba}} = \frac{AE}{BA}$$

ba = 5.25 m/s on measuring from the diagram.

$$\therefore \mathbf{ae} = 5.25 \times \frac{0.45}{1.60} = 1.48 \text{ m/s}$$

v_e = **oe** = <u>10.2 m/s</u>

(iii) To locate a point F on the connecting rod which has the least velocity relative to the crankshaft or has the least absolute velocity, draw **of** $\perp$ **ab** through **o.**

Locate the point F on AB such the $\dfrac{AF}{AB} = \dfrac{\mathbf{af}}{\mathbf{ab}}$

or

$$AF = 1.60 \times \frac{2.76}{5.25} = 0.84 \text{ m}$$

v_f = **of** = <u>9.4 m/s</u>

(iv) $\omega_{ba} = \dfrac{v_{ba}}{AB} = \dfrac{5.25}{1.60} = 3.28$ rad/s clockwise

(v) (a) Velocity of rubbing at the pin of the crankshaft (at O)

$= \omega_{ao} r_o = 20 \times 0.04 = 0.8$ m/s

$$\left(r_o \frac{80}{2} = 40 \text{ mm} \right)$$

(b) ω_{oa} is counter-clockwise and ω_{ba} is clockwise.

Velocity of rubbing at the crank pin $A = (\omega_{oa} + \omega_{ba})\, r_a$

$= (20 + 3.28) \times 0.03$

$= 0.698$ m/s

(c) At the cross-head, the slider does not rotate and only the connecting rod has the angular motion.

Velocity of rubbing at the cross-head pin at B

$= \omega_{ab} r_b = 3.28 \times 0.05 =$ <u>0.164 m/s</u>

Example 2.3 *Figure 2.11a shows a mechanism in which OA = QC = 100 mm, AB = QB = 300 mm and CD = 250 mm. The crank OA rotates at 150 rpm in the clockwise direction. Determine the*

(i) *velocity of the slider at D*

(ii) *angular velocities of links QB and AB*

(iii) *rubbing velocity at the pin B which is 40 mm in diameter*

Solution $\omega_{ao} = \dfrac{2\pi \times 150}{60} = 15.7$ rad/s

$$v_{ao} = \omega_{ao} \times OA = 15.7 \times 0.1 = 1.57 \text{ m/s}$$

The vector equation for the mechanism $OABQ$,

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$

or $\mathbf{v}_{bq} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$ or $\mathbf{qb} = \mathbf{oa} + \mathbf{ab}$

Take the vector $\mathbf{v}_{ao}$ to a convenient scale in the proper direction and sense [Fig. 2.11 (b)].

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line $\perp AB$ through **a;**

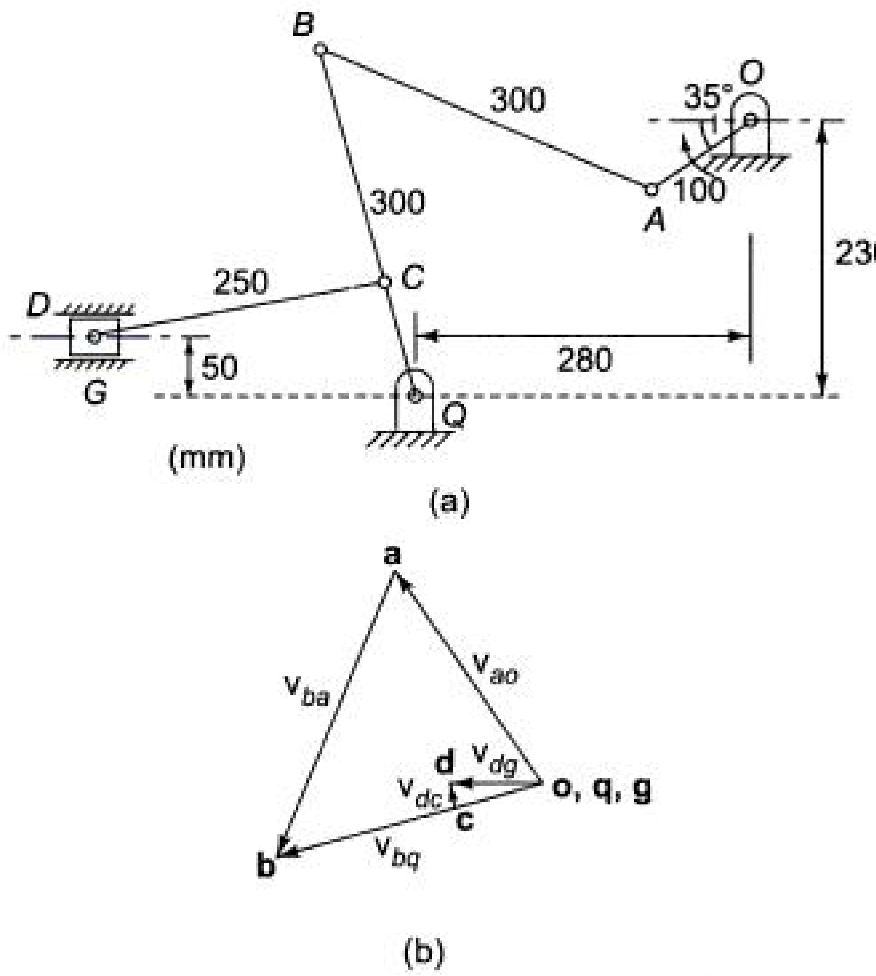


Fig. 2.11

$\mathbf{v}_{bq}$ is $\perp QB$, draw a line $\perp QB$ through **q;**

The intersection of the two lines locates the point **b.**

Locate the point **c** on **qb** such that

$$\frac{\mathbf{qc}}{\mathbf{qb}} = \frac{100}{300} = 0.3$$

The vector equation for the mechanism *QCD*,

$$\mathbf{v}_{dq} = \mathbf{v}_{dc} + \mathbf{v}_{cq} \quad \text{or} \quad \mathbf{v}_{dg} = \mathbf{v}_{cq} + \mathbf{v}_{dc}$$

or $\quad \mathbf{gd} = \mathbf{qc} + \mathbf{cd}$

$\mathbf{v}_{dc}$ is $\perp DC$, draw a line $\perp DC$ through **c;**

For $\mathbf{v}_{dg}$, draw a line through **g,** parallel to the line of stroke of the slider in the guide *G*.

The intersection of the two lines locates the point **d.**

(i) The velocity of slider at *D*, v_d = **gd** = 0.56 m/s

(vi) $\omega_{bq} = \dfrac{v_{bq}}{QB} = \dfrac{1.69}{0.3} = 5.63$ rad/s counter-clockwise

(vii) $\omega_{ba} = \dfrac{v_{ba}}{AB} = \dfrac{1.89}{0.3} = 6.3$ rad/s counter-clockwise

As both the links connected at *B* have counter-clockwise angular velocities,

velocity of rubbing at the crank pin

$$B = (\omega_{ba} - \omega_{bq})\, r_b$$
$$= (6.3 - 5.63) \times 0.04 = 0.0268 \text{ m/s}$$

Example 2.4 *An engine crankshaft drives a reciprocating pump through a mechanism as shown in Fig. 2.12(a). The crank rotates in the clockwise direction at 160 rpm. The diameter of the pump piston at F is 200 mm. Dimensions of the various links are*

OA = 170 mm (crank)	*CD = 170 mm*
AB = 660 mm	*DE = 830 mm*
BC = 510 mm	

For the position of the crank shown in the diagram, determine the

(i) velocity of the crosshead E

(ii) velocity of rubbing at the pins A, B, C and D, the diameters being 40, 30, 30 and 50 mm respectively

(iii) torque required at the shaft O to overcome a pressure of 300 kN/m² at the pump piston at F

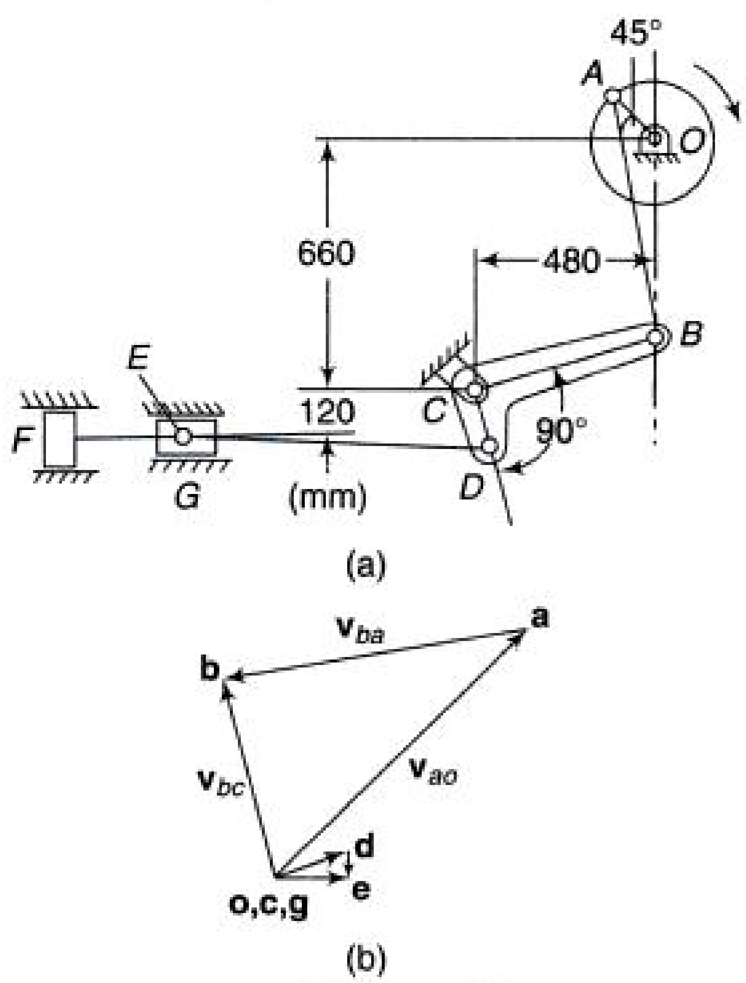


Fig. 2.12

Solution:

$$\omega_{ao} = \frac{2\pi N}{60} = \frac{2\pi \times 160}{60} = 16.76 \text{ rad/s}$$

$$v_{ao} = \omega_{ao} \times OA = 16.76 \times 0.17 = 2.85 \text{ m/s}$$

Writing the vector equation for the mechanism *OABC*,

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$

or

$$\mathbf{v}_{bc} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

or

$$\mathbf{cb} = \mathbf{oa} + \mathbf{ab}$$

Take the vector $\mathbf{v}_{ao}$ to a convenient scale [Fig. 2.12(b)]

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line $\perp AB$ through **a;**

$\mathbf{v}_{bc}$ is $\perp BC$, draw a line $\perp BC$ through **c.**

The intersection of the two lines locates the point ***b***. Velocity of *B* relative to *C* is upwards for the given configuration. Therefore, the link *BCD* moves counter-clockwise about the pivot *C*.

$$\frac{v_{dc}}{v_{bc}} = \frac{DC}{BC}$$

or $v_{dc} = 1.71 \times \dfrac{0.17}{0.51} = 0.57$ m/s $\quad (\perp DC)$

Writing the vector equation for the mechanism *CDE*,

$$\mathbf{v}_{ec} = \mathbf{v}_{ed} + \mathbf{v}_{dc}$$

or

$$\mathbf{v}_{eg} = \mathbf{v}_{dc} + \mathbf{v}_{ed}$$

or

$$\mathbf{ge} = \mathbf{cd} + \mathbf{de}$$

Take $\mathbf{v}_{dc}$ in the proper direction and sense from **c** assuming *D* in the configuration diagram as an offset point on link *CB:*

$\mathbf{v}_{ed}$ is $\perp DE$, draw a line $\perp DE$ through **d**.

For $\mathbf{v}_{eg}$, draw a line through **g,** parallel to the direction of motion of the slider *E* in the guide *G*.

This way the point **e** is located.

(i) The velocity of the crosshead,

$$v_e = \mathbf{oe} = \underline{0.54 \text{ m/s}}$$

(ii) (a) ω_{oa} and ω_{ba} both are clockwise.

$$\omega_{ba} = \frac{\mathbf{ab}}{AB} = \frac{2.49}{0.66} = 3.77 \text{ rad/s}$$

Velocity of rubbing at the pin $A = (\omega_{oa} - \omega_{ba})\, r_a$

$$= (16.76 - 3.77) \times \frac{0.04}{2}$$

$$= \underline{0.26 \text{ m/s}}$$

(b) ω_{ab} is clockwise and ω_{cd} is counter-clockwise.

$$\omega_{cb} = \frac{v_{cb}}{CB} = \frac{1.71}{0.51} = 3.35 \text{ rad/s}$$

Velocity of rubbing at $B = (\omega_{ab} + \omega_{cb})\, r_b$

$= (3.77 + 3.35) \times 0.015$ .... $(\omega_{ab} = \omega_{ba})$

$= \underline{0.107 \text{ m/s}}$

(c) Velocity of rubbing at $C = \omega_{bc} . r_c$

$$= 3.35 \times \frac{0.03}{2} = \underline{0.05 \text{ m/s}}$$

(d) ω_{cd} and ω_{ed}, both are counter-clockwise

$\omega_{cd} = \omega_{bc} = 3.35$ rad/s ... (*BCD* is one link)

$$= \omega_{ed} = \frac{v_{ed}}{ED} = \frac{0.15}{0.83} = 0.18 \text{ rad/s}$$

Velocity of rubbing at $D = (\omega_{cd} - \omega_{ed})\, r_d$

$$= (3.35 - 0.18) \times \frac{0.05}{2}$$

$$= \underline{0.079 \text{ m/s}}$$

(iii) Work input = work output

$T.\omega = F.v$

where T = torque on the crankshaft

ω = angular velocity of the crank

F = force on the piston

v = velocity of the piston = $v_f = v_e$

Thus, neglecting losses,

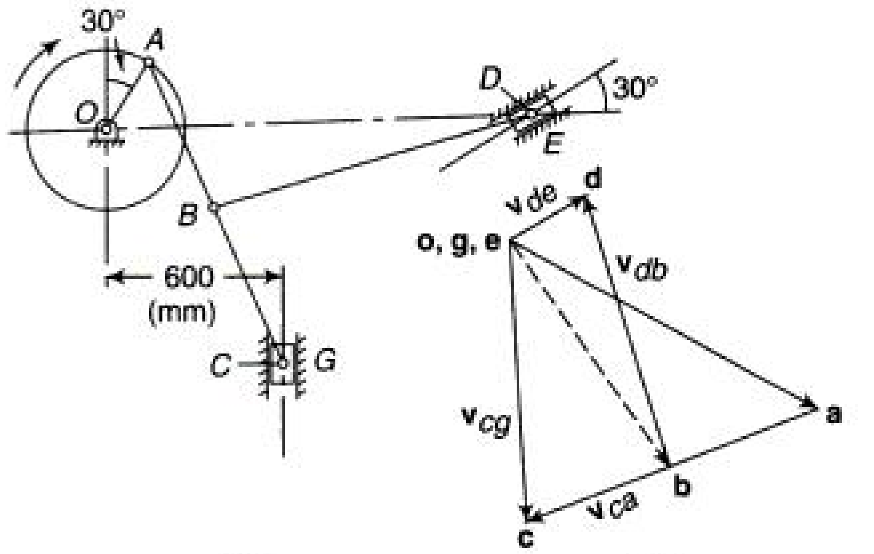
$$T = \frac{F.v}{\omega} = \frac{\pi}{4}(0.02)^2 \times 300 \times 10^3 \times \frac{0.54}{16.76}$$

$= \underline{303.66 \text{ N.m}}$

Example 2.5 *Figure 2.13(a) shows a mechanism in which OA = 300 mm, AB = 600 mm, AC = BD = 1.2 m. OD is horizontal for the given configuration. If OA rotates at 200 rpm in the clockwise direction, find*

(iv) linear velocities of C and D

(v) angular velocities of links AC and BD



(a) (b)

Fig. 2.13

Solution: $\omega_a = \dfrac{2\pi \times 200}{60} = 20.94$ rad/s

$$v_a = \omega_a OA = 20.94 \times 0.3 = 6.28 \text{ m/s}$$

Writing the vector equation for the mechanism *OAC*,

$$\mathbf{v}_{co} = \mathbf{v}_{ca} + \mathbf{v}_{ao}$$

or

$$\mathbf{v}_{cg} = \mathbf{v}_{ao} + \mathbf{v}_{ca}$$

or

$$\mathbf{gc} = \mathbf{oa} + \mathbf{ac}$$

Take the vector $\mathbf{v}_{ao}$ to a convenient scale [Fig. 2.13(b)].

$\mathbf{v}_{ca}$ is $\perp AC$, draw a line $\perp AC$ through **a**;

$\mathbf{v}_{cg}$ is vertical, draw a vertical line through **g** (or **o**).

The intersection of the two lines locates the point **c**. Locate the point **b** on **ac** as usual. Join **ob** which gives $\mathbf{v}_{bo}$. Writing the vector equation for the mechanism *OABD*,

$$\mathbf{v}_{do} = \mathbf{v}_{db} + \mathbf{v}_{bo}$$

or

$$\mathbf{v}_{de} = \mathbf{v}_{bo} + \mathbf{v}_{db}$$

or

$$\mathbf{ed} = \mathbf{ob} + \mathbf{bd}$$

$\mathbf{v}_{db}$ is $\perp BD$, draw a line $\perp BD$ through **b**;

For $\mathbf{v}_{de}$, draw a line through **e**, parallel to the line of stroke of the piston in the guide *E*.

The intersection locates the point **d**.

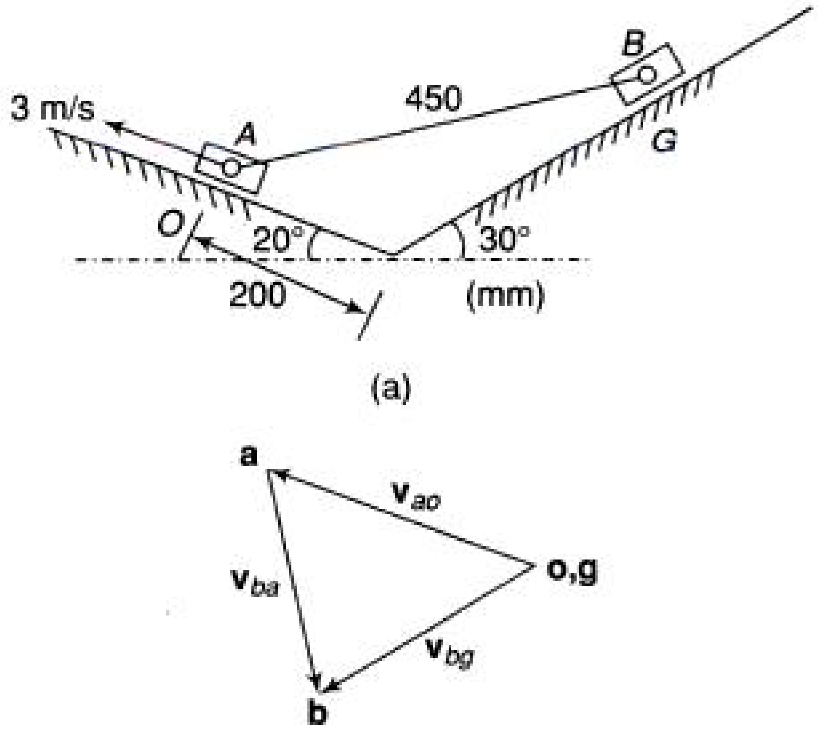
v_c = **oc** = <u>5.2 m/s</u>

v_d = **od** = <u>1.55 m/s</u>

$$\omega_{ac} = \omega_{ca} = \frac{v_{ca}}{AC} = \frac{5.7}{1.20} = \underline{4.75 \text{ rad/s}} \text{ clockwise}$$

$$\omega_{bd} = \omega_{db} = \frac{v_{db}}{BD} = \frac{5.17}{1.20} = \underline{4.31 \text{ rad/s}} \text{ clockwise}$$

Example 2.6 *For the position of the mechanism shown in Fig. 2.14(a), find the velocity of the slider B for the given configuration if the velocity of the slider A is 3 m/s.*



(a)

(b)

Fig. 2.14

Solution The velocity vector equation is

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$

or $\mathbf{v}_{bg} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$

or **gb = oa + ab**

Take the vector $\mathbf{v}_{ao}$ (= 3 m/s) to a convenient scale [Fig. 2.14(b)]

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line AB through **a**;

For $\mathbf{v}_{bg}$, draw a line through **g** parallel to the line of stroke of the slider B on the guide G.

The intersection of the two lines locates the point **b**.

Velocity of B = **gb** = 2.67 m/s.

Example 2.7 *In a mechanism shown in Fig. 2.15(a), the angular velocity of the crank OA is 15 rad/s and the slider at E is constrained to move at 2.5 m/s. The motion of both the sliders is vertical and the link BC is horizontal in the position shown. Determine the*

(i) rubbing velocity at B if the pin diameter is 15 mm

(ii) velocity of slider D.

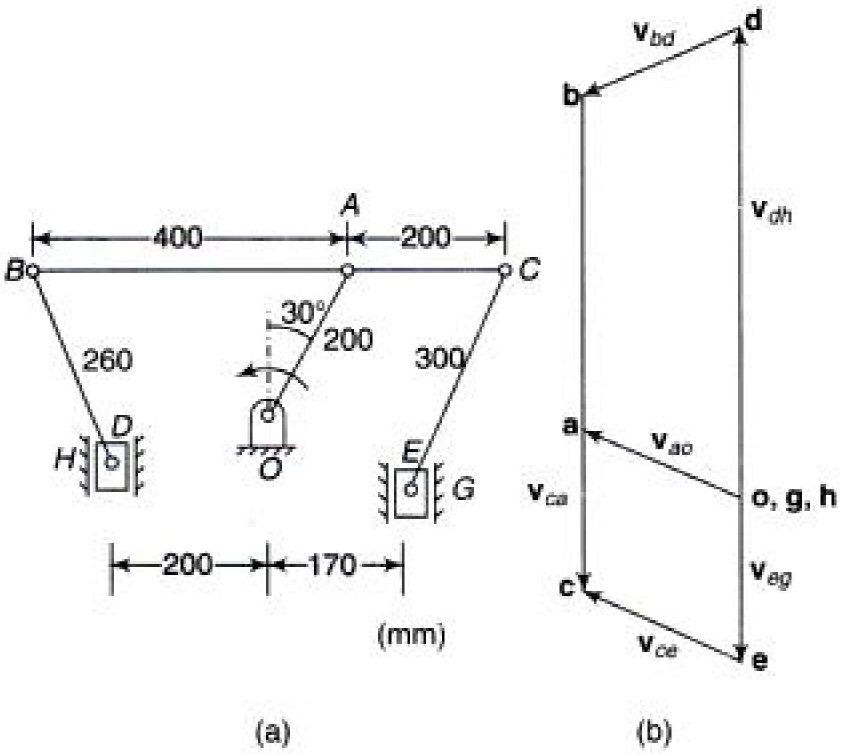


Fig. 2.15

Solution $v_a = \omega_a OA = 15 \times 0.2 = 3$ m/s

Draw the velocity diagram as follows:

- Take vector **oa** to a suitable scale (2.15b).
- Consider two points G and H on the guides of sliders E and F respectively. In the velocity diagram, the points **g** and **h** coincide with **o**. Through **g**, take a vector **ge** parallel to direction of motion of the slider E and equal to 2.5 m/s using some scale.
- Through **e** draw a line $\perp EC$ and through **a**, a line $\perp AC$, the intersection of these two lines locates the point **c**.
- Locate the point **b** on the vector **ca** so that **ca**/**cb** = CA/CB.
- Through **b**, draw a line $\perp BD$ and through **h**, a line parallel to direction of motion of the slider D, the intersection of these two lines locates the point **d**.

(i) Angular velocity of link BD = **bd**/BD = 2.95/0.26 =11.3 rad/s (counter-clockwise)
Angular velocity of link BC = **bc**/BC = 8.4/0.6 =14 rad/s (clockwise)
Thus velocity of rubbing at

$$B = (\omega_{bd} + \omega_{bc})r_b = (11.3 + 14) \times 0.015 = \underline{0.38 \text{ m/s}}$$

(ii) The velocity of the slider D = **hd** = $\underline{8.3 \text{ m/s}}$

Example 2.8 *The lengths of various links of a mechanism shown in Fig. 2.16(a) are as follows:*

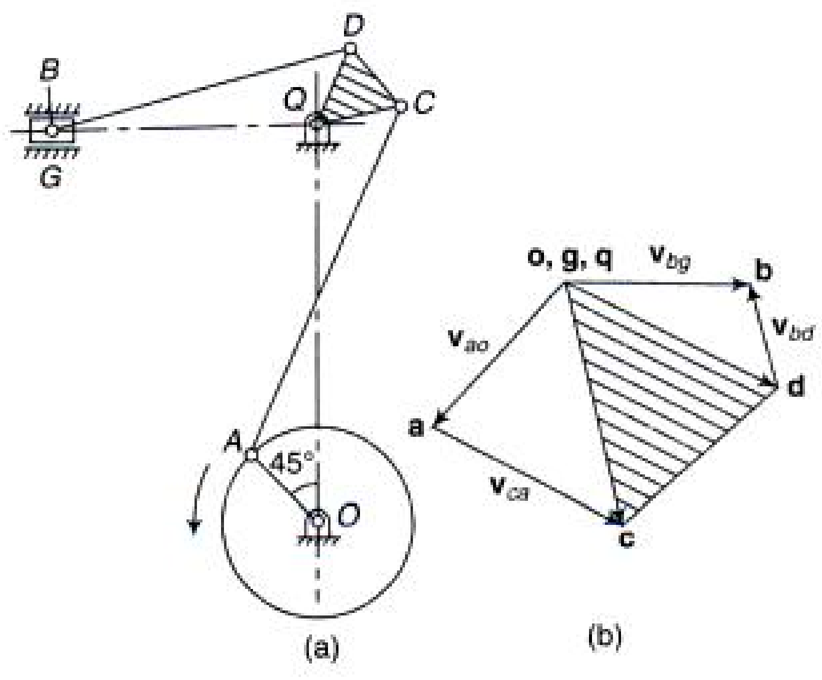


Fig. 2.16

OA = 150 mm	*CD = 125 mm*
AC = 600 mm	*BD = 500 mm*
CQ = QD = 145 mm	*OQ = 625 mm*

The crank OA rotates at 60 rpm in the counter-clockwise direction. Determine the velocity of the slider B and the angular velocity of the link BD when the crank has turned an angle of 45° with the vertical.

Solution

$$v_a = \frac{2\pi N}{60} \times OA = \frac{2\pi \times 60}{60} \times 0.15 = 0.94 \text{ m/s}$$

Take the vector $\mathbf{v}_a$, to a convenient scale [Fig. 2.16(b)] and complete the velocity diagram for the mechanism *OACQ*.

Now *CQD* is one link. Make Δ **cqd** similar to Δ *CQD* such that **cqd** reads clockwise as *CQD* is clockwise. This locates the point **d**. Complete the velocity diagram for the mechanism *QDB*.

v_b = **ob** = 0.9 m/s

$$\omega_{bd} = \frac{v_{bd}}{BD} = \frac{0.49}{0.50} = \underline{0.98 \text{ rad/s}} \text{ clockwise}$$

Example 2.9 *The configuration diagram of a wrapping machine is given in Fig. 2.17(a). The crank OA rotates at 6 rad/s clockwise. Determine the*

(i) velocity of the point P on the bell-crank lever DCP

(ii) angular velocity of the bell-crank lever DCP

(iii) velocity of rubbing at B if the pin diameter is 20 mm

Solution

$$v_a = 6 \times 0.15 = 0.9 \text{ m/s}$$

Take the vector $\mathbf{v}_a$, to a convenient scale [Fig. 2.17(b)] and complete the velocity diagram for the mechanism *OABQ*.

Now locate point **e** on the vector **ab**.

$\mathbf{v}_{de}$ is ⊥ *DE*, draw **de** ⊥ *DE* through **e;**

$\mathbf{v}_{dc}$ is ⊥ *CD*, draw **cd** ⊥ *CD* through **c.**

The intersection locates the point **d.**

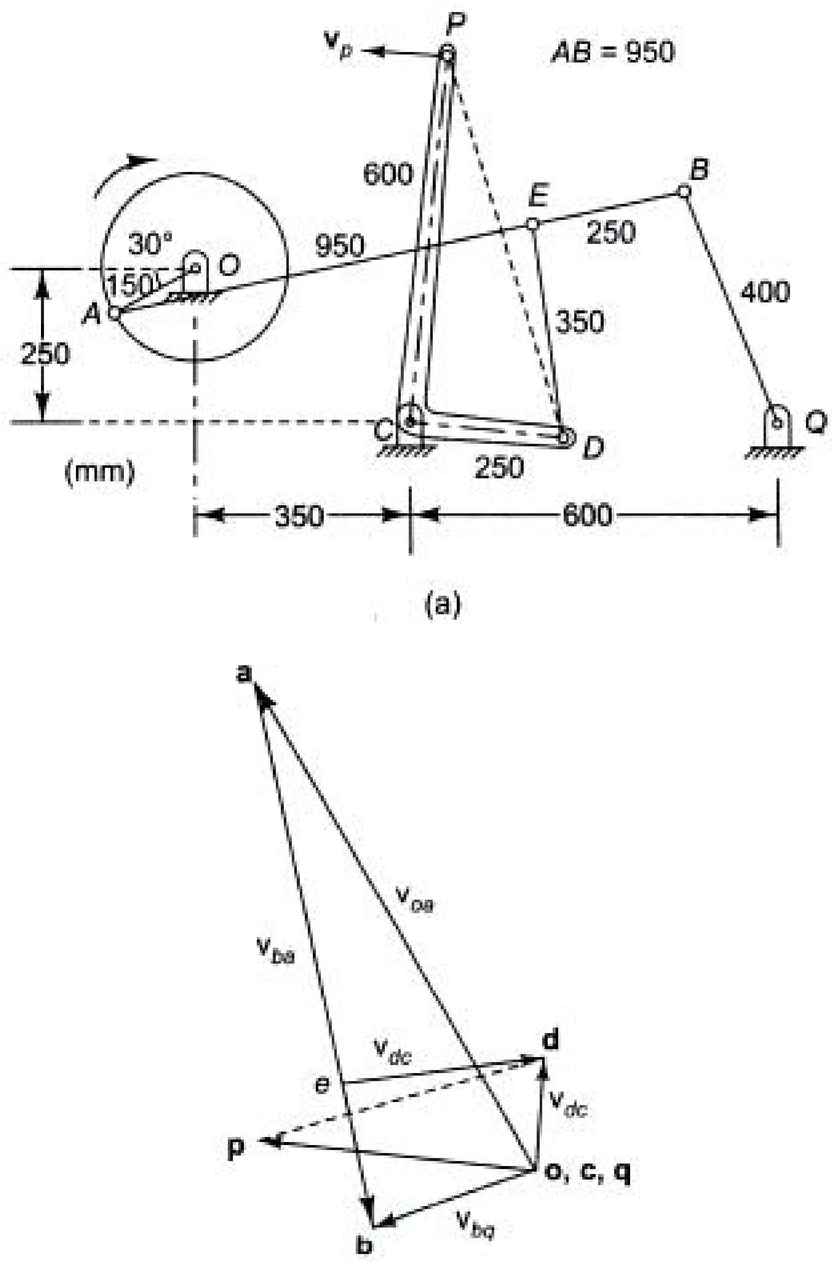


Fig. 2.17

Now, *DCP* is one link. Make Δ**dcp** similar to Δ*DCP* such that **dcp** reads clockwise as *DCP* is clockwise. This locates the point **p**. Then

(i) v_p = **cp** = 0.44 m/s

(ii) $$\omega_{cd} = \frac{v_{cd}}{CD} = \frac{0.182}{0.25} = \underline{0.73 \text{ rad/s}}$$

counter clockwise

(iii) $$\omega_{ab} = \frac{v_{ab}}{AB} = \frac{0.91}{0.95} = \underline{0.96 \text{ rad/s}} \text{ clockwise}$$

$$\omega_{qb} = \frac{v_{qb}}{QB} = \frac{0.28}{0.4} = \underline{0.7 \text{ rad/s}}$$

counter-clockwise

Thus, velocity of rubbing at $B = (\omega_{ab} + \omega_{bq})r_b$

$= (0.96 + 0.7) \times 0.02 = \underline{0.0332 \text{ m/s}}$

Example 2.10 *Figure 2.18(a) shows an Andrew variable-stroke-engine mechanism. The lengths of the cranks OA and QB are 90 mm and 45 mm respectively. The diameters of wheels with centres O and Q are 250 mm and 120 mm respectively. Other lengths are shown in the diagram in mm. There is a rolling contact between the two wheels. If OA rotates at 100 rpm, determine the*

(i) velocity of the slider D

(ii) angular velocities of links BC and CD

(iii) torque at QB when force required at D is 3 kN

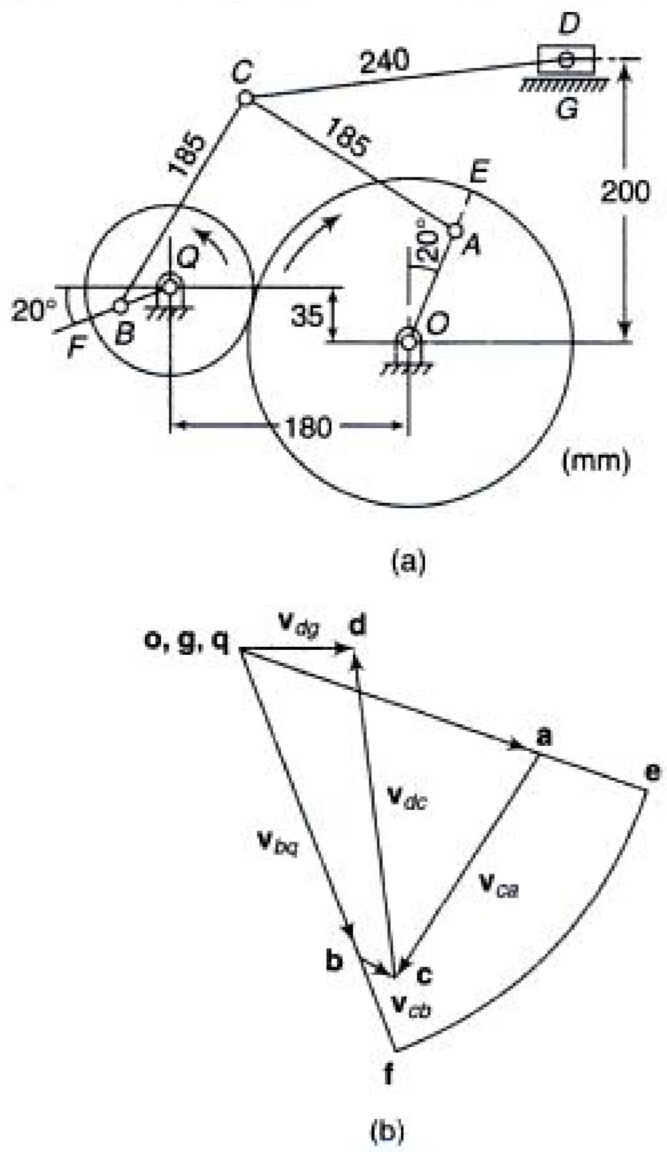


Fig. 2.18

Solution

$$v_a = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60} \times 0.09 = 0.943 \text{ m/s}$$

$$v_e = v_a \frac{OE}{OA} = 0.943 \times \frac{0.125}{0.09} = 1.309 \text{ m/s}$$

$$v_f = v_e = 1.309 \text{ m/s}$$

$$v_b = v_f \cdot \frac{QB}{QF} = 1.309 \times \frac{0.045}{0.06} = 0.982 \text{ m/s}$$

v_b can also be obtained graphically as follows:

Take vector $\mathbf{v}_a$ to a convenient scale [Fig. 2.18(b)]. Produce **oa** to **e** such that **oe/oa** = OE/OA. Rotate **oe** to **of** so that **of** is perpendicular to QF. Mark the point **b** on **qf** such that **qb/qf** = QB/Qf.

Now, $\mathbf{v}_{co} = \mathbf{v}_{cq}$

$$\mathbf{v}_{ca} + \mathbf{v}_{ao} = \mathbf{v}_{cb} + \mathbf{v}_{bq}$$

or

$$\mathbf{v}_{ao} + \mathbf{v}_{ca} = \mathbf{v}_{bq} + \mathbf{v}_{cb}$$

or

oa + ac = qb + bc

$\mathbf{v}_{ao}$ and $\mathbf{v}_{bq}$ are already there in the velocity diagram.

$\mathbf{v}_{ca}$ is $\perp AC$, draw a line $\perp AC$ through **a**;

$\mathbf{v}_{cb}$ is $\perp BC$, draw a line $\perp BC$ through **b**;

Thus, the point **c** is located.

Further, $\mathbf{v}_{do} = \mathbf{v}_{dc} + \mathbf{v}_{co}$

or

$$\mathbf{v}_{dg} = \mathbf{v}_{co} + \mathbf{v}_{dc}$$

or

gd = oc + cd

$\mathbf{v}_{co}$ already exists in the diagram.

$\mathbf{v}_{dc}$ is $\perp CD$, draw **cd** $\perp CD$ through **c**;

$\mathbf{v}_{dg}$ is horizontal. Draw a horizontal line through **g** (or **o**) and locate the point **d**.

(i) v_d = **od** = <u>0.34 m/s</u>

(ii) $$\omega_{bc} = \frac{v_{bc}}{BC} = \frac{0.12}{0.185} = \underline{0.649 \text{ rad/s}} \text{ clockwise}$$

$$\omega_{cd} = \frac{v_{dc}}{DC} = \frac{1.0}{0.24} = \underline{4.17 \text{ rad/s}}$$

counter-clockwise

(iii) $T.\omega = F_d . v_d$

$F_d = 3000$ N

v_d = **od** (=**gd**) = 0.34 m/s

$$T = \frac{3000 \times 0.34}{(2\pi \times 100)/60} = \underline{97.4 \text{ N.m}}$$

Example 2.11 *The mechanism of a stone-crusher is shown in Fig. 2.19a along with various dimensions of links in mm. If crank OA rotates at a uniform velocity of 120 rpm, determine the velocity of the point K (jaw) when the crank OA is inclined at an angle of 30° to the horizontal. What will be the torque required at the crank OA to overcome a horizontal force of 40 kN at K?*

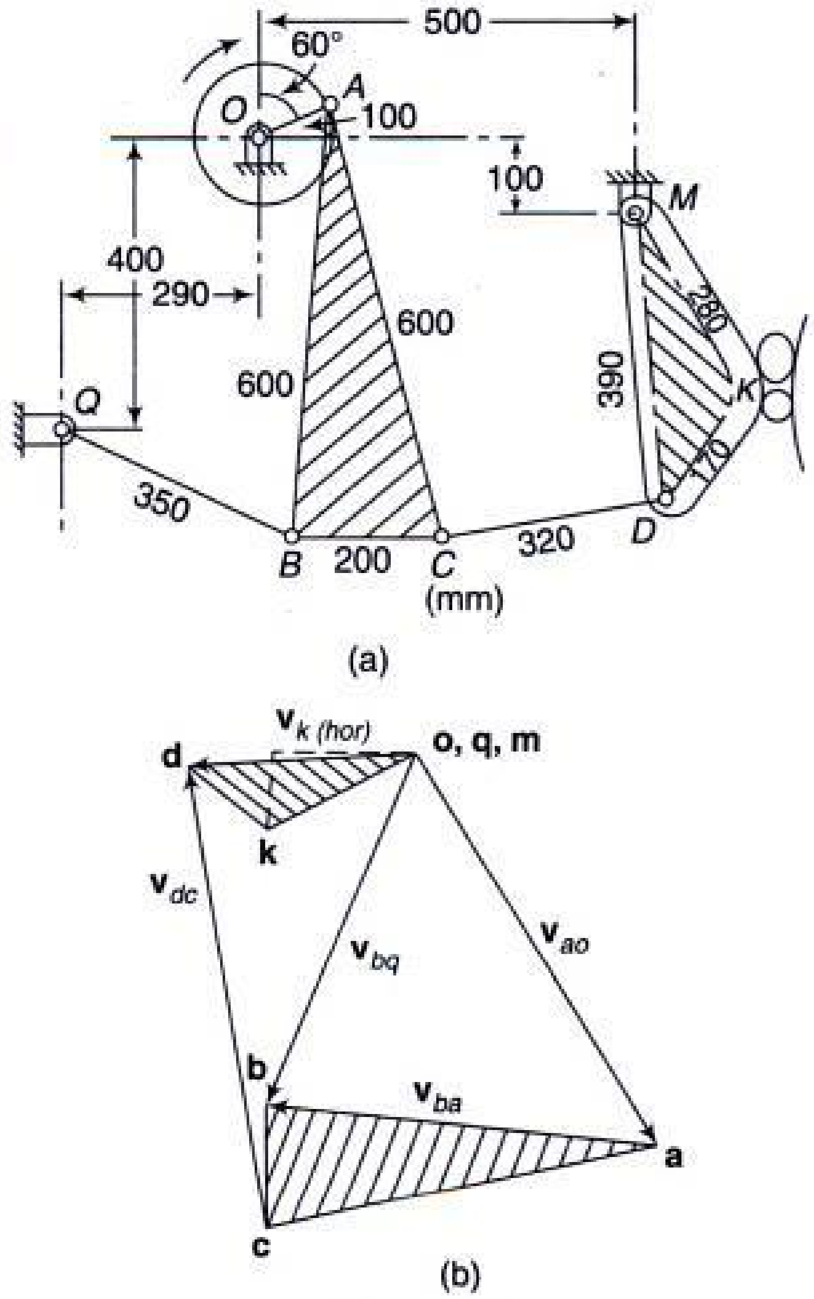


Fig. 2.19

Solution $\omega_{ao} = \dfrac{2\pi \times 120}{60} = 12.6$ rad/s

$v_{ao} = \omega_{ao} \times OA = 12.6 \times 0.1 = 1.26$ m/s

Write the vector equation for the mechanism *OABQ* and complete the velocity diagram as usual [(Fig. 2.19(b)]. Make Δ**bac** similar to ΔBAC (both are read clockwise).

Write the vector equation for the mechanism *OACDM* and complete the velocity diagram. Make Δ**dmk** similar to ΔDMK (both are read clockwise).

v_k = **ok** = 0.45 m/s

v_k (horizontal) = 0.39 m/s

$T.\omega_{ao} = F_k\, v_k$ (horizontal)

$$T = \frac{40\,000 \times 0.39}{12.6} = \underline{1242 \text{ N.m}}$$

2.10 CRANK- AND SLOTTED-LEVER MECHANISM

While analysing the motions of various links of a mechanism, sometimes we are faced with the problem of describing the motion of a moveable point on a link which has some angular velocity. For example, the motion of a slider on a rotating link. In such a case, the angular velocity of the rotating link along with the linear velocity of the slider may be known and it may be required to find the absolute velocity of the slider.

A crank and slotted-lever mechanism, which is a form of quick-return mechanism used for slotting and shaping machines, depicts the same form of motion [Fig. 2.20(a)]. *OP* is the crank rotating at an angular velocity of ω rad/s in the

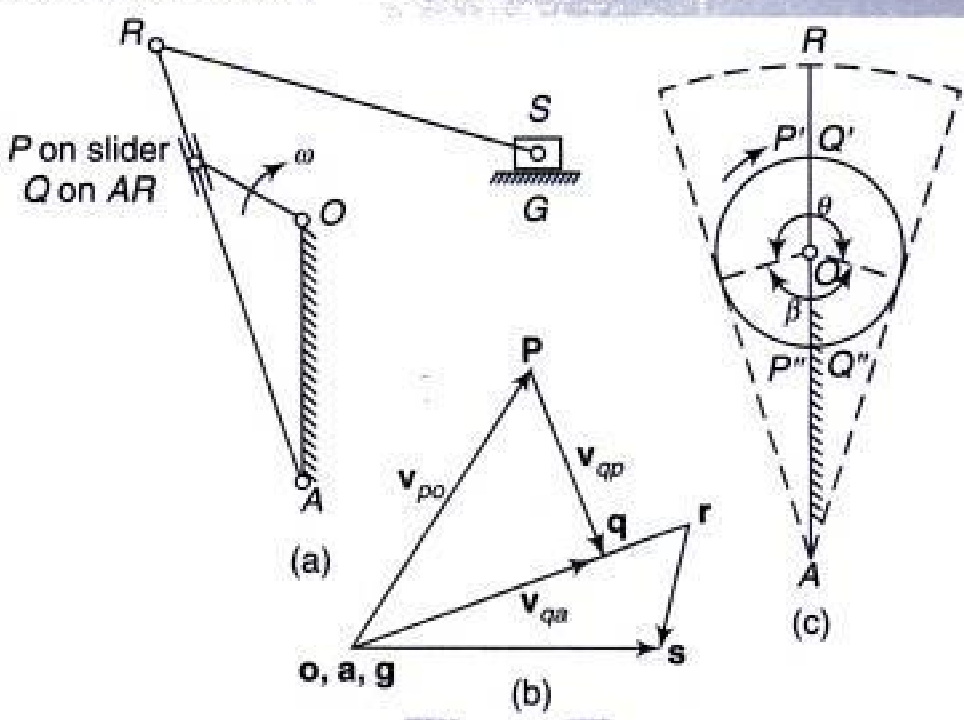


Fig. 2.20

clockwise direction about the centre O. At the end of the crank, a slider P is pivoted which moves on an oscillating link AR.

In such problems, it is convenient if a point Q on the link AR immediately below P is assumed to exist (P and Q are known as coincident points). As the crank rotates, there is relative movement of the points P and Q along AR.

Writing the vector equation for the mechanism OPA,

Vel. of Q rel. to O = Vel. of Q rel. to P + Vel. of P rel. to O

$$\mathbf{v}_{qo} = \mathbf{v}_{qp} + \mathbf{v}_{po}$$

or

$$\mathbf{v}_{qa} = \mathbf{v}_{po} + \mathbf{v}_{qp}$$

or

$$\mathbf{aq} = \mathbf{op} + \mathbf{pq}$$

In this equation,

$\mathbf{v}_{po}$ or **op** = $\omega.OP$; $\perp$ to OP

$\mathbf{v}_{qp}$ or **pq** is unknown in magnitude ; | | to AR

$\mathbf{v}_{qa}$ or **aq** is unknown in magnitude ; $\perp$ to AR

Take the vector $\mathbf{v}_{po}$ which is fully known [Fig. 2.20 (b)].

$\mathbf{v}_{qp}$ is | | AR, draw a line | | to AR through **p;**

$\mathbf{v}_{qa}$ is $\perp AR$, draw a line $\perp AR$ through **a** (or **o**).

The intersection locates the point **q.**

The vector equation for the above could also have been written as

Vel. of P rel. to A = Vel. of P rel. to Q + Vel. of Q rel. to A

$$\mathbf{v}_{pa} = \mathbf{v}_{pq} + \mathbf{v}_{qa}$$

or

$$\mathbf{v}_{po} = \mathbf{v}_{qa} + \mathbf{v}_{pq}$$

or

$$\mathbf{op} = \mathbf{aq} + \mathbf{qp}$$

Take the vector $\mathbf{v}_{po}$ which is completely known.

$\mathbf{v}_{qa}$ is $\perp AR$, draw a line $\perp AR$ through **a;**

$\mathbf{v}_{pq}$ is | | AR, draw a line | | AR through **p.**

The intersection locates the point **q.** Observe that the velocity diagrams obtained in the two cases are the same except that the direction of $\mathbf{v}_{pq}$ is the reverse of that of $\mathbf{v}_{qp}$.

As the vectors **oq** and **qp** are perpendicular to each other, the vector $\mathbf{v}_{po}$ may be assumed to have two components, one perpendicular to AR and the other parallel to AR.

The component of velocity along AR, i.e., **qp** indicates the relative velocity between Q and P or the velocity of sliding of the block on link AR.

Now, the velocity of R is perpendicular to AR. As the velocity of Q perpendicular to AR is known, the point **r** will lie on vector **aq** produced such that **ar**/**aq** = AR/AQ

To find the velocity of ram S, write the velocity vector equation,

$$\mathbf{v}_{so} = \mathbf{v}_{sr} + \mathbf{v}_{ro}$$

or

$$\mathbf{v}_{sg} = \mathbf{v}_{ro} + \mathbf{v}_{sr}$$

or

$$\mathbf{gs} = \mathbf{or} + \mathbf{rs}$$

$\mathbf{v}_{ro}$ is already there in the diagram. Draw a line through **r** perpendicular to RS for the vector $\mathbf{v}_{sr}$ and a line through **g**, parallel to the line of motion of the slider S on the guide G, for the vector $\mathbf{v}_{sg}$. In this way the point **s** is located.

The velocity of the ram S = **os** (or **gs**) towards right for the given position of the crank.

Also, $\omega_{rs} = \dfrac{v_{rs}}{RS}$ clockwise

Usually, the coupler RS is long and its obliquity is neglected.

Then **or** ≈ **os**

Referring Fig. 2.20 (c),

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\theta}{\beta}$$

When the crank assumes the position OP' during the cutting stroke, the component of velocity along AR (i.e., **pq**) is zero and **oq** is maximum (= **op**)

Let r = length of crank (= OP)
l = length of slotted lever (= AR)

c = distance between fixed centres (= AO)

ω = angular velocity of the crank

Then, during the cutting stroke,

$$v_{s\,\max} = \omega \times OP' \times \frac{AR}{AQ} = \omega r \times \frac{l}{c+r}$$

This is by neglecting the obliquity of the link RS, i.e. assuming the velocity of S equal to that of R.

Similarly, during the return stroke,

$$v_{s\,\max} = \omega \times OP'' \times \frac{AR}{AQ''} = \omega r \times \frac{l}{c-r}$$

$$\frac{v_{s\,\max}\,(\text{cutting})}{v_{s\,\max}\,(\text{return})} = \frac{\omega r \dfrac{1}{c+r}}{\omega r \dfrac{1}{c-r}} = \frac{c-r}{c+r}$$

Example 2.12 *Figure 2.21(a) shows the link mechanism of a quick return mechanism of the slotted lever type, the various dimensions of which are*

OA = 400 mm, OP = 200 mm, AR = 700 mm, RS = 300 mm

For the configuration shown, determine the velocity of the cutting tool at S and the angular velocity of the link RS. The crank OP rotates at 210 rpm.

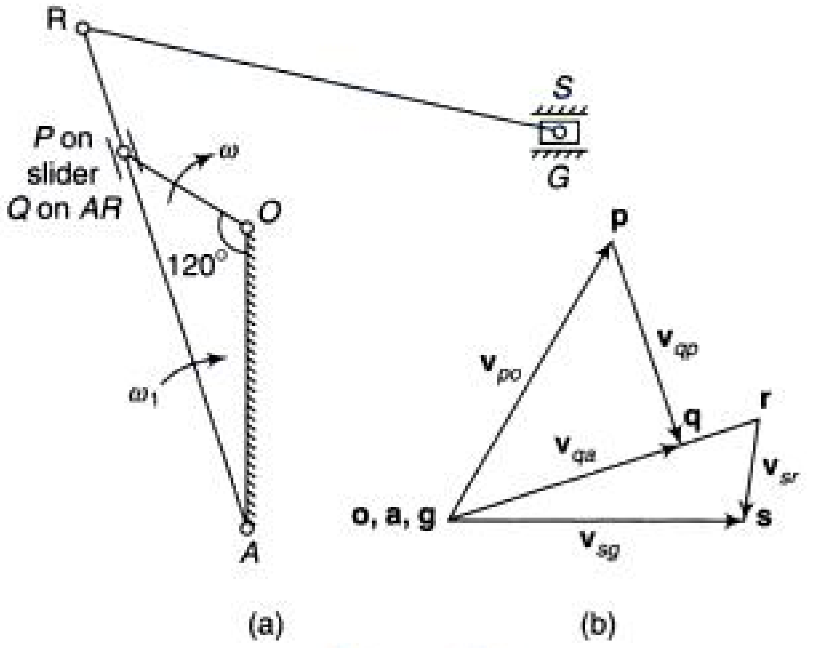


Fig. 2.21

Solution $\omega_{po} = \dfrac{2\pi \times 210}{60} = 22$ rad/s

Draw the configuration to a suitable scale. The vector equation for the mechanism *OPA*,

$$\mathbf{v}_{qa} = \mathbf{v}_{po} + \mathbf{v}_{qp} \quad \text{or} \quad \mathbf{aq} = \mathbf{op} + \mathbf{pq}$$

In this equation,

$$\mathbf{v}_{po} \text{ or } \mathbf{op} = \omega . OP = 22 \times 0.2 = 4.4 \text{ m/s}$$

Take the vector $\mathbf{v}_{po}$ which is fully known [Fig. 2.21(b)].

$\mathbf{v}_{qp}$ is | | *AR*, draw a line | | to *AR* through **p;**

$\mathbf{v}_{qa}$ is $\perp AR$, draw a line $\perp AR$ through **a** (or **o**).

The intersection locates the point **q.** Locate the point **r** on the vector **aq** produced such that **ar**/**aq** = *AR/AQ*.

Draw a line through **r** perpendicular to *RS* for the vector $\mathbf{v}_{sr}$ and a line through **g**, parallel to the line of motion of the slider *S* on the guide *G*, for the vector $\mathbf{v}_{sg}$. In this way the point **s** is located.

The velocity of the ram *S* = **os** (or **gs**) = 4.5 m/s

It is towards right for the given position of the crank.

Angular velocity of link *RS*,

$$\omega_{rs} = \frac{v_{rs}}{RS} = \frac{1.4}{0.3} = 4.67 \text{ rad/s clockwise}$$

Example 2.13 *For the inverted slider-crank mechanism shown in Fig. 2.22(a), find the angular velocity of the link QR and the sliding velocity of the block on the link QR. The crank OA is 300 mm long and rotates at 20 rad/s in the clockwise direction. OQ is 650 mm and QOA = 40°*

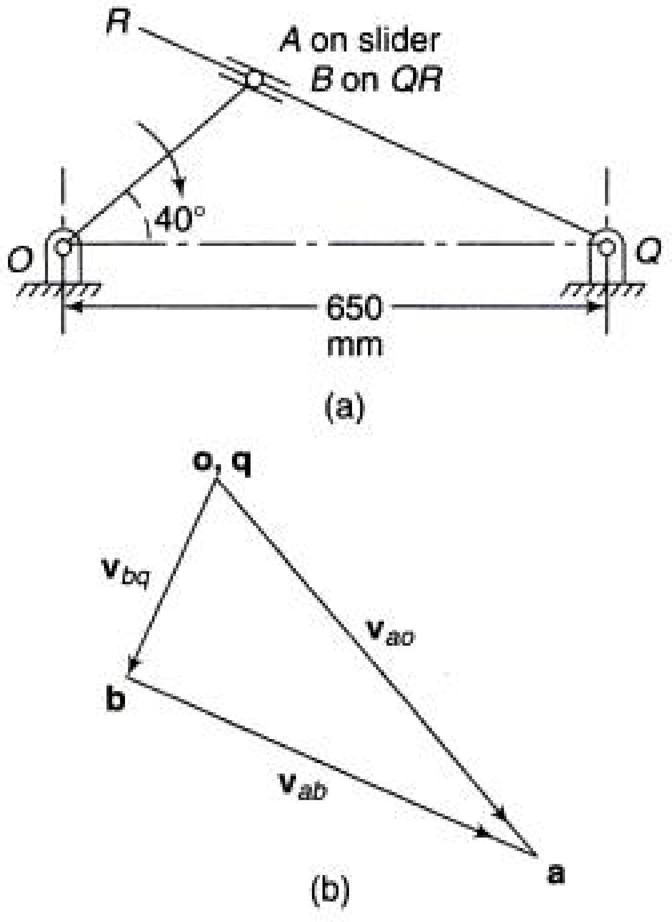


Fig. 2.22

Solution The velocity vector equation can be written as usual.

$$\mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq} \quad \text{or} \quad \mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao}$$

$$\mathbf{v}_{ao} = \mathbf{v}_{bq} + \mathbf{v}_{ab} \qquad \mathbf{v}_{bq} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

$$\mathbf{oa} = \mathbf{qb} + \mathbf{ba} \qquad \mathbf{qb} = \mathbf{oa} + \mathbf{ab}$$

$\mathbf{v}_{ao}$ is fully known and after taking this vector, draw lines for $\mathbf{v}_{bq}$ and $\mathbf{v}_{ab}$ (or $\mathbf{v}_{ba}$) and locate the point **b**. obviously, the direction-sense of $\mathbf{v}_{ab}$ is opposite to that of $\mathbf{v}_{ba}$. Figure 2.22 (b) shows the solution of the first equation.

$$\omega_{qr} = \omega_{qb} = \frac{v_{qb} \text{ or } v_{bq}}{BQ}$$

$$= \frac{2.55}{0.46} \quad (BQ = 0.46 \text{ m on measuring})$$

$$= \underline{5.54 \text{ rad/s}} \text{ counter – clockwise}$$

Sliding velocity of block = v_{ba} or **ab** = 5.45 m/s

Example 2.14 *For the position of the mechanism shown in Fig. 2.23(a), calculate the angular velocity of the link AR. OA is 300 mm long and rotates at 20 rad/s in the clockwise direction. OQ = 650 mm and QOA = 40°*

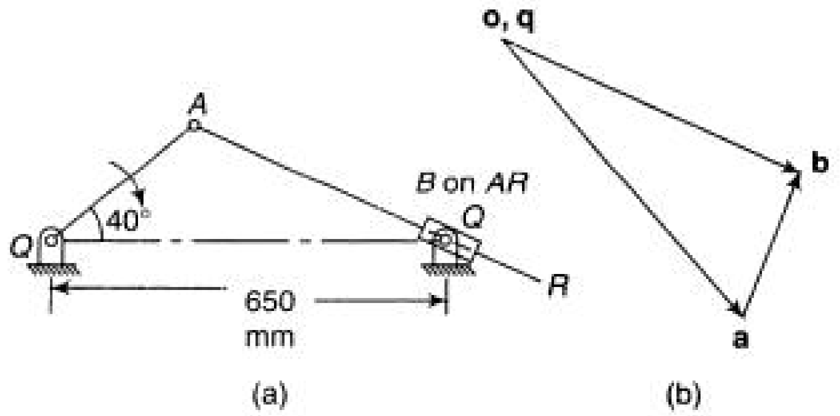


Fig. 2.23

Solution $v_{ao} = 20 \times 0.3 = 6$ m/s

Writing the vector equation,

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao} \quad \text{or} \quad \mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq}$$

Solving the first one,

$$\mathbf{v}_{bq} = \mathbf{v}_{ao} + \mathbf{v}_{ba}$$

or **qb = oa + ab**

Take $\mathbf{v}_{ao}$ to a convenient scale [Fig. 2.23(b)].

$\mathbf{v}_{ba}$ is $\perp AB$, draw a line $\perp AB$ through **a**;

$\mathbf{v}_{bq}$ is along AB, draw a line || to AB through **q**.

The intersection locates the point **b**.

$$\omega_{ar} = \omega_{ab} = \frac{v_{ab} \text{ or } v_{ba}}{AB} = \frac{2.55}{0.46}$$

= $\underline{5.54 \text{ rad/s}}$ counter–clockwise

Example 2.15 *In the pump mechanism shown in Fig. 2.24(a), OA = 320 mm, AC = 680 mm and OQ = 650 mm. For the given configuration, determine the*

(i) angular velocity of the cylinder

(ii) sliding velocity of the plunger

(iii) absolute velocity of the plunger

The crank OA rotates at 20 rad/s clockwise.

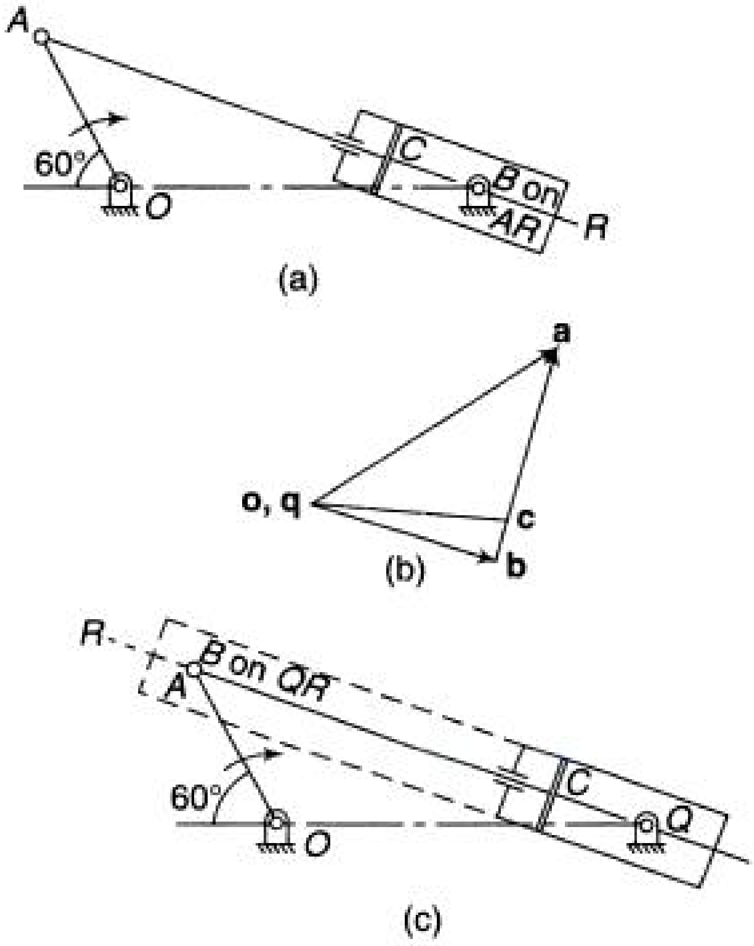


Fig. 2.24

Solution $v_{ao} = 0.32 \times 20 = 6.4$ m/s

Method I Produce AC to R. Line AC passes through the pivot Q. Let B be a point on AR beneath Q.

Writing the vector equation,

$$\mathbf{v}_{bo} = \mathbf{v}_{ba} + \mathbf{v}_{ao} \quad \text{or} \quad \mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq}$$

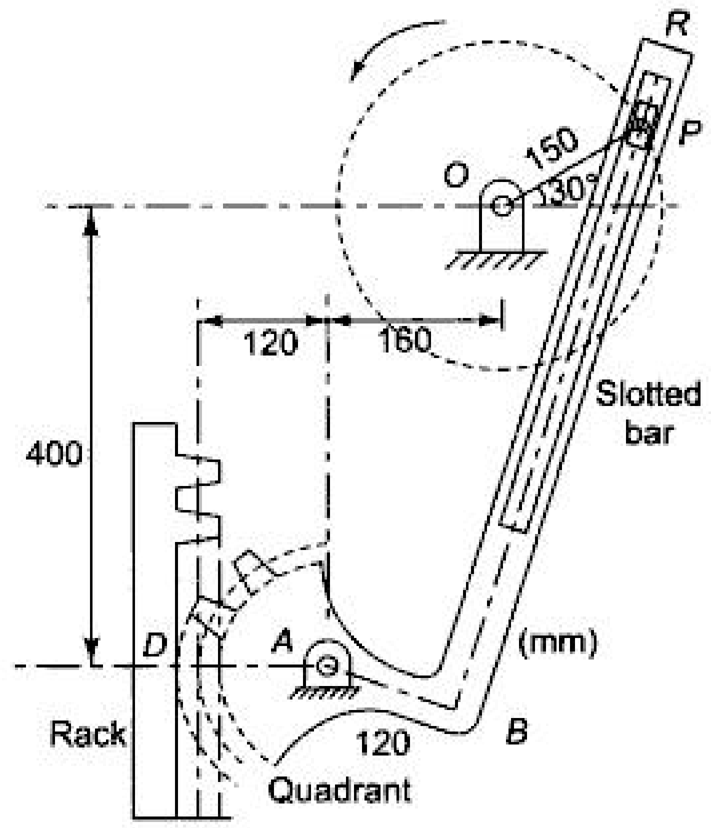
Solving any of these equations leads to same velocity diagram except for the direction of $\mathbf{v}_{ba}$ and $\mathbf{v}_{ab}$.

Taking the latter equation,

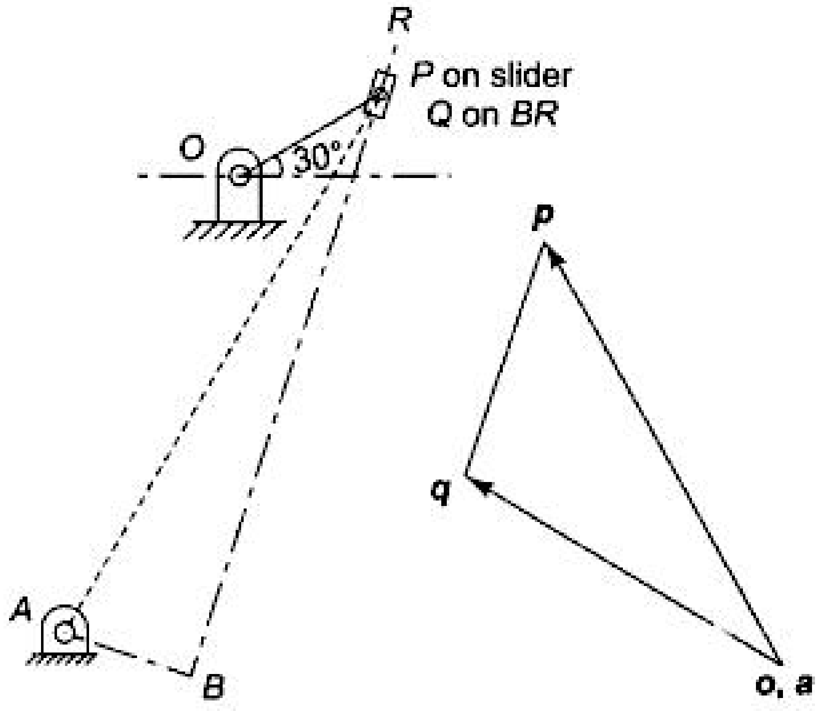
$$\mathbf{v}_{aq} = \mathbf{v}_{ab} + \mathbf{v}_{bq}$$

or

$$\mathbf{v}_{ao} = \mathbf{v}_{bq} + \mathbf{v}_{ab}$$



(a)



(b) (c)

Fig. 2.26

Solution $\omega_{po} = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 210}{60} = 22$ rad/s

$$v_{po} = 22 \times 0.15 = 3.3 \text{ m/s}$$

Draw the configuration diagram to a suitable scale [Fig. 2.26(b)].

Locate a point Q on BR beneath point P on the slider.

Then the vector equation is

$$\mathbf{v}_{qo} = \mathbf{v}_{qp} + \mathbf{v}_{po} \quad \text{or} \quad \mathbf{v}_{qa} = \mathbf{v}_{po} + \mathbf{v}_{qp}$$

Take the vector $\mathbf{v}_{po}$ to a convenient scale in the proper direction and sense [Fig. 2.26(c)].

$\mathbf{v}_{qp}$ is along BR, draw a line parallel to BR through **p**;

Now, Q is a point on the link ABR which is pivoted at point A. The direction of velocity of any point on the link is perpendicular to the line joining that point with the pivoted point A.

$\mathbf{v}_{qa}$ is $\perp QA$, draw a line $\perp QA$ through **a**;

The intersection of the two lines locates the point **q**.

Now angular velocity of the quadrant and the lever ABQ,

$$\omega_{aq} = \frac{v_{aq}}{AQ} = \frac{2.5}{0.577} = 4.33 \text{ rad/s}$$

counter-clockwise

(i) The linear velocity of the rack will be equal to the tangential velocity of the quadrant at the teeth, i.e.,

$$v_r = \omega \times AD = \omega \times 120 = 4.33 \times 120 = \underline{519.6 \text{ mm/s}}$$

(ii) The reciprocating rack changes the direction when the crank OP assumes a position such that the tangent at P to the circle at O is also a tangent to the circle at A with radius AB as shown in Fig. 2.27. The rack is lowered during the rotation of the crank from P to P' and is raised when P' moves to P counter-clockwise.

Thus,

$$\frac{\text{Time of lowering}}{\text{Time of raising}} = \frac{\theta}{\beta} = \frac{215^\circ}{135^\circ} = 1.59$$

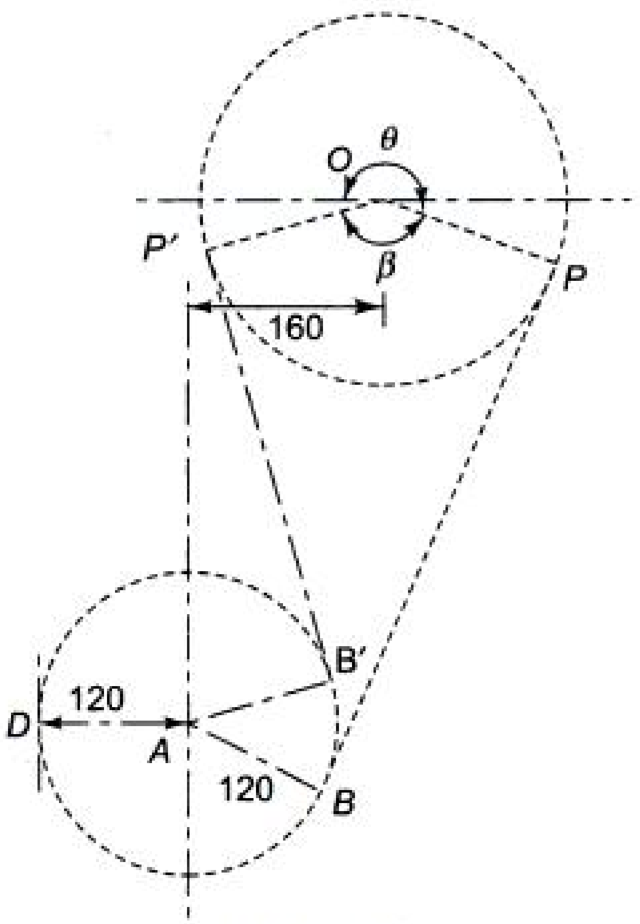


Fig. 2.27

(iii) Stroke of the rack = angular displacement of the quadrant × its radius

= angle $BAB' \times AB$

$$= 44 \times \frac{\pi}{180} \times 120 = 92.2 \text{ mm}$$

($\angle BAB' = 44°$ by measurement)

Example 2.18 *In the swiveling-joint mechanism shown in Fig. 2.28(a), AB is the driving crank rotating at 300 rpm clockwise.*

The lengths of the various links are

AD = 650 mm, AB = 100 mm, BC = 800 mm, DC = 250 mm, BE = CE, EF = 400 mm, OF = 240 mm, FS = 400 mm

For the given configuration of the mechanism, determine the

(i) velocity of the slider block S

(ii) angular velocity of the link EF

(iii) velocity of the link EF in the swivel block

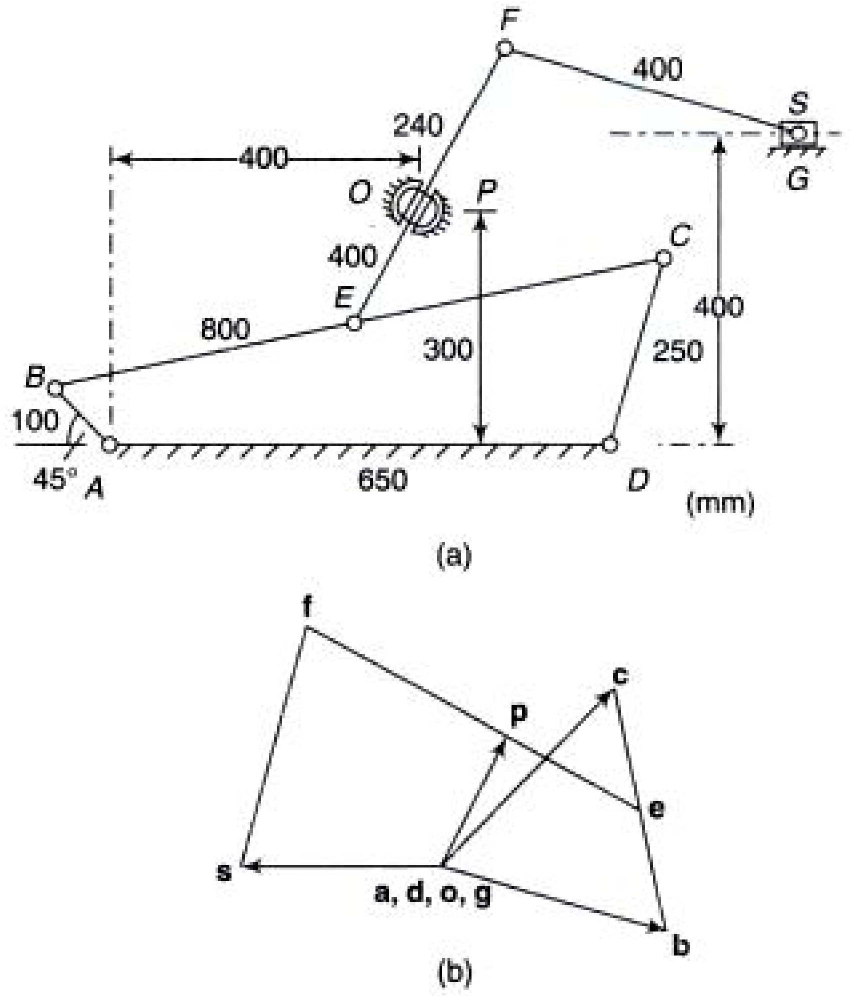


Fig. 2.28

Solution $$\omega_{ba} = \frac{2\pi \times 300}{60} = 31.4 \text{ rad/s}$$

$$v_b = 31.4 \times 0.1 = 3.14 \text{ m/s}$$

The velocity diagram is completed as follows:

- Draw the velocity diagram of the four-link mechanism *ABCD* as usual starting with the vector **ab** as shown in Fig. 2.28(b).
- Locate the point **e** in the velocity diagram at the midpoint of **bc** as the point *E* is the midpoint of *BC*. Let *Q* be a point on the link *EF* at the joint *O*. Draw a line ⊥ *EQ* through **e**, a point on which will represent the velocity of *Q* relative to *E*.
- The sliding velocity of link *EF* in the joint at the instant is along the link. Draw a line parallel to *EF* through **o**, the intersection of which with the previous line locates the point **q**.
- Extend the vector **eq** to **f** such that **ef/eq** = *EF/EQ*.

- Through **f** draw a line $\perp FS$, and through **g** a line parallel to line of stroke of the slider. The intersection of the two lines locates the point **s**.

Thus, the velocity diagram is completed.

(i) The velocity of slider S = **gs** = 2.6 m/s

(ii) The angular velocity of the link EF

$$= \frac{v_{fe}}{EF} = \frac{\mathbf{ef}}{EF} = \frac{4.9}{0.4} = 12.25 \text{ rad/s (ccw)}$$

(iii) The velocity of the link EF in the swivel block = **oq** =1.85 m/s

2.11 ALGEBRAIC METHODS

Vector Approach

In Sec. 2.10, the concept of coincident points was introduced. However, complex algebraic methods provide an alternative formulation for the kinematic problems. This also furnishes an excellent means of obtaining still more insight into the meaning of the term *coincident points*.

Let there be a plane moving body having its motion relative to a fixed coordinate system xyz (Fig. 2.29). Also, let a moving coordinate system $x'y'z'$ be attached to this moving body. Coordinates of the origin A of the moving system are known relative to the absolute reference system. Assume that the moving system has an angular velocity ω also.

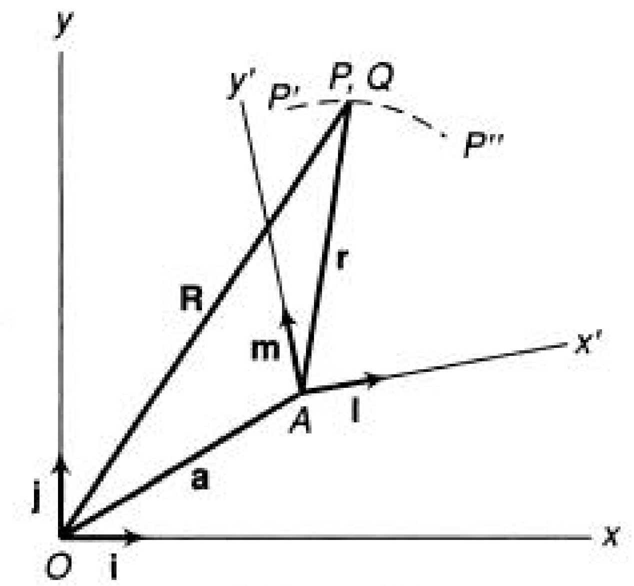


Fig. 2.29

Let

i,j,k	unit vectors for the absolute system
l,m,n	unit vectors for the moving system
ω	angular velocity of rotation of the moving system
R	vector relative to fixed system
r	vector relative to moving system

Let a point P move along path $P'PP''$ relative to the moving coordinate system $x'y'z'$. At any instant, the position of P relative to the fixed system is given by the equation

$$\mathbf{R} = \mathbf{a} + \mathbf{r} \qquad \text{(i)}$$

where $\mathbf{r} = x'\mathbf{l} + y'\mathbf{m} + z'\mathbf{n}$

Thus, (i) may be written as, $\mathbf{R} = \mathbf{a} + x'\mathbf{l} + y'\mathbf{m} + z'\mathbf{n}$

Taking the derivatives with respect to time to find the velocity,

$$\dot{\mathbf{R}} = \dot{\mathbf{a}} + (\dot{x}'\mathbf{l} + \dot{y}'\mathbf{m} + \dot{z}'\mathbf{n}) + (x'\dot{\mathbf{l}} + y'\dot{\mathbf{m}} + z'\dot{\mathbf{n}}) \qquad \text{(ii)}$$

The first term in this equation indicates the velocity of the origin of the moving system. The second term refers to the velocity of P relative to the moving system. The third term is due to the fact that the reference system has also rotary motion with angular velocity ω.

Also, $\dot{\mathbf{l}} = \omega \times \mathbf{l}, \quad \dot{\mathbf{m}} = \omega \times \mathbf{m}, \quad \dot{\mathbf{n}} = \omega \times \mathbf{n}$

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Solution The slider-crank mechanism is shown in Fig. 2.38(a). Name the four links as 1, 2, 3 and 4. Locate the various I-centres as follows:

(i) Locate I-centres 12, 23 and 34 by inspection. They are at the pivots joining the respective links. As the line of stroke of the slider is horizontal, the I-centre 14 lies vertically upwards or downwards at infinity as shown in Fig. 2.38(b).

(ii) Take four points in the form of a square and mark them as 1, 2, 3 and 4. Join 12, 23, 34 and 14 by firm lines as these have been located by inspection.

(iii) I-centre 24 lies at the intersection of lines joining the I-centres 12, 14 and 23, 34 by Kennedy's theorem. Joining of 12 and 14 means a vertical line through 12. This I-centre can be shown in the square by a dotted line to indicate that this has been located by inspection.

(iv) I-centre 13 lies at the intersection of lines joining the I-centres 12, 23 and 14, 34. Joining of 34 and 14 means a vertical line through 34. Show this I-centre in the square by a dotted line.

Thus, all the I-centres are located.

As velocity of the link 2 is known and the velocity of the link 4 is to be found, consider the I-centre 24. The point 24 has the same velocity whether it is assumed to lie in link 2 or 4. First, assume 24 to lie on the link 2 which rotates at angular velocity of 40 rad/s.

Linear velocity of I-centre 24 = 40 × (12–24) = 40 × 0.123

= 4.92 m/s in the horizontal direction

Now, when this point is assumed in the link 4, it will have the same velocity which means the linear velocity of the slider is the same as of the point 24.

Thus, linear velocity of the slider = <u>4.92 m/s</u>

Example 2.20 *Figure 2.39(a) shows a six-link mechanism. The dimensions of the links are OA = 100 mm, AB = 580 mm, BC = 300 mm, QC = 100 mm and CD = 350 mm. The crank OA rotates clockwise at 150 rpm. For the position when the crank OA makes an angle of 30° with the horizontal, determine the*

(i) linear velocities of the pivot points B, C and D

(ii) angular velocities of the links AB, BC and CD

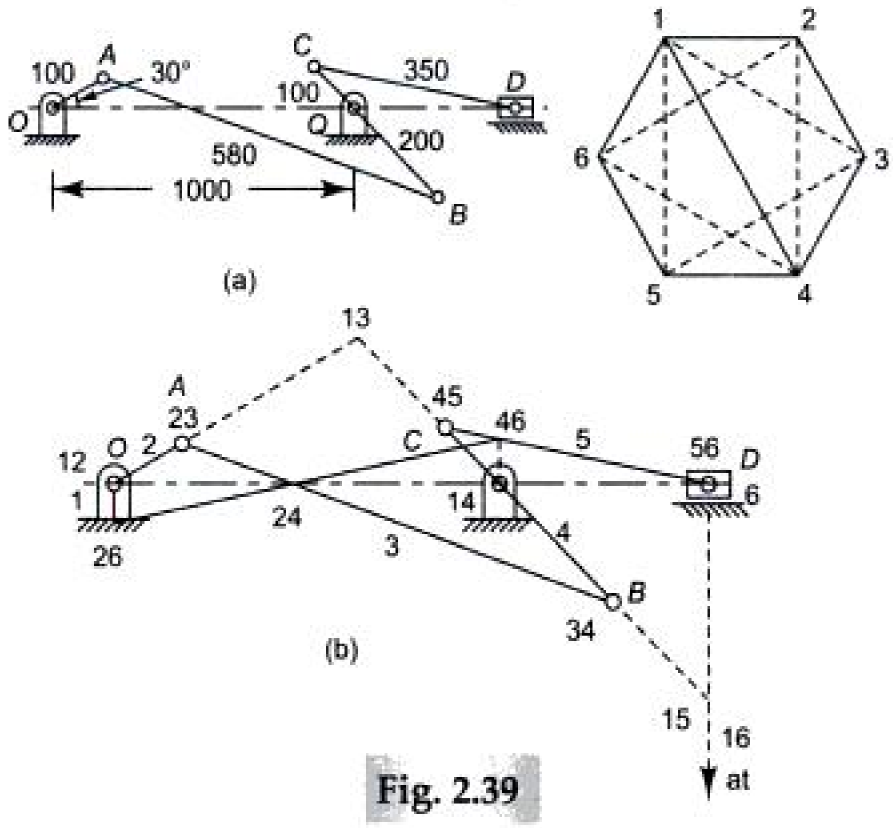


Fig. 2.39

Solution $\omega_2 = \dfrac{2\pi \times 150}{60} = 15.7$ rad/s

$$v_a = \omega_2 .OA = 15.7 \times 0.1 = 1.57 \text{ m/s}$$

Locate I-centres 12, 23, 34, 45, 56, 16 and 14 by inspection.

- Locate 13 which lies on the intersection of 12–23 and 14–34 [Fig. 2.39(b)]
- Locate 15 which lies on the intersection of 14–45 and 56–16 (16 is at ∞)

(i) Now, at the instance, the link 3 rotates about the I-centre 13.

$$\text{Thus, } \frac{v_b}{v_a} = \frac{13-34}{13-23} \text{ or } v_b = \frac{453}{265} \times 1.57 = 2.66 \text{ m/s}$$

$$\text{and } \frac{v_c}{v_b} = \frac{QC}{QB} \text{ or } v_c = \frac{100}{200} \times 2.66 = 1.33 \text{ m/s}$$

At the instance, the link 5 rotates about the I-centre 15.

Thus, $\frac{v_d}{v_c} = \frac{15-56}{15-45}$ or

$$v_d = \frac{300}{506} \times 1.33 = 0.788 \quad \text{m/s}$$

(ii) $\omega_{ab} = \frac{v_a}{13-A} = \frac{1.57}{0.267} = 5.88$ rad/s

$$\omega_{bc} = \frac{v_b}{BQ} = \frac{2.66}{0.2} = 13.3 \text{ rad/s}$$

$$\omega_{cd} = \frac{v_c}{15-c} = \frac{1.33}{0.499} = 2.66 \text{ rad/s}$$

* In case it is required to find the velocity of the slider *D* only, then as the velocity of a point *A* on the link 2 is known and the velocity of a point on the link 6 is to be found, locate the I-centre 26 as follows:

- Locate 24 which lies on the intersection of 21–14 and 23–34.
- Locate 46 which lies on the intersection of 45–56 and 14–16 (16 is at ∞).
- Locate 26 which is the intersection of 24–46 and 21–16.

First, imagine the link 2 to be in the form of a flat disc containing the point 26 and revolving about *O* with an angular velocity of 15.7 rad/s.

Then, $v_{26} = \omega_2 \times (12\text{–}26) = 15.7 \times 50 = 785$ mm/s or 0.785 m/s

The velocity of the point 26 is in the horizontally left direction if *OA* rotates clockwise.

Now, imagine the link 6 (slider) to be large enough to contain the point 26. The slider can have motion in the horizontal direction only and the velocity of a point 26 on it is known; it implies that all the points on the slider move with the same velocity.

Thus, velocity of the slider, $v_d = v_{26} = \underline{785 \text{ mm/s}}$

Example 2.21 *Figure 2.40(a) shows a six-link mechanism. The dimensions of the links are OA = 100 mm, AB = 450 mm, BD = 200 mm, QB = 400 mm, DE = 200 mm, CE = 200 mm. Find the angular velocity of the link CE by the instantaneous centre method if the link OA rotates at 20 rad/s.*

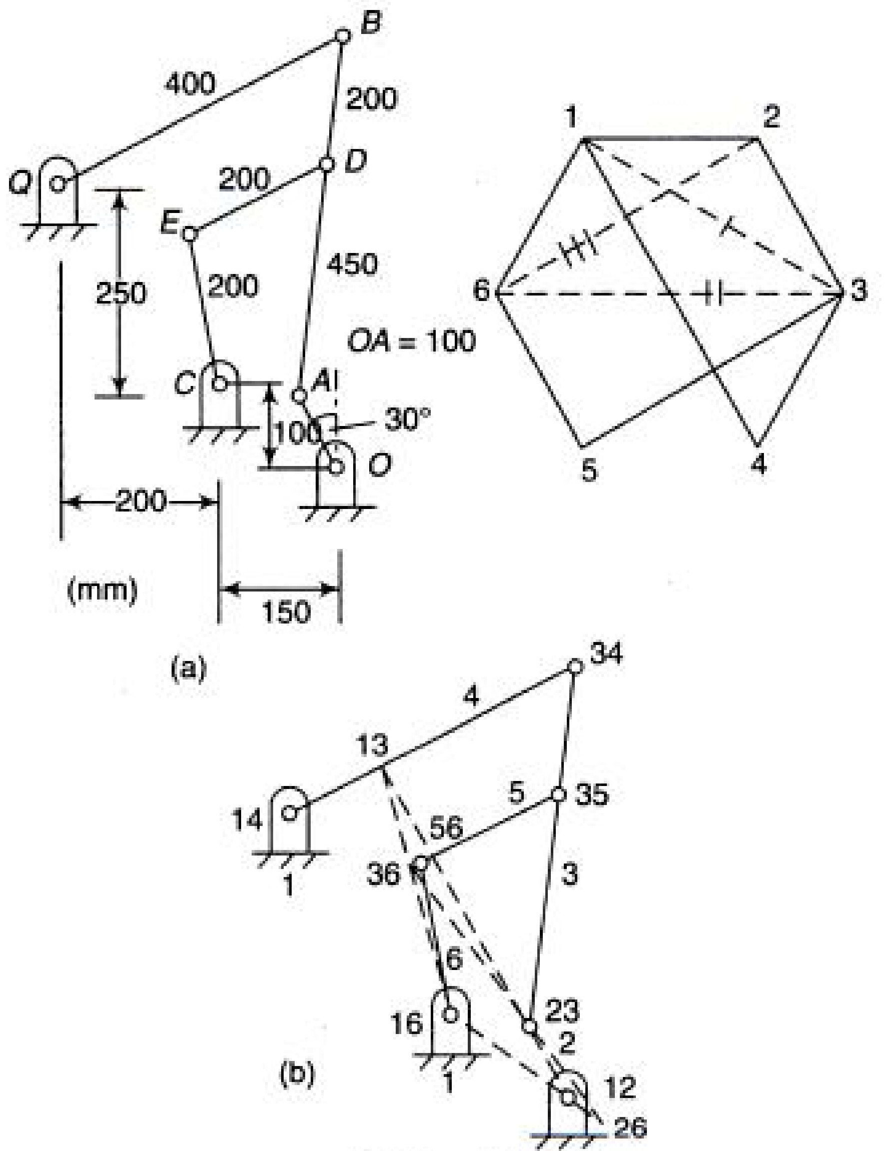


Fig. 2.40

Solution Name the six links by numbers as 1, 2, 3, 4, 5 and 6 as shown in Fig. 2.40(b).

The velocity of the link *OA* (2) is known and the velocity of the link *CE* (6) is to be found. Therefore, the I-centre 26 is required to be located.

- First mark the I-centres which can be located by inspection. They are 12, 23, 34, 56 16 and 14.
- Locate the I-centre 13 which is at the intersection of lines joining I-centres 12, 23 and 16, 36. Similarly, Locate the I-centre 36 which is at the intersection of lines joining I-centres 13,16 and 35, 56.
- Locate the I-centre 26 at the intersection of lines joining I-centres 12, 16 and 23, 36.
- Now, as the velocity of the I-centre 26 is the same whether it is considered to lie on link 2 or 6,

intersection of lines joining I-centres 12, 16 and 25, 56.

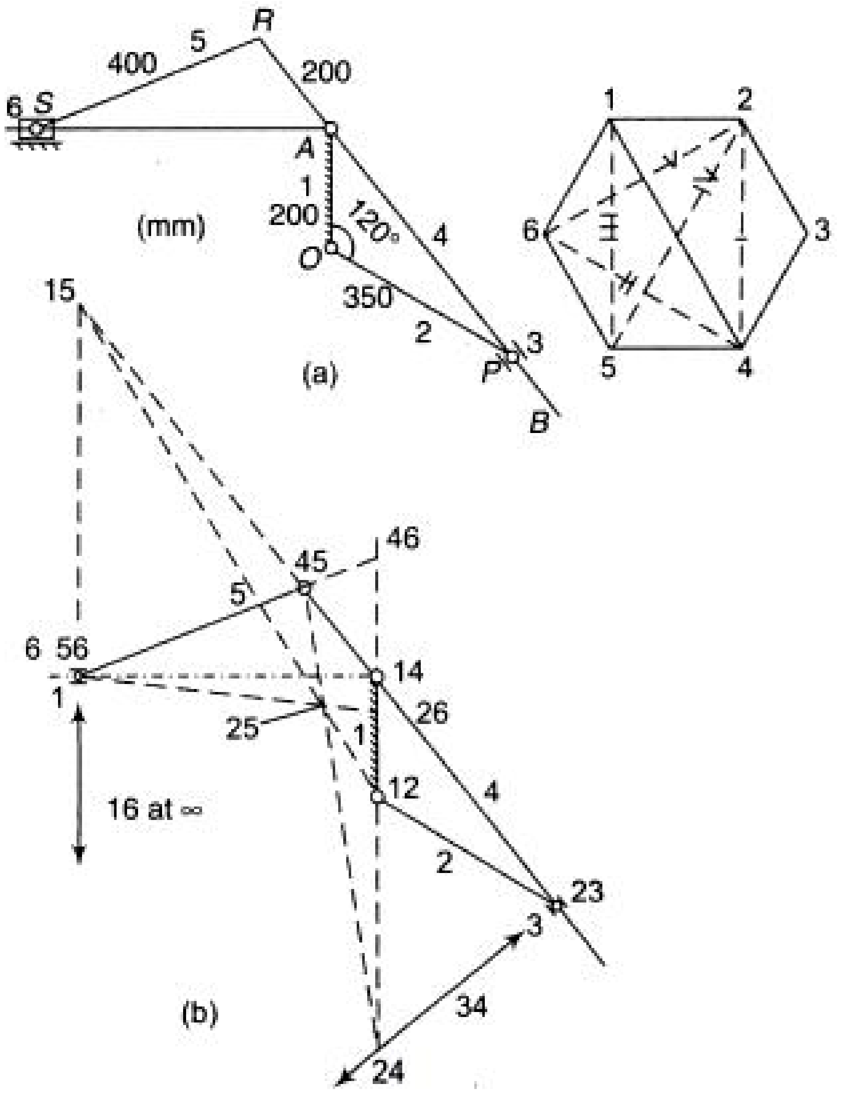


Fig. 2.42

Now, as the velocity of the I-centre 26 is the same whether it is considered to lie on the link 2 or 6,

$v_{26} = \omega_2 .(12–26) = v_s$

or $v_s = \omega_2 .(12–26) = 10 \times 0.137 = 1.37$ m/s

Example 2.24 *Solve Example 2.4 by the instantaneous centre method.*

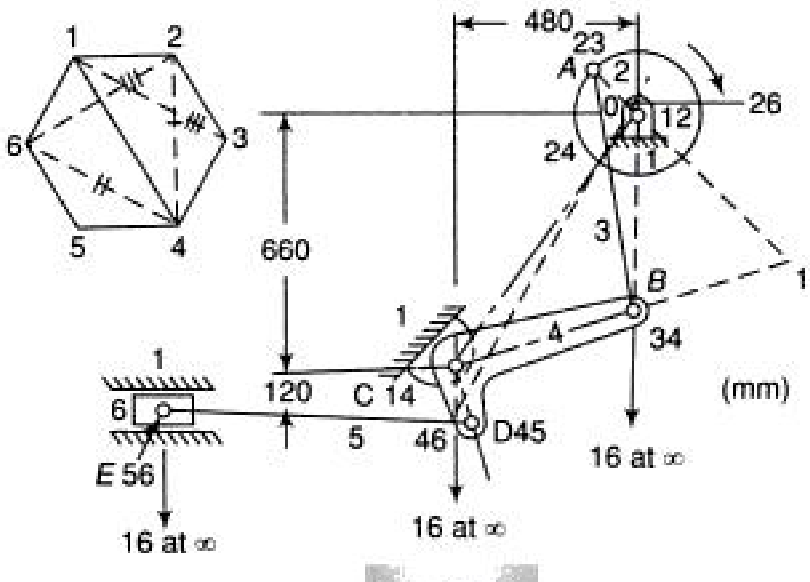


Fig. 2.43

Solution Draw the configuration to a suitable scale as shown in Fig. 2.43.

(a) To find the velocity of E or the link 6, it is required to locate the I-centre 26 as the velocity of a point A on the link 2 is known. After locating I-centres by inspection, locate I-centres 24, 46 and 26 by Kennedy's theorem.

First consider 26 to be on the crank 2.

$v_{26} = \omega(12–26) = 16.76 \times 0.032 = 0.536$ m/s (horizontal)

When the point 26 is considered on the link 6, all points on it will have the same velocity as the point 26.

Velocity of the crosshead = <u>0.536 m/s</u>

(b) (i) To find the velocity of rubbing at A (or 23), ω_2 and ω_3 are required.

Locate I-centre 13. Then

$$\omega_3(23–13) = \omega_2 (23–12)$$

$$\therefore \quad \omega_3 = 16.76 \times \frac{0.17}{0.756} = \underline{3.77 \text{rad/s}}$$

ω_3 is clockwise as 13 and 12 lie on the same side of 23.

Velocity of rubbing at $A = (\omega_3 - \omega_2)\, r_a$

$$= (16.76 - 3.77) \times \frac{0.04}{2} = \underline{0.26 \text{ m/s}}$$

(ii) For velocity of rubbing at B, ω_2 and ω_4 are required. ω_3 was calculated above.

$$\omega_4(34 - 14) = \omega_3(34 - 13)$$

$$\omega_4 = 3.77 \times \frac{0.45}{0.51} = 3.33 \text{ rad/s}$$

ω_4 is counter-clockwise as 14 and 13 lie on the opposite sides of 34 and ω_3 is clockwise.

Thus, velocity of rubbing at B can be calculated.

(iii) As ω_4 is known, the velocity of rubbing at C can be known.

Similarly, locate the I-centre 15 and obtain ω_5 from the relation,

$$\omega_5 = \omega_4 \left(\frac{45-14}{45-15} \right) \text{ and determine the}$$

velocity of rubbing at *D*.

Torque is determined in the same way as in Example 2.3.

Example 2.25 *The configuration diagram of a wrapping machine is given in Fig. 2.44(a). Determine the velocity of the point P on the bell-crank lever DCP if the crank OA rotates at 80 rad/s.*

Solution ω_2 is known, ω_6 is required.

Locate the I-centre 26 by first finding the 13 and 16 by Kennedy's theorem. [Fig. 2.44(b)].

Then $\omega_6(26-16) = \omega_2(26-12)$

$$\omega_6 = \omega_2 \times \frac{26-12}{26-16} = 80 \times \frac{47}{383} = 9.82 \text{ rad/s}$$

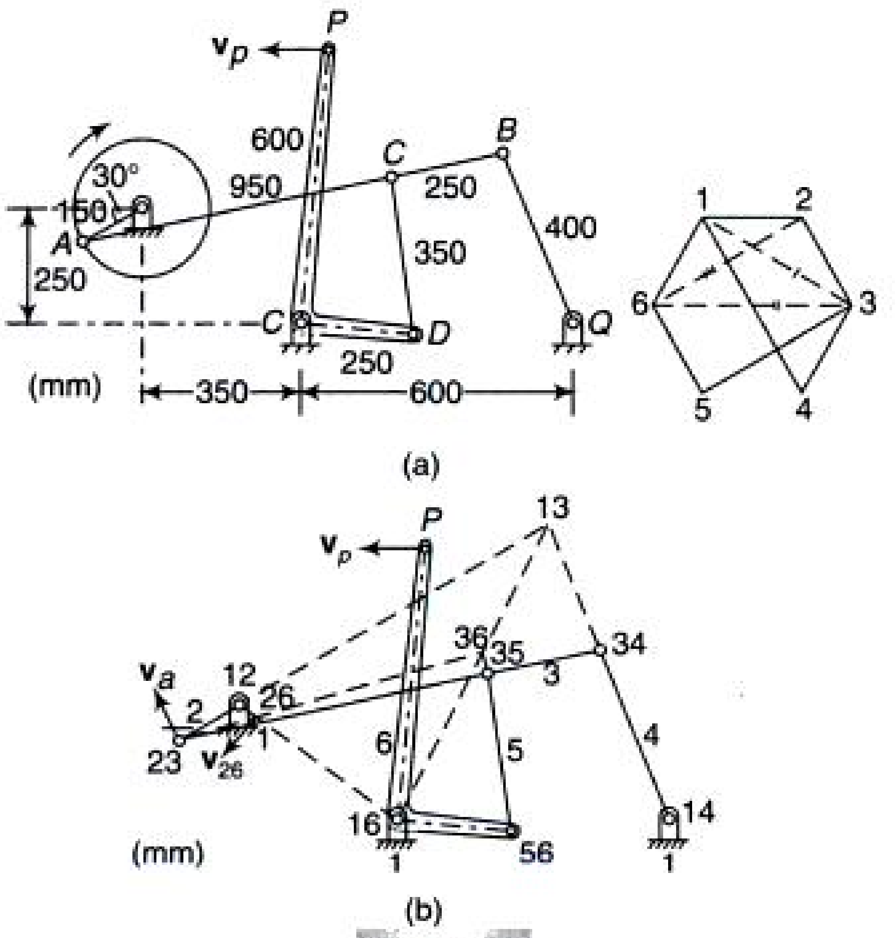


Fig. 2.44

It is counter-clockwise as 16 and 12 lie on the opposite sides of 26 and ω_2 is clockwise.

Thus $v_p = \omega_6 \times (16 - P) = 9.82 \times 600 =$ <u>5.89 m/s</u>

Example 2.26 *Figure 2.45(a) shows the mechanism of a sewing machine needle box. For the given configuration, find the velocity of the needle fixed to the slider D when the crank OA rotates at 40 rad/s.*

Solution Locate the I-centre 26 (Fig. 2.45b).

Consider 26 to lie on the link 2.

$v_{26} = \omega_2 \times (12 - 26) = 40 \times 22.4 = 896$ mm/s vertically downwards.

Consider 26 to lie on the link 6.

Velocity of needle = Velocity of slider = v_{26} = <u>896 mm/s</u>

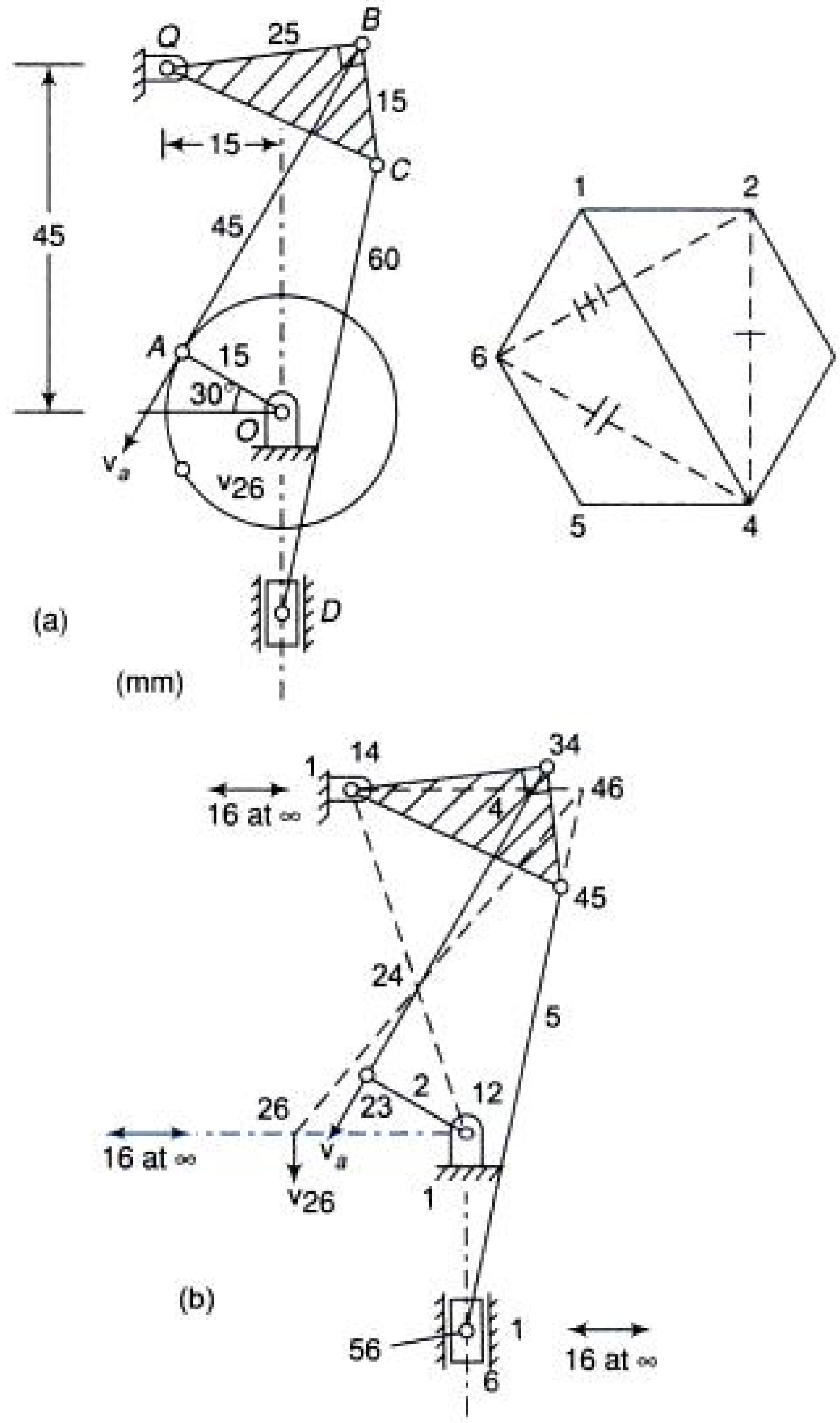


Fig. 2.45

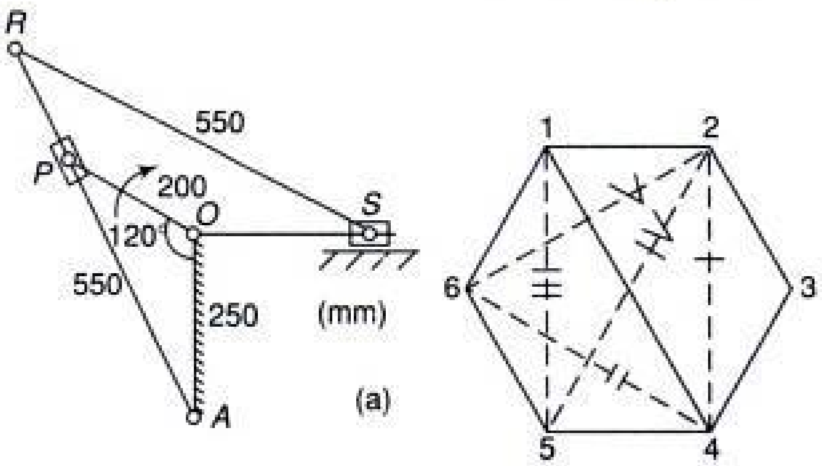
Example 2.27 *Figure 2.46(a) represents a shaper mechanism. For the given configuration, what will be the velocity of the cutting tool at S and the angular velocities of the links AR and RS. Crank OP rotates at 10 rad/s.*

Solution Locate the I-centre 26 [Fig. 2.46(b)]

(i) $v_6 = v_{26} = \omega_2 \times (12 - 26) = 10 \times 0.166 = 1.66$ m/s

(ii) $\omega_4 = \omega_2 \left(\dfrac{24-12}{24-14}\right) = 10 \times \left(\dfrac{183}{430}\right)$

$= 4.25$ rad/s (clockwise)



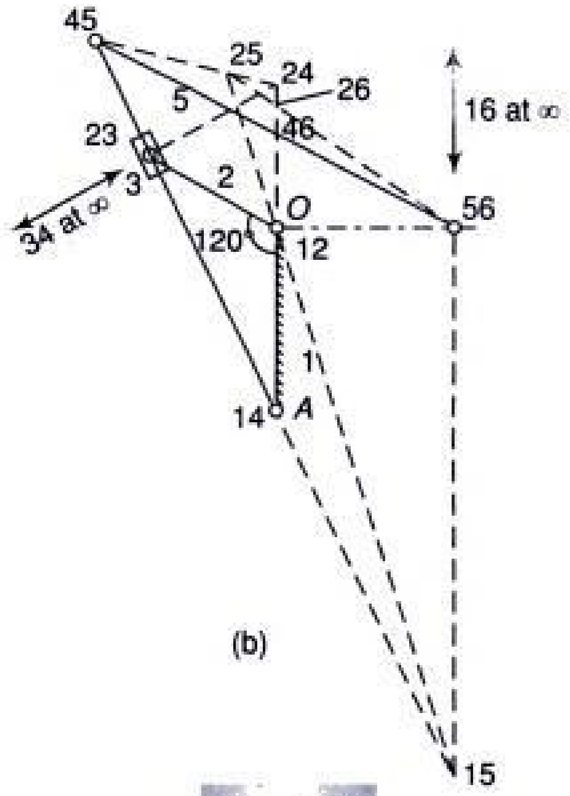


Fig. 2.46

Similarly,

$$\omega_5 = \omega_2 \left(\frac{25-12}{25-15}\right) = 10 \times \left(\frac{210}{1060}\right)$$

$= 1.98$ rad/s (clockwise)

or

$$\omega_5 = \omega_4 \left(\frac{45-14}{45-15}\right) = 4.25 \left(\frac{552}{1187}\right) =$$

1.97 rad/s (clockwise)

2.16 CENTRODE

An I-centre is defined only for an instant and changes as the mechanism moves. A *centrode* is the locus of the I-centre of a plane body relative to another plane body for the range of motion specified or during a finite period of time.

There are two types of centrodes:

1. Space Centrode (or Fixed Centrode) of a Moving Body

It is the locus of the I-centre of the moving body relative to the fixed body.

2. Body Centrode (or Moving Centrode) of a Moving Body

It is the locus of the I-centre of the fixed body relative to the movable body, i.e., the locus of the

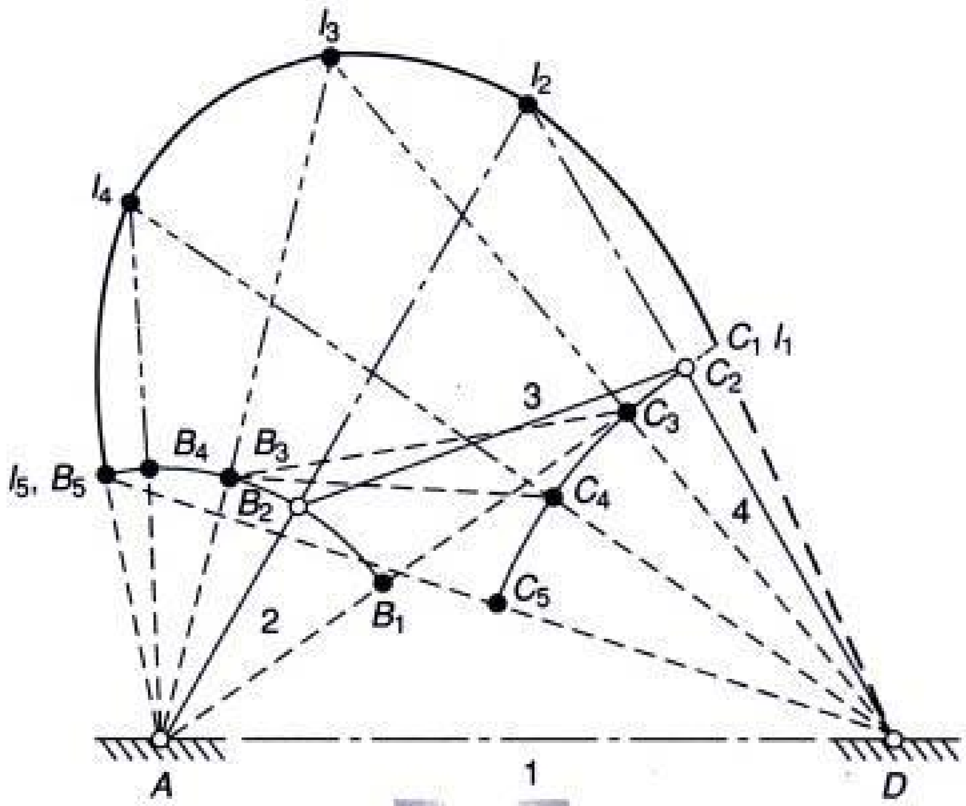


Fig. 2.47

I-centre assuming the movable body to be fixed and the fixed body to be movable.

In a four-link mechanism shown in Fig. 2.47, the link 1 is fixed. The locus of the I-centre of links 1 and 3 over a range of motion of the link 3 is the space centrode. Five positions of the I-centre, i.e., I_1, I_2, I_3, I_4 and I_5 have been obtained and joined with a smooth curve which is the space centrode. If the link 3 is assumed to be fixed and 1 movable, the locus of the I-centre of 1 and 3 is the body centrode. This has been shown in Fig. 2.43 for five positions of the link 1.

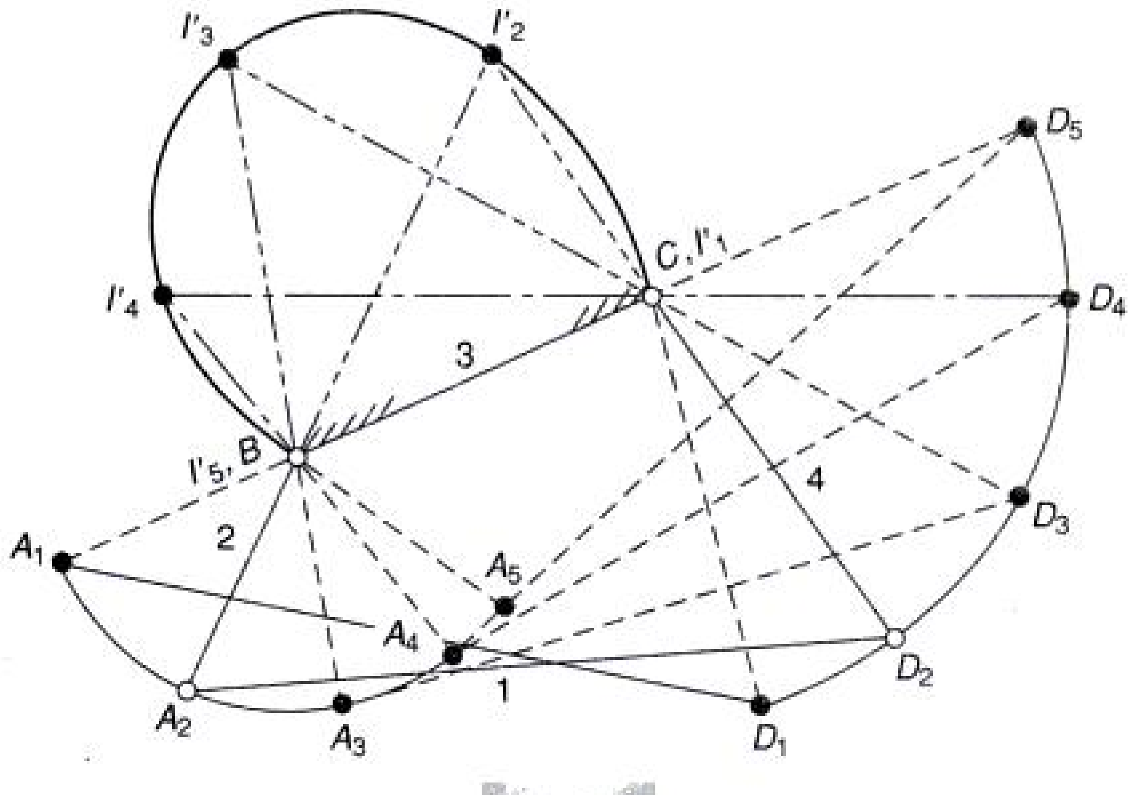


Fig. 2.48

Comparing Figs 2.47 and 2.48, observe that the first position AB_1C_1 (I_1) D of Fig. 2.47 is exactly similar to the first position A_1BC $(I_1')D_1$ of Fig. 2.48. The second positions of the two figures are also similar. Similarly, the third position $AB_3I_3C_3D$ of Fig. 2.47 is exactly similar to the third position $A_3BI_3'CD_3$ of Fig. 2.48, and so on. Thus, Δs $B_2C_2I_2$, $B_3C_3I_3$ and $B_4C_4I_4$ are similar to Δs BCI_2', BCI_3' and BCI_4' respectively. This implies that the positions of the I-centre of Fig. 2.43 can be obtained directly by constructing on B_2C_2 Δs, similar to $B_2C_2I_2$ (already exists), $B_3C_3I_3$ and $B_4C_4I_4$. I_1' lies on C_2 and I_5' on B_2.

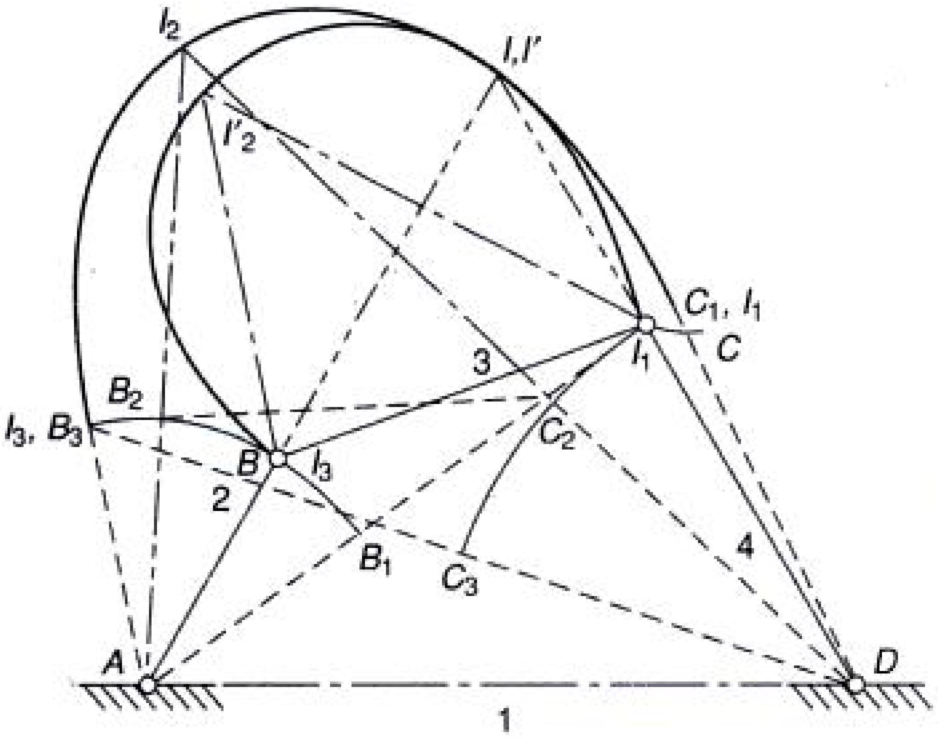


Fig. 2.49

In Fig. 2.49, space and body centrodes of the link 3 relative to 1 have been obtained in the same diagram considering four positions of the link 3.

Figure 2.50 shows the fixed centode (attached to the fixed link 1) and the moving centrode (attached to the moving link 3) with links 2 and 4 removed entirely. Now, if the moving centrode is made to roll on the fixed centrode without slip, the coupler link 3 will exactly traverse the same motion as it had in the original mechanism. This is because a point of rolling contact is always an I-centre in different positions of the link 3.

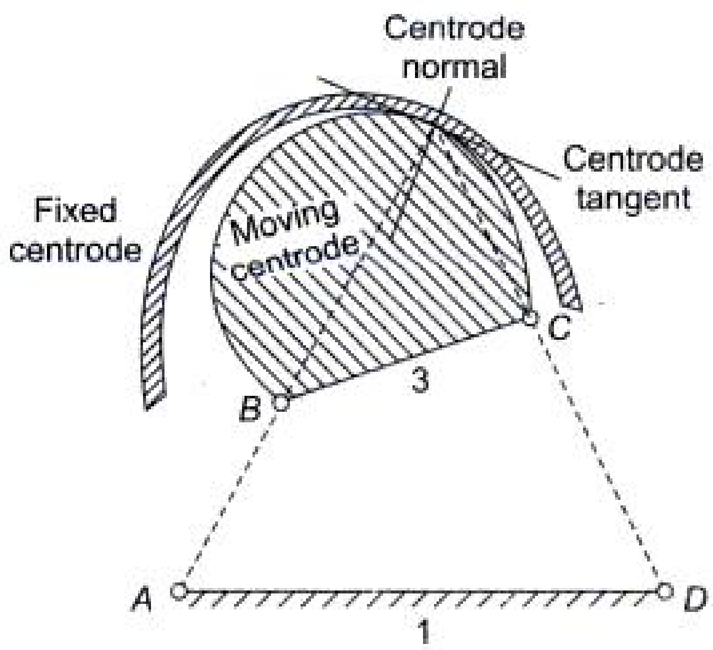


Fig. 2.50

Thus, *the plane motion of a rigid body relative to another rigid body is equivalent to the rolling motion of one centrode on the other.*

The instant point of rolling contact is the instantaneous centre. The common tangent and the common normal to the two centrodes are known as the *centrode tangent* and the *centrode normal* respectively.

Summary

1. A machine or a mechanism, represented by a skeleton or a line diagram, is commonly knows as a *configuration diagram*.
2. Velocity is the derivative of displacement with respect to time and is proportional to the slope of the tangent to the displacement–time curve at any instant.
3. A vector is a line which represents a vector quantity such as force, velocity and acceleration.
4. The magnitude of the instantaneous linear velocity of a point on a rotating body is proportional to its distance from the axis of rotation.
5. The velocity of an intermediate point on any of the links can be found easily by dividing the corresponding velocity vector in the same ratio as the point divides the link.
6. *Velocity images* are found to be very helpful devices in the velocity analysis of complicated linkages. The order of the letters in the velocity image is the same as in the configuration diagram.
7. The angular velocity of a link about one extremity is the same as the angular velocity about the other.
8. The *instantaneous centre of rotation* of a body relative to another body is the centre about which the body rotates at the instant.
9. In a mechanism, the number of I-centres is given by $N = n(n - 1)/2$
10. If three plane bodies have relative motion among themselves, their I-centres must lie on a straight line. This is known as *Kennedy's theorem*.
11. When the angular velocity of a link is known and it is required to find the angular velocity of another link, locate their common I-centre. The velocity of this I-centre relative to a fixed third link is the same whether the I-centre is considered on the first or the second link.
12. A *centrode* is the locus of the I-centre of a plane body relative to another plane body for the range of motion specified or during a finite period of time.
13. The plane motion of a rigid body relative to another rigid body is equivalent to the rolling motion of one centrode on the other.

Exercises

1. What is a configuration diagram? What is its use?
2. Describe the procedure to construct the diagram of a four-link mechanism.
3. What is a velocity image? State why it is known as a helpful device in the velocity analysis of complicated linkages.
4. What is velocity of rubbing? How is it found?
5. What do you mean by the term 'coincident points'?
6. What is *instantaneous centre of rotation?* How do you know the number of *instantaneous centres* in a mechanism?
7. State and prove Kennedy's theorem as applicable to instantaneous centres of rotation of three bodies. How is it helpful in locating various instantaneous centres of a mechanism?
8. State and explain *angular-velocity-ratio theorem* as applicable to mechanisms.
9. What do you mean by *centrode* of a body? What are its types?
10. What are fixed centrode and moving centrode? Explain.
11. Show that the plane motion of a rigid body relative to another rigid body is equivalent to the rolling motion of one centrode on the other.
12. In a slider-crank mechanism, the stroke of the slider is one-half the length of the connecting rod. Draw a diagram to give the velocity of the slider at any instant assuming the crankshaft to turn uniformly.
13. In a four-link mechanism, the crank *AB* rotates at 36 rad/s. The lengths of the links are *AB* = 200 mm, *BC* = 400 mm, *CD* = 450 mm and *AD* = 600 mm. *AD* is the fixed link. At the instant when *AB* is at right angles to *AD*, determine the velocity of
    (i) the midpoint of link *BC*
    (ii) a point on the link *CD*, 100 mm from the pin connecting the links *CD* and *AD*.
    (6.55 m/s; 1.45 m/s)
14. For the mechanism shown in Fig. 2.51, determine the velocities of the points *C*, *E* and *F* and the angular velocities of the links *BC*, *CDE* and *EF*.

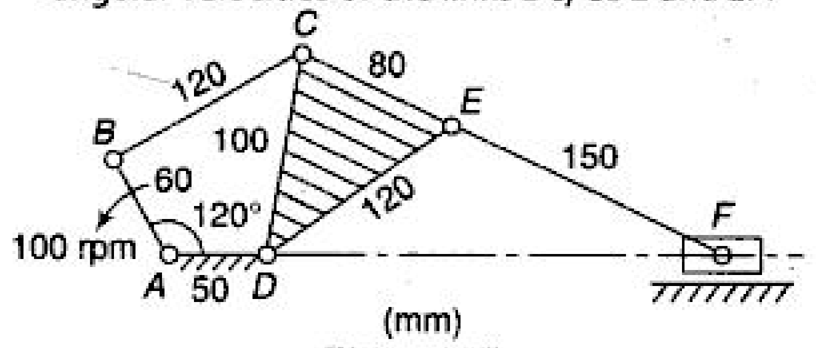


Fig. 2.51

(0.83 m/s; 0.99 m/s; 0.81 m/s; 5.4 rad/s ccw; 8.3 rad/s ccw; 6.33 rad/s ccw)

15. For the four-link mechanism shown in Fig. 2.52, find the linear velocities of sliders *C* and *D* and the angular velocities of links *AC* and *BD*.

(2.1 m/s; 0.38 m/s; 1.14 rad/s; 5.93 rad/s)

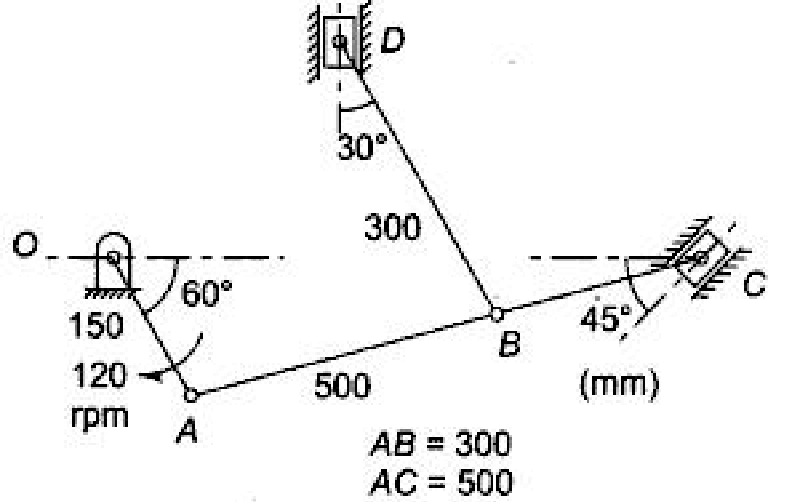


Fig. 2.52

16. An offset slider-crank mechanism is shown in Fig. 2.53. The crank is driven by the slider *B* at a speed of 15 m/s towards the left at given instant. Find the velocity of the offset point *D* on the coupler *AB* and the angular velocities of links *OA* and *AB*.

(18 m/s; 60 rad/s; 16.4 rad/s)

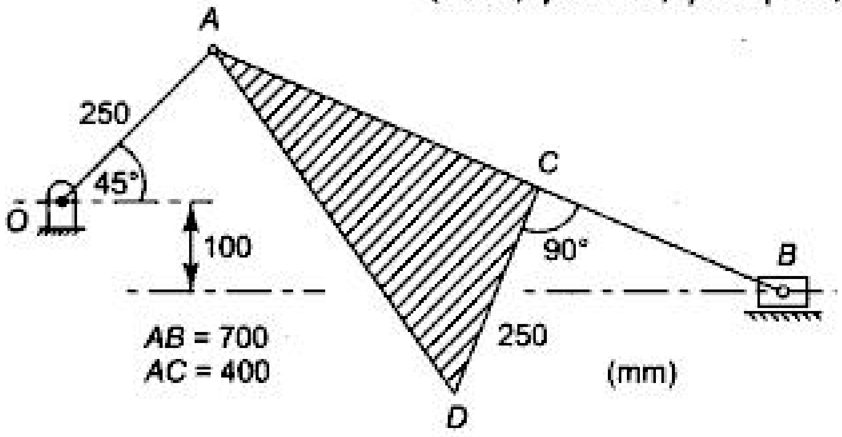


Fig. 2.53

17. A toggle mechanism is shown in Fig. 2.54 along with the dimensions of the links in mm. Find the velocities of the points *B* and *C* and the angular velocities of links *AB*, *BQ* and *BC*. The crank rotates at 50 rpm in the clockwise direction.

(0.13 m/s; 0.105 m/s; 0.74 rad/s ccw; 1.3 rad/s ccw; 1.33 rad/s cw)

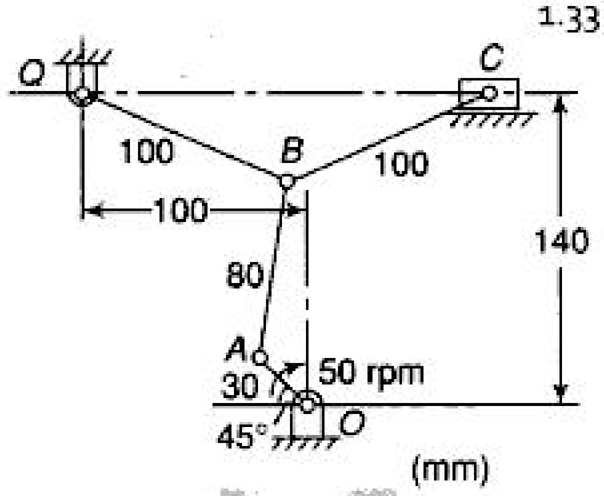


Fig. 2.54

18. In the mechanism shown in Fig. 2.55, the link *OA* has an angular velocity of 10 rad/s. Determine the velocities of points *B*, *C* and *D* and the angular velocities of *ABC* and *QD*.

(2.34 m/s; 4.87 m/s; 4.87 m/s; 1.87 rad/s; 4.06 rad/s)

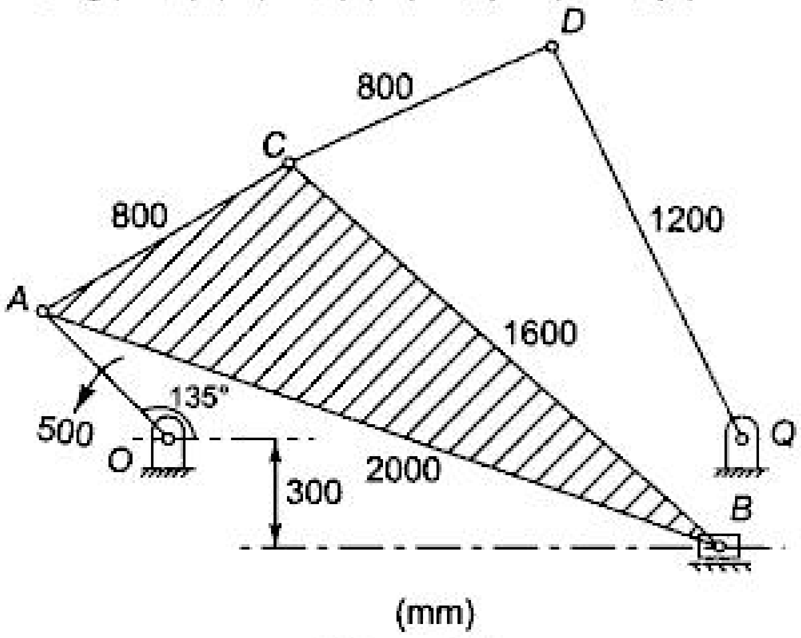


Fig. 2.55

19. Draw the velocity polygon for the mechanism shown in Fig. 2.56. Find the angular velocity of link *AB*. (1.75 rad/s cw)

(*Hint*: Assume the length of vector $\mathbf{v}_{ef}$ and complete the velocity polygon. Determine the velocity scale.)

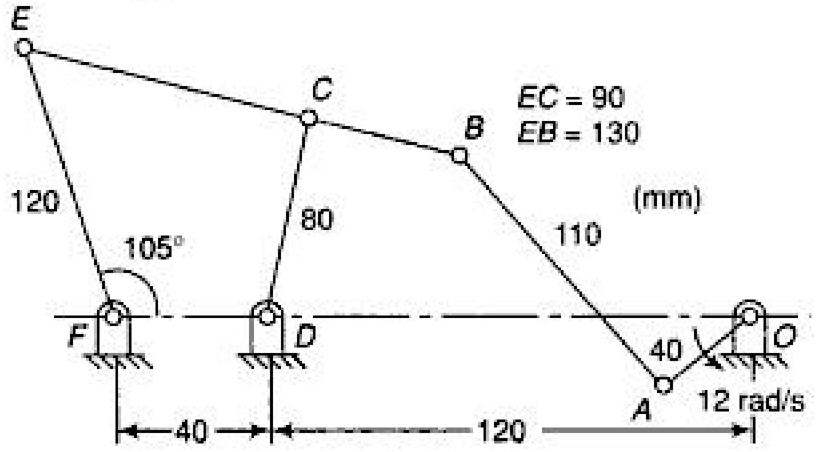


Fig. 2.56

20. For the mechanism shown in Fig. 2.57, determine the angular velocity of link *AB*.

(60.3 rad/s ccw)

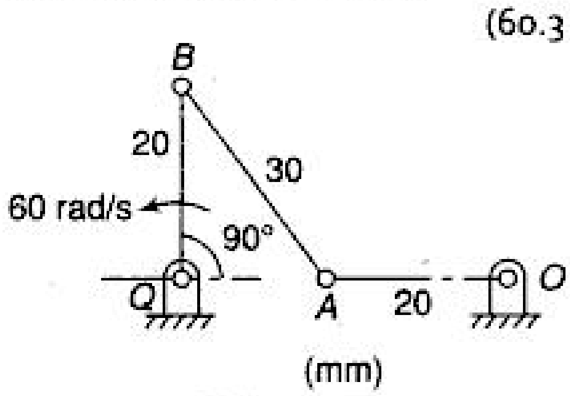


Fig. 2.57

21. In the mechanism shown in Fig. 2.58, O and A are fixed. CD = 200 mm, OA = 60 mm, AC = 50 mm and OB(crank) = 150 mm. OAD = 90°. Determine the velocity of the slider D for counter-clockwise rotation of OB at 80 rpm. (0.32 m/s)

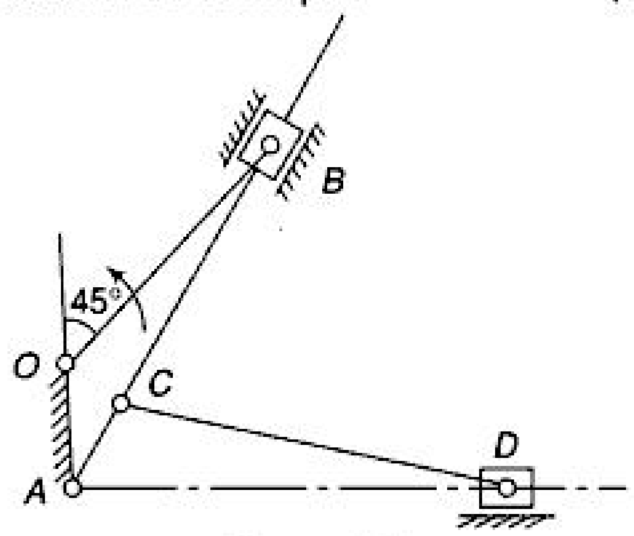


Fig. 2.58

22. The crank OP of a crank- and slotted-lever mechanism (Fig. 2.59) rotates at 100 rpm in the counter-clockwise direction. Various lengths of the links are OP = 90 mm, OA = 300 mm, AR = 480 mm and RS = 330 mm. The slider moves along an axis perpendicular to AO and is 120 mm from O. Determine the velocity of the slider when the AOP is 135°. Also, find the maximum velocity of the slider during cutting & return strokes. (0.975 m/s; 1.16 m/s, 2.15 m/s)

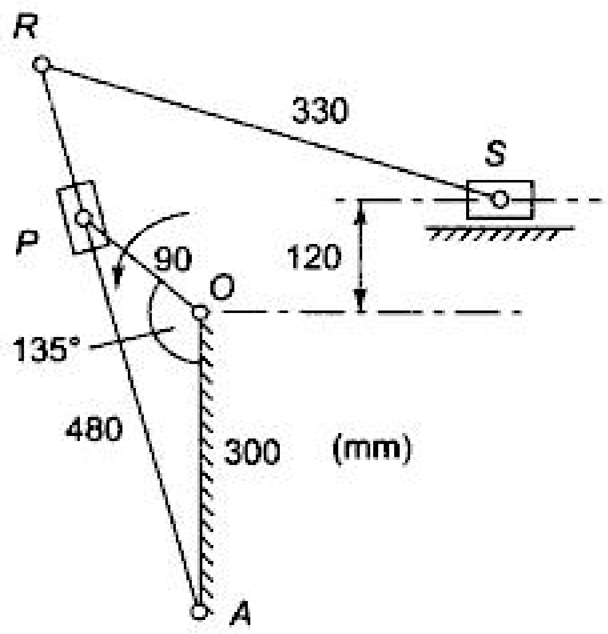


Fig. 2.59

23. For the four-link mechanism shown in Fig. 2.60, find the angular velocities of the links BC and CD using the instantaneous centre method. (1.3 rad/s, 3.07 rad/s)

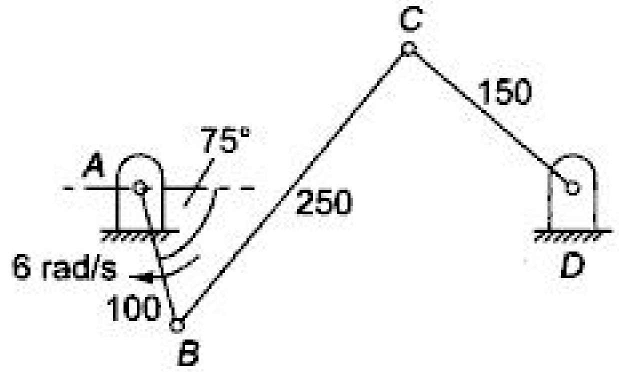


Fig. 2.60

24. Solve Problem 17 (Fig. 2.54) using the I-centre method.
25. Solve Problem 21 (Fig. 2.58) using the I-centre method.
26. Solve Example 2.5 (Fig. 2.13) using the I-centre method.
27. Solve Example 2.11 (Fig. 2.19) using the I-centre method.

3 ACCELERATION ANALYSIS

Introduction

Velocity of a moving body is a vector quantity having magnitude and direction. A change in the velocity requires any of the following conditions to be fulfilled:

- A change in the magnitude only
- A change in the direction only
- A change in both magnitude and direction

The rate of change of velocity with respect to time is known as *acceleration* and it acts in the direction of the change in velocity. Thus acceleration is also a vector quantity. To find linear acceleration of a point on a link, its linear velocity is required to be found first. Similarly, to find the angular acceleration of a link, its angular velocity has to be found. Apart from the graphical method, algebraic methods are also discussed in this chapter. After finding the accelerations, it is easy to find inertia forces acting on various parts of a mechanism or machine.

3.1 ACCELERATION

Let a link OA, of length r, rotate in a circular path in the clockwise direction as shown in Fig. 3.1(a). It has an instantaneous angular velocity ω and an angular acceleration α in the same direction, i.e., the angular velocity increases in the clockwise direction.

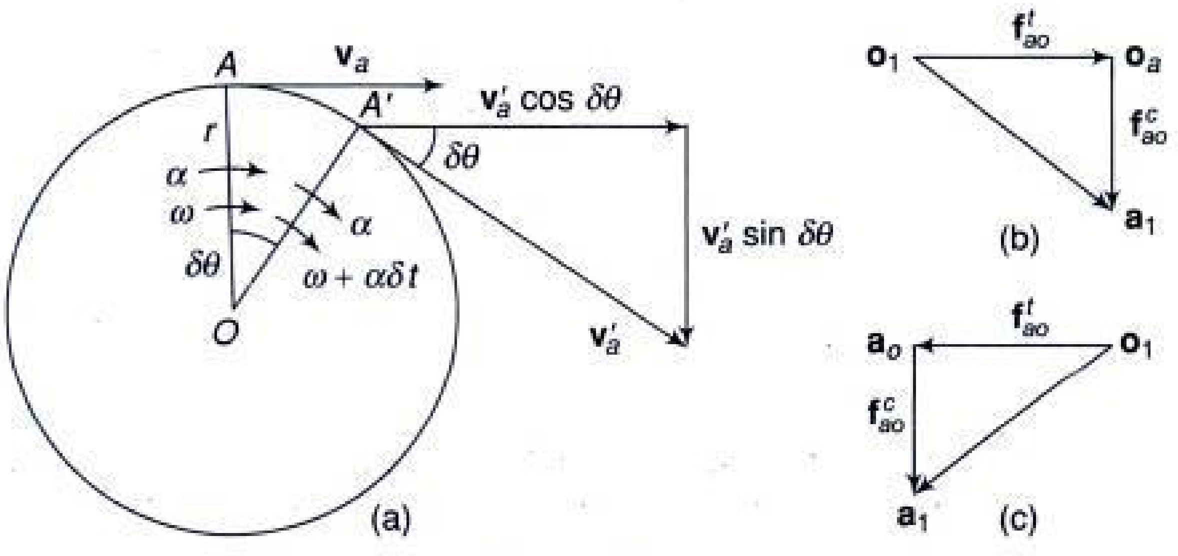


Fig. 3.1

Tangential velocity of A, $v_a = \omega r$

In a short interval of time δt, let OA assume the new position OA' by rotating through a small angle $\delta\theta$.

Angular velocity of OA', $\omega'_a = \omega + \alpha\,\delta t$

Tangential velocity of A', $v'_a = (\omega + \alpha\,\delta t)\,r$

The tangential velocity of A' may be considered to have two components; one perpendicular to OA and the other parallel to OA.

Change of Velocity Perpendicular to OA

Velocity of $A \perp$ to $OA = v_a$

Velocity of $A' \perp$ to $OA = v'_a \cos \delta\theta$

$\therefore$ change of velocity $= v'_a \cos \delta\theta - v_a$

$$\text{Acceleration of } A \perp \text{ to } OA = \frac{(\omega + \alpha.\delta t) r \cos \delta\theta - \omega r}{\delta t}$$

In the limit, as $\delta t \to 0$, $\cos \delta\theta \to 1$

$\therefore$ acceleration of $A \perp$ to $OA = \alpha\, r$

$$= \left(\frac{d\omega}{dt}\right) r \qquad \ldots\left(\alpha = \frac{d\omega}{dt}\right)$$

$$= \frac{dv}{dt} \tag{3.1}$$

This represents the rate of change of velocity in the tangential direction of the motion of A relative to O, and thus is known as the *tangential acceleration* of A relative to O. It is denoted by f^t_{ao}.

Change of Velocity Parallel to OA

Velocity of A parallel to $OA = 0$

Velocity of A' parallel to $OA = v'_a \sin \delta\theta$

$\therefore$ change of velocity $= v'_a \sin \delta\theta - 0$

$$\text{Acceleration of } A \text{ parallel to } OA = \frac{(\omega + \alpha\,\delta t) r \sin \delta\theta}{\delta t}$$

In the limit, as $\delta t \to 0$, $\sin \delta\theta \to \delta\theta$

$$\text{Acceleration of } A \text{ parallel to } OA = \omega r \frac{d\theta}{dt}$$

$$= \omega\, r.\omega \qquad \ldots\left(\omega = \frac{d\theta}{dt}\right)$$

$$= \omega^2 r \tag{3.2}$$

$$= \frac{v^2}{r} \ldots\ldots \quad (v = \omega\, r) \tag{3.3}$$

This represents the rate of change of velocity along OA, the direction being from A towards O or towards the centre of rotation. This acceleration is known as the *centripetal* or the *radial acceleration* of A relative to O and is denoted by f^c_{ao}.

Figure 3.1(b) shows the centripetal and the tangential components of the acceleration acting on A. Note the following:

1. When $\alpha = 0$, i.e., OA rotates with uniform angular velocity, $f_{ao}^{t} = 0$ and thus f_{ao}^{c} represents the total acceleration.
2. When $\omega = 0$, i.e., A has a linear motion, $f_{ao}^{c} = 0$ and thus the tangential acceleration is the total acceleration.
3. When α is negative or the link OA decelerates, tangential acceleration will be negative or its direction will be as shown in Fig. 3.1(c).

Total acceleration vectors are denoted by small letters with a subscript '1' attached. The meeting point of its components may be denoted by any of the small letters used for the total acceleration with a subscript of the other.

For example, components of the total acceleration $\mathbf{o_1a_1}$ can be written in either of the two ways:

1. $\mathbf{o_1o_a}$ and $\mathbf{o_aa_1}$ as in Fig. 3.1 (b)
2. $\mathbf{o_1a_0}$ and $\mathbf{a_0a_1}$ as in Fig. 3.1 (c)

Note that the centripetal acceleration is denoted by the same letters as are in the configuration diagram but the positions are changed.

3.2 FOUR-LINK MECHANISM

The configuration and the velocity diagrams of a four-link mechanism discussed in Sec. 2.5 have been reproduced in Figs 3.2(a) and (b). Let α = angular acceleration of AB at this instant, assumed positive, i.e., the speed increases in the clockwise direction.

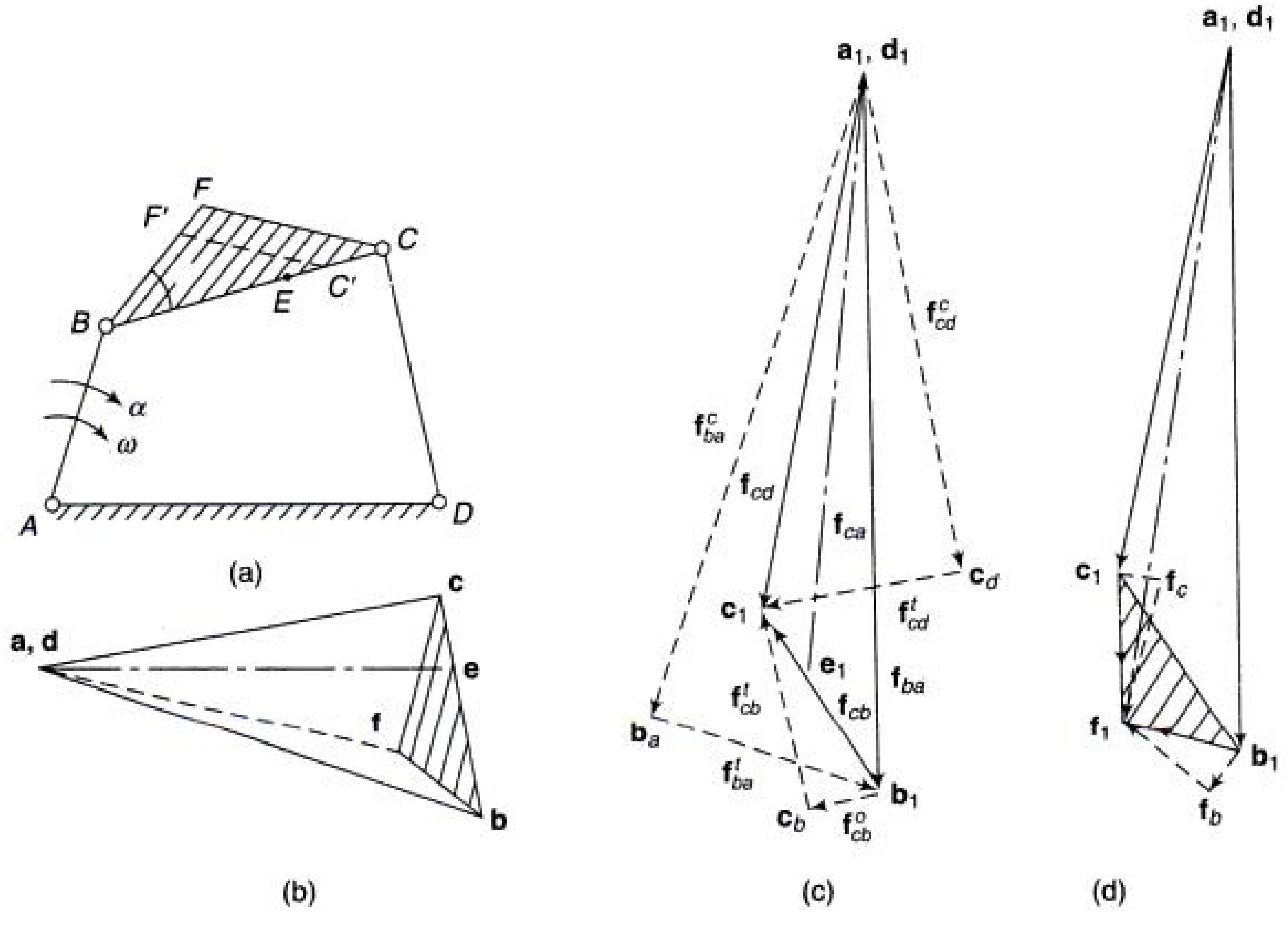


Fig. 3.2

Now, in Δs $\mathbf{b_1f_1c_1}$ and BFC,

$$\angle 3 = \angle 1$$

and
$$\frac{\mathbf{b_1 f_1}}{\mathbf{b_1 c_1}} = \frac{BF}{BC} = k$$

Therefore, the two triangles are similar.

Thus, to find the acceleration of an offset point on a link, a triangle similar to the one formed in the configuration diagram can be made on the acceleration image of the link in such a way that the sequence of letters is the same, i.e., $\mathbf{b_1f_1c_1}$ is clockwise, so should be BFC.

An easier method of making the triangle $\mathbf{b_1f_1}$ similar to BFC is by marking BC' on BC equal to $\mathbf{b_1c_1}$ and drawing a line parallel to CF, meeting BF in F'. $BC'F'$ is the exact size of the triangle to be made on $\mathbf{b_1c_1}$. Take $\mathbf{b_1f_1} = BF'$ and $\mathbf{c_1f_1} = C'F'$.
Thus, the point $\mathbf{f_1}$ is obtained.

3.4 SLIDER-CRANK MECHANISM

The configuration and the velocity diagrams of a slider-crank mechanism discussed in Sec. 2.8 have been reproduced in Figs. 3.4(a) and (b).

Writing the acceleration equation,

Acc. of B rel. to O = Acc. of B rel. to A + Acc. of A rel. to O

$$\mathbf{f}_{bo} = \mathbf{f}_{ba} + \mathbf{f}_{ao}$$

$$\mathbf{f}_{bg} = \mathbf{f}_{ao} + \mathbf{f}_{ba} = \mathbf{f}_{ao} + \mathbf{f}^c_{ba} + \mathbf{f}^t_{ba}$$

$$\mathbf{g_1\ b_1} = \mathbf{o_1\ a_1} + \mathbf{a_1\ b_a} + \mathbf{b_a\ b_1}$$

The crank OA rotates at a uniform velocity, therefore, the acceleration of A relative to O has only the centripetal component. Similarly, the slider moves in a linear direction and thus has no centripetal component.

Setting the vector table:

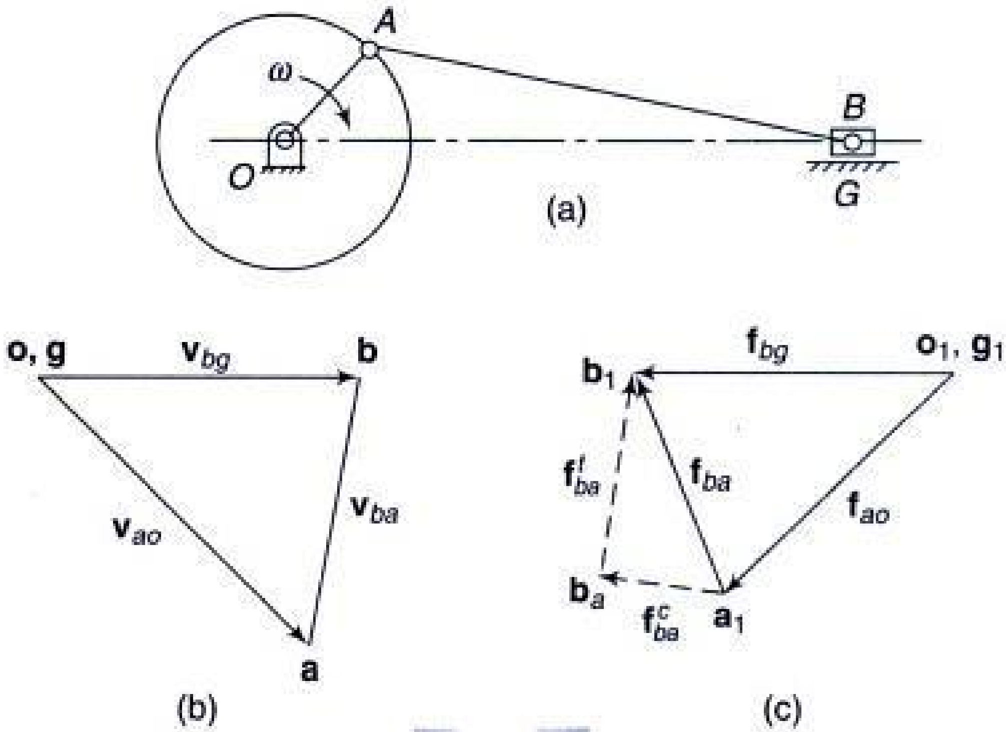


Fig. 3.4

SN	Vector	Magnitude	Direction	Sense
1.	$\mathbf{f}_{ao}$ or $\mathbf{o_1 a_1}$	$\frac{(\mathbf{oa})^2}{OA}$	‖ OA	→ O
2.	$\mathbf{f}^c_{ba}$ or $\mathbf{a_1\ b_a}$	$\frac{(\mathbf{ab})^2}{AB}$	‖ AB	→ A
3.	$\mathbf{f}^t_{ba}$ or $\mathbf{b_a\ b_1}$	-	⊥ AB	-
4.	$\mathbf{f}_{bg}$ or $\mathbf{g_1 b_1}$	-	‖ to line of motion of B	-

Construct the acceleration diagram as follows:

1. Take the first vector $\mathbf{f}_{ao}$.
2. Add the second vector to the first.
3. For the third vector, draw a line ⊥ to *AB* through the head $\mathbf{b_a}$ of the second vector.
4. For the fourth vector, draw a line through $\mathbf{g_1}$ parallel to the line of motion of the slider.

   This completes the velocity diagram.

   Acceleration of the slider $B = \mathbf{o_1\, b_1}$ (or $\mathbf{g_1\, b_1}$)

   Total acceleration of B relative to $A = \mathbf{a_1\, b_1}$

Note that for the given configuration of the mechanism, the direction of the acceleration of the slider is opposite to that of the velocity. Therefore, the acceleration is negative or the slider decelerates while moving to the right.

Example 3.1 *Figure 3.5(a) shows the configuration diagram of a four-link mechanism along with the lengths of the links in mm. The link AB has an instantaneous angular velocity of 10.5 rad/s and a retardation of 26 rad/s² in the counter-clockwise direction. Find*

(i) the angular accelerations of the links BC and CD

(ii) the linear accelerations of the points E, F and G

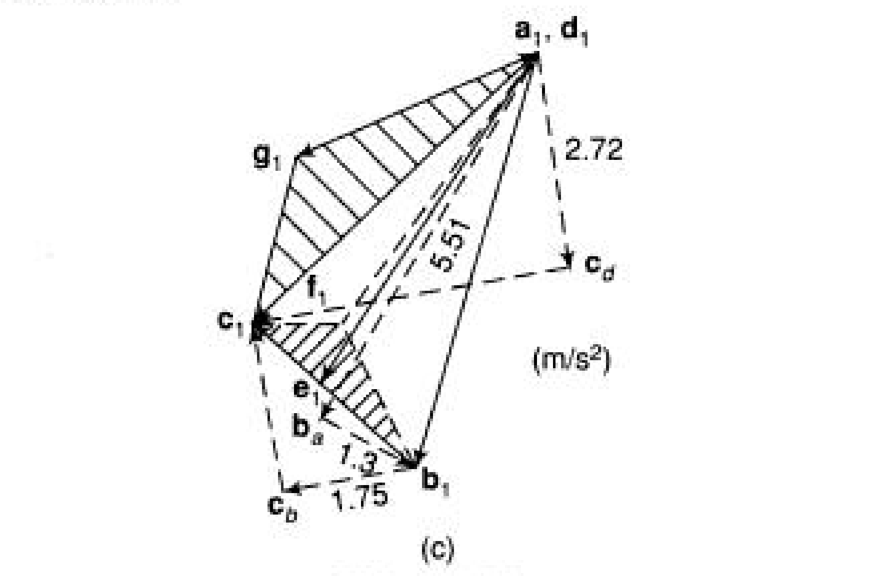


Fig. 3.5

Solution $v_b = 10.5 \times 0.05 = 0.525$ m/s

Complete the velocity diagram [Fig. 3.5(b)] as explained in Example 2.1.

Writing the vector equation for acceleration,

Acc. of *C* rel. to *A* = Acc. of *C* rel. to *B* + Acc. of *B* rel. to *A*

$$\mathbf{f}_{ca} = \mathbf{f}_{cb} + \mathbf{f}_{ba}$$

or $$\mathbf{f}_{cd} = \mathbf{f}_{ba} + \mathbf{f}_{cb}$$

or $$\mathbf{d_1\, c_1 = a_1\, b_1 + b_1\, c_1}$$

Each vector has a centripetal and a tangential component,

∴ $$\mathbf{f}^c_{cd} + \mathbf{f}^t_{cd} = \mathbf{f}^c_{ba} + \mathbf{f}^t_{ba} + \mathbf{f}^c_{cb} + \mathbf{f}^t_{cb}$$

or $$\mathbf{d_1\, c_d + c_d\, c_1 = b_a + b_a\, b_1 + b_1\, c_b + c_b\, c_1}$$

Set the vector table (Table 1) on the next page.

Draw the acceleration diagram as follows:

(i) Take the pole point $\mathbf{a_1}$ or $\mathbf{d_1}$ [Fig. 3.5(c)].

(ii) Starting from $\mathbf{a_1}$, take the first vector $\mathbf{a_1\, b_a}$.

(iii) To the first vector, add the second vector and to the second vector, add the third.

(iv) The vector 4 is known in direction only. Therefore, through the head $\mathbf{c_b}$ of the third vector, draw a line, ⊥ to *BC*. The point $\mathbf{c_1}$ of the fourth vector is to lie on this line.

(v) Start with $\mathbf{d_1}$ and take the fifth vector $\mathbf{d_1\, c_d}$.

Table 1

SN	Vector	Magnitude (m/s²)	Direction	Sense
1.	f^c_{ba} or $\mathbf{a_1 b_a}$	$\frac{(ab)^2}{AB} = \frac{(0.525)^2}{0.05} = 5.51$	$\parallel AB$	$\rightarrow A$
2.	f^t_{ba} or $\mathbf{b_a b_1}$	$\alpha \times AB = 26 \times 0.05 = 1.3$	$\perp AB$ or $\parallel$ **ab**	$\rightarrow$ a
3.	f^t_{cb} or $\mathbf{b_1 c_b}$	$\frac{(bc)^2}{BC} = \frac{(0.34)^2}{0.066} = 1.75$	$\parallel BC$	$\rightarrow B$
4.	f^t_{cb} or $\mathbf{b_b c_1}$	-	$\perp B$	-
5.	f^c_{cd} or $\mathbf{d_1 c_d}$	$\frac{(dc)^2}{DC} = \frac{(0.39)^2}{0.56} = 2.72$	$\parallel DC$	$\rightarrow D$
6.	f^t_{cd} or $\mathbf{c_d c_1}$	-	$\perp B$	-

(vi) The sixth vector is known in direction only. Draw a line ⊥ to DC through head $\mathbf{c_d}$ of the fifth vector, the intersection of which with the line in the step (d) locates the point $\mathbf{c_1}$.

(vii) Join $\mathbf{a_1b_1}$, $\mathbf{b_1c_1}$ and $\mathbf{d_1c_1}$.

Now, $\mathbf{a_1b_1}$ represents the total accelerations of the point B relative to the point A.

Similarly, $\mathbf{b_1c_1}$ is the total acceleration of C relative to B and $\mathbf{d_1c_1}$ is the total acceleration of C relative to D.

[Note In the acceleration diagram shown in Fig. 2.5c, the arrowhead has been put on the line joining points $\mathbf{b_1}$ and $\mathbf{c_1}$ in such a way that it represents the vector for acceleration of C relative to B. This satisfies the above equation. As the same equation

$$\mathbf{f}_{cd} = \mathbf{f}_{ba} + \mathbf{f}_{cb}$$

can also be put as

$$\mathbf{f}_{cd} + \mathbf{f}_{bc} = \mathbf{f}_{ba}$$

$$\mathbf{d_1 c_1 + c_1 b_1 = a_1 b_1}$$

This shows that on the same line joining $\mathbf{b_1}$ and $\mathbf{c_1}$, the arrowhead should be marked in the other direction so that the vector represents the acceleration of B relative to C to satisfy the latter equation.

This implies that in case both the ends of a link are in motion, the arrowhead may be put in either direction or no arrowhead is put at all. This is because every time it is not necessary to write the acceleration equation. The acceleration equation is helpful only at the initial stage for better comprehension.]

(i) *Angular accelerations*

$$\alpha_{bc} = \frac{\mathbf{f}^t_{cb} \text{ or } \mathbf{c_b\, c_1}}{BC} = \frac{2.25}{0.066} = \underline{34.09 \text{ rad/s}^2}$$

counter-clockwise

$$\alpha_{cd} = \frac{\mathbf{f}^t_{cd} \text{ or } \mathbf{c_d\, c_1}}{CD} = \frac{4.43}{0.056}$$

$= \underline{79.11 \text{ rad/s}^2}$ counter-clockwise

(ii) *Linear accelerations*

(a) Locate point $\mathbf{e_1}$ on $\mathbf{b_1\, c_1}$ such that

$$\frac{\mathbf{b_1 e_1}}{\mathbf{b_1 c_1}} = \frac{BE}{BC}$$

$$f_e = \mathbf{a_1 e_1} = \underline{5.15 \text{ m/s}^2}$$

(b) Draw Δ $\mathbf{b_1c_1f_1}$ similar to Δ BCF keeping in mind that BCF as well as $\mathbf{b_1c_1f_1}$ are read in the same order (clockwise in this case).

$$f_f = \mathbf{a_1 f_1} = \underline{4.42 \text{ m/s}^2}$$

(c) Linear acceleration of the point G can also be found by drawing the acceleration image of the triangle DCG on $\mathbf{d_1c_1}$ in the acceleration diagram such that the order of the letters remains the same.

$$f_g = \mathbf{d_1 g_1} = \underline{3.9 \text{ m/s}^2}$$

Example 3.2 *For the configuration of a slider-crank mechanism shown in Fig. 3.6(a), calculate the*

(i) acceleration of the slider at B
(ii) acceleration of the point E
(iii) angular acceleration of the link AB
OA rotates at 20 rad/s counter-clockwise.

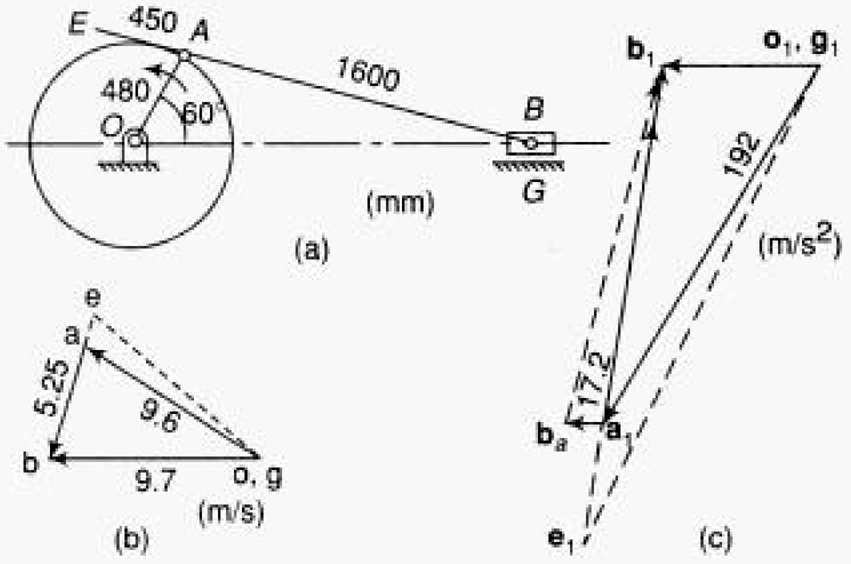


Fig. 3.6

Solution $v_a = 20 \times 0.48 = 9.6$ m/s

Complete the velocity diagram as shown in Fig. 3.6(b).

Writing the vector equation,

$$\mathbf{f}_{bo} = \mathbf{f}_{ba} + \mathbf{f}_{ao}$$

or $$\mathbf{f}_{bg} = \mathbf{f}_{ao} + \mathbf{f}_{ba} = \mathbf{f}^c_{ao} + \mathbf{f}^c_{ba} + \mathbf{f}^t_{ba}$$

or $$\mathbf{g_1 b_1} = \mathbf{o_1 a_1} + \mathbf{a_1 b_a} + \mathbf{b_a b_1}$$

Set the vector table (Table 2) as given below.

The acceleration diagram is drawn as follows:

(a) Take the pole point $\mathbf{o_1}$ or $\mathbf{g_1}$ [Fig. 3.6 (c)].

(b) Take the first vector $\mathbf{o_1a_1}$ and add the second vector.

(c) For the third vector, draw a line ⊥ to *AB* through the head $\mathbf{b_a}$ of the second vector.

(d) For the fourth vector, draw a line || to the line of motion of the slider through $\mathbf{g_1}$. The intersection of this line with the line drawn in the step (d) locates point $\mathbf{b_1}$.

(e) Join $\mathbf{a_1\ b_1}$.

(i) $f_b = \mathbf{g_1b_1} = 72$ m/s²

As the direction of acceleration $\mathbf{f}_b$ is the same as of $\mathbf{v}_b$, this means the slider is accelerating at the instant.

(ii) Locate point $\mathbf{e_1}$ on $\mathbf{b_1a_1}$ produced such that

$$\frac{\mathbf{b_1\ e_1}}{\mathbf{b_1\ a_1}} = \frac{\mathbf{BE}}{\mathbf{BA}}$$

$f_e = \mathbf{o_1e_1} =$ <u>236 m/s²</u>

(iii) $$\alpha_{ab} = \frac{\mathbf{f}^t_{ba}}{AB} = \frac{\mathbf{b_a\ b_1}}{AB} = \frac{167}{1.6}$$

= <u>104 rad/s²</u> counter-clockwise

Example 3.3 *Figure 3.7(a) shows configuration of an engine mechanism. The dimensions are the following:*

Crank OA = 200 mm; Connecting rod AB = 600 mm; distance of centre of mass from crank end, AD = 200 mm. At the instant, the crank has an angular velocity of 50 rad/s clockwise and an angular acceleration of 800 rad/s². Calculate the

(i) velocity of D and angular velocity of AB
(ii) acceleration of D and angular acceleration of AB

Table 2

SN	Vector	Magnitude (m/s²)	Direction	Sense
1.	$\mathbf{f}^c_{ao}$ or $\mathbf{o_1\ a_1}$	$\frac{(\mathbf{oa})^2}{OA} = \frac{(9.6)^2}{0.48} = 192$	|| *OA*	→ *O*
2.	$\mathbf{f}^c_{ba}$ or $\mathbf{a_1\ b_a}$	$\frac{(\mathbf{ab})^2}{AB} = \frac{(5.25)^2}{1.60} = 17.2$	|| *AB*	→ *A*
3.	$\mathbf{f}^t_{ba}$ or $\mathbf{b_a\ b_1}$	-	⊥*AB*	-
4.	$\mathbf{f}_{bg}$ or $\mathbf{g_1\ b_1}$	-	|| to slider motion	-

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Note that in the present case, the sliding acceleration $\mathbf{a_1 q_a}$ is in the opposite direction to the sliding velocity **qp**. This signifies that the slider is decelerating.

Also,

$$\mathbf{f}_{sa} = \mathbf{f}_{sr} + \mathbf{f}_{ra}$$
$$\mathbf{f}_{sg} = \mathbf{f}_{ra} + \mathbf{f}_{sr}$$
$$= \mathbf{f}_{ra} + \mathbf{f}^{c}_{sr} + \mathbf{f}^{t}_{sr}$$
$$\mathbf{g_1 s_1 = a_1 r_1 + r_1 s_r + s_r s_1}$$

This equation can be solved as usual.

Total acc. of *S* relative to *R*, $\mathbf{f}_{sr} = \mathbf{r_1 s_1}$

Acceleration of $S = \mathbf{g_1 s_1}$ or $\mathbf{a_1 s_1}$ or $\mathbf{o_1 s_1}$

The direction of $\mathbf{g_1 s_1}$ is opposite to the direction of motion of the slider *S* indicating that the slider is decelerating.

Example 3.9 *Figure 3.16(a) shows a slider moving outwards on a rod with a velocity of 4 m/s when its distance from the point O is 1.5 m. At this instant, the velocity of the slider is increasing at a rate of 10 m/s². The rod has an angular velocity of 6 rad/s counter-clockwise about O and an angular acceleration of 20 rad/s² clockwise. Determine the absolute acceleration of the slider.*

Solution

Writing the acceleration vector equation,

$$\mathbf{f}_{po} = \mathbf{f}_{pq} + \mathbf{f}_{qo} = \mathbf{f}_{qo} + \mathbf{f}_{pq} = \mathbf{f}^{c}_{qo} + \mathbf{f}^{t}_{qo} + \mathbf{f}^{s}_{pq} + \mathbf{f}^{cr}_{pq}$$

or $\mathbf{o_1 p_1 = o_1 q_o + q_o q_1 + q_1 p_q + p_q p_1}$

Set the following vector table (Table 12):

Figure 3.16(b) shows how to obtain the direction of the coriolis component. The velocity vector of the slider is rotated through 90° in the angular direction of the rod.

Draw the acceleration diagram as follows [Fig. 3.16(c)]:

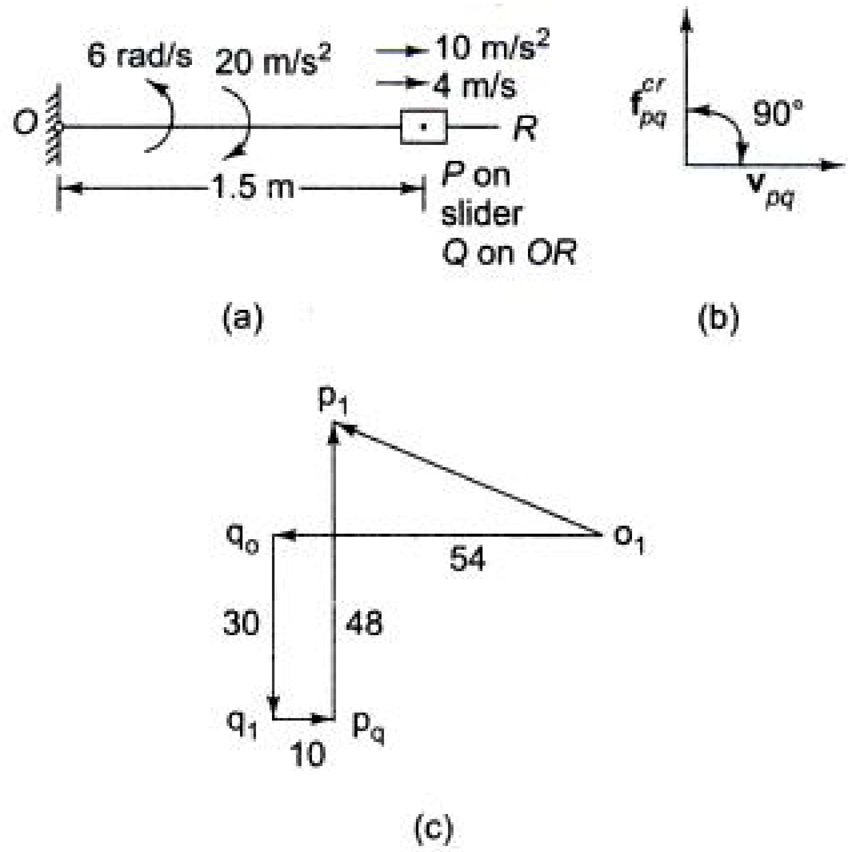

Fig. 3.16

1. From the pole point $\mathbf{o_1}$, take the first vector $\mathbf{o_1 q_o}$.
2. Add to it the second vector $\mathbf{q_o q_1}$.

Table 12

SN	Vector	Magnitude (m/s²)	Direction	Sense
1.	$\mathbf{f}^{c}_{qo}$ or $\mathbf{o_1 q_o}$	$\omega^2.r = 6^2 \times 1.5 = 54$	∥ *OR*	←
2.	$\mathbf{f}^{t}_{qo}$ or $\mathbf{q_o q_1}$	$\alpha_{or} \times OQ = 20 \times 1.5 = 30$	⊥ *OR*	↓
3.	$\mathbf{f}^{s}_{pq}$ or $\mathbf{q_1 p_q}$	10	∥ *OR*	→
4.	$\mathbf{f}^{cr}_{pq}$ or $\mathbf{p_q p_1}$	$2\omega.v_{pq} = 2 \times 6 \times 4 = 48$	⊥ *OR*	

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or

$$A \tan^2\left(\frac{\varphi}{2}\right) + B \tan\left(\frac{\varphi}{2}\right) + C = 0$$

where

$$A = k - a\,(d - c)\cos\theta - cd$$
$$B = -\,2ac\sin\theta$$
$$C = k - a\,(d + c)\cos\theta + cd$$

Equation (4.6) is a quadratic in $\tan\left(\frac{\varphi}{2}\right)$. Its two roots are

$$\tan\left(\frac{\varphi}{2}\right) = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

or

$$\varphi = 2\tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right] \tag{4.7}$$

Thus, the position of the output link, given by angle φ, can be calculated if the magnitude of the links and the position of the input link are known, i.e., a, b, c, d and θ are known.

A relation between the coupler link position β and the input link position θ can also be found as below:

Equations (4.1) and (4.3) can be written as,

$$c\cos\varphi = a\cos\theta + b\cos\beta - d \tag{4.8}$$

$$c\sin\varphi = a\sin\theta + b\sin\beta \tag{4.9}$$

Squaring and adding the two equations,

$$c^2 = a^2 + b^2 + d^2 + 2ab\cos\theta\cos\beta - 2bd\cos\beta - 2ad\cos\theta + 2ab\sin\theta\sin\beta$$

Put $\quad a^2 + b^2 - c^2 + d^2 = 2k'$

$$-\,2bd\cos\beta + 2ab\cos\theta\cos\beta + 2ab\sin\theta\sin\beta - 2ad\cos\theta + 2k' = 0$$

$$-\,bd\cos\beta + ab\cos\theta\cos\beta + ab\sin\theta\sin\beta - ad\cos\theta + k' = 0 \tag{4.10}$$

Equation (4.10) is identical to Eq. 4.6 and can be obtained from the same by substituting β for φ, $-b$ for c and k' for k.

Thus, the solution of Eq. (4.10) will be,

$$\beta = 2\tan^{-1}\left[\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right] \tag{4.11}$$

where $\quad D = k' - a(d + b)\cos\theta + bd$

$$E = 2ab\sin\theta$$

$$F = k' - a(d - b)\cos\theta - bd$$

β can also be found directly from relation (4.3) after calculating φ.

$$= \frac{50 \times 10.5 \sin(10.29 - 60^\circ)}{56 \sin(10.29 - 100.35)} = 7.15 \text{ rad/s}$$

$$\omega_b = -\frac{a\omega_a \sin(\varphi - \theta)}{b \sin(\varphi - \beta)} =$$

$$-\frac{50 \times 10.5 \sin(100.35^o - 60^o)}{66 \times \sin(100.35^o - 10.29^o)} = -5.15 \text{ rad/s}$$

$$\alpha_c = \frac{a\alpha_a \sin(\beta - \theta) - a\omega_a^2 \cos(\beta - \theta) - b\omega_b^2 + c\omega_c^2 \cos(\beta - \varphi)}{c \sin(\beta - \varphi)}$$

$$= \frac{50 \times (-26)\sin(10.29^\circ - 60^\circ) - 50 \times 10.5^2 \cos(10.29^\circ - 60^\circ) - 66 \times (5.15)^2 + 56^2 \cos(10.29^\circ - 100.35^\circ)}{56 \sin(10.29^\circ - 100.35^\circ)}$$

$$= 77.26 \text{ rad/s}^2$$

$$\alpha_b = \frac{a\alpha_a \sin(\varphi - \theta) - a\omega_a^2 \cos(\varphi - \theta) - b\omega_b^2 \cos(\varphi - \beta) + c\omega_c^2}{b \sin(\beta - \varphi)}$$

$$= \frac{50 \times (-26)\sin(100.35^\circ - 60^\circ) - 50 \times 10.5^2 \cos(100.35^\circ - 60^\circ) - 66 \times (5.15)^2 \cos(100.35^\circ - 10.29^\circ) + 56 \times 7.15^2)}{56 \sin(10.29^\circ - 100.35^\circ)}$$

$$= 32.98 \text{ rad/s}^2$$

Using the other value of ϕ, ($\phi = -160.3^\circ$), another set of values of velocities and accelerations can be obtained.

The results obtained using the program of Fig. 4.2 are given in Fig. 4.3.

```
Enter values of a, b, c, d, vela, acca, theta, limit
50  66  56  100  10.5  -26  60  0
thet    vela     acca      phi      beta      velc     velb     accc     accb
60      10.5    -26.00   -160.35   -70.29    -7.15     5.15    50.04    94.32
60      10.50   -26.00    100.35    10.29     7.15    -5.15    77.26    32.98
```

Fig. 4.3

[Compare these values of ω_b, ω_c, α_b and α_c at 60° with the values obtained graphically in Examples 2.1 and 3.1]

Example 4.2 *In a four-link mechanism, the dimensions of the links are as under: AB = 20 mm, BC = 66 mm, CD = 56 mm and AD = 80 mm AD is the fixed link. The crank AB rotates at uniform angular velocity of 10.5 rad/s in the counter-clockwise direction. Determine using the program of Fig. 4.2, the angular displacements, angular velocities and angular accelerations of the output link DC and the coupler BC for a complete revolution of the crank at an interval of 40°.*

Solution The results obtained using the program of Fig. 4.2 are given in Fig. 4.4.

```
Enter values of a, b, c, d, vela, acca, theta, limit
20  66  56  80  10.5  40  0
thet    vela    acca      phi      beta     velc     velb    aqccc     accb
00      10.5    0.0    -110.74   -52.51   -3.50    -3.50   -37.58    18.56
00      10.5    0.0     110.74    52.51   -3.50    -3.50    37.58   -18.56
40      10.5    0.0    -126.30   -61.47   -4.06    -0.83    15.50    50.99
40      10.5    0.0     103.82    38.99    0.07    -3.15    56.46    20.96
80      10.5    0.0    -139.02   -58.74   -2.51     2.03    26.17    31.04
80      10.5    0.0     110.16    29.87    2.92    -1.62    27.30    22.42
```

(contd.)

DC_2 and DC_3 as its angular displacements are known.

3. Find the points C'_2 and C'_3 after rotating AC_2 and AC_3 about A through angles θ_{12} and θ_{13} respectively in the counter-clockwise direction.
4. Intersection of the midnormals of $C_1 C'_2$ and $C_1 C'_3$ locates the point B_1.

Then AB_1C_1D is the required four-link mechanism. Figure 5.16(b) shows the mechanism in the required three positions. The mechanism could also have been obtained by drawing the input link AB in three positions as stated earlier.

Example 5.4 *Design a slider-crank mechanism to coordinate three positions of the input and of the slider for the following data by inversion method:*

$\theta_{12} = 30°$ $s_{12} = 40$ mm
$\theta_{13} = 60°$ $s_{13} = 96$ mm
Eccentricity = 20 mm

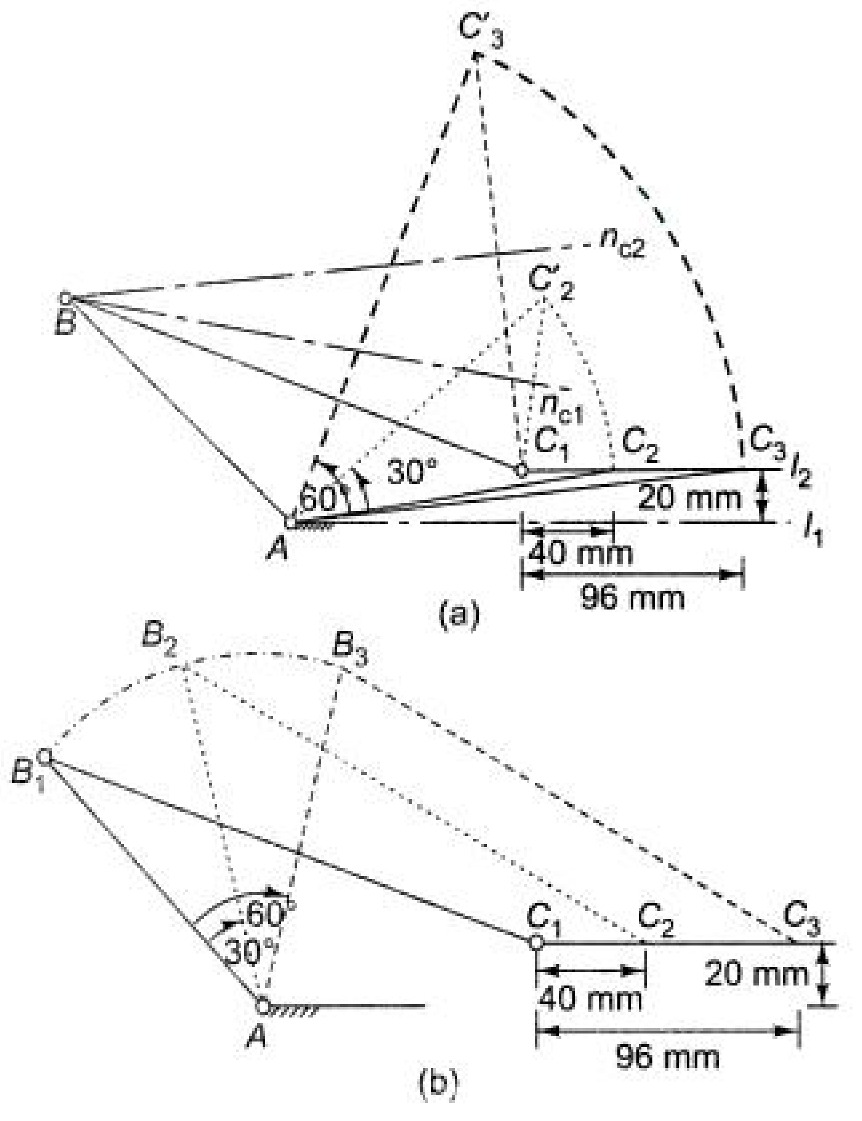

Fig. 5.17

Solution For the given two angular displacements of the input link and the two linear displacements of the slider along with the eccentricity e, the required slider-crank mechanism is obtained as follows:

1. Draw two parallel lines l_1 and l_2 at a distance of 20 mm apart [Fig. 5.17(a)].
2. Take an arbitrary point A on the line l_1 for the fixed pivot and three points C_1, C_2 and C_3 on the line l_2, at distances 40 mm and 96 mm apart for the initial and subsequent positions of the slider.
3. Rotate the point C_2 about A through an angle 30° in the counter-clockwise direction to obtain the point C'_2. Similarly, rotate the point C_3 about A through an angle 60° to obtain the point C'_3
4. Join $C_1C'_2$ and $C_1C'_3$ and draw their midnormals to intersect at point B.

Then ABC_1 is the required slider-crank mechanism. Figure 5.17(b) shows the mechanism in the required three positions.

Example 5.5 *Design a four-link mechanism to coordinate four positions of the input and the output links for the following angular displacements of the input link and the output link respectively:*

$\theta_{12} = 50°$ $\varphi_{12} = 30°$
$\theta_{13} = 80°$ $\varphi_{13} = 80°$
$\theta_{13} = 100°$ $\varphi_{13} = 120°$

Solution Make the following construction:

1. Draw a line segment AD of suitable length to be the distance between the fixed pivots [Fig. 5.18(a)].
2. Locate the position of the relative pole by rotating AD about A through angle 25° ($= \theta_{12}/2$) and about D through an angle 15° ($= \varphi_{12}/2$) both in counter-clockwise direction. Take this as point B_1.

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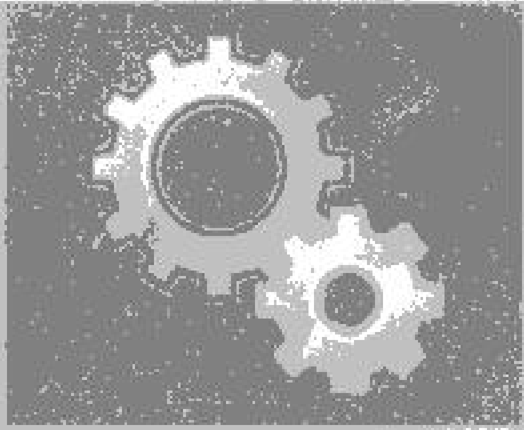
```
{
 del=pl*(t2-t3)+tl*(p3-p2)+(p2*t3-p3*t2);
 dell=cl*(t2-t3)+tl*(c3-c2)+(c2*t3-c3*t2);
 del2=pl*(c2-c3)+cl*(p3-p2)+(p2*c3-p3*c2);
 del3=pl*(t2*c3-t3*c2)+tl*(c2*p3-c3*p2)
 +cl*(p2*t3-p3*t2);
 akl=dell/del;
 ak2=del2/del;
 ak3=del3/del;
  if (k==0)
  {
   ala=akl;
   alg=ak2;
   alk=ak3;
   cl=2*cos(tll-gg);
   c2=2*cos(t22-gg):
   c3=2*cos(t33-gg):
  }
 }
ama=akl;
amg=ak2:
amk=ak3:
aa=ama*amg:
bb=ala*amg+alg*ama-l:
cc=ala*alg;
squ=bb*bb-4*aa*cc:
 if (squ>0)
 {
  all=sqrt(squ):
  all=(-bb-all)/(2*aa):
  a12=(-bb+all)/(2*aa):
  al=ala+all*ama:
  gl=alg+all*amg:
  a2=ala+a12*ama;
  g2=alg+a12*amg:
  el=sqrt(alk+all*amk+al*al+gl*gl):
  e2=sqrt(alk+a12*amk+a2*a2+g2*g2):
  if(j==0) ( printf("       g        e       a")
        printf("    h          c     f\n"):   }
  if(j==l) (printf("%8.2f %8.2f %8.2f %8.2f %8.2f
  %8.2f \n",g12,a12,e12,gl,el,al):
  printf("%8.2f %8.2f %8.2f %8.2f %8.2f %8.2f \n",
  g12,a12,e12,g2,e2,a2): }
  if(j==2){printf("%8.2f %8.2f %8.2f %8.2f %8.2f
  %8.2f \n",g21,a21,e21,gl,el,al):
  printf("%8.2f %8.2f %8.2f %8.2f %8.2f %8.2f \n",
  g21,a21,e21,g2,e2,a2);}
  if (j==0)
  {
        g12=gl:
```

6 LOWER PAIRS



Introduction

In the chapter of mechanisms and machines, basic mechanisms with their inversions were introduced. In this chapter, some more mechanisms of the lower pair category will be discussed. Lower pairs usually comprise turning (pivoted) and sliding pairs. Mechanisms with pivoted links are widely used in machines and the required movements of links are produced by using them in a variety of forms and methods. In this chapter, some of the more common mechanisms will be studied. Pantographs are used to copy the curves on reduced or enlarged scales. Some pivoted-link mechanisms are used to guide reciprocating parts either exactly or approximately in straight paths to eliminate the friction of the straight guides of the sliding pairs. However, these days, sliders are also being used to get linear motions.

An exact straight-line mechanism guides a reciprocating part in an exact straight line. On the other hand, an approximate straight-line mechanism is designed in such a way that the middle and the two extreme positions of the guided point are in a straight line and the intermediate positions deviate as little as possible from the line.

Although this chapter will be restricted to the more elementary aspects of the analysis of mechanisms, the possibilities of their use in the mechanisms and the machine of daily use can easily be glimpsed. Moreover, systematic design techniques are being developed so that these mechanisms can be used for accurate control of the processes and the machines being needed in modern technology.

6.1 PANTOGRAPH

A pantograph is a four-bar linkage used to produce paths exactly similar to the ones traced out by a point on the linkage. The paths so produced are, usually, on an enlarged or reduced scale and may be straight or curved ones.

The four links of a pantograph are arranged in such a way that a parallelogram *ABCD* is formed (Fig. 6.1). Thus, $AB = DC$ and $BC = AD$. If some point *O* in one of the links is made fixed and three other points *P*, *Q* and *R* on the other three links are located in such a way that *OPQR* is a straight line, it can be shown that the points *P*, *Q* and *R* always move parallel and similar to each other over any path, straight or curved. Their motions will be proportional to their distances from the fixed point.

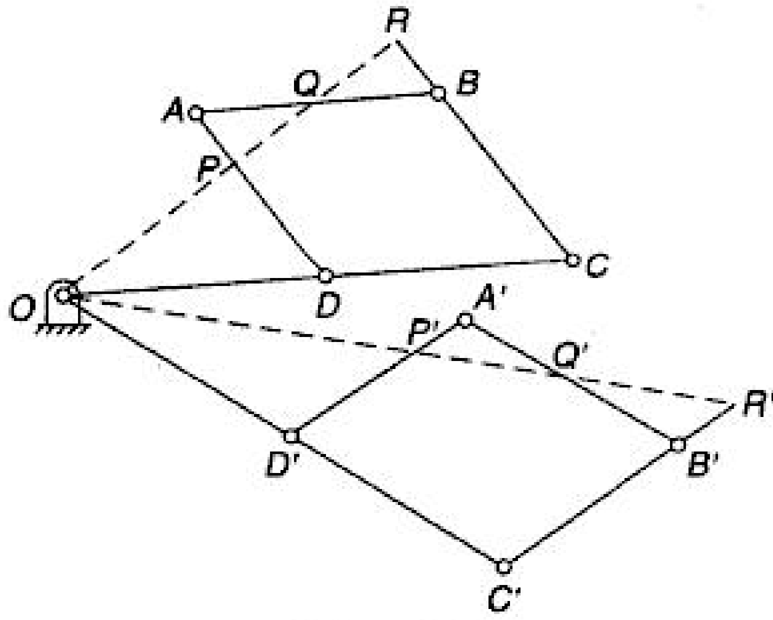


Fig. 6.1

In ΔABD $\dfrac{AE}{AB} = \dfrac{AQ}{AD}$ (Given)

Therefore, EQ is parallel to BD and thus parallel to AC.

In ΔABC $\dfrac{AE}{AB} = \dfrac{CP}{CB}$ (Given)

Therefore, EP is parallel to AC and thus parallel to BD.

Now, EQ and EP are both parallel to AC and BD and have a point E in common; therefore, EQP is a straight line.

Δs AEQ and ABD are similar ($\because EQ \parallel BD$).

$$\therefore \quad \frac{EQ}{BD} = \frac{AE}{AB} \text{ or } EQ = BD \times \frac{AE}{AB} \qquad \text{(ii)}$$

Δs BEP and BAC are similar ($\because EP \parallel AC$).

$$\therefore \quad \frac{EP}{AC} = \frac{BE}{BA} \text{ or } EP = AC \times \frac{BE}{AB} \qquad \text{(iii)}$$

Δs EQQ' and $EP'P$ are similar, because $\angle QEQ'$ or $\angle PEP'$ is common and $\angle EQQ' = \angle QP'P = 90°$.

$$\therefore \quad \frac{EQ}{EP'} = \frac{EQ'}{EP}$$

or
$$EQ' \times EP' = EQ \times EP$$
$$= \left(BD \times \frac{AE}{AB}\right)\left(AC \times \frac{BE}{AB}\right) \qquad \text{[from (ii) and (iii)]}$$

or
$$EP' = \frac{AE \times BE}{(EQ')(AB)^2}[(BD)(AC)]$$
$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BF + FD)(BF - FD)\right]$$
$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BF)^2 - (FD)^2\right]$$
$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[\{(BC)^2 - (CF)^2\} - \{(CD)^2 - (CF)^2\}\right]$$
$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BC)^2 - (CD)^2\right]$$

= constant, as all the parameters are fixed.

Thus, EP' is always constant. Therefore, the projection of P on EO produced is always the same point or P moves in a straight line perpendicular to EO.

Example 6.2 *A circle with EQ' as diameter has a point Q on its circumference. P is a point on EQ produced such that if Q turns about E, $EQ.EP$ is constant. Prove that the point P moves in a straight line perpendicular to EQ'.*

Solution Let PP' be perpendicular to EQ' produced (Fig. 6.5).

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$$yh - h^2 \tan\theta = hy - hx + y^2 \tan\theta - xy \tan\theta$$

$$hx = (y^2 - xy + h^2)\tan\theta$$

$$\tan\theta = \frac{hx}{y^2 - xy + h^2}$$

Also, $\tan(\alpha + \varphi) = \dfrac{y + x}{h}$

and it can be proved that $\tan\varphi = \dfrac{hx}{y^2 + xy + h^2}$

For correct steering action, $\cot\varphi - \cot\theta = \dfrac{w}{l}$

or

$$\frac{y^2 + xy + h^2}{hx} - \frac{y^2 - xy + h^2}{hx} = \frac{w}{l}$$

or

$$\frac{2xy}{hx} = \frac{w}{l}$$

or

$$\frac{y}{h} = \frac{w}{2l}$$

or

$$\tan\alpha = \frac{w}{2l} \qquad (6.2)$$

The usual value of w/l is between 0.4 to 0.5 and that of α from 11 or 14 degrees.

Example 6.5 *The ratio between the width of the front axle and that of the wheel base of a steering mechanism is 0.44. At the instant when the front inner wheel is turned by 18°, what should be the angle turned by the outer front wheel for perfect steering?*

Solution

$w/l = 0.44$ $\quad\theta = 18°$

As $\quad\cot\varphi - \cot\theta = \dfrac{w}{l}$

$\therefore\quad \cot\varphi - \cot 18° = 0.44$

$\cot\phi = 0.44 + 3.078 = 3.518$

or $\quad\phi = \underline{15.9°}$

Example 6.6 *The distance between the steering pivots of a Davis steering gear is 1.3 m. The wheel base is 2.75 m. What will be the inclination of the track arms to the longitudinal axis of the vehicle if it is moving in a straight path?*

Solution

$w = 1.3$ m $\qquad l = 2.75$ m

$$\tan\alpha = \frac{w}{2l} = \frac{1.3}{2 \times 2.75} = 0.236$$

$\therefore\quad \alpha = 13.3°$ or $13° 18'$

Example 6.7 *The track arm of a Davis steering gear is at a distance of 192 mm from the front main axle whereas the difference between their lengths is 96 mm. If the distance between steering pivots of the main axle is 1.4 m, determine the length of the chassis between the front and the rear wheels. Also, find the inclination of the track arms to the longitudinal axis of the vehicle.*

Solution

$w = 1.4$ m $\qquad h = 192$ mm $\qquad y = 96/2 = 48$ mm

$$\tan\alpha = \frac{y}{h} = \frac{48}{192} = 0.25$$

$\therefore\quad \alpha = \underline{14°}$

Also $\quad\tan\alpha = \dfrac{w}{2l}$

$\therefore\quad \tan 14° = \dfrac{1.4}{2l}$

or $\quad l = \underline{2.8 \text{ m}}$

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25. Figure 6.32 shows a Robert straight-line mechanism in which $ABCD$ is a four-bar linkage. The cranks AB and DC are equal and the connecting rod BC is one-half as long as the line of centres AD. P is a point rigidly attached to the connecting rod and lying on the midpoint of AD when BC is parallel to AD. Show that the point P moves in an approximately straight line for small displacement of the cranks.
(Note: *For better results take AB or $DC > 0.6\ AD$*)

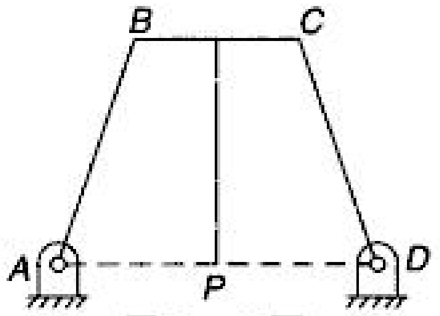

Fig. 6.32

26. In the Robert mechanism (Fig. 6.32) if $AB = BC = CD = AD/2$, locate the point P on the central vertical arm that approximately describes a straight line. (At a length 1.3 BC below BC)
27. In a Watt parallel motion (Fig. 6.10), the links OA and QB are perpendicular to the link AB in the mean position. The lengths of the moving links are OA = 120 mm, QB = 200 mm and AB = 175 mm.
Locate the position of a point P on AB to trace approximately a straight line motion. Also, trace the locus of P for all possible movements. (AP = 109.3 mm)
28. In a Watt mechanism of the type shown in Fig. 6.33, the links OA and QB are perpendicular to the link AB in the mean position. If OA = 45 mm, QB = 90 mm and AB = 60 mm, find the point P on the link AB produced for approximate straight-line motion of point P.

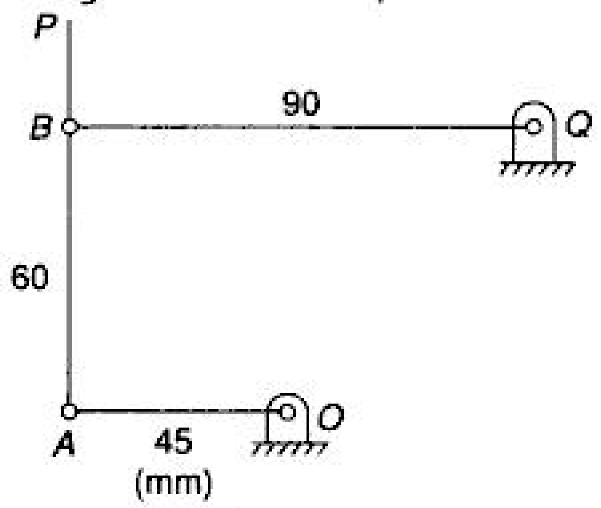

Fig. 6.33

(AP = 120 mm)

29. In a Davis steering gear, the length of the car between axles is 2.4 m, and the steering pivots are 1.35 m apart. Determine the inclination of the track arms to the longitudinal axis of the car when the car moves in a straight path. (15°42′)
30. In a Hooke's joint, the angle between the two shafts is 15°. Find the angles turned by the driving shaft when the velocity of the driven shaft is maximum, minimum and equal to that of the driving shaft. Also, determine when the driven shaft will have the maximum acceleration and retardation.
(Max. vel. at 0° and 180°; min. at 90° and 270°; equal to 44°30′. 135°30′, 224°30′ and 315°30′; Max. acc. at 137° and 317°; and Max. ret. at 43° and 223°)
31. The driving shaft of a Hooke's joint has a uniform angular speed of 280 rpm. Determine the maximum permissible angle between the axes of the shafts to permit a maximum variation in speed of the driven shaft by 8% of the mean speed. (22.6°)
32. The two shafts of a Hooke's coupling have their axes inclined at 20°. The shaft A revolves at a uniform speed of 1000 rpm. The shaft B carries a flywheel of mass 30 kg. If the radius of gyration of the flywheel is 100 mm, find the maximum torque in shaft B. (411 N.m)
33. In a double universal coupling joining two shafts, the intermediate shaft is inclined at 10° to each. The input and the output forks on the intermediate shaft have been assembled inadvertently at 90° to one another. Determine the maximum and the least velocities of the output shaft if the speed of the input shaft is 500 rpm. Also, find the coefficient of fluctuation in speed. (515.5 rpm; 484.9 rpm; 0.06)

1. ***Rise-Return-Rise*** **(R-R-R)** In this, there is alternate rise and return of the follower with no periods of dwells (Fig. 7.9a). Its use is very limited in the industry. The follower has a linear or an angular displacement.
2. ***Dwell-Rise-Return-Dwell (D-R-R-D)*** In such a type of cam, there is rise and return of the follower after a dwell [Fig. 7.9(b)]. This type is used more frequently than the R-R-R type of cam.
3. ***Dwell-Rise-Dwell-Return-Dwell*** **(D-R-D-R-D)** It is the most widely used type of cam. The dwelling of the cam is followed by rise and dwell and subsequently by return and dwell as shown in Fig. 7.9(c). In case the return of the follower is by a fall [Fig. 7.9(d)], the motion may be known as Dwell-Rise-Dwell (D-R-D).

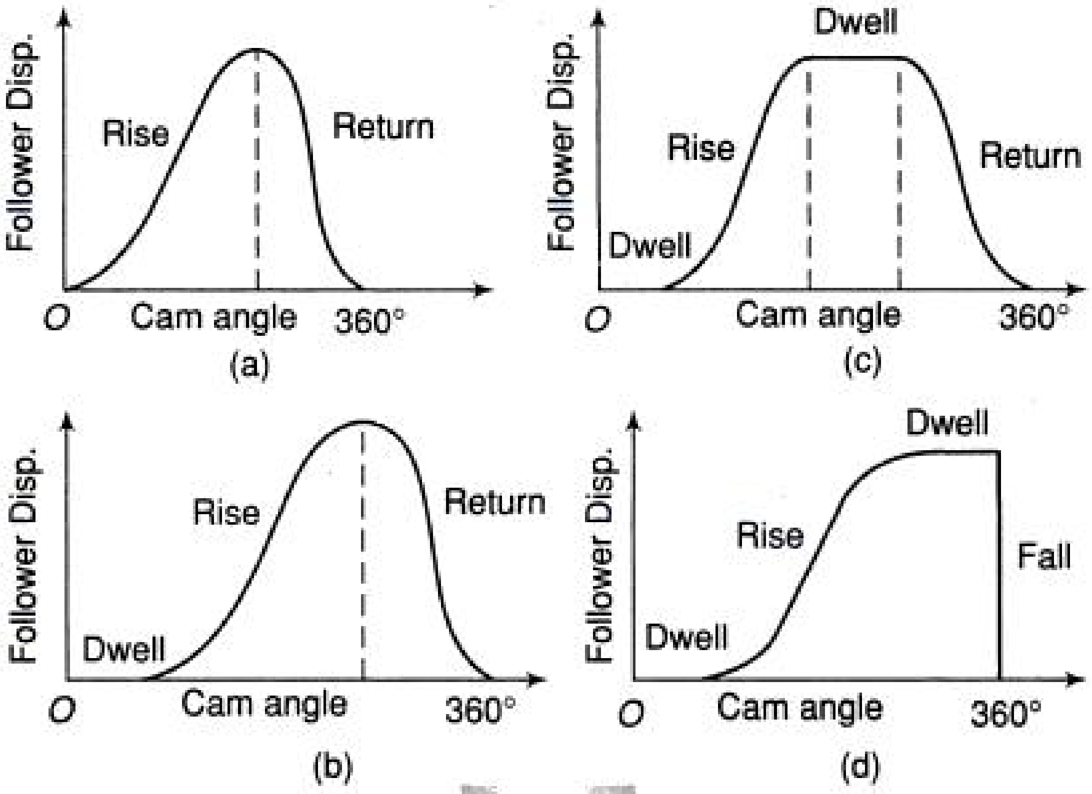

Fig. 7.9

According to Manner of Constraint of the Follower

To reproduce exactly the motion transmitted by the cam to the follower, it is necessary that the two remain in touch at all speeds and at all times. The cams can be classified according to the manner in which this is achieved.

1. ***Pre-loaded Spring Cam*** A pre-loaded compression spring is used for the purpose of keeping the contact between the cam and the follower [Figs 7.1(a) and (b), 7.3(a), 7.5(b) and 7.8].
2. ***Positive-drive Cam*** In this type, constant touch between the cam and the follower is maintained by a roller follower operating in the groove of a cam [Figs 7.2, 7.3(b). 7.4, 7.5(a) and 7.7]. The follower cannot go out of this groove under the normal working operations. A constrained or positive drive is also obtained by the use of a conjugate cam (Fig. 7.6).
3. ***Gravity Cam*** If the rise of the cam is achieved by the rising surface of the cam and the return by the force of gravity or due to the weight of the cam, the cam is known as a gravity cam. Figure 7.2(c) shows such a cam. However, these cams are not preferred due to their uncertain behaviour.

7.2 TYPES OF FOLLOWERS

Cam followers are classified according to the

1. shape,
2. movement, and
3. location of line of movement.

According to Shape

1. ***Knife-edge Follower*** It is quite simple in construction. Figure 7.1(a) shows such a follower. However, its use is limited as it produces a great wear of the surface at the point of contact.
2. ***Roller Follower*** It is a widely used cam follower and has a cylindrical roller free to rotate about a pin joint [Figs 7.1(b), 7.2, 7.5, 7.8]. At low speeds, the follower has a pure rolling action, but at high speeds, some sliding also occurs.

The second derivative is

$$\ddot{s} = \frac{d^2 s}{dt^2} = \omega^2 \frac{d^2 s}{d\theta^2}$$

It represents the *acceleration* of the follower. A higher value of acceleration means a higher inertia force. A third derivative is known as the *jerk*.

$$\dddot{s} = \frac{d^3 s}{dt^3} = \omega^3 \frac{d^3 s}{d\theta^3}$$

For smooth movement of the follower, even the high values of the jerk are undesirable in case of high-speed cams.

7.6 HIGH-SPEED CAMS

A real follower always has some mass and when multiplied by acceleration, inertia force of the follower is obtained. This force is always felt at the contact point of the follower with the cam surface and at the bearings. An acceleration curve with abrupt changes exerts abrupt stresses on the cam surfaces and at the bearings accompanied by detrimental effects such as surface wear and noise. All this may lead to an early failure of the cam system. Thus, it is very important to give due consideration to velocity and acceleration curves while choosing a displacement diagram. They should not have any step changes.

In low-speed applications, cams with discontinuous acceleration characteristics may not show any undesirable characteristic, but at higher speeds such cams are certainly bound to show the same. The higher the speed, the higher is the need for smooth curves. At very high speeds, even the jerk (related to rate of change of acceleration or force) is made continuous as well. For most of the applications, however, this may not be needed. In Section 7.8, standard cam motions have been discussed from which some comparison can easily be made for suitable selection.

7.7 UNDERCUTTING

Sometimes, it may happen that the prime circle of a cam is proportioned to provide a satisfactory pressure angle; still the follower may not be completing the desired motion. This can happen if the curvature of the pitch curve is too sharp.

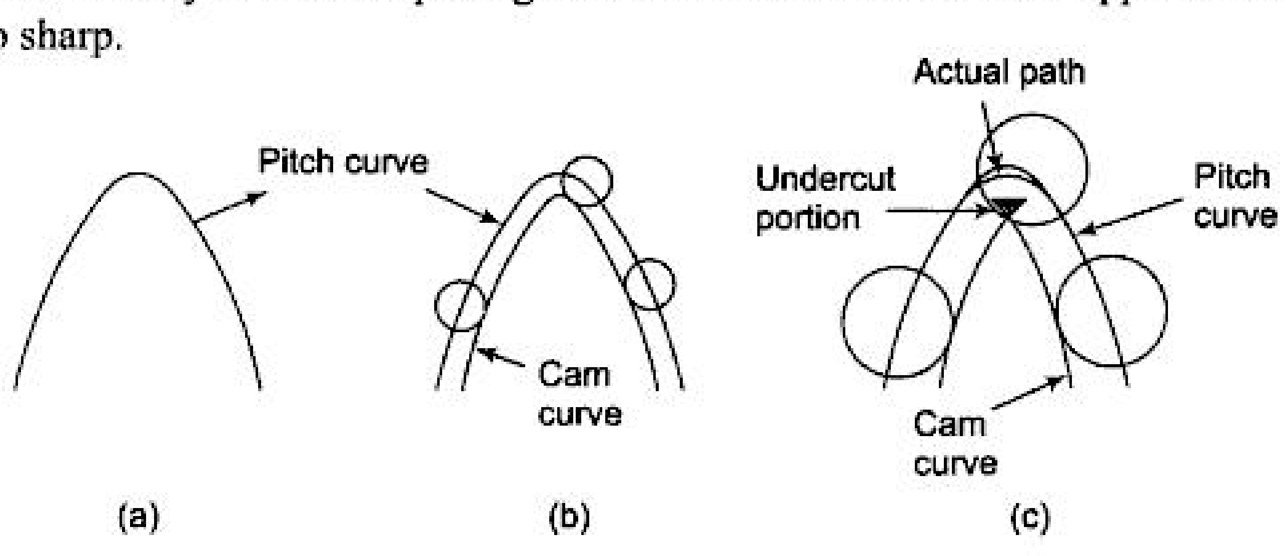

Fig. 7.14

Figure 7.14(a) shows the pitch curve of a cam. In Fig. 7.14(b), a roller follower is shown generating this curve. In Fig. 7.14(c), a larger roller is shown trying to generate this curve. It can easily be observed that the

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7.9 LAYOUT OF CAM PROFILES

A cam profile is constructed on the principle of kinematic inversion, i.e., considering the cam to be stationary and the follower to be rotating about it in the opposite direction of the cam rotation. In general, the following procedure is adopted for laying out the profile for a reciprocating follower:

1. Draw the displacement diagram of the follower according to the given follower motion by dividing the cam displacement interval into n equal parts as has been discussed in the previous section. The usual number taken is 6, 8, 10 or 12 depending upon the angular displacement and convenience. Remember that the scale of the displacement interval does not affect the cam profile whereas the follower displacement does.
2. Draw the prime circle of the cam with radius
   (a) r_c if it is a knife-edge or mushroom follower (Ex. 7.1, 7.2 and 7.5)
   (b) $r_c + r_r$ if it is a roller follower (Ex. 7.3 and 7.4)
3. Divide the prime circle into segments as follows:
   (a) In case of a radial follower, divide the circle from the vertical position indicating the angles of ascent, dwell period and angle of descent, etc., in the opposite direction of the cam rotation (Ex. 7.1, 7.3 and 7.5).
   (b) In case of an offset follower, draw another circle with radius equal to the offset of the follower and assume the initial position on the prime circle where the tangent to the horizontal radius of the circle meets the prime circle (Ex. 7.2 and 7.4).
4. Further, divide each segment of ascent and descent into the same number of angular parts as is done in the displacement diagram.
5. On the radial lines produced, mark distances equal to the lift of the follower beyond the circumference of the prime circle (Ex. 7.1, 7.3 and 7.5). In case of offset follower, the distances are marked on the tangents drawn to the circle with radius equal to the offset (Ex. 7.2 and 7.4). It can be visualized that with rotation of the cam, each radial or tangential line so obtained merges with the axis of the follower at successive intervals of time and the marked points are the various positions of the tracing point of the follower.
6. Obtain the cam profile as follows:
   (a) For a knife-edge follower, draw a smooth curve passing through the marked points which is the required cam profile (Ex. 7.1 and 7.2).
   (b) In case of a roller follower, draw a series of arcs of radii equal to r_r on the inner side and draw a smooth curve tangential to all the arcs to get the required cam profile (Ex. 7.3 and 7.4).
   (c) For a mushroom follower, draw the follower in all the positions by drawing perpendiculars to the radial or tangent lines and draw a smooth curve tangential to the flat-faces of the follower representing the cam profile (Ex. 7.5).

Example 7.1 *Draw the profile of a cam operating a knife-edge follower having a lift of 30 mm. The cam raises the follower with SHM for 150° of the rotation followed by a period of dwell for 60°. The follower descends for the next 100° rotation of the cam with uniform velocity, again followed by a dwell period. The cam rotates at a uniform velocity of 120 rpm and has a least radius of 20 mm. What will be the maximum velocity and acceleration of the follower during the lift and the return?*

Solution:

$h = 30$ mm $\varphi_a = 150°$
$N = 120$ rpm $\delta_1 = 60°$

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these arcs as shown in the diagram. It is on the assumption that for small angular displacements, the linear displacements on the arcs and on the straight lines are the same.

(viii) With 1′, 2′, 3′, etc., draw a series of arcs of radii equal of r_r.

(ix) Draw a smooth curve tangential to all the arcs and obtain the required cam profile.

7.10 CAMS WITH SPECIFIED CONTOURS

It is always desired that a cam is made to provide a smooth motion of the follower and for that a follower motion programme is always selected first. However, sometimes it becomes difficult to manufacture the cams in large quantities of the specified contours. Under such circumstances, it becomes necessary that the cam is designed first and then some improvements are made in that if possible. Such cams are generally made up of some combination of curves such as straight lines, circular arcs, etc. In the present section, some cams with specified contours are analysed.

1. Tangent Cam (with Roller Follower)

A tangent cam is symmetrical about the centre line. It has straight flanks (such as AK in Fig. 7.31) with a circular nose. The centre of the cam is at O and that of the nose at Q. A tangent cam is used with a roller cam since there is no meaning of using flat-faced followers with straight flanks.

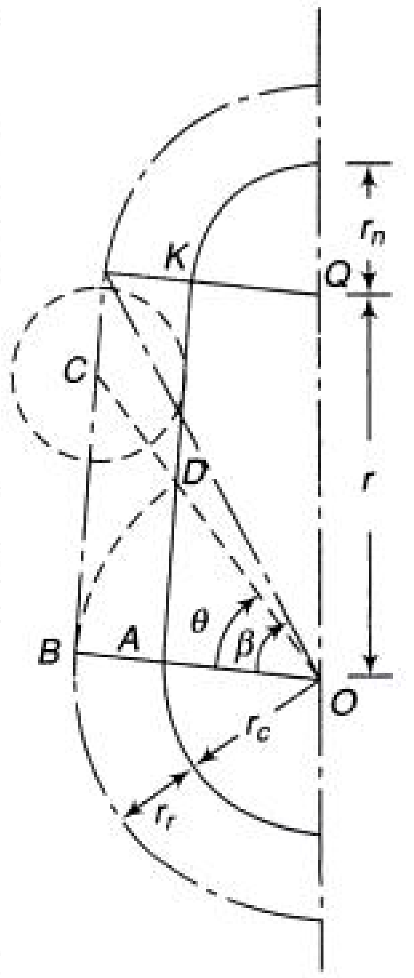

Fig. 7.31

Let r_c = least radius of cam

r_n = radius of nose

r_r = radius of roller

r = distance between the cam and the nose centres.

Roller on the Flank When the roller is on the straight flank, the centre of the roller is at C on the pitch profile as shown in Fig. 7.31.

Let θ = angle turned by the cam from the beginning of the follower motion

Let,

$$x = OC - OD = OC - OB = \frac{OB}{\cos\theta} - OB = OB\left(\frac{1}{\cos\theta} - 1\right)$$

or

$$x = (r_c + r_r)\left(\frac{1}{\cos\theta} - 1\right) \quad (7.17)$$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\,\frac{d\theta}{dt} = (r_c + r_r)\left(\frac{\sin\theta}{\cos^2\theta} - 0\right)\omega$$

$$= \omega\,(r_c + r_r)\frac{\sin\theta}{\cos^2\theta} \quad (7.18)$$

$\sin\theta$ increases with the increase in θ whereas $\cos\theta$ decreases. Hence the velocity increases with θ and it is maximum when θ is maximum. This will happen when the point of contact leaves the straight flank.

Let β = angle turned by the cam when the roller leaves the flank.

$$\therefore \quad v_{max} = \omega\,(r_c + r_r)\frac{\sin\beta}{\cos^2\beta} \text{ and } v_{min} = 0 \text{ at } \theta = 0$$

The same result is obtained if relation of Eq. (7.28) is used, i.e.,

$$v_{max} = \omega r \sin(\alpha - \theta)$$

Maximum acceleration is when $\theta = 0$,

$f_{max} = \omega^2 (r_f - r_c) = (157.08)^2 (92.2 - 25)$
$= 1658\ 090$ mm/s² or 1658.09 m/s²

Maximum retardation is when $\alpha - \theta = 0$,

$f_{max} = \omega^2 r = (157.08)^2 \times 34 = 838\ 920$ mm/s² or 838.92 m/s²

Spring force is needed to maintain contact during the retardation of the follower.

Minimum force, $F = m \times f$
$= 0.3 \times 838.92 = 251.7$ N

7.11 ANALYSIS OF A RIGID ECCENTRIC CAM

Analysis of a rigid eccentric cam involves the determination of the contact force, the spring force and the cam shaft torque for one revolution of the cam. In simplified analysis, all the components of the cam system are assumed to be rigid and the results are applicable to low-speed systems. However, if the speeds are high and the members are elastic, an elastic body analysis must be made. The elasticity of the members may be due to extreme length of the follower or due to use of elastic materials in the system. In such cases, noise, excessive wear, chatter, fatigue failure of some of the parts are the usual things.

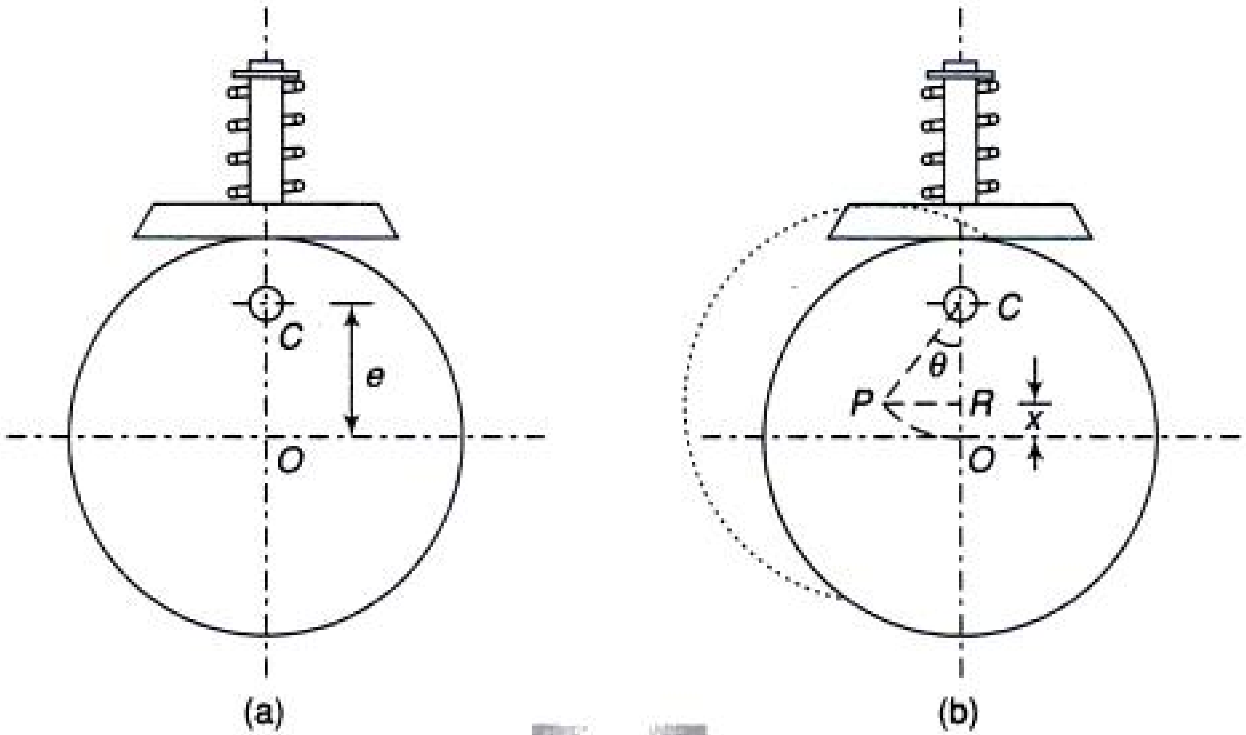

Fig. 7.40

A circular disc cam with the cam shaft hole drilled off centre is known as an *eccentric plate cam*. Figure 7.40(a) shows a simplified reciprocating eccentric cam system consisting of a plate cam, a flat face follower and a retaining spring.

Let e = eccentricity, the distance between the centre of the disc and of the shaft
m = mass of the follower
s = stiffness of the retaining spring
ω = angular velocity of the cam rotation
P = preload including the weight of the follower or the force on the cam at $x = 0$
x = motion of the follower (zero at the bottom of the stroke)

If the disc is rotated through an angle θ [Fig. 7.40(b)], the mass m is displaced by a distance x, so that

$$x = e - e\cos\theta \tag{7.33}$$

Velocity,
$$\dot{x} = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = e\omega\sin\theta \tag{7.34}$$

Acceleration,
$$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{d\theta}\frac{d\theta}{dt} = e\omega^2\cos\theta \tag{7.35}$$

Now, as the follower is displaced in the upward direction, the acceleration $\ddot{x}$ of the follower is in the upward direction indicating the inertia force to be in the downward direction. The spring force is also exerted in the downward direction. The force exerted by the cam on the follower F, however, will be in the upward direction.

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When $t = 0$, $x = \dot{x} = 0$ and $y = 0$

$\therefore$ from (i), $0 = A + 0 + 0$

or $A = 0$

From (ii),
$$0 = 0 + B\omega_n + \frac{s_2}{m\omega_n^{\ 2}}\dot{y}$$
$$B = -\frac{s_2}{m\omega_n^{\ 2}}\dot{y}$$

$\therefore$ (i) becomes,
$$x = 0 - \frac{s_2}{m\omega_n^2}\dot{y}\sin\omega_n t + \frac{s_2}{m\omega_n^2}y \tag{7.40}$$

The particular integral $\frac{s_2}{m\omega_n^{\ 2}}y$ is called the *follower command.*

$$x = \frac{s_2}{m\omega_n^{\ 2}}\left(y - \frac{\dot{y}}{\omega_n}\sin\omega_n t\right)$$
$$= \frac{s_2}{m\omega_n^{\ 2}}\left(\frac{h}{\varphi}\theta - \frac{h\omega}{\varphi\omega_n}\sin\omega_n t\right) \quad \cdots\left(\dot{y} = \frac{h}{\varphi}\omega\right)$$
$$\therefore \quad x = \frac{s_2 h}{m\omega_n^{\ 2}\varphi}\left(\omega t - \frac{\omega}{\omega_n}\sin\omega_n t\right)$$
$$= \frac{\omega \cdot s_2 h}{m\omega_n^2\varphi}\left(t - \frac{\sin\omega_n t}{\omega_n}\right)$$
$$= \frac{s_2 h}{rm\omega_n\varphi}\left(t - \frac{\sin\omega_n t}{\omega_n}\right)$$

where $r = \omega_n/\omega$

$$\dot{x} = \frac{s_2 h}{rm\omega_n\varphi}\left(1 - \frac{\omega_n}{\omega_n}\cos\omega_n t\right)$$
$$= \frac{s_2 h}{rm\omega_n\varphi}(1 - \cos r\theta)$$

At the peak,
$$x = x_1,\ \theta = \varphi$$
$$x_1 = \frac{s_2 h}{rm\omega_n^{\ 2}\varphi}(r\varphi - \sin r\varphi)$$
$$\dot{x}_1 = \frac{s_2 h}{rm\omega_n\varphi}(1 - \cos\varphi)$$

These become the initial conditions for dwell period.

For dwell period,
$$x = A\cos\omega_n t + B\sin\omega_n t + \frac{s_2}{m\omega_n^{\ 2}}h$$

Roller diameter = 44 mm
Angle of ascent = 60°

Calculate the acceleration of a follower at the beginning of the lift. Also, find its values when the roller just touches the nose and is at the apex of the circular nose. Sketch the variation of displacement, velocity and acceleration during ascent.

(88.12 m/s^2; 164 m/s^2; and –92.6 m/s^2; –111 m/s^2)

23. A flat-ended valve tappet is operated by a symmetrical cam with circular arcs for flank and nose profiles. The total angle of action is 150°, base circle diameter is 125 mm and the lift is 25 mm. During the lift, the period of acceleration is half that of the declaration. The speed of cam shift is 1250 rpm. The straight-line path of the tappet passes through the cam axis. Find
    (i) radii of the nose and the flank, and
    (ii) maximum acceleration and declaration during the lift.

(40.3 mm, 148 mm; 1465 m/s^2; 808.8 m/s^2)

24. In a four-stroke petrol engine, the crank angle is 5° after t.d.c. when the suction valve opens and 53° after b.d.c. when the suction valve closes. The lift is 8 mm, the nose radius is 3 mm and the least radius of the cam is 18 mm. The shaft rotates at 800 rpm. The cam is of the circular type with a circular nose and flanks while the follower is flat-faced. Determine the maximum velocity and the maximum acceleration and retardation of the valve. When is the minimum force exerted by the springs to overcome the inertia of moving parts weighting 250 g.

(1.3 m/s; 433.7 m/s; 161.4 m/s^2; 40.35 N)

25. A symmetrical circular cam operates a flat-faced follower with a lift of 30 mm. The minimum radius of the cam is 50 mm and the nose radius is 12 mm. The angle of lift is 80°. If the speed of the cam is 210 rpm, find the main dimensions of the cam and the acceleration of the follower at (i) the beginning of the lift (ii) the end of contact with the circular flank (iii) the beginning of contact with the nose, and (iv) the apex of nose.

(r = 68 mm, 29.38 m/s^2, 23.8 m/s^2, 26.2 m/s^2, 32.9 m/s^2)

26. A circular disc cam with diameter of 80 mm with its centre displaced at30 mm from the camshaft is used with a flat surface follower. The line of action of the follower is vertical and passes through the shaft axis. The mass of the follower is 2.5 kg and is pressed downwards with a spring of stiffness 4 N/mm. In the lowest position, the spring force is 50 N. Derive an expression for the acceleration of the follower as a function of cam rotation from the lowest position of the follower. Also, find the speed at which the follower begins to lift from the cam surface.

(618.4 rpm)

$$= \frac{\sin\alpha}{\sin(\alpha+\theta)} \frac{\sin\theta\sin(\alpha+\theta)\left[\frac{\cos(\alpha+\varphi)}{\sin(\alpha+\varphi)} - \frac{\cos\theta}{\sin\theta}\right]}{\sin\theta\sin\alpha\left[\frac{\cos\alpha}{\sin\alpha} - \frac{\cos\theta}{\sin\theta}\right]}$$

$$= \frac{\cot(\alpha+\varphi) - \cot\theta}{\cot\alpha - \cot\theta} \tag{8.5}$$

If $\theta = 90°$, i.e., if the direction of the applied force is horizontal,

$$\eta = \frac{\cot(\alpha+\varphi) - \cot 90°}{\cot\alpha - \cot 90°}$$

$$= \frac{\cot(\alpha+\varphi)}{\cot\alpha}$$

$$= \frac{\tan\alpha}{\tan(\alpha+\varphi)} \tag{8.6}$$

3. Motion Down the Plane

When the body moves down the plane, the force of friction $\mathbf{F}'$ ($=\mu\mathbf{R}_n$) acts in the upwards direction and the reaction $\mathbf{R}$, i.e., the combination of $\mathbf{R}_n$ and $\mathbf{F}'$ is inclined backwards as shown in Fig. 8.5(a). Assume that $\mathbf{F}$ acts downwards.

Applying Lami's theorem as before [Fig. 8.5(b)],

$$\frac{F}{\sin[\pi - (\varphi - \alpha)]} = \frac{W}{\sin[\theta + (\varphi - \alpha)]}$$

$$F = \frac{W\sin(\varphi - \alpha)}{\sin[\theta + (\varphi - \alpha)]} \tag{8.7}$$

The equation suggests that F is positive only for $\varphi > \alpha$ and when $\varphi = \alpha$, the force required to slide the body down is zero, i.e., the body is on the point of moving down under its own weight W.

When $\varphi < \alpha$, i.e., the angle of friction is lesser than the angle of the inclined plane, F will be negative meaning that a force equal to F is to be applied in the opposite direction to resist the motion.

However, for a given value of α, F is minimum when the denominator of Eq. (8.7) is maximum

$$F_{min} = W\sin(\varphi - \alpha) \tag{8.8}$$

If friction is neglected, i.e., $\varphi = 0$.

$$F_o = \frac{W\sin(-\alpha)}{\sin(\theta - \alpha)} = \frac{-W\sin\alpha}{\sin(\theta - \alpha)} \tag{8.9}$$

The force is negative indicating that in the absence of force of friction,

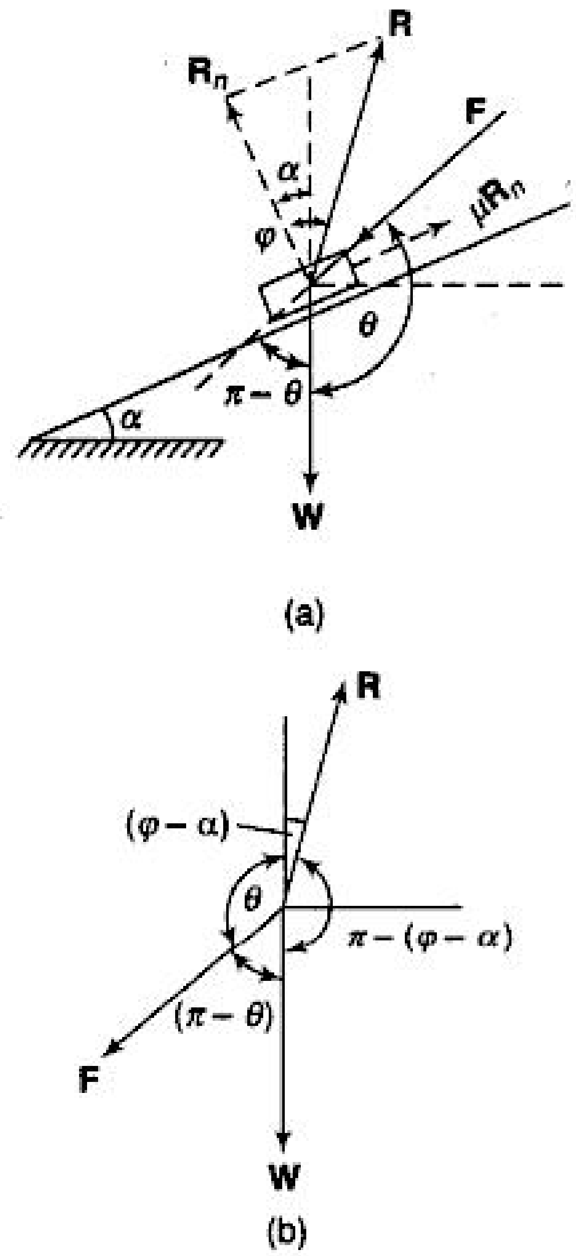

Fig. 8.5

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11. The reversal of the nut is avoided if the efficiency of the thread is less than 50%
12. A wedge is used to raise loads like a screw jack.
13. Efficiency of a wedge

$$= \frac{\cos\varphi' \tan\alpha}{\sin(\alpha + 2\varphi')} \times \frac{\cos(\alpha + \varphi + \varphi')}{\cos\varphi}$$

If $\varphi = \varphi'$, $\eta = \frac{\tan\alpha}{\tan(\alpha + 2\varphi)}$

14. A collar bearing or simply a collar is provided at any position along the shaft and bears the axial load on a mating surface.
15. When the axial load is taken by the end of the shaft which is inserted in a recess to bear the thrust, it is called a *pivot bearing* or simply a *pivot*. It is also known as *footstep bearing*.
16. In uniform pressure theory, pressure is assumed to be uniform over the surface area.
17. For uniform wear over an area, the intensity of pressure varies inversely proportional to the elementary areas and the product of the normal pressure and the corresponding radius is constant. Pressure intensity p at a radius r of the collar,

$$p = \frac{F}{2\pi r(R_o - R_i)}$$

18. For flat collars, friction torque is

$$T = \frac{2\mu F(R_o^3 - R_i^3)}{3(R_o^2 - R_i^2)} \text{ with uniform pressure theory}$$

$$= \frac{\mu F}{2}(R_o^2 + R_i^2) \text{ with uniform wear theory}$$

19. For conical collars, friction torque is increased by $1/\sin\alpha$ times from that for flat collars.
20. Expressions for torque in case of pivots can directly be obtained from the expressions for collars by inserting the values $R_i = 0$ and $R_o = R$.
21. To be on safer side, friction torque in clutches is calculated on the basis of uniform wear theory and in bearings on the basis of uniform pressure theory.
22. A clutch is a device used to transmit the rotary motion of one shaft to another when desired. The axes of the two shafts are coincident.
23. In a multi-plate clutch, the number of frictional linings and the metal plates is increased which increases the capacity of the clutch to transmit torque.
24. Ball and roller bearings are known as *anti-friction bearings*.
25. The property of a lubricant to form a layer of molecular thickness (adsorbed film) on a metallic surface is known as its *oiliness*.
26. Greasy or boundary friction occurs in heavily loaded, slow-running bearings.
27. A circle drawn with μr as radius is known as the *friction circle* of the journal.
28. Friction couple (torque) = $Wr\mu$
29. During rotation, the positions of the crank where the reaction at the crankshaft bearing and the friction axis of the connecting rod become aligned in the same straight line are known as *dead centre positions*.
30. For a journal rotating in a bearing under the film lubrication conditions, the frictional resistance is proportional to the area, the viscosity of the lubricant, the speed and is independent of the pressure and the materials of the journal and the bearing.

Exercises

1. What is friction? Is it a blessing or curse? Justify your answer giving examples.
2. What are various kinds of friction? Discuss each in brief.
3. What are the laws of solid dry friction?
4. Define the terms *coefficient of friction* and *limiting angle of friction*.
5. Deduce an expression for the efficiency of an inclined plane when a body moves
   (i) up a plane
   (ii) down a plane
6. Find expression for the screw efficiency of a square thread. Also, determine the condition for maximum efficiency.
7. Show that the reversal of the nut is avoided if the efficiency of square thread is less than 50% (approximately).
8. What is a wedge? Deduce an expression for its efficiency.
9. What are uniform pressure and uniform wear theories? Deduce expressions for the friction torque considering both the theories for a flat collar.
10. In what way are the expressions for the friction

effective radius of rotation of a pulley is obtained by adding half the belt thickness to the radius of the pulley.

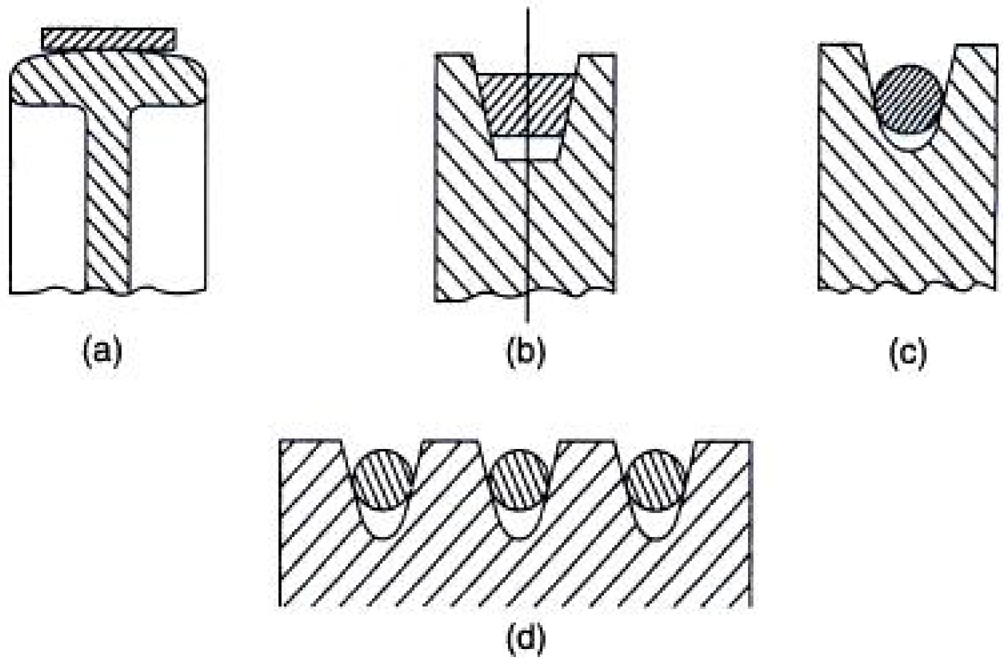

Fig. 9.2

Belt Drive A belt may be of rectangular section, known as a *flat belt* [Fig. 9.2(a)] or of trapezoidal section, known as a *V-belt* [Fig. 9.2(b)]. In case of a flat belt, the rim of the pulley is slightly *crowned* which helps to keep the belt running centrally on the pulley rim. The groove on the rim of the pulley of a V-belt drive is made deeper to take the advantage of the wedge action. The belt does not touch the bottom of the groove. Owing to wedging action, V-belts need little adjustment and transmit more power, without slip, as compared to flat belts. Also, a multiple V-belt system, using more than one belt in the two pulleys, can be used to increase the power transmitting capacity. Generally, these are more suitable for shorter centre distances.

Some advantages of V-belts are

- Positive drive as slip between belt and pulley is negligible
- No joint troubles as V-belts are made endless
- Operation is smooth and quite
- High velocity ratio up to 10 can be obtained
- Due to wedging action in the grooves, limiting ratio of tensions is higher and thus, more power transmission
- Multiple V-belt drive increases the power transmission manifold
- May be operated in either direction with tight side at the top or bottom
- Can be easily installed and removed.

Disadvantages of V-belts are

- Cannot be used for large centre distances
- Construction of pulleys is not simple
- Not as durable as flat belts
- Costlier as compared to flat belts.

Rope Drive For power transmission by ropes, grooved pulleys are used [Fig. 9.2(c)]. The rope is gripped on its sides as it bends down in the groove reducing the chances of slipping. Pulleys with several grooves can also be employed to increase the capacity of power transmission [Fig. 9.2(d)]. These may be connected in either of the two ways:

1. Using a continuous rope passing from one pulley to the other and back again to the same pulley in the next groove, and so on.
2. Using one rope for each pair of grooves.

The advantage of using continuous rope is that the tension in it is uniformly distributed. However, in case of belt failure, the whole drive is put out of action. Using one rope for each groove poses difficulty in tightening the ropes to the same extent but with the advantage that the system can continue its operation even if a rope fails. The repair can be undertaken when it is convenient.

Rope drives are, usually, preferred for long centre distances between the shafts, ropes being cheaper as compared to belts. These days, however, long distances are avoided and thus, the use of ropes has been limited.

9.6 MATERIAL FOR BELTS AND ROPES

Choice of materials for the belts and ropes is influenced by climate or environmental conditions along with the service requirements. The common materials are as given below:

1. Flat Belts

Usual materials for flat belts are leather, canvas, cotton and rubber. These belts are used to connect shafts up to 8–10 m apart with speeds as high as 22 m/s.

Leather belts are made from 1.2 to 1.5 m long strips. The thickness of a belt may be increased by cementing the strips together. The belts are specified by the number of layers, i.e., single, double or triple ply. The leather belts are cleaned and dressed periodically with suitable oils to keep them soft and flexible.

Fabric belts are made by folding cotton or canvas layers to three or more layers and stitching together. The belts are made waterproof by impregnating with linseed oil. These are mostly used in belt conveyors and farm machinery.

Rubber belts are very flexible and are destroyed quickly on coming in contact with heat, grease or oil. Usually, these are made endless. Rubber belts are used in paper and saw mills as these can withstand moisture.

2. V-Belts

These are made of rubber impregnated fabric with the angle of V between 30 to 40 degrees. These are used to connect shafts up to 4 m apart. Speed ratios can be up to 7 to 1 and belt speeds up to 24 m/s.

3. Ropes

The materials for ropes are cotton, hemp, manila or wire. Ropes may be used to connect shafts up to 30 m apart with operating speed less than 3 m/s.

Hemp and *manila* fibres are rough and thus, the ropes made from such materials are not very flexible. Manila ropes are stronger as compared to hemp ropes. Generally, the rope fibres are lubricated with tar, tallow or graphite to prevent sliding of fibres when the ropes are bent over the pulleys. The cotton ropes are soft and smooth and do not require lubrication. These are not as strong and durable as manila ropes.

Wire ropes are used when the power transmitted is large over long distances, may be up to 150 m such as cranes, conveyors, elevators, etc. Wire ropes are lighter in weight, have silent operation, do not fail suddenly, more reliable and durable, less costly and can withstand shock loads.

9.7 CROWING OF PULLEYS

As mentioned is Section 9.2, the rim of the pulley of a flat-belt drive is slightly crowned to prevent the slipping off the belt from the pulley. The crowing can be in the form of conical surface or a convex surface.

Assume that somehow a belt comes over the conical portion of the pulley and takes the position as shown in Fig. 9.5(a), i.e., its centre line remains in a plane, the belt will touch the rim surface at its one edge only. This is impractical. Owing to the pull, the belt always tends to stick to the rim surface. The belt also has a lateral stiffness. Thus, a belt has to bend in the way shown in Fig. 9.5(b) to be on the conical surface of the pulley.

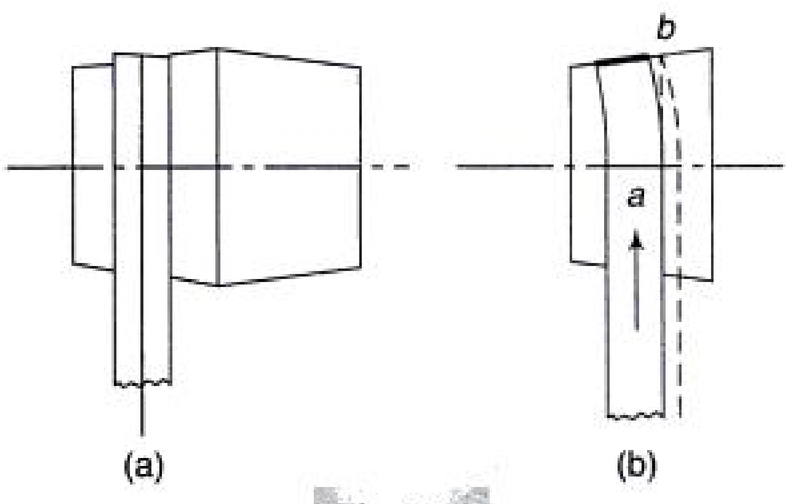

Fig. 9.5

As CD is tangent to two circles, AC and BD both are perpendicular to CD or AN.

Now, $AB \perp BK$ and $AN \perp BD$.

$\therefore \quad \angle DBK = \angle NAB = \beta$

Similarly, as $BA \perp AJ$, $NA \perp AC$

$\angle CAJ = \angle NAB = \beta$

$$L_o = 2\,[\text{Arc } GC + CD + \text{arc } DH]$$

$$= 2\left[\left(\frac{\pi}{2} - \beta\right)r + AN + \left(\frac{\pi}{2} + \beta\right)R\right]$$

$$= 2\left[\left(\frac{\pi}{2} - \beta\right)r + C\cos\beta + \left(\frac{\pi}{2} + \beta\right)R\right]$$

$$= \pi(R + r) + 2\beta(R - r) + 2C\cos\beta \qquad (9.4)$$

This relation gives the exact length of belt required for an open belt drive. In this relation,

$$\beta = \sin^{-}\left(\frac{R - r}{C}\right) \qquad (9.5)$$

An approximate relation for the length of belt can also be found in terms of R, r and C eliminating β, if β is small, i.e., if the difference in radii of the two pulleys is small and the centre distance is large.

For small angle of β, $\sin\beta \approx \beta$

$$\therefore \qquad \beta = \frac{R - r}{C}$$

and

$$\cos\beta = \sqrt{1 - \sin^2\beta}$$

$$= (1 - \sin^2\beta)^{1/2}$$

$$= \left(1 - \frac{1}{2}\sin^2\beta + \ldots\ldots\right) \qquad \text{[By binomial theorem]}$$

or

$$\cos\beta = \left(1 - \frac{1}{2}\beta^2\right) = 1 - \frac{1}{2}\left(\frac{R - r}{C}\right)^2$$

$$L_o = \pi(R + r) + 2\left(\frac{R - r}{C}\right)(R - r) + 2C\left[1 - \frac{1}{2}\left(\frac{R - r}{C}\right)^2\right]$$

$$= \pi(R + r) + 2\frac{(R - r)^2}{C} + 2C - \frac{2C}{2}\frac{(R - r)^2}{C^2}$$

$$= \pi(R + r) + 2\frac{(R - r)^2}{C} - \frac{(R - r)^2}{C} + 2C$$

$$= \pi(R + r) + \frac{(R - r)^2}{C} + 2C \qquad (9.6)$$

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9.16 INITIAL TENSION

When a belt is first fitted to a pair of pulleys, an initial tension T_0 is given to the belt when the system is stationary. When transmitting power, the tension on the tight side increases to T_1 and that on slack side decreases to T_2. If it is assumed that the material of the belt is perfectly elastic, i.e., the strain in the belt is proportional to stress in it and the total length of the belt remains unchanged, the tension on the tight side will increase by the same amount as the tension on the slack side decreases. If this change in the tension is δT then

tension on tight side, $T_1 = T_o + \delta T$
tension on slack side, $T_2 = T_o - \delta T$

$$\therefore \quad T_o = \frac{T_1 + T_2}{2}$$

= mean of the tight and the slack side tensions. (9.20)

Initial Tension with Centrifugal Tension

Total tension on tight side $= T_1 + T_c$
Total tension on slack side $= T_2 + T_c$

$$\therefore \quad T_o = \frac{(T_1 + T_c) + (T_2 + T_c)}{2}$$

$$= \frac{T_1 + T_2}{2} + T_c$$

or $\quad T_1 + T_2 = 2\,(T_o - T_c)$

Let $\quad \frac{T_1}{T_2} = e^{\mu\theta} = k$

Therefore,

$$kT_2 + T_2 = 2(T_o - T_c)$$

$$T_2 = \frac{2(T_o - T_c)}{k+1}$$

and

$$T_1 = \frac{2k(T_o - T_c)}{k+1} \qquad \text{(i)}$$

$$T_1 - T_2 = \frac{2k(T_o - T_c)}{k+1} - \frac{2(T_o - T_c)}{k+1}$$

$$= \frac{2(k-1)(T_o - T_c)}{k+1}$$

Power transmitted,

$$P = (T_1 - T_2).v$$

$$= \frac{2(k-1)(T_o - T_c)}{k+1}v = \frac{2(k-1)(T_o - mv^2)}{k+1}v$$

$$= \frac{2(k-1)(T_o v - mv^3)}{k+1}$$

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10.1 CLASSIFICATION OF GEARS

Gears can be classified according to the relative positions of their shaft axes as follows:

1. Parallel Shafts

Regardless of the manner of contact, uniform rotary motion between two parallel shafts is equivalent to the rolling of two cylinders, assuming no slipping. Depending upon the teeth of the equivalent cylinders, i.e., straight or helical, the following are the main types of gears to join parallel shafts:

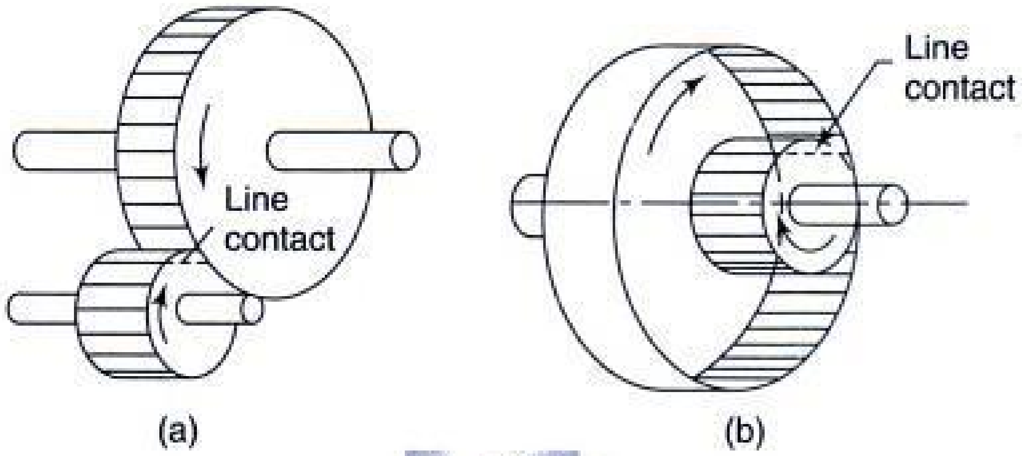

Fig. 10.2

Spur Gears They have straight teeth parallel to the axes and thus are not subjected to axial thrust due to tooth load [Fig. 10.2(a)].

At the time of engagement of the two gears, the contact extends across the entire width on a line parallel to the axes of rotation. This results in sudden application of the load, high impact stresses and excessive noise at high speeds.

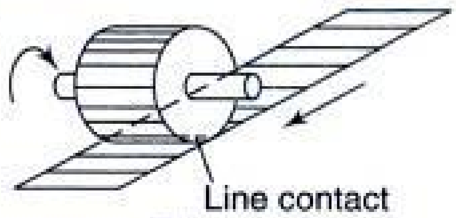

Fig. 10.3

Further, if the gears have external teeth on the outer surface of the cylinders, the shafts rotate in the opposite direction [Fig. 10.2(a)]. In an internal spur gear, the teeth are formed on the inner surface of an annulus ring. An internal gear can mesh with an external pinion (smaller gear) only and the two shafts rotate in the same direction as shown in [Fig. 10.2(b)].

Spur Rack and Pinion Spur rack is a special case of a spur gear where it is made of infinite diameter so that the pitch surface is a plane (Fig. 10.3). The spur rack and pinion combination converts rotary motion into translatory motion, or vice-versa. It is used in a lathe in which the rack transmits motion to the saddle.

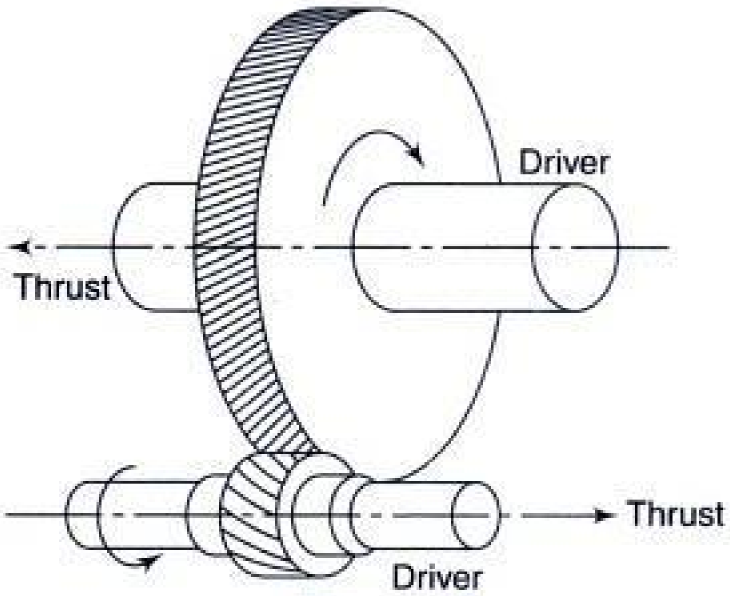

Fig. 10.4

Helical Gears or Helical Spur Gears In helical gears, the teeth are curved, each being helical in shape. Two mating gears have the same helix angle, but have teeth of opposite hands (Fig. 10.4).

At the beginning of engagement, contact occurs only at the point of leading edge of the curved teeth. As the gears rotate, the contact extends along a diagonal line across the teeth. Thus, the load application is gradual which results in low impact stresses and reduction in noise. Therefore, the helical gears can be used at higher velocities than the spur gears and have greater load-carrying capacity.

Helical gears have the disadvantage of having end thrust as there is a force component along the gear axis. The bearings and the assemblies mounting the helical gears must be able to withstand thrust loads.

Double-helical and Herringbone Gears A *double-helical* gear is equivalent to a pair of helical gears secured together, one having a right-hand helix and the other a left-hand helix. The teeth of the two rows are separated by a groove used for tool run out. Axial thrust which occurs in case of single-helical gears is

As seen earlier, P is also the point of contact of two pitch circles which divides the line of centres in the inverse ratio of the angular velocities of the two circles and is the pitch point.

Thus, for constant angular velocity ratio of the two gears, the common normal at the point of contact of the two mating teeth must pass through the pitch point.

Also, as the Δs AEP and BFP are similar,

$$\frac{BP}{AP} = \frac{FP}{EP}$$

or
$$\frac{\omega_1}{\omega_2} = \frac{FP}{EP} \quad \text{or} \quad \omega_1 EP = \omega_2 FP \qquad (10.4)$$

10.4 VELOCITY OF SLIDING

If the curved surfaces of the two teeth of the gears 1 and 2 are to remain in contact, one can have a sliding motion relative to the other along the common tangent $t - t$ at C or D (Fig. 10.17).

Component of v_c along $t - t = v_c \sin\alpha$

Component of v_d along $t - t = v_d \sin\alpha$

Velocity of sliding $= v_c \sin\alpha - v_d \sin\beta$

$$= \omega_1 AC \frac{EC}{AC} - \omega_2 BD \frac{FD}{BD}$$

$$= \omega_1 EC - \omega_2 FD$$

$$= \omega_1 (EP + PC) - \omega_2 (FP - PD)$$

$$= \omega_1 EP + \omega_1 PC - \omega_2 FP + \omega_2 PC$$

(C and D are the coinciding points)

$$= (\omega_1 + \omega_2)\, PC + \omega_1 EP - \omega_2 FP$$

$$= (\omega_1 + \omega_2)\, PC \qquad [\omega_1 EP = \omega_2 FP, \text{ Eq. } (10.4)]$$

= sum of angular velocities × distance between the pitch point and the point of contact

10.5 FORMS OF TEETH

Two curves of any shape that fulfill the law of gearing can be used as the profiles of teeth. In other words, an arbitrary shape of one of the mating teeth can be taken and applying the law of gearing the shape of the other can be determined. Such gear are said to have *conjugate* teeth. However, it will be very difficult to manufacture such gears and the cost will be high. Moreover, on wearing, it will be very difficult to replace them with the available gears. Thus, there arises the need to standardize gear teeth.

Common forms of teeth that also satisfy the law of gearing are

1. Cycloidal profile teeth
2. Involute profile teeth

10.6 CYCLOIDAL PROFILE TEETH

In this type, the faces of the teeth are epicycloids and the flanks are the hypocycloids.

A *cycloid* is the locus of a point on the circumference of a circle that rolls without slipping on a fixed straight line.

(i) $n = \dfrac{\text{Arc of contact}}{\text{Circular pitch}} = \left(\dfrac{\text{Path of contact}}{\cos\varphi}\right)$

$$\times \frac{1}{\pi m} = \frac{\text{Path of approach} + \text{Path of recess}}{\cos\varphi \times \pi m}$$

$$= \frac{\left[\begin{array}{l}\sqrt{R_a^2 - R^2\cos^2\varphi} - R\sin\varphi \\ +\sqrt{r_a^2 - r^2\cos^2\varphi} - r\sin\varphi\end{array}\right]}{\cos\varphi \times \pi m}$$

$$\frac{\left[\begin{array}{l}\sqrt{(236)^2 - (228^2\cos^2 20^\circ} - 228\sin 20^\circ \\ +\sqrt{(100)^2 - (92)^2\cos^2 20^\circ} - 92\sin 20^\circ\end{array}\right]}{\cos 20^\circ \pi \times 8}$$

$$= \frac{20.97 + 18.79}{\cos 20^\circ \times \pi \times 8} = 42.31 \times \frac{1}{\pi \times 8} = \underline{1.68}$$

(ii) Angle of action, $\delta_p = \dfrac{\text{Arc of contact}}{r} = \dfrac{42.31}{92}$

$= 0.46$ rad or $0.46 \times 180/\pi = 26.3^\circ$

$$\delta_g = \frac{\text{Arc of contact}}{R} = \frac{42.31}{228} = 0.1856 \text{ rad}$$

or $0.1856 \times 180/\pi = 10.63^\circ$

(iii) (a) $\dfrac{\text{Sliding velocity}}{\text{Rolling velocity}}$

$$= \frac{(\omega_p + \omega_g) \times \text{Path of approach}}{\text{Pitch line velocity } (= \omega_p \times r)}$$

$$= \frac{\left(\omega_p + \frac{23}{57}\omega_p\right) \times 20.97}{\omega_p \times 92} = \underline{0.32}$$

(b) $\dfrac{\text{Sliding velocity}}{\text{Rolling velocity}} = \dfrac{(\omega_p + \omega_g) \times 0}{\text{Pitch line velocity}} = 0$

(c) $\dfrac{\text{Sliding velocity}}{\text{Rolling velocity}}$

$$= \frac{\left(\omega_p + \frac{23}{57}\omega_p\right) \times \text{Path of recess}}{\omega_p \times r}$$

$$= \frac{\left(1 + \frac{23}{57}\right) \times 18.79}{92} = \underline{0.287}$$

Example 10.7 *Two 20° gears have a module pitch of 4 mm. The number of teeth on gears 1 and 2 are 40 and 24 respectively. If the gear 2 rotates at 600 rpm, determine the velocity of sliding when the contact is at the tip of the tooth of gear 2. Take addendum equal to one module.*

Also, find the maximum velocity of sliding.

Solution 1 is the gear wheel and 2 is the pinion.

$\varphi = 20^\circ$; $T = 40$; $N_p = 600$ mm; $t = 24$; $m = 4$ mm

Addendum = 1 module = 4 mm

$$R = \frac{mT}{2} = \frac{4 \times 40}{2} = 80 \text{ mm}; \; R_a = 80 + 4 = 84 \text{ mm}$$

$$r = \frac{mt}{2} = \frac{4 \times 24}{2} = 48 \text{ mm}; \; r_a = 48 + 4 = 52 \text{ mm}$$

$$N_g = N_p \times \frac{t}{T} = 600 \times \frac{24}{40} = 360 \text{ rpm}$$

(i) Let pinion (gear 2) be the driver.

The tip of the driving wheel is in contact with a tooth of the driven wheel at the end of engagement. Thus, it is required to find the path of recess which is obtained from the dimensions of the driving wheel.

Path of recess $= \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi$

$= \sqrt{(52)^2 - (48\cos 20^\circ)^2} - 48\sin 20^\circ$

$= 9.458$ mm

Velocity of sliding $= (\omega_p + \omega_g) \times$ Path of recess

$= 2\pi(N_p + N_g) \times 9.458$

$= 2\pi(600 + 360) \times 9.458$

$= 57\,049$ mm/min

$= \underline{950.8 \text{ mm/s}}$

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10.23 TERMINOLOGY OF WORM GEARS

Refer Fig. 10.35,

(i) Axial Pitch **(p_a)** It is the distance between corresponding points on adjacent teeth measured along the direction of the axis.

(ii) Lead **(L)** The distance by which a helix advances along the axis of the gear for one turn around is known as lead.

In a single helix, the axial pitch is equal to lead. In a double helix, this is one-half the lead, in a triple helix, one third of lead, and so on.

(iii) Lead Angle **(λ)** It is the angle at which the teeth are inclined to the normal to the axis of rotation. Obviously, the lead angle is the complement of the helix angle.

i.e., $\psi + \lambda = 90°$

In case of worms, the lead angle is very small and the helix angle approaches 90°.

As the shaft axes of worm and worm gear are at 90°,

$$\psi_1 + \psi_2 = 90° \qquad \text{(1 denotes worm)}$$

$$(90° - \lambda_1) + \psi_2 = 90°$$

or $$\lambda_1 = \psi_2$$

i.e., lead angle of worm = helix angle of the gear wheel

Also, p_n of worm = p_n of wheel

$$p_{a1}\cos\lambda_1 = p_2\cos\psi_2$$

but $$\lambda_1 = \psi_2$$

$\therefore$ $$p_{a1} = p_2$$

i.e., axial pitch of worm = circular pitch of wheel

10.24 VELOCITY RATIO AND CENTRE DISTANCE OF WORM GEARS

Velocity Ratio As a worm may be multistart, the velocity is not calculated from the number of teeth.

Assume that a worm rotates through one revolution about its axis. Then the angle turned by it will be 2π.

The lead of the worm is equal to the axial distance advanced by a thread in one revolution of the worm.

The lead is also the distance moved by the pitch circle of the gear wheel. Thus, angle turned by it during the same time will be l/R_2 or $2l/d_2$.

$$VR = \frac{\text{Angle turned by the gear}}{\text{Angle turned by the worm}} = \frac{2l/d_2}{2\pi} = \frac{l}{\pi d_2} \qquad (10.15)$$

Centre Distance

$$C = \frac{m_n}{2}\left(\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right)$$

$$= \frac{m_2\cos\psi_2}{2}\frac{1}{\cos\psi_2}\left(\frac{\cos\psi_2}{\cos\psi_1}.T_1 + T_2\right)$$

$$= \frac{m_2}{2}\left[\frac{\cos\lambda_1}{\cos(90° - \lambda_1)}T_1 + T_2\right] \qquad [\psi_2 = \lambda_1,\ \psi_1 = 90° - \lambda_1]$$

so that the pressure angle is increased to 22°?

(222 mm)

32. A pinion has 24 teeth and drives a gear with 64 teeth. The teeth are of involute type with 20° pressure angle. The addendum and the module are 8 mm and 10 mm respectively. Determine path of contact, arc of contact and the contact ratio.

(41.08 mm, 43.72, 1.39)

33. Two gears in mesh have a module of 10 mm and a pressure angle of 25°. The pinion has 20 teeth and the gear has 52. The addendum on both the gears is equal to one module. Determine the
   (i) number of pairs of teeth in contact
   (ii) angles of action of the pinion and the wheel
   (iii) ratio of the sliding velocity to the rolling velocity at the pitch point and at the beginning and end of engagement

(1.475; δ_p = 26°36′, δ_g = 10°13′; zero, 0.304, 0.278)

34. The number of teeth on the gear and the pinion of two spur gears in mesh are 30 and 18 respectively. Both the gears have a module of 6 mm and a pressure angle of 20°. If the pinion rotates at 400 rpm, what will be the sliding velocity at the moment the tip of the tooth of pinion has contact with the gear flank? Take addendum equal to one module. Also, find the maximum velocity of sliding.

(908 mm/s; 981.5 mm/s)

35. Two 20° involute spur gears have a module of 6 mm. The larger wheel has 36 teeth and the pinion has 16 teeth. If the addendum be equal to one module, will the interference occur? What will be the effect if the number of teeth on the pinion is reduced to 14?

(No; interference occurs)

36. The addendum on each wheel of two mating gears is to be such that the line of contact on each side of the pitch point is half the maximum possible length. The number of teeth on the two gears is 24 and 48. The teeth are of 20° pressure angle involute with a module of 12 mm. Determine the addendum for the pinion and the gear. Also, find the arc of contact and the contact ratio.

(23.4 mm, 9.3 mm, 78.6 mm, 2.08)

37. The following data refer to two meshing gears having 20° involute teeth:
   Number of teeth of gear wheel = 52
   Number o teeth of pinion =20
   Speed of pinion = 360 rpm
   Module = 8 mm
   If the addendum of each gear is such that the path of approach and path of recess are half of their maximum possible values, determine the addendum for the gear and the pinion and the length of arc of contact.

(5.07 mm; 18.04 mm; 52.4 mm)

38. Determine the minimum number of teeth and the arc of contact (in terms of module) to avoid interference in the following cases:
   (a) Gear ratio is unity.
   (b) Gear ratio is 3.
   (c) Pinion gears with a rack.
   The addendum of the teeth is 0.88 module and the power component is 0.94 times the normal thrust.

(11, 3.96 m; 14, 4.48 m; 16, 4.24 m)

39. Two 20° involute spur gears having a velocity ratio of 2.5 mesh externally. The module is 4 mm and the addendum is equal to 1.23 module. The pinion rotates at 150 rpm. Find the
   (i) minimum number of teeth on each wheel to avoid interference
   (ii) number of pairs of teeth in contact.

(45, 18; 1.95)

40. If the angle of obliquity of a pair of gear wheels is 20°, and the arc of approach or recess not less than the pitch, what will be the least number of teeth on the pinion?

(18)

41. Two 20° full-depth involute spur gears having 30 and 48 teeth are in mesh. The pinion rotates at 800 rpm. The module is 4 mm. Find the sliding velocities at the engagement and at the disengagement of a pair of teeth and the contact ratio. If the interference is just avoided, find (i) the addenda on the wheel and the pinion, (ii) the path of contact, (iii) the maximum velocity of sliding at engagement and disengagement of a pair of teeth, and (iv) contact ratio.

(8.8 mm, 17.6 mm; 53.35 mm;
2.934 m/s, 4.695 m/s; 4.52)

42. A rack is driven by a pinion having 24 involute teeth and a 140 mm pitch circle diameter. The addendum of both pinion and the rack is 6 mm. Determine the least pressure angle which can be used to avoid interference. For this pressure angle, find the minimum number of teeth in contact at a time.

(17.02°, 2.11)

43. The centre distance between two meshing spiral gears is 150 mm and the angle between the shafts is 60°. The gear ratio is 2 and the normal circular pitch is 10 mm. The driven gear has a helix angle of 25°, determine the

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20. In an epicyclic gear (Fig. 11.32), the wheel A fixed to S_1 has 30 teeth and rotates at 500 rpm. B gears with A and is fixed rigidly to C, both being free to rotate on S_2. The wheels B, C and D have 50, 70 and 90 teeth respectively. If D rotates at 80 rpm in a direction opposite to that of A, find the speed of the shaft S_2. (104.5 rpm in same direction)

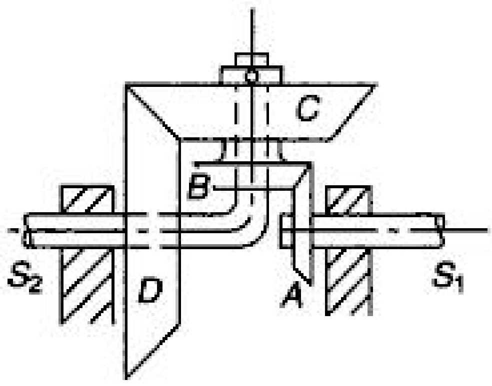


Fig. 11.32

21. In an indexing mechanism of a milling machine (Fig. 11.30), the drive is from gear wheels fixed to shafts S_1 and S_2 to the bevel A through the gear train. The number of teeth of A, B, D and F are 30, 60, 28 and 24 respectively. Each gear has a module of 10 mm.
Determine the number of revolutions of S (or A) for one revolution of S_1 when
(a) S_1 and S_2 have same speed in the same direction
(b) S_1 and S_2 have same speed in the opposite direction
(c) S_1 makes 48 rpm and S_2 is at rest
(d) S_1 makes 48 rpm and S_2 24 rpm in the same direction
(2; zero; 48 rpm; 72 rpm)

22. Show that in a Humpage reduction gear (Fig. 11.24), the wheel E rotates in the same direction as the wheel B if T_C/T_D is more than T_F/T_E and in the opposite direction if the same is less than T_F/T_E. Gear F is the fixed frame.

23. In a sun and planet gear train, the sun gear wheel having 60 teeth is fixed to the frame. Determine the numbers of teeth on the planet and the annulus wheels if the annulus rotates 130 times and the arm rotates 100 times, both in the same direction.
(70; 200)

24. A four-speed sliding gear box of an automobile is to be designed to give approximate speed ratios of 4, 2.4, 1.4 and 1 for the first, second, third and top gears respectively. The input and the output shafts have the same alignment. Horizontal central distance between them and the lay shaft is 98 mm. The teeth have a module of 4 mm. No wheel has less than 16 teeth. Calculate suitable number of teeth on each wheel and find the actual speed ratios attained.

25. In the pre-selective gear-box shown in Fig. 11.28, the number of teeth are
$T_{A1} = T_{A2} = 80$ $T_{S1} = T_{S2} = 24$
$T_{A3} = 68$ $T_{S3} = 21$
$T_{A4} = 90$ $T_{S4} = 41$
If the input shaft E rotates at a uniform speed of 640 rpm, determine the speeds of the output shaft F when different gears are engaged.
(147.7 rpm, 261.3 rpm, 423 rpm, 640 rpm, 101.4 rpm)

26. In the differential gear of a car shown in Fig. 11.30, the number of teeth on the pinion A on the propeller shaft is 24 whereas the crown gear B has 128 teeth. If the propeller shaft rotates at 800 rpm and the wheel attached to the shaft S_2 has a speed of 175 rpm, determine the speed of the wheel attached to shaft S_1 when the vehicle takes a turn.
(125 rpm)

15 N and cam force F_5 of unknown magnitude act on it along the lines of action as shown. F_3 and F_4 are the bearing reactions. Determine the magnitudes of the forces F_3, F_4 and F_5. Assume no friction.

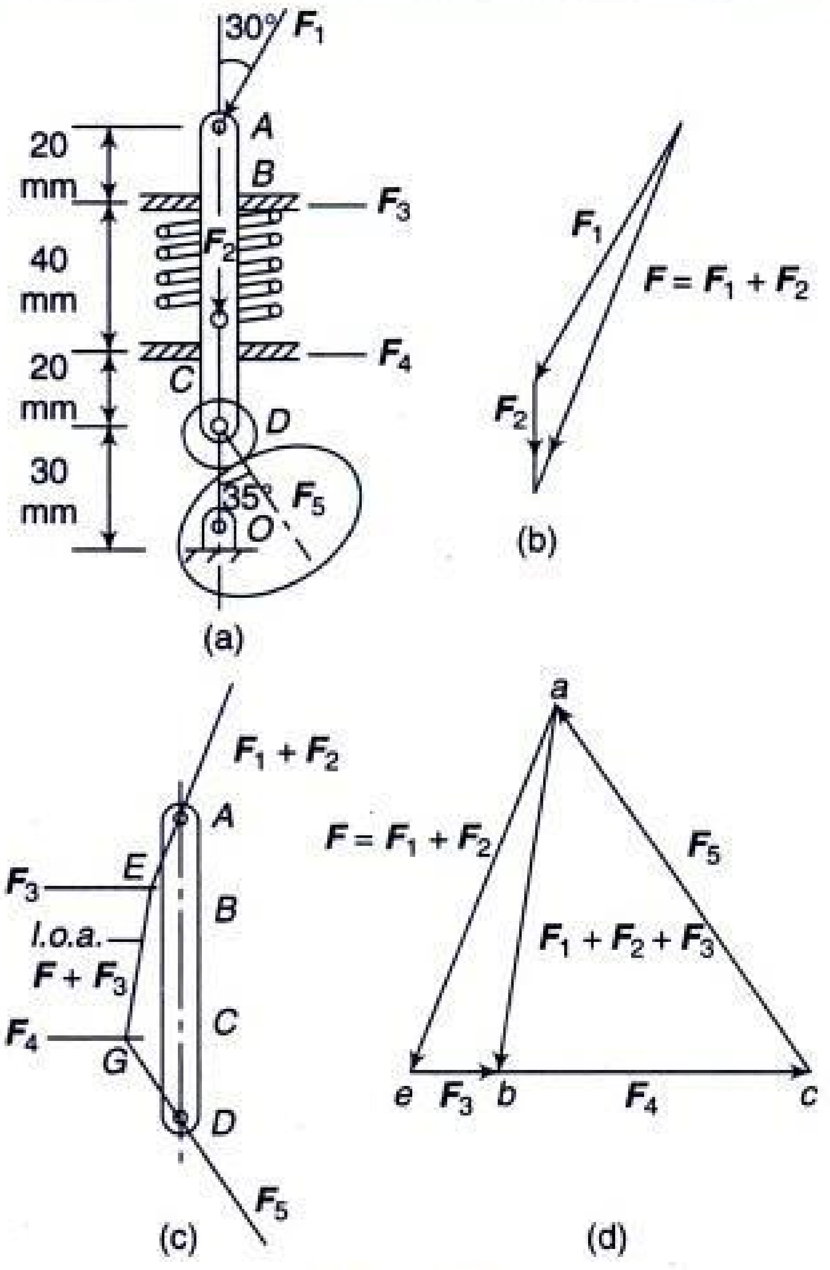


Fig. 12.5

Solution As in the previous example, forces $\mathbf{F}_1$ and $\mathbf{F}_2$ can be combined into a single force **F** by obtaining their resultant [Figs 12.5(b)]. Their resultant must pass through point A, the point of intersection of $\mathbf{F}_1$ and $\mathbf{F}_2$. Thus, the number of forces acting on the body is reduced to four.

Now, assume that the magnitude of force $\mathbf{F}_3$ is known and the force **F** is to be combined with it. Then the resultant must pass through their point of intersection, i.e., the point E [Fig. 12.5(c)]. This way, the body becomes under the action of three forces which must be concurrent for the equilibrium of the body. Thus, the resultant of **F** and $\mathbf{F}_3$ must pass through the point G, the point of intersection of the forces $\mathbf{F}_4$ and $\mathbf{F}_5$. Therefore, the line of action of the resultant of **F** and $\mathbf{F}_3$ is EG.

Now since the force **F** is completely known and the lines of action of $\mathbf{F}_3$ and their resultant are known, the force diagram can be made. First take the force **F** and then to add $\mathbf{F}_3$ draw a line parallel to its line of action through the head of **F** [Fig. 12.5(d)]. Through the tail of vector **F** draw a line parallel to the line of action of the resultant. The triangle aeb thus provides the magnitude of the force $\mathbf{F}_3$ as well as resultant of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$.

Now the number of forces acting on the body is reduced to three. One force is completely known and the lines of action of the other two are known. A triangle of forces can be drawn and magnitudes of $\mathbf{F}_3$, $\mathbf{F}_4$ and $\mathbf{F}_5$ can be found.

Magnitude of $\mathbf{F}_3$ = 12 N
Magnitude of $\mathbf{F}_4$= 42 N
Magnitude of $\mathbf{F}_5$ = 60 N

12.6 FORCE CONVENTION

The force exerted by the member i on the member j is represented by $\mathbf{F}_{ij}$.

12.7 FREE-BODY DIAGRAMS

A free-body diagram is a sketch or diagram of a part isolated from the mechanism in order to determine the nature of forces acting on it.

Figure 12.6(a) shows a four-link mechanism. The free-body diagrams of its members 2, 3 and 4 are shown in Figs 12.6 (b) (c) and (d) respectively. Various forces acting on each member are also shown. As the mechanism is in static equilibrium, each of its members must be in equilibrium individually.

Example 12.7 *For the static equilibrium of the mechanism of [Fig. 12.14(a)], find the torque to be applied on link AB.*

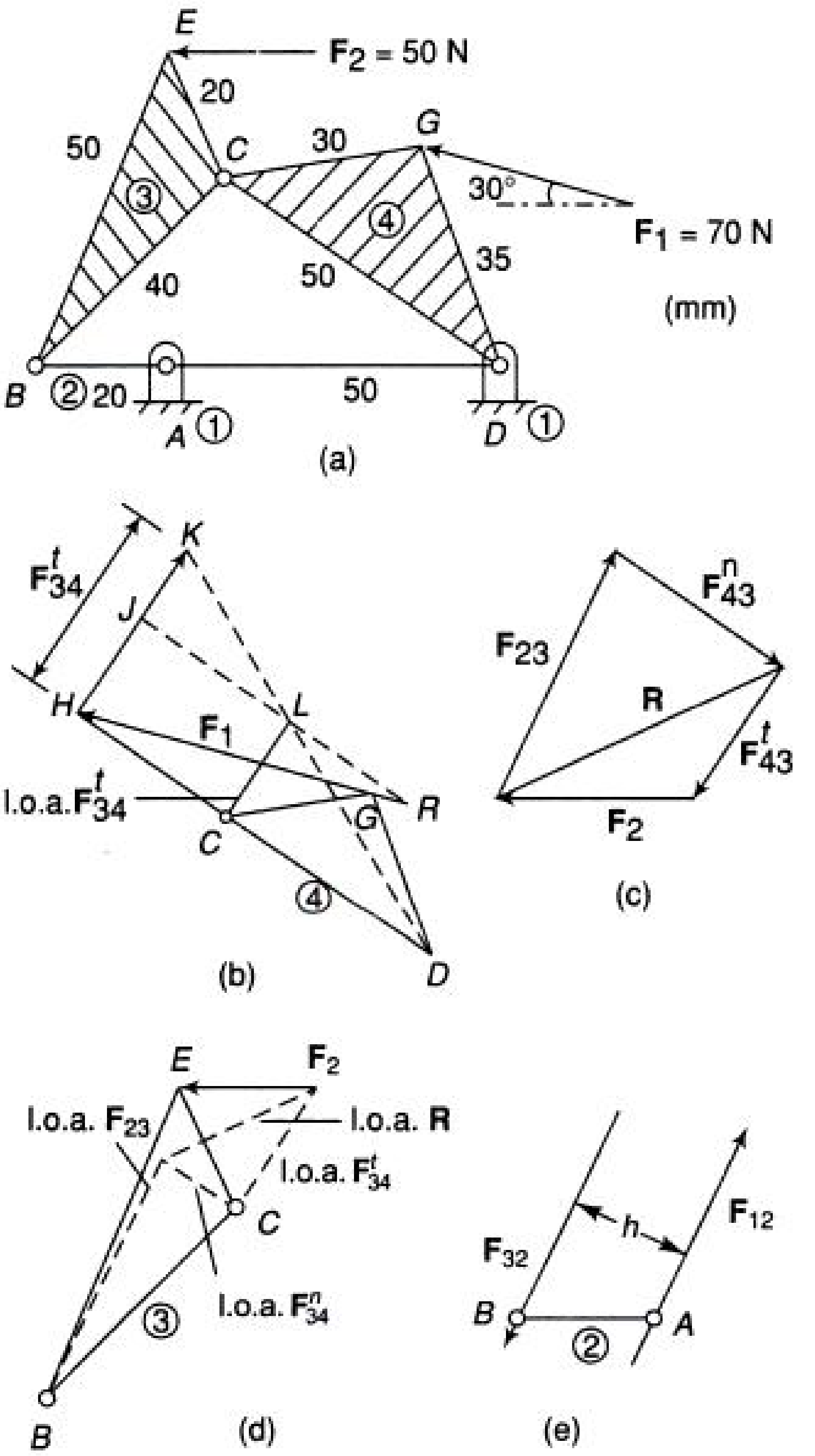


Fig. 12.14

Solution The point of action of force $\mathbf{F}_1$ on the link 4 is an offset point *G*. If *DC* is extended and let the line of action of force $\mathbf{F}_1$ meet at *H* then the force $\mathbf{F}_1$ may be considered to be acting on a virtual point *H* on the link *DC* as the magnitude of force as well as the magnitude couple effect is not going to vary.

Now, the problem can be solved by adopting the procedure given in the previous example. In brief:

- Take vector *RH* to represent force $\mathbf{F}_1$ to some scale.
- Find force $\mathbf{F}^t_{34}$. Its magnitude is given by *HK* and it acts through *C*.
- Find the resultant of $\mathbf{F}_2$ and $\mathbf{F}^t_{43}$ and its point of application in the free body diagram.
- Through point *C*, draw line for the vector $\mathbf{F}^n_{43}$ and then find the line of application of $\mathbf{F}_{23}$.
  From force diagram,
  $F_{23} = 68.9$ N
  Now, $F_{32} = -F_{23} = -49.4$
  Member 2 will be in equilibrium if $\mathbf{F}_{12}$ is equal, parallel and opposite to $\mathbf{F}_{32}$ and
  $T = -F_{32} \times h = -68.9 \times 18.65 = -1285$ N.mm
  The input torque has to be equal and opposite to this couple, i.e.,
  $T = 1.285$ N.m (clockwise)
- The example can also be worked out by the graphical method using the principle of superposition.

Example 12.8 *For the static equilibrium of the quick-return mechanism shown in Fig. 12.15a, determine the input torque T_2 to be applied on the link AB for a force of 300 N on the slider D. The dimensions of the various links are*

OA = 400 mm, AB = 200 mm, OC = 800 mm, CD = 300 mm

Solution The slider at *D* or the link 6 is a three-force member. Lines of action of the forces are [Fig.12.15(b)]

- **F**, 300 N as given
- $\mathbf{F}_{56}$ along *CD*, as link 5 is a two force member
- $\mathbf{F}_{16}$, normal reaction, perpendicular to slider motion

Draw the force diagram and determine the direction sense of forces $\mathbf{F}_{56}$ and $\mathbf{F}_{16}$. From the force $\mathbf{F}_{56}$, the directions of forces $\mathbf{F}_{65}$, $\mathbf{F}_{35}$ and $\mathbf{F}_{53}$ are known. Now, the link 3 is a three-force member. Lines of action of the forces are

- $\mathbf{F}_{53}$, known completely through *C*

Consider a slider-crank mechanism shown in Fig. 12.25. It is acted upon by the external piston force **F**, the external crankshaft torque **T** and the force at the bearings. As the crank rotates through a small angular displacement $\delta\theta$, the corresponding displacement of the piston is δx. the various forces acting on the system are

- Bearing reaction at O (performs no work)
- Force of cylinder on piston, perpendicular to piston displacement (produces no work)
- Bearing forces at A and B, being equal and opposite (AB is a two-force member), no work is done
- Work done by torque $T = T\delta\theta$
- Work done by force $F = F\,\delta x$

Work done is positive if a force acts in the direction of the displacement and negative if it acts in the opposite direction.

According to the principle of virtual work,

$$W = T\,\delta\theta + F\,\delta x = 0 \tag{12.7}$$

As virtual displacement must take place during the same interval δt,

$$\therefore \qquad T\frac{d\theta}{dt} + F\frac{dx}{dt} = 0$$

or

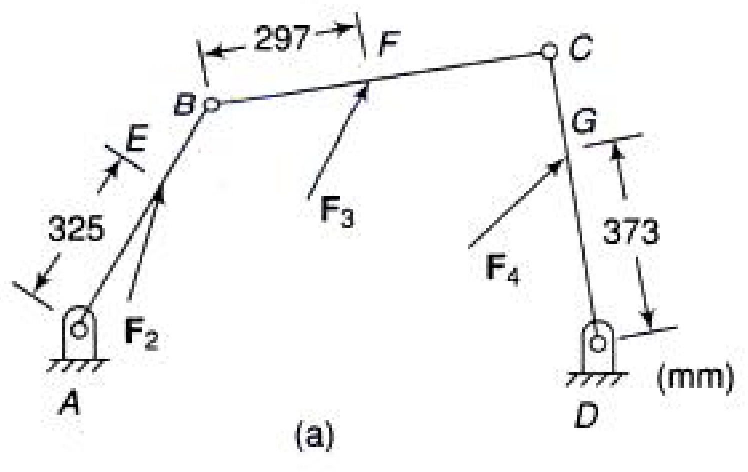
$$T\omega + Fv = 0 \tag{12.8}$$

where ω is the angular velocity of the crank and v, the linear velocity of the piston.

$$T = -\frac{F}{\omega}v$$

The negative sign indicates that for equilibrium, **T** must be applied in the opposite direction to the angular displacement.

Example 12.12 *Solve Example 12.9 by using the principle of virtual work.*



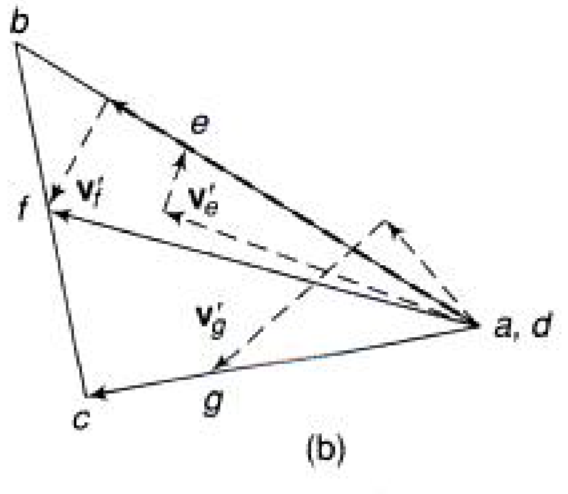


Fig. 12.26

Solution Assume that the line AB has an instantaneous angular velocity of ω rad/s counter-clockwise. Then $v_b = 0.5\,\omega$ m/s.

$$\Sigma \mathbf{T} + \mathbf{C}_i = 0 \tag{13.4}$$

These equations are similar to the equation of a body in static equilibrium, i.e., $\Sigma\mathbf{F} = 0$ and $\Sigma\mathbf{T} = 0$.

This suggests that first the magnitudes and the directions of inertia forces and couples can be determined, after which they can be treated just like static loads on the mechanism. Thus, a dynamic analysis problem is reduced to one requiring static analysis.

13.2 EQUIVALENT OFFSET INERTIA FORCE

In plane motions involving accelerations, the inertia force acts on a body through its centre of mass. However, if the body is acted upon by forces such that their resultant does not pass through the centre of mass, a couple also acts on the body. In graphical solutions, it is possible to replace inertia force and inertia couple by an equivalent offset inertia force which can account for both. This is done by displacing the line of action of the inertia force from the centre of mass. The perpendicular displacement h of the force from the centre of mass is such that the torque so produced is equal to the inertia couple acting on the body,

i.e. $\quad T_i = C_i$

or $\quad F_i \times h = C_i$

or
$$h = \frac{C_i}{F_i} = \frac{-I_g\alpha}{-mf_g} = \frac{mk^2\alpha}{mf_g} = \frac{k^2\alpha}{f_g} \tag{13.5}$$

h is taken in such a way that the force produces a moment about the centre of mass, which is opposite in sense to the angular acceleration α.

13.3 DYNAMIC ANALYSIS OF FOUR-LINK MECHANISMS

For dynamic analysis of four-link mechanisms, the following procedure may be adopted:

1. Draw the velocity and acceleration diagrams of the mechanism from the configuration diagram by usual methods.
2. Determine the linear acceleration of the centres of masses of various links, and also the angular accelerations of the links.
3. Calculate the inertia forces and inertia couples from the relations $\mathbf{F}_i = -m\mathbf{f}_g$ and $\mathbf{C}_i = -I_g\alpha$.
4. Replace $\mathbf{F}_i$ with equivalent offset inertia force to take into account $\mathbf{F}_i$ as well as $\mathbf{C}_i$.
5. Assume equivalent offset inertia forces on the links as static forces and analyse the mechanism by any of the methods outlined in Chapter 12.

Example 13.1 *The dimensions of a four-link mechanism are AB = 500 mm, BC = 660 mm, CD = 560 mm and AD = 1000 mm.*

The link AB has an angular velocity of 10.5 rad/s counter-clockwise and an angular retardation of 26 rad/s^2 at the instant when it makes an angle of 60° with AD, the fixed link.

The mass of the links BC and CD is 4.2 kg/m length. The link AB has a mass of 3.54 kg, the centre of which lies at 200 mm from A and a moment of inertia of 88 500 kg.mm^2.

Neglecting gravity and friction effects, determine the instantaneous value of the drive torque required to be applied on AB to overcome the inertia forces.

This means that by considering the two masses at A and B instead of at D and B, the inertia torque is increased from the actual value ($T = I\alpha_c$). The error is corrected by incorporating a correction couple.

Then,

$$\begin{aligned}\text{correction couple, } \Delta T &= \alpha_c\,(mab - mbd)\\ &= mb\alpha_c\,(a - d)\\ &= mb\alpha_c\,[(a + b) - (b + d)]\\ &= mb\alpha_c\,(l - L) \qquad \text{(taking } b + d = L) \qquad (13.25)\end{aligned}$$

This correction couple must be applied in the opposite direction to that of the applied inertia torque. As the direction of the applied inertia torque is always opposite to the direction of the angular acceleration, the direction of the correction couple will be the same as that of angular acceleration, i.e., in the direction of the decreasing angle β.

The correction couple will be produced by two equal, parallel and opposite forces F_y acting at the gudgen pin and crankpin ends perpendicular to the line of stroke (Fig. 13.8). The force at B is taken by the reaction of guides.

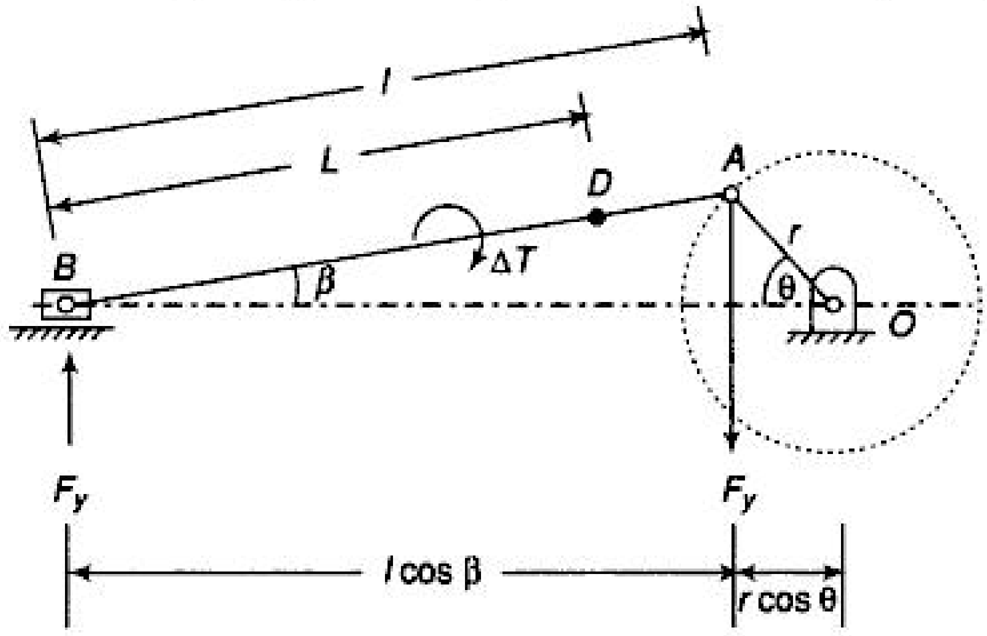


Fig. 13.8

Turning moment at crankshaft due to force at A or correction torque,

$$\begin{aligned}T_c &= F_y \times r\cos\theta\\ &= \frac{\Delta T}{l\cos\beta}\times r\cos\theta \qquad (\because \Delta T = F_y l\cos\beta)\\ &= \frac{\Delta T}{(l/r)}\frac{\cos\theta}{\cos\beta}\\ &= \Delta T\frac{\cos\theta}{n\frac{1}{n}\sqrt{n^2 - \sin^2\theta}}\\ &= \Delta T\frac{\cos\theta}{\sqrt{n^2 - \sin^2\theta}} \qquad (13.26)\end{aligned}$$

This correction torque is to be deducted from the inertia torque acting on the crankshaft.

Also, due to the weight of the mass at A, a torque is exerted on the crankshaft which is given by

$$T_a = (m_a g) r\cos\theta \qquad (13.27)$$

In case of vertical engines, a torque is also exerted on the crankshaft due to the weight of mass at B and the expression will be similar to Eq. (13.21), i.e.,

$$T_b = (m_b g)\, r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right) \qquad (13.28)$$

The net torque or turning moment on the crankshaft will be the algebraic sum of the

(i) turning moment due to the force of gas pressure (T)

(ii) inertia torque due to the inertia force at the piston as a result of inertia of the reciprocating mass including the mass of the portion of the connecting rod (T_b)

represents the acceleration of A relative to O, i.e., 355 m/s^2, therefore, f_b can be obtained from

$$f_b = 355 \times \frac{\text{length } b_1O}{\text{length } OA}$$

It is found to be $f_b = 343.2$ m/s^2

Similarly, $f_g = 345$ m/s^2

$$\therefore \quad F_b = m_b \times f_b = 120 \times 343.2 = 41186 \text{ N}$$

$$F_i = m \times f_g = 90 \times 345 = 31050 \text{ N}$$

Complete the diagram of Fig. 13.12(a) as discussed in Section 13.10. Taking moments about I,

$$F_t \times IA = F_b \times IB + F_i \times IP + mg \times IQ$$

$$F_t \times 515 = 41\,186 \times 300 + 31\,050 \times 152 + 90 \times 9.81 \times 268$$

$$F_t = 33615.5 \text{ N.m}$$

$$\therefore \quad T = F_t \times r = 33615.5 \times 0.9 = \underline{3025.4 \text{ N.m}}$$

Instead of taking moments about the I-centre, the principle of virtual work can also be applied to obtain the torque as follows:

On the velocity diagram [Fig. 13.12(b)], locate the points b, h and g corresponding to B, H and G respectively and take the components of velocities in the directions of forces F_b, F_i and mg. In Klein's construction, the velocity diagram in turned through 90°. Then

$$T \times \omega = F_b \times v_b + F_i \times v_h + mg \times v_g$$

$$T \times 62.8 = 41\,186 \times 3.29 + 31\,050 \times 1.67 + 90 \times 9.81 \times 2.94$$

$$T = 2157.6 + 825.7 + 41.3$$

$$= \underline{3024.6 \text{ N.m}}$$

If it is desired to find the resultant force on the crank, complete the force diagram as shown in Fig. 13.12(c).

Resultant force on the crank pin, $R = \underline{70\,000}$ N at 0°

Example 13.8 *The connecting rod of a vertical reciprocating engine is 2 m long between centres and weighs 250 kg. The mass centre is 800 mm from the big end bearing. When suspended as a pendulum from the gudgeon pin axis, it makes 8 complete oscillations in 22 seconds. Calculate the radius of gyration of the rod about an axis through its mass centre. The crank is 400 mm long and rotates at 200 rpm. Find the inertia torque exerted on the crankshaft when the crank has turned through 40° from the top dead centre and the piston is moving downwards.*

Solution

Analytical method

Divide the mass of the rod into two parts (Fig. 13.13),

Mass at the crank pin,

$$m_a = 250 \times \frac{2.0 - 0.8}{2.0} = 150 \text{ kg}$$

Mass at the gudgeon pin,

$$m_b = 250 - 150 = 100 \text{ kg}$$

$$F = mr\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

$$= 100 \times 0.4 \times \left(\frac{2\pi \times 200}{60} \right)^2 \left(\cos 40° + \frac{\cos 80°}{2/0.4} \right)$$

$$= 100 \times 0.4 \times 438.6 \times 0.8$$

$$= 14\,049 \text{ N}$$

As it is a vertical engine, the weight (force) of the portion of the connecting rod at the piston pin also can be combined with this force, i.e.,

Net force = 14 049 – 100 × 9.81 = 13 068 N (upwards)

$$T_b = Fr \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$

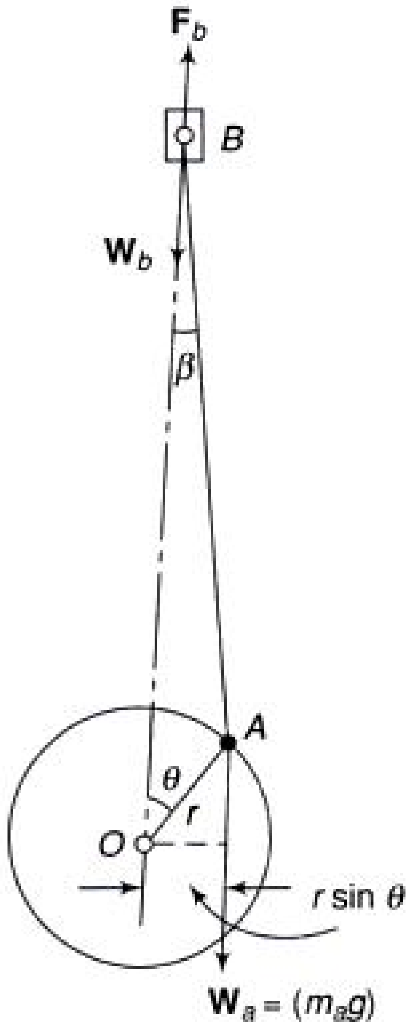


Fig. 13.13

The excess work is represented by the area *fgh*. This excess work increases the speed of the engine and is stored in the flywheel.

During the crank travel from *ob* or *oc*, the work needed for the external resistance is proportional to *bhjc* whereas the work produced by the engine is represented by the area under *hpj*. Thus, during this period, more work has been taken from the engine that is produced. The loss is made up by the flywheel which gives up some of its energy and the speed decreases during this period.

Similarly, during the period of crank travel from *oc* to *od*, excess work is again developed and is stored in the flywheel and the speed of the engine increases. During the crank travel from *od* to *oa*, the loss of work is made up by the flywheel and the speed again decreases.

The areas *fgh, hpj, jkl* and *lqf* represent fluctuations of energy of the flywheel. When the crank is at *b*, the flywheel has absorbed energy while the crank has moved from *a* to *b* and thereby, the speed of the engine is maximum. At *c*, the flywheel has given out energy while the crank has moved from *b* to *c* and thus the engine has a minimum speed. Similarly, the engine speed is again maximum at *d* and minimum at *a*. Thus, there are two maximum and two minimum speeds for the turning-moment diagram.

The greatest speed is the greater of the two maximum speeds and the least speed is the lesser of the two minimum speeds.

The difference between the greatest and the least speeds of the engine over one revolution is known as the *fluctuation of speed.*

2. Single-Cylinder Four-stroke Engine

In case of a four-stroke internal combustion engine, the diagram repeats itself after every two revolutions instead of one revolution as for a steam engine. It can be seen from the diagram (Fig. 13.19) that for the majority of the suction stroke, the turning moment is negative but becomes positive after the point *p*. During the compression stroke, it is totally negative. It is positive throughout the expansion stroke and again negative for most of the exhaust stroke.

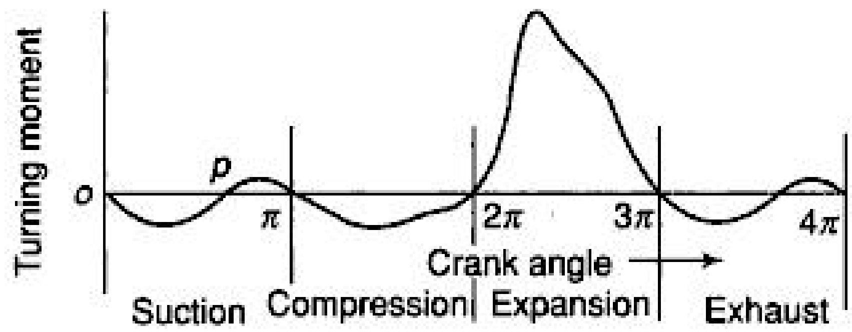


Fig. 13.19

3. Multi-Cylinder Engines

As observed in the foregoing paragraphs, the turning-moment diagram for a single-cylinder engine varies considerably and a greater variation of the same is observed in case of a four-stroke, single-cylinder engine. For engines with more than one cylinder, the total crankshaft torque at any instant is given by the sum of the torques developed by each cylinder at the instant.

For example, if an engine has two cylinders with cranks at 90°, the resultant turning moment diagram has a less variation than that for a single cylinder. In a three-cylinder engine having its cranks at 120°, the variation is still less.

Figure 13.20 shows the turning-moment diagram for a multicylinder engine. The mean torque line *ab* intersects the turning moment curve at *c, d, e, f, g* and *h*. The area under the wavy curve is equal to the area *oabk*. As discussed earlier, the

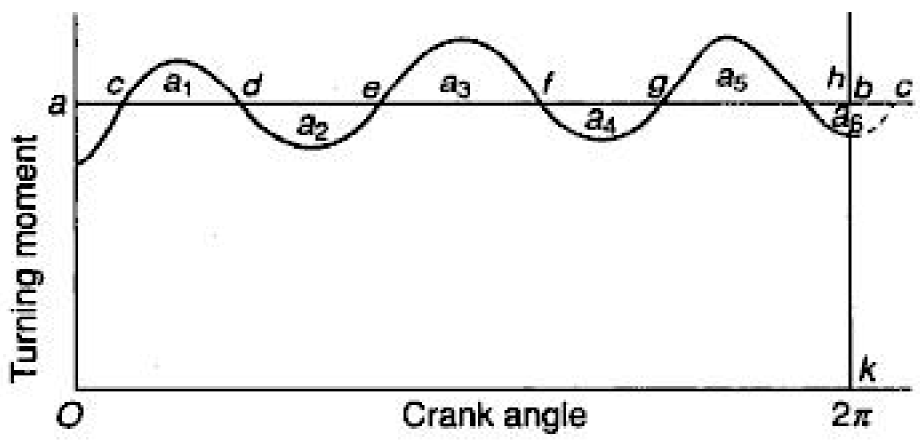


Fig. 13.20

(ii) coefficient of fluctuation of speed if the mass of the flywheel is 10 kg and radius of gyration is 88 mm

(iii) coefficient of fluctuation of energy

(iv) maximum angular acceleration of flywheel

Solution The turning-moment diagram for each cylinder is shown in Fig. 13.22(a) and the resultant-turning moment diagram for the three combined cylinders is shown in Fig. 13.22(b).

(i) Work done/cycle = Area of three triangles

$$= 3 \times (60 \times \pi/2) = 90\pi$$

$$\text{Mean torque} = \frac{\text{Work done /cycle}}{\text{Angle turned}} = \frac{90\pi}{2\pi} = 45 \text{ N.m}$$

$$P = T\omega = 45 \times \frac{2\pi \times 400}{60} = 1885 \text{ W}$$

or <u>1.885 kW</u>

(ii) As the area above or below the mean torque line is the maximum fluctuation of energy,

$$\therefore \quad e_{max} = \frac{60 \times \pi}{180} \times (60 - 45) \times \frac{1}{2}$$

$$= 2.5\pi \text{ N.m}$$

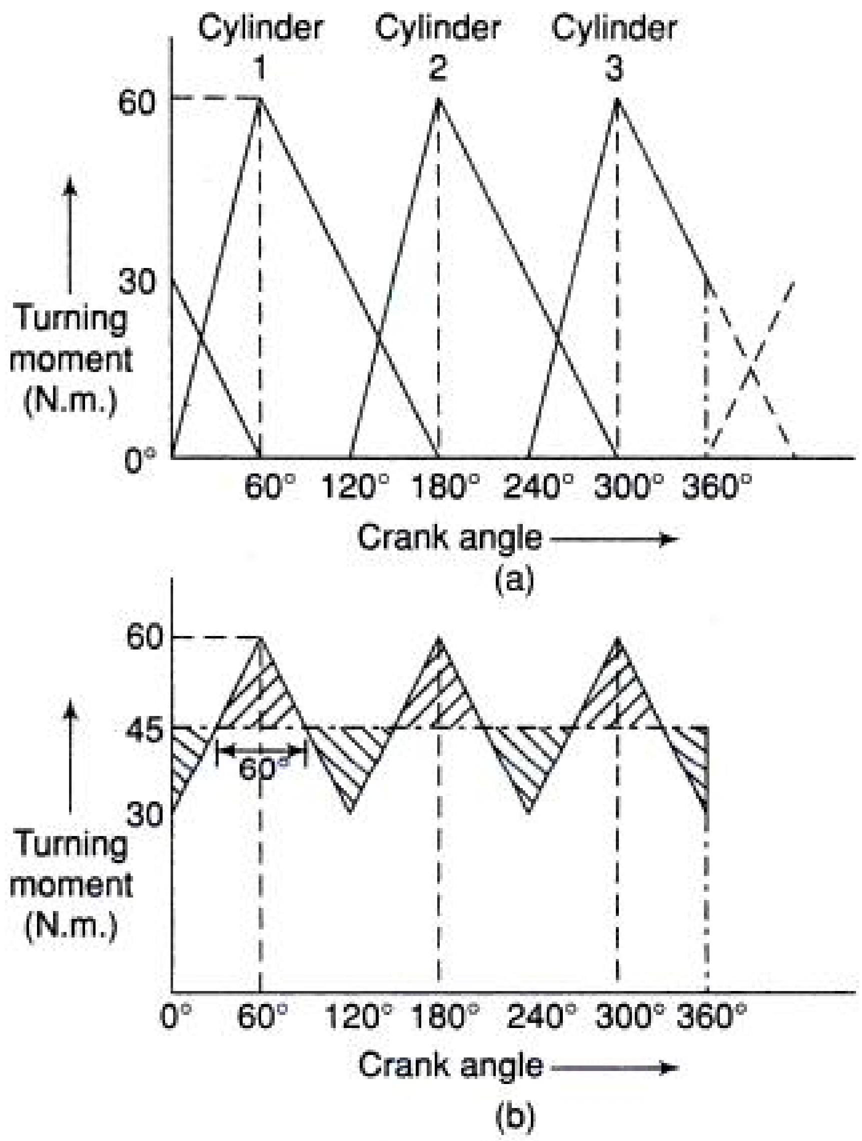


Fig. 13.22

$$K = \frac{e}{I\omega^2} = \frac{e}{mk^2\omega^2}$$

$$= \frac{2.5\pi}{10 \times 0.088^2 \left(\frac{2\pi \times 400}{60}\right)^2}$$

$$= 0.0578 \quad \text{or } \underline{5.78\%}$$

(iii) Coefficient of fluctuation of energy,

$$K_e = \frac{\text{Maximum fluctuation of energy}}{\text{work done/cycle}}$$

$$= \frac{2.5\pi}{90\pi}$$

$$= \underline{0.0278}$$

(iv) Maximum fluctuation of torque

$$= 60 - 45 = 15 \text{ N.m}$$

$\therefore \quad \Delta T = 15$ N.m

or $\quad I\alpha = mk^2 \alpha = 15$

or $\quad 10 \times (0.088)^2 \times \alpha = 15$

or $\quad \alpha = 193.7$ rad/s²

Example 13.18 *In a single-acting four-stroke engine, the work done by the gases during the expansion stroke is three times the work done during the compression stroke. The work done during the suction and exhaust strokes is negligible. The engine develops 14 kW at 280 rpm. The fluctuation of speed is limited to 1.5% of the mean speed on either side. The turning-moment diagram during the compression and the expansion strokes may be assumed to be triangular in shape. Determine the inertia of the flywheel.*

Solution

$P = 14$ kW, $N = 280$ rpm, $K = 1.5\%$,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 280}{60} = 29.32 \text{ rad/s}$$

It is a four-stroke engine, Thus, a cycle is completed in 4π radians. Thus the number of working strokes per minute is half the rpm, i.e., 140. The turning-moment diagram is shown in Fig. 13.23.

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ΔT is zero when $180 \sin 3\theta = 0$

or when $\sin 3\theta = 0$

or $3\theta = 0°$ or $180°$

or $\theta = 0°$ or $60°$

$$e_{max} = \int_{0°}^{60°} \Delta T dt$$

$$= \int_{0°}^{60°} (180 \sin 3\theta)\, d\theta$$

$$= \left[\frac{180 \cos 3\theta}{3}\right]_{0°}^{60°}$$

$$= 120 \text{ N.m}$$

$$K = \frac{e}{mk^2\omega^2} = \frac{120}{350 \times (0.22)^2 \times (41.89)^2}$$

$= 0.00404$ or 0.404%

(b) $\Delta T = T$ of engine $- T$ of machine

$= (800 + 180 \sin 3\theta) - (800 + 80 \sin\theta)$

$= 180 \sin 3\theta - 80 \sin \theta$

ΔT is zero when $180 \sin 3\theta - 80 \sin \theta = 0$

or $180 \sin 3\theta = 80 \sin \theta$

or $180 (3 \sin \theta - 4 \sin^3 \theta) = 80 \sin \theta$

or $3 - 4\sin^2\theta = \dfrac{80}{100} = 0.4444$

or $\sin^2\theta = 0.639$

or $\sin \theta = \pm 0.799$

or $\theta = \pm 53°$ and $\pm 127°$

$$e_{max} = \int_{53°}^{127°} \Delta T d\theta = \int_{53°}^{127°} (180 \sin 3\theta - 80 \sin \theta)\, d\theta$$

$$= \left[-\frac{180 \cos 3\theta}{3} + 80 \cos \theta\right]_{53°}^{127°}$$

$= -60 \cos 381° + 80 \cos 127° + 60 \cos 159° - 80 \cos 53°$

$= -208.3$ N.m

$$K = \frac{e}{mk^2\omega^2} = \frac{208.3}{350 \times (0.22)^2 \times (41.89)^2} = 0.007$$

$= 0.7\%$

Example 13.22 *The torque delivered by a two-stroke engine is represented by $T = (1200 + 1400 \sin\theta + 210 \sin 2\theta + 21 \sin 3\theta)$ N.m where θ is the angle turned by the crank from the inner-dead centre. The engine speed is 210 rpm. Determine the power of the engine and the minimum mass of the flywheel if its radius of gyration is 800 mm and the maximum fluctuation of speed is to be ± 1.5% of the mean.*

Solution

$k = 800$ mm $\qquad N = 210$ rpm

$K = 0.015 + 0.015 = 0.03$

The expression for torque being a function of θ, 2θ and 3θ the cycle is repeated after every 360° of the crank rotation (Fig. 13.27).

(i) $$T_{mean} = \frac{1}{\pi}\int_0^{\pi} T d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (1200 + 1400 \sin\theta + 210 \sin 2\theta + 21 \sin 3\theta)\, d\theta$$

$$= \frac{1}{2\pi}\left[1200\,\theta + 1400 \cos\theta + \frac{210}{2}\cos 2\theta + \frac{21}{2}\cos 3\theta\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}[(2400\pi + 1400 + 105 + 10.5) - (0 + 1400 + 105 + 10.5)]$$

$$= 1200 \text{ N.m}$$

$$P = T\omega = 1200 \times \frac{2\pi \times 210}{60} = 26\,390 \text{ W}$$

or 26.39 kW

(ii) At any instant, $\Delta T = T - T_{mean}$

$= (1200 + 1400 \sin\theta - 210 \sin 2\theta + 21 \sin 3\theta) - 1200$

$= 1400 \sin\theta + 210 \sin 2\theta + 21 \sin 3\theta$

ΔT is zero when

$1400 \sin\theta + 210 \sin 2\theta + 21 \sin 3\theta = 0$

This will be so when θ is 180° or 360°. This can be easily seen from the plot of the turning moment diagram.

the torque available is constant but the load varies during the cycle. Figure 13.32 shows the sketch of a punching press. It is a slider-crank mechanism in which a punch replaces the slider. A motor provides a constant torque to the crankshaft through a flywheel. It may be observed that the actual punching process is performed only during the downward stroke of the punch and that also for a limiting period when the punch travels through the thickness of the plate. Thus, the load is applied during the actual punching process only and during the rest of the downward stroke and the return stroke, there is no load on the crankshaft. In the absence of a flywheel, the decrease in the speed of the crankshaft will be very large during the actual punching period whereas it will increase to a much higher value during the no-load period as the motor will continue to supply the energy all the time.

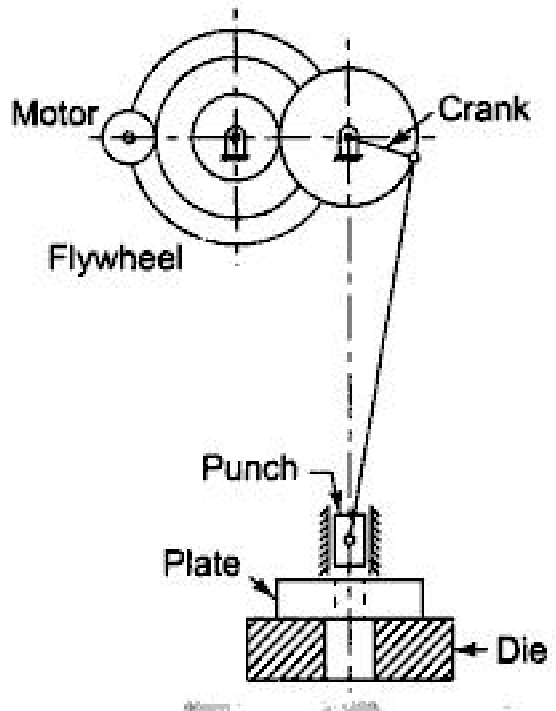

Fig. 13.32

Example 13.26 *A riveting machine is driven by a motor of 3 kW. The actual time to complete one riveting operation is 1.5 seconds and it absorbs 12 kN.m of energy. The moving parts including the flywheel are equivalent to 220 kg at 0.5 m radius. Determine the speed of the flywheel immediately after riveting if it is 360 rpm before riveting. Also, find the number of rivets closed per minute.*

Solution

$P = 3$ kW, $m = 220$ kg, $k = 0.5$ m,

$$\omega_1 = \frac{2\pi N}{60} = \frac{2\pi \times 360}{60} = 37.7 \text{ rad/s}$$

Energy required/riveting = 12 000 N.m

Energy supplied by the motor in 1 seconds = 3000 N.m

∴ energy supplied by the motor in 1.5 seconds = 3000 × 1.5 = 4500 N.m

Energy supplied by the flywheel

e = energy required/hole – energy supplied by the motor in 1.5 s

= 12000 – 4500 = 7500 N.m

Also $e = \frac{1}{2} I (\omega_1^2 - \omega_2^2) = \frac{1}{2} mk^2 (\omega_1^2 - \omega_2^2)$

or $\quad 7500 = \frac{1}{2} \times 220 \times 0.5^2 (37.7^2 - \omega_2^2)$

or $\quad 37.7^2 - \omega_2^2 = 272.7$ or $\omega_2 = 33.89$ rad/s

or $\quad N_2 = \frac{33.89 \times 60}{2\pi}$ 323.6 rpm

Now, energy supplied by the motor in one minute = 3000 × 60 N.m

Energy required/riveting = 12 000 N.m

∴ number of rivets closed /minute

$$= \frac{3000 \times 60}{12\,000} = 15$$

Example 13.27 *A punching machine carries out 6 holes per minute. Each hole of 40-mm diameter in 35-mm thick plate requires 8 N.m of energy/mm² of the sheared area. The punch has a stroke of 95 mm. Find the power of the motor required if the mean speed of the flywheel is 20 m/s. If total fluctuation of speed is not to exceed 3% of the mean speed, determine the mass of the flywheel.*

Solution

$d = 40$ mm $\quad K = 0.03$

$t = 35$ mm $\quad$ Stroke = 95 mm

$v = 20$ m/s

As 6 holes are punched in one minute, time required to punch one hole is 10 s.

Energy required/hole or energy supplied by the motor in 10 seconds

= area of hole × energy required /mm²

$= \pi dt \times 8$

Figure 14.2 shows a rigid rotor revolving with a constant angular velocity of ω rad/s. A number of masses, say three, are depicted by point masses at different radii in the same transverse plane. They may represent different kinds of rotating masses such as turbine blades, eccentric discs, etc. If m_1, m_2 and m_3 are the masses revolving at radii r_1, r_2 and r_3 respectively in the same plane, then each mass produces a centrifugal force acting radially outwards from the axis of rotation. Let **F** be the vector sum of these forces,

$$\mathbf{F} = m_1\mathbf{r}_1\omega^2 + m_2\mathbf{r}_2\omega^2 + m_3\mathbf{r}_3\omega^2$$

The rotor is said to be statically balanced if the vector sum **F** is zero.

If **F** is not zero, i.e., the rotor is unbalanced, then introduce a counterweight (balance weight) of mass m_c, at radius $\mathbf{r}_c$ to balance the rotor so that

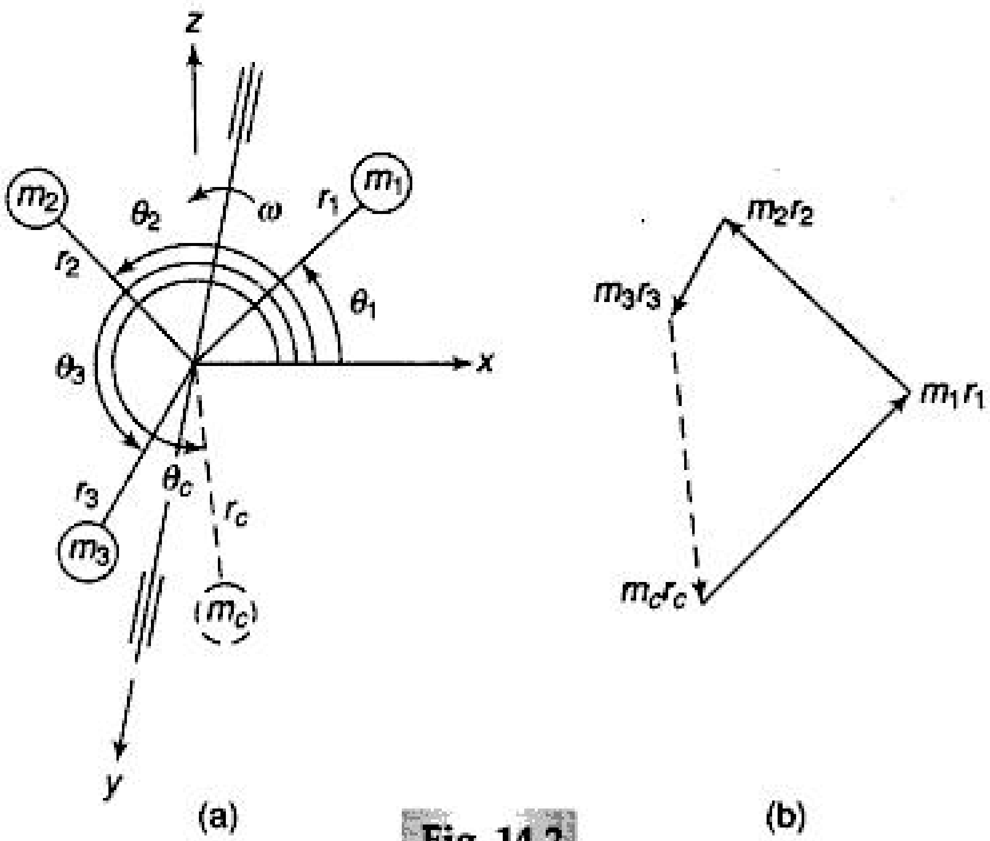

Fig. 14.2

$$m_1\mathbf{r}_1\omega^2 + m_2\mathbf{r}_2\omega^2 + m_3\mathbf{r}_3\omega^2 + m_c\mathbf{r}_c\omega^2 = 0 \tag{14.1}$$

or

$$m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + m_3\mathbf{r}_3 = 0 \tag{14.1a}$$

The magnitude of either m_c or $\mathbf{r}_c$ may be selected and of the other can be calculated.

In general, if $\sum m\mathbf{r}$ is the vector sum of $m_1\mathbf{r}_1$, $m_2\mathbf{r}_2$, $m_3\mathbf{r}_3$, $m_4\mathbf{r}_4$, etc., then

$$\sum m\mathbf{r} + m_c\mathbf{r}_c = 0 \tag{14.2}$$

The equation can be solved either mathematically or graphically. To solve it mathematically, divide each force into its x and z components,

i.e.,
$$\sum mr\cos\theta + m_cr_c\cos\theta_c = 0$$

and
$$\sum mr\sin\theta + m_cr_c\sin\theta_c = 0$$

or

$$m_cr_c\cos\theta_c = -\sum mr\cos\theta \tag{i}$$

and

$$m_cr_c\sin\theta_c = -\sum mr\sin\theta \tag{ii}$$

Squaring and adding (i) and (ii),

$$m_cr_c = \sqrt{(\sum mr\cos\theta)^2 + (\sum mr\sin\theta)^2} \tag{14.3}$$

Dividing (ii) by (i),

$$\tan\theta_c = \frac{-\sum mr\sin\theta}{-\sum mr\cos\theta} \tag{14.4}$$

The signs of the numerator and denominator of this function identify the quadrant of the angle.

In graphical solution, vectors, $m_1\mathbf{r}_1$, $m_2\mathbf{r}_2$, $m_3\mathbf{r}_3$, etc., are added. If they close in a loop, the system is balanced. Otherwise, the closing vector will be giving $m_c\mathbf{r}_c$. Its direction identifies the angular position of the countermass relative to the other masses.

Example 14.3 *A rotating shaft carries three unbalanced masses of 4 kg, 3 kg and 2.5 kg at radial distances of 75 mm, 85 mm and 50 mm and at the angular positions of 45°, 135° and 240° respectively. The second and the third masses are in the planes at 200 mm and 375 mm from the plane of the first mass. The angular positions are measured counter-clockwise from the reference line along x-axis and viewing the shaft from the first mass end.*

The shaft length is 800 mm between bearings and the distance between the plane of the first mass and the bearing at that end is 225 mm. Determine the amount of the countermasses in planes at 75 mm from the bearings for the complete balance of the shaft. The first countermass is to be in a plane between the first mass and the bearing and the second mass in a plane between the third mass and the bearing at that end.

Solution Figure 14.7(a) shows the planes of unbalanced masses as well as the planes of the countermasses. Plane C_1 is to be taken as the reference plane and the various distances are to be considered from this plane.

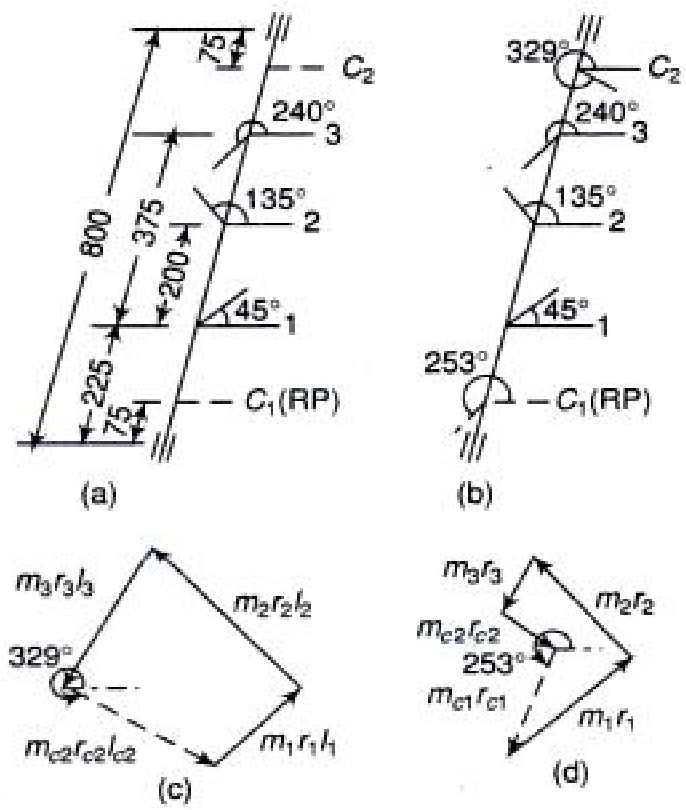

Fig. 14.7

Analytical solution

$$l_{c2} = (800 - 75 \times 2) = 650 \text{ mm}$$
$$l_1 = 225 - 75 = 150 \text{ mm}$$
$$l_2 = 150 + 200 = 350 \text{ mm}$$
$$l_3 = 150 + 375 = 525 \text{ mm}$$

$$m_1 r_1 l_1 = 4 \times 75 \times 150 = 45\,000 \quad m_1 r_1 = 4 \times 75 = 300$$
$$m_2 r_2 l_2 = 3 \times 85 \times 350 = 89\,250 \quad m_2 r_2 = 3 \times 85 = 255$$
$$m_3 r_3 l_3 = 2.5 \times 50 \times 525 = 65\,625 \quad m_3 r_3 = 2.5 \times 50 = 125$$
$$\Sigma m\mathbf{r}l + m_{c2}\mathbf{r}_{c2}\, l_{c2} = 0$$

or $4500 \cos 45° + 89\,250 \cos 135° + 65\,625 \cos 240° + m_{c2}\mathbf{r}_{c2} l_{c2} \cos\theta_{c2} = 0$

and $45000 \sin 45° + 89\,250 \sin 135° + 65\,625 \sin 240° + m_{c2}\mathbf{r}_{c2} l_{c2} \sin\theta_{c2} = 0$

Squaring, adding and then solving,

$$m_{c2} r_{c2} l_{c2} = \left[\begin{array}{l}\left(\begin{array}{l}45\,000 \cos 45° + 89\,250 \\ \cos 135° + 65\,625 \cos 240°\end{array}\right)^2 \\ + \left(\begin{array}{l}45\,000 \sin 45° + 89\,250 \\ \sin 135° + 65\,625 \sin 240°\end{array}\right)^2\end{array}\right]^{1/2}$$

$$= [(-64\,102)^2 + (38\,096)^2]^{1/2}$$

or $\quad m_{c2} \times 40 \times 650 = 74\,568$

$$m_{c2} = \underline{2.868 \text{ kg}}$$

$$\tan\theta_{c2} = \frac{-38\,096}{-(-64\,102)} = -0.594$$

$$\theta_{c2} = 329.3° \text{ or } \underline{329°18'}$$

Now,

$$\Sigma m\mathbf{r} + m_{c1}\mathbf{r}_{c1} + m_{c2}\mathbf{r}_{c2} = 0$$

or $\quad 300 \cos 45° + 255 \cos 135° + 125 \cos 240° + m_{c1}\mathbf{r}_{c1} \cos\theta_1 + 2.868 \times 40 \cos 329.3 = 0$

and $\quad 300 \sin 45° + 255 \sin 135° + 125 \sin 240° + m_{c1}\mathbf{r}_{c1} \sin\theta_1 + 2.868 \times 40 \sin 329.3 = 0$

Squaring, adding and then solving,

$$m_{c1} r_{c1} = \left[\begin{array}{l}(300 \cos 45° + 255 \cos 135° \\ + 125 \cos 240° + 2.868 \\ \times 40 \cos 329.3°)^2 + \\ (300 \sin 45° + 255 \sin 135° \\ + 125 \sin 240° + 2.868 \\ \times 40 \sin 329.3°)^2\end{array}\right]^{1/2}$$

$$m_{c1} \times 75 = [(67.96)^2 + (225.62)^2]^{1/2} = 235.63$$

$$m_{c1} = \underline{3.14 \text{ kg}}$$

$$\tan\theta_{c1} = \frac{-225.62}{-67.96} = 3.32;\ \theta_{c1} = 253.2° \text{ or } \underline{253°12'}$$

14.5 FORCE BALANCING OF LINKAGES

Balancing of a linkage implies that the total centre of its mass remains stationary so that the vector sum of all the frame forces always remains zero. Figure 14.12 shows a four-link mechanism. a, b, c and d represent the magnitudes of the links AB, BC, CD and DA respectively. The link masses are m_a, m_b and m_c, located at G_a, G_b and G_c respectively. Let the coordinates g_i, ϕ_i describe the position of these points within each link.

As in any configuration of the mechanism, the links of the mechanism can be considered as vectors. Thus,

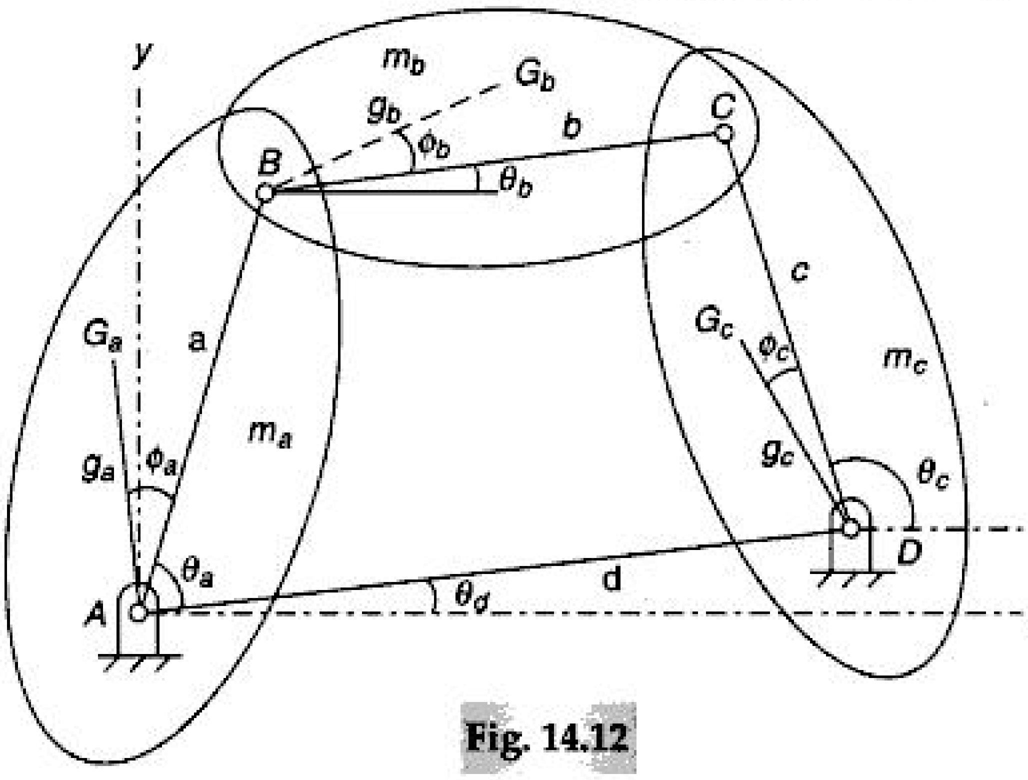

Fig. 14.12

$$ae^{i\theta_a} + be^{i\theta_b} - ce^{i\theta_c} - de^{i\theta_d} = 0 \qquad (14.15)$$

or

$$e^{i\theta_b} = \frac{1}{b}(de^{i\theta_d} - ae^{i\theta_a} + ce^{i\theta_c}) \qquad (14.15a)$$

Let G be the centre of mass for the system of moving links and let $\mathbf{g}$ define the position of G with respect to origin A.

Total mass of moving links, $m = m_a + m_b + m_c$ (14.16)

Then for the centre of mass of the entire system to remain stationary at a point, the following expression must be a constant (acceleration due to weight is constant).

$$m\mathbf{g} = m_a\mathbf{g}_a + m_b\mathbf{g}_b + m_c\mathbf{g}_c \qquad (14.17)$$

where $\mathbf{g}_a$, $\mathbf{g}_b$ and $\mathbf{g}_c$ are the vectors representing the positions of masses m_a, m_b and m_c respectively, w.r.t. A.

$$m\mathbf{g} = m_a g_a e^{i(\theta_a+\varphi_a)} + m_b[ae^{i\theta_a} + g_b e^{i(\theta_b+\varphi_b)}] + m_c[de^{i\theta_d} + g_c e^{i(\theta_c+\varphi_c)}]$$

$$= m_a g_a e^{i\theta_a} e^{i\varphi_a} + m_b a e^{i\theta_a} + m_b g_b e^{i\theta_b} e^{i\varphi_b} + m_c d e^{i\theta_d} + m_c g_c e^{i\theta_c} e^{i\varphi_c}$$

Inserting the value of $e^{i\theta_b}$ from (14.15a)

$$mg = m_a g_a e^{i\theta_a} e^{i\varphi_a} + m_b a e^{i\theta_a} + m_b g_b \frac{1}{b}(de^{i\theta_d} - ae^{i\theta_a} + ce^{i\theta_c})e^{i\varphi_b} + m_c d e^{i\theta_d} + m_c g_c e^{i\theta_c} e^{i\varphi_c}$$

$$= \left(m_a g_a e^{i\varphi_a} + m_b a - m_b g_b \frac{a}{b} e^{i\varphi_b} \right) e^{i\theta_a} + \left(m_c g_c e^{i\varphi_c} + m_b g_b \frac{c}{b} e^{i\varphi_b} \right) e^{i\theta_c} + \left(m_c d + m_b g_b \frac{d}{b} e^{i\varphi_b} \right) e^{i\theta_d}$$

The centre of mass can be made stationary at the position $\mathbf{g} = \left(m_c d + m_b g_b \frac{d}{b} e^{i\varphi_b} \right) e^{i\theta_d}$ if the remaining two terms in the brackets can be made zero. Let the vector $\mathbf{g}_a'$ represent the position of the countermass m_a' to be added to the input link and vector $\mathbf{g}_c'$ represent the position of the countermass m_c' to be added to the output link to have complete force balancing.

Thus the equations may be written as

15 BRAKES AND DYNAMOMETERS

Introduction

A *brake* is an appliance used to apply frictional resistance to a moving body to stop or retard it by absorbing its kinetic energy. In general, in all types of motion, there is always some amount of resistance which retards the motion and is sufficient to bring the body to rest. However, the time taken and the distance covered in this process is usually too large. By providing brakes, the external resistance is considerably increased and the period of retardation shortened.

A *dynamometer* is a brake incorporating a device to measure the frictional resistance applied. This is used to determine the power developed by the machine, while maintaining its speed at the rated value.

The functional difference between a clutch and a brake is that a clutch connects two moving members of a machine whereas a brake connects a moving member to a stationary member.

15.1 TYPES OF BRAKES

The following are the main types of mechanical brakes:

(i) Block or shoe brake
(ii) Band brake
(iii) Band and block brake
(iv) Internal expanding shoe brake

15.2 BLOCK OR SHOE BRAKE

A block or shoe brake consists of a block or shoe which is pressed against a rotating drum. The force on the drum is increased by using a lever [Fig. 15.1(a)]. If only one block is used for the purpose, a side thrust on the bearing of the shaft supporting the drum will act. This can be prevented by using two blocks on the two sides of the drum [Fig.15.1(b)]. This also doubles the braking torque.

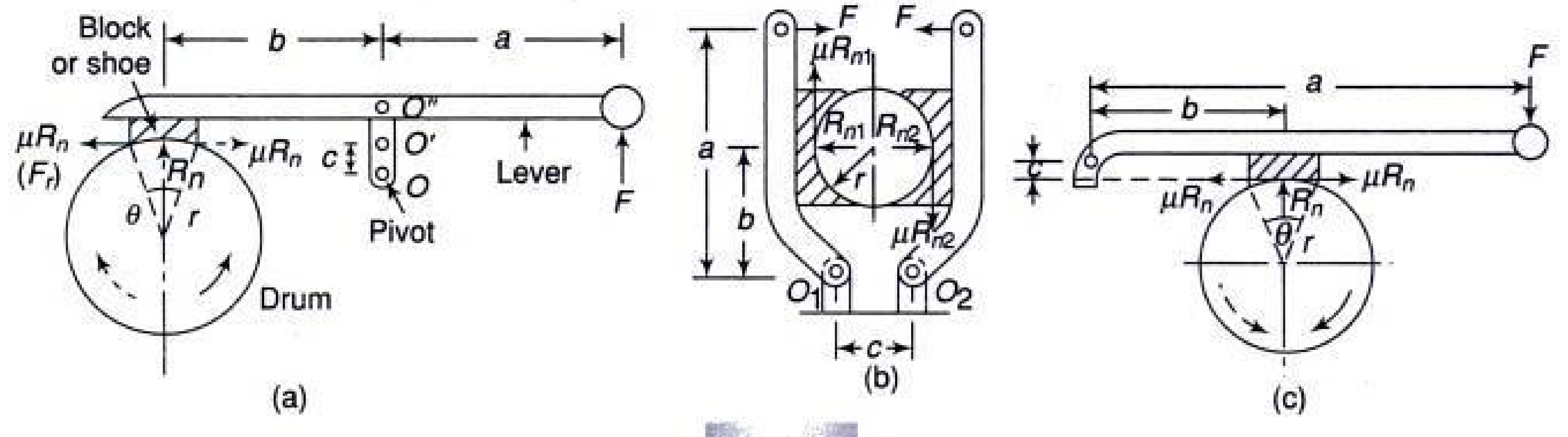

Fig. 15.1

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or $\frac{1}{2}mv^2 = T_B.\omega$

or $\frac{160}{2} \times \left(\frac{\pi \times 0.44 \times 1500}{60}\right)^2 = 86.8 \times 2\pi n$

or $80 \times 1194.2 = 545.4\,n$ or $n = 175$

Time taken, $t = \frac{n}{N} = \frac{175}{1500/60} = 7$ s

Example 15.4 *A spring-operated pivoted shoe brake shown in Fig. 15.4 (a) is used for a wheel diameter of 500 mm. The angle of contact is 90° and the coefficient of friction is 0.3. The force applied by the spring on each arm is 5 kN. Determine the brake torque on the wheel.*

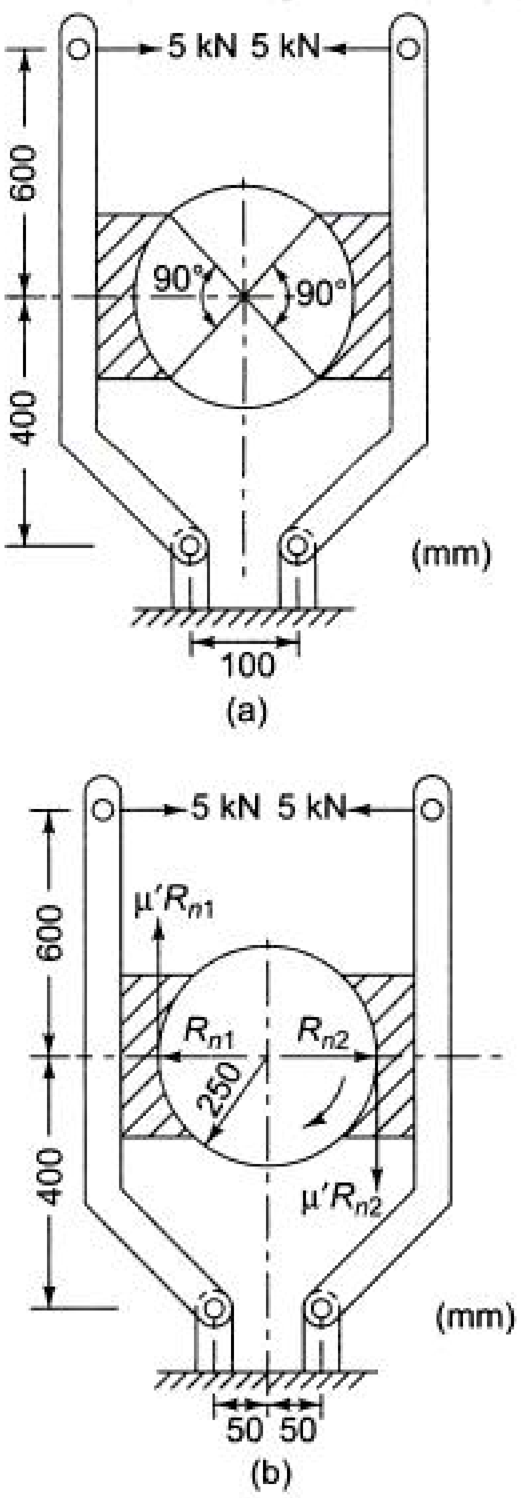

Fig. 15.4

Solution

$\mu = 0.3$;

$F = 5000$ N;

$d = 500$ mm,

$r = 250$ mm

Assuming the rotation to be clockwise, the various forces acting on the two blocks are shown in Fig. 15.4(b).

$$\text{Now, } \mu' = \mu\left(\frac{4\sin(\theta/2)}{\theta + \sin\theta}\right)$$

$$= 0.3\left(\frac{4\sin 45°}{(\pi/2) + \sin 90°}\right) = 0.33$$

For the left-hand side block, taking moments about O_1,

$$F \times 1 - R_{n1} \times 0.4 + \mu' R_{n1} \times (0.25 - 0.05) = 0$$

$$5000 \times 1 - R_{n1} \times 0.4 + 0.33 \times R_{n1} \times 0.2 = 0$$

$$R_{n1} = 14\,970 \text{ N}$$

For the right-hand side block, taking moments about O_2,

$$5000 \times 1 - R_{n2} \times 0.4 - 0.33 \times R_{n2} \times 0.2 = 0$$

$$R_{n2} = 10\,730 \text{ N}$$

Maximum braking torque, $T_B = \mu' (R_{n1} + R_{n2})\, r$

$= 0.33\,(14\,970 + 10\,730) \times 0.25$

$= 2120$ N.m

Example 15.5 *Figure 15.5(a) shows an arrangement of a double block shoe brake. The force to each block is applied by means of a turn buckle with right and left-handed threads of six-start with a lead of 40 mm. The diameter of the turn buckle is 20 mm and it is rotated by a lever. The angle subtended by each block is 80°. The coefficient of friction for the brake blocks is 0.3 and for the screw and the nut, 0.18. Determine the brake torque applied by a force of 80 N at the end of the lever.*

Let T_0 = tension on the slack side
T_1 = tension on the tight side after one block
T_2 = tension on the tight side after two blocks
....
....
T_n = tension on the tight side after n blocks
μ = coefficient of friction
R_n = normal reaction on the block

The forces on one block of the brake are shown in Fig.15.15(b).
For equilibrium,

$$(T_1 - T_0) \cos \theta = \mu R_n$$

$$(T_1 + T_0) \sin \theta = R_n$$

or
$$\frac{T_1 - T_0}{T_1 + T_0} \frac{1}{\tan \theta} = \mu$$

or
$$\frac{T_1 - T_0}{T_1 + T_0} = \frac{\mu \tan \theta}{1}$$

or
$$\frac{(T_1 - T_0) + (T_1 + T_0)}{(T_1 - T_0) - (T_1 + T_0)} = \frac{\mu \tan \theta + 1}{\mu \tan \theta - 1}$$

$$-\frac{2T_1}{2T_0} = -\frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

$$\frac{T_1}{T_0} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

Similarly,
$$\frac{T_2}{T_1} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}, \text{ and so on.}$$

....
....

and
$$\frac{T_n}{T_n - 1} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

$$\frac{T_n}{T_0} = \frac{T_n}{T_{n-1}} \frac{T_{n-1}}{T_{n-2}} \cdots \frac{T_2}{T_1} \frac{T_1}{T_0}$$

$$= \left(\frac{1 + \mu \tan \theta}{1 - \mu \tan \theta} \right)^n \qquad (15.8)$$

Example 15.11 *A band and block brake has 14 blocks. Each block subtends an angle of 14° at the centre of the rotating drum. The diameter of the drum is 750 mm and the thickness of the blocks is 65 mm. The two ends of the band are fixed to the pins on the lever at distances of 50 mm and 210 mm from the fulcrum on the opposite sides. Determine the least force required to be applied at the lever*

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Brakes applied to front wheels only

$$R_A + R_B = Mg \cos \alpha \qquad \text{(iv)}$$

$$\mu R_A + Mg \sin \alpha = Mf \qquad \text{(v)}$$

Taking moments about G,

$$R_B x + \mu R_A \times h - R_A (l - x) \qquad \text{(vi)}$$

From (iv) and (vi),

$$(Mg \cos \alpha - R_A) x + \mu R_A \times h - R_A (l - x) = 0$$

or

$$Mg\, x \cos \alpha = R_a (x - \mu h + l - x)$$

$$R_A = \frac{Mgx \cos \alpha}{l - \mu h}$$

Therefore (v) becomes, $\mu = \dfrac{Mgx \cos \alpha}{l - \mu h} + Mg \sin \alpha = Mf$

or

$$f = g \cos \alpha \left[\frac{\mu x}{l - \mu h} + \tan \alpha \right] \qquad (15.15)$$

On a level road, $\alpha = 0$, and therefore

$$f = g \frac{\mu x}{l - \mu h} \qquad (15.16)$$

On a down plane,

$$f = g \cos \alpha \left(\frac{\mu x}{l - \mu h} - \tan \alpha \right) \qquad (15.17)$$

Brakes applied to all four wheels

$$R_A + R_B = Mg \cos \alpha \qquad \text{(vii)}$$

$$\mu R_A + \mu R_B + Mg \sin \alpha = Mf \qquad \text{(viii)}$$

or

$$\mu (R_A + R_B) + Mg \sin \alpha = Mf$$

or

$$\mu Mg \cos \alpha + Mg \sin \alpha = Mf$$

or

$$f = g \cos \alpha (\mu + \tan \alpha) \qquad (15.18)$$

On a level road, $\alpha = 0$. Therefore

$$f = g \mu \qquad (15.19)$$

On a down plane,

$$f = g \cos \alpha (\mu - \tan \alpha) \qquad (15.20)$$

Example 15.14 *A vehicle having a wheel base of 3.2 m has its centre of mass at 1.4 m from the rear wheels and 55 mm from the ground level. It moves on a level ground at a speed of 54 km/h. Determine the distance moved by the car before coming to rest on applying the brakes to the*

(i) rear wheels
(ii) front wheels
(iii) all the four wheels

The coefficient of friction between the tyres and the road is 0.5.

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extent. When the speed decreases, the balls rotate at a smaller radius and the valve is opened according to the requirement.

(ii) Inertia Governor

In this type, the positions of the balls are affected by the forces set up by an angular acceleration or deceleration of the given spindle in addition to centrifugal forces on the balls. Using suitable linkages and springs, the change in position of the balls is made to open or close the throttle valve.

Thus, whereas the balls are operated by the actual change of engine speed in the case of centrifugal governors, it is by the rate of change of speed in case of inertia governors. Therefore, the response of inertia governors is faster than that of centrifugal types.

16.2 WATT GOVERNOR (SIMPLE CONICAL GOVERNOR)

Figure 16.2 shows three forms of a simple centrifugal or a Watt governor. In this, a pair of balls (masses) is attached to a spindle with the help of links. In Fig.16.2(a), the upper links are pinned at point O. In the Fig.16.1(b), the upper links are connected by a horizontal link and the governor is known as the *open-arm* type Watt governor. On extending the upper arms, they still meet at O. In Fig.16.2(c), the upper links cross the spindle and are connected by a horizontal link and the governor is known as a *crossed-arm* Watt governor. In this type also, the two links intersect at O. The lower links in every case are fixed to a sleeve free to move on the vertical spindle.

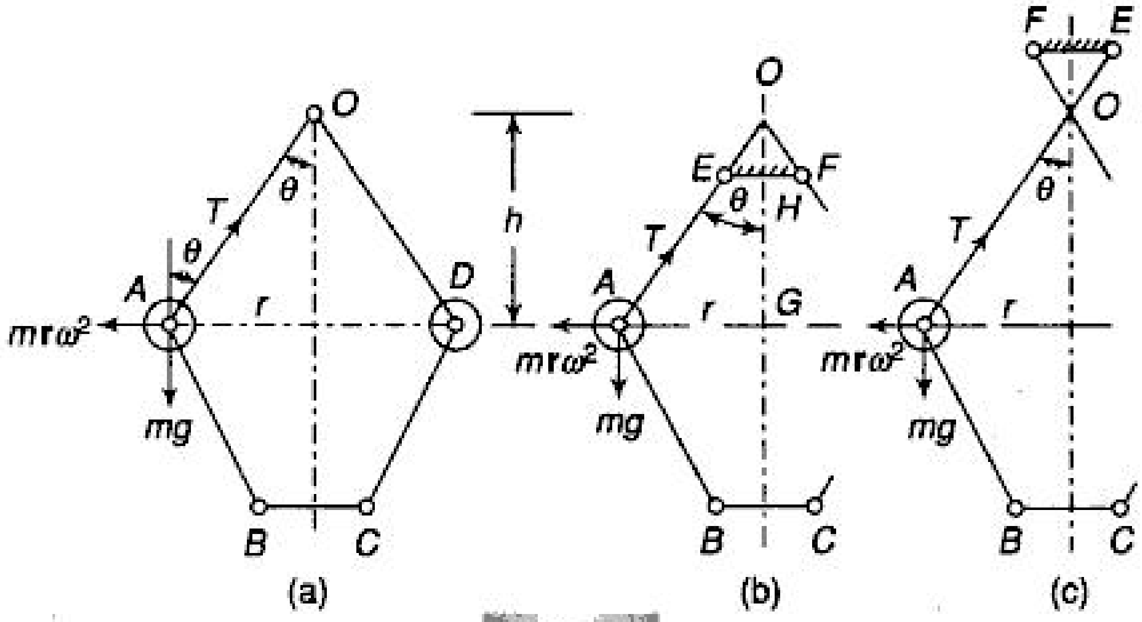

Fig. 16.2

As the spindle rotates, the balls take up a position depending upon the speed of the spindle. If it lowers, they move near to the axis due to reduction in the centrifugal force on the balls and the ability of the sleeve to slide on the spindle. The movement of the sleeve is further taken to the throttle of the engine by means of a suitable linkage to decrease or increase the fuel supply.

The vertical distance from the plane (horizontal) of rotation of the balls to the point of intersection of the upper arms along the axis of the spindle is called the *height of the governor*. The height of the governor decreases with increase in speed, and increases with decrease in speed.

Let m = mass of each ball

h = height of governor

w = weight of each ball (= mg)

ω = angular velocity of the balls, arms and the sleeve

T = tension in the arm

r = radial distance of ball-centre from spindle-axis

Assuming the links to be massless and neglecting the friction of the sleeve, the mass m at A is in static equilibrium under the action of

- Weight **w** (= mg)
- Centrifugal force $mr\omega^2$
- Tension **T** in the upper link

If the sleeve is massless and also friction is neglected, the lower links will be tension free.

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17 GYROSCOPE

Introduction

If the axis of a spinning or rotating body is given an angular motion about an axis perpendicular to the axis of spin, an angular acceleration acts on the body about the third perpendicular axis. The torque required to produce this acceleration is known as the *active gyroscopic torque*. A reactive gyroscopic torque or couple also acts similar to the concept of centripetal and centrifugal forces on a rotating body. The effect produced by the reactive gyroscopic couple is known as the *gyroscopic effect*. Thus aeroplanes, ships, automobiles, etc., that have rotating parts in the form of wheels or rotors of engines experience this effect while taking a turn, i.e., when the axes of spin is subjected to some angular motion.

17.1 ANGULAR VELOCITY

The angular velocity of a rotating body is specified by

- the magnitude of velocity
- the direction of the axis of rotor
- the sense of rotation of the rotor, i.e., clockwise or counter-clockwise

Angular velocity is represented by a vector in the following manner:

(i) Magnitude of the velocity is represented by the length of the vector.

(ii) Direction of axis of the rotor is represented by drawing the vector parallel to the axis of the rotor or normal to the plane of the angular velocity.

(iii) Sense of rotation of the rotor is denoted by taking the direction of the vector in a set rule. The general rule is that of a right-handed screw, i.e., if a screw is rotated in the clockwise direction, it goes away from the viewer and vice-versa.

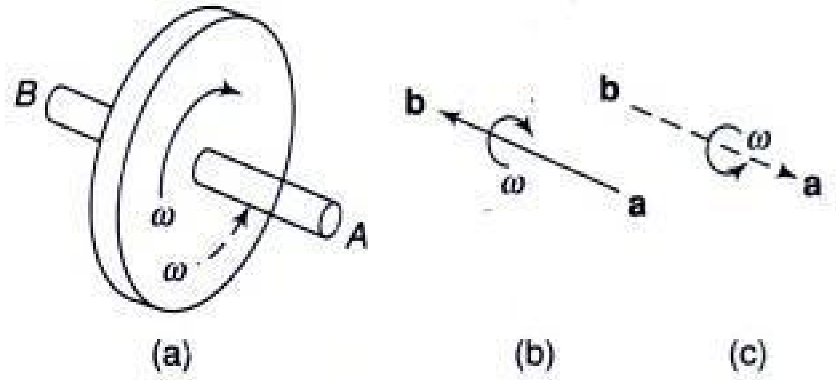

Fig. 17.1

For example, Fig.17.1(a) shows a rotor which rotates in the clockwise direction when viewed from the end A. Its angular motion has been shown vectorially in Fig.17.1(b). The vector has been taken to a scale parallel to the axis of the rotor. The sense of direction of the vector is from **a** to **b** according to the screw rule. However, if the direction of rotation of the rotor is reversed, it would be from **b** to **a** [Fig.17.1(c)].

17.2 ANGULAR ACCELERATION

Let a rotor spin (rotate) about the horizontal axis Ox at a speed of ω rad/s in the direction as shown in Fig.17.2(a). Let **oa** represent its angular velocity [Fig.17.2(b)].

Solution

$m = 2200$ kg $\quad N = 1800$ rpm

$R = 250$ m $\quad v = 25$ km/h

$k = 0.32$ m $\quad = \dfrac{25 \times 1000}{3600}$

$= 6.94$ m/s

$I = mk^2 = 2200 \times (0.32)^2$

$= 225.3$ kg.m^2

$$\omega = \frac{2\pi \times 1800}{60} = 188.5 \text{ rad/s}$$

$$\omega_p = \frac{v}{R} = \frac{6.94}{250} = 0.0278 \text{ rad/s}$$

(i) $C = I\,\omega\omega_p$

$= 225.3 \times 188.5 \times 0.0278$

$= 1180$ N.m

The effect is to lower the bow (fore) and raise the stern (aft) when the ship turns right [Fig. 17.7(b)].

(ii) $\omega_p = 0.8$ rad/s

$C = I\,\omega\omega_p = 225.3 \times 188.5 \times 0.8$

$= 33\,972$ N.m

The effect of the reaction couple when the bow is rising, is to turn the ship towards right or towards starboard.

(iii) $\omega_p = 0.1$ rad/s

$C = 225.3 \times 188.5 \times 0.1 = 4246.5$ N.m

As the axis of spin is always parallel to the axis of precession for all positions, there is no gyroscopic effect on the ship.

Example 17.5 *The rotor of the turbine of a ship has a mass of 2500 kg and rotates at a speed of 3200 rpm counter-clockwise when viewed from stern. The rotor has radius of gyration of 0.4 m. Determine the gyroscopic couple and its effect when*

(i) the ship steers to the left in a curve of 80-m radius at a speed of 15 knots (1 knot = 1860 m/h)

(ii) the ship pitches 5 degrees above and 5 degrees below the normal position and the bow is descending with its maximum velocity—the pitching motion is simple harmonic with a periodic time of 40 seconds

(iii) the ship rolls and at the instant, its angular velocity is 0.4 rad/s clockwise when viewed from stern

Also find the maximum angular acceleration during pitching.

Solution

$m = 2500$ kg $\quad N = 1800$ rpm

$k = 0.4$ m $\quad v = \dfrac{15 \times 1860}{3600} = 7.75$ m/s

$I = mk^2 = 2500 \times (0.4)^2 = 400$ kg.m^2

$$\omega = \frac{2\pi \times 3200}{60} = 335 \text{ rad/s}$$

(i) $R = 80$ m

$$\omega_p = \frac{v}{R} = \frac{775}{80} = 0.097 \text{ rad/s}$$

$C = 400 \times 335 \times 0.097$

$= 12\,981$ N.m

The effect is to lower the bow and raise the stern [Figs 17.8 (a) and (b)].

(ii) $\varphi = 5° = 5 \times \dfrac{\pi}{180} = 0.0873$ rad

$T = 40$ s

$$\therefore \quad \omega_0 = \frac{2\pi}{40} = 0.157 \text{ rad/s}$$

$\omega_p = \varphi\,\omega_0 = 0.0873 \times 0.157 = 0.0137$ rad/s

$C = I\,\omega\,\omega_p = 400 \times 335 \times 0.0137 = 1837.5$ N.m

As the bow descends during pitching, the ship would turn towards right or starboard [Figs 17.8(a) and (c)].

(iii) $\omega_p = 0.04$ rad/s

$C = 400 \times 335 \times 0.04 = 5360$ N.m

No gyroscopic effect is there as discussed earlier.

Maximum angular acceleration during pitching

$$\alpha_{max} = \varphi\omega_0^2 = 0.0873 \times (0.157)^2 = \underline{0.00215 \text{rad/s}^2}$$

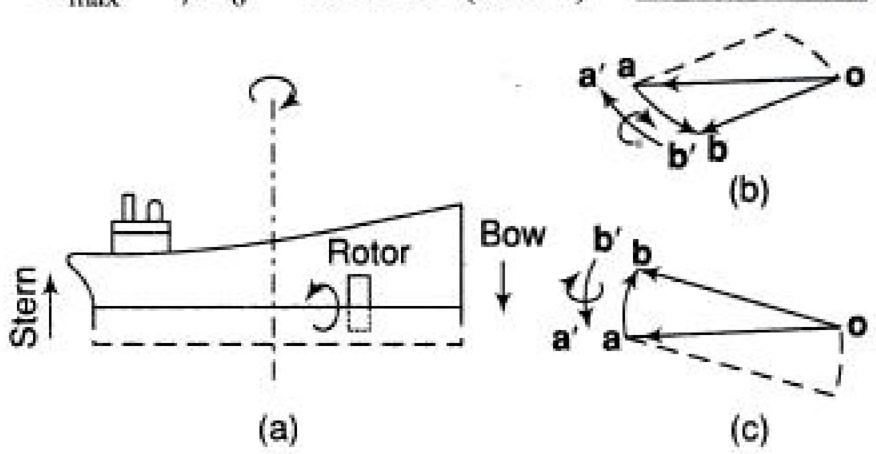

Fig. 17.8

centrifugal force make the rider of a two-wheeler tilt on one side? Derive a relation for the limiting speed of the vehicle.

10. A flywheel having a mass of 20 kg and a radius of gyration of 300 mm is given a spin of 500 rpm about its axis which is horizontal. The flywheel is suspended at a point that is 250 mm from the plane of rotation of the flywheel. Find the rate of precession of the wheel. (0.52 rad/s)
11. A disc supported between two bearings on a shaft of negligible weight has a mass of 80 kg and a radius of gyration of 300 mm. The distances of the disc from the bearings are 300 mm to the right from the left-hand bearing and 450 mm to the left from the right-hand bearing. The bearings are supported by thin vertical cords. When the disc rotates at 100 rad/s in the clockwise direction looking from the left-hand bearing, the cord supporting the left-hand side bearing breaks. Find the angular velocity of precession at the instant the cord is cut and discuss the motion of the disc. (0.327 rad/s)
12. The moment of inertia of an aeroplane air screw is 20 kg.m² and the speed of rotation is 1000 rpm clockwise when viewed from the front. The speed of the flight is 200 km per hour. Find the gyroscopic reaction of the air screw on the aeroplane when it makes a left-handed turn on a path of 150-m radius. (775.5 N.m)
13. The rotor of a marine turbine has a moment of inertia of 750 kg.m² and rotates at 3000 rpm clockwise when viewed from aft. If the ship pitches with angular simple harmonic motion having a periodic time of 16 seconds and an amplitude of 0.1 radian, find the
    (i) maximum angular velocity of the rotor axis
    (ii) maximum value of the gyroscopic couple
    (iii) gyroscopic effect as the bow dips

    (0.0393 rad/s; 9261 N.m; bow swings to port (left) as it dips)
14. The turbine rotor of a sea vessel having a mass of 950 kg rotates at 1200 rpm clockwise while looking from the stern. The vessel pitches with an angular velocity of 1.2 rad/s. What will be the gyroscopic couple transmitted to the hull when the bow rises? The radius of gyration of the rotor is 300 mm. (12.89 kN.m)
15. A ship is propelled by a turbine rotor having a mass of 6 tonnes and a speed of 2400 rpm. The direction of rotation of the rotor is clockwise when viewed from the stern. The radius of gyration of the rotor is 450 mm. Determine the gyroscopic effect when the
    (i) ship steers to the left in a curve of 60 m radius at a speed of 18 knots (1 knots = 1860 m/h)
    (ii) ship pitches 7.5 degrees above and 7.5 degrees below the normal position and the bow is descending with its maximum velocity; the pitching motion is simple harmonic with a periodic time of 18 seconds
    (iii) ship rolls and at the instant, its angular velocity is 0.035 rad/s counter-clockwise when viewed from the stern

    Also, find the maximum angular acceleration during pitching.

    (47.33 kN.m, bow is raised; 13.96 kN.m, ship turns towards port side; 10. 69 kN.m, no gyroscopic effect; 0.016 rad/s²)
16. A rear engine automobile is travelling along a curved track of 120 m radius. Each of the four wheels has a moment of inertia of 2.2 kg/m² and an effective diameter of 600 mm. The rotating parts of the engine have a moment of inertia of 1.25 kg.m². The gear ratio of the engine to the back wheel is 3.2. The engine axis is parallel to the rear axle and the crankshaft rotates in the same sense as the road wheels. The mass of the vehicle is 2050 kg and the centre of mass is 520 mm above the road level. The width of the track is 1.6 m. What will be the limiting speed of the vehicle if all the four wheels maintain contact with the road surface? (150.2 km/h)
17. The moment of inertia of a pair of locomotive driving wheels with the axle is 200 kg.m². The distance between the wheel centres is 1.6 m and the diameter of the wheel treads is 1.8 m. Due to defective ballasting, one wheel falls by 5 mm and rises again in a total time of 0.12 seconds while the locomotive travels on a level track at 100 km/h. Assuming that the displacement of the wheel takes place with simple harmonic motion, determine the gyroscopic couple produced and the reaction between the wheel and rail due to this couple. (505 N.m; 315.6 N)
18. Each road wheel of a motor cycle is of 600 mm diameter and has a moment of inertia of 1.1 kg.m². The motorcycle and the rider together weigh 220 kg and the combined centre of mass is 620 mm above the ground level when the motor cycle is upright. The moment of inertia of the rotating parts of the engine is 0.18 kg/m². The engine rotates at 4.5 times the speed of road wheels in the same sense. Find the angle of heel necessary when the motor cycle is taking a turn of 35 m radius at a speed of 72 kg/h. (38.6°)

SECTION-1 (LONGITUDINAL VIBRATIONS)

18.5 FREE LONGITUDINAL VIBRATIONS

The natural frequency of a vibrating system may be found by any of the following methods.

1. Equilibrium Method

It is based on the principle that whenever a vibratory system is in equilibrium, the algebraic sum of forces and moments acting on it is zero. This is in accordance with D' Alembert's principle that the sum of the inertia forces and the external forces on a body in equilibrium must be zero.

Figure 18.5(a) shows a helical spring suspended vertically from a rigid support with its free end at *A-A*.

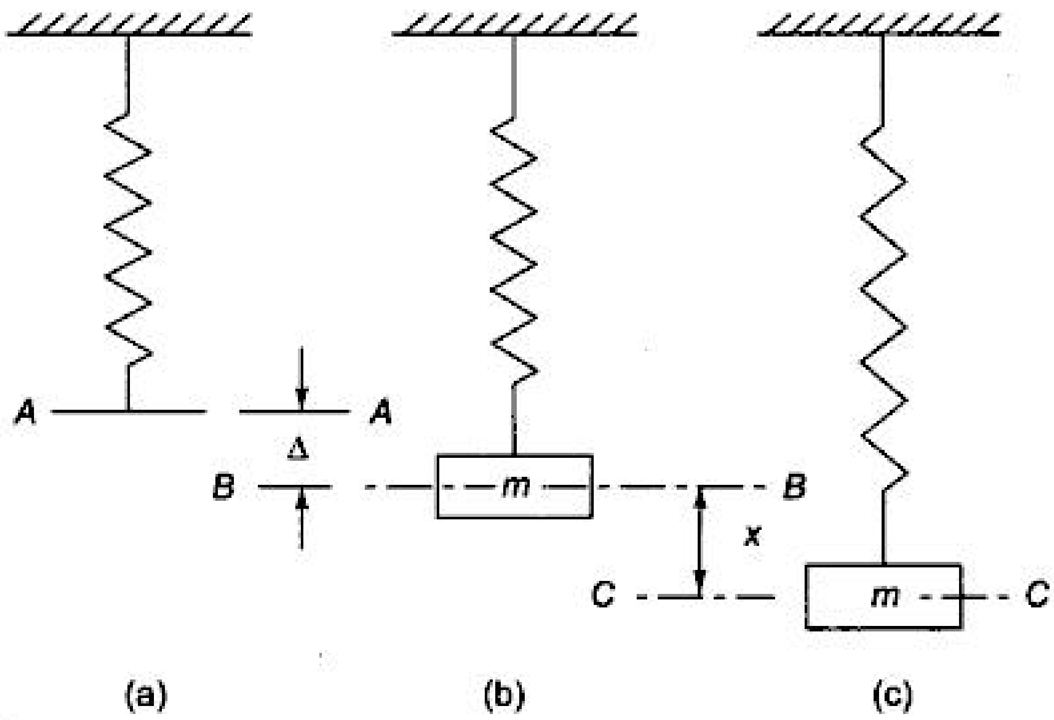

Fig. 18.5

If a mass m is suspended from the free end, the spring is stretched by a distance Δ and *B-B* becomes the equilibrium position [Fig.18.5(b)]. Thus Δ is the static deflection of the spring under the weight of the mass m.

Let s = stiffness of the spring under the weight of the mass m.

In the static equilibrium position,

upward force = downward force

$$s \times \Delta = mg \tag{18.1}$$

Now, if the mass m is pulled farther down through a distance x [Fig.18.5(c)], the forces acting on the mass will be

inertia force $= m\ddot{x}$ (upwards)

spring force (restoring force) $= sx$ (upwards)

(x is downward and thus velocity $\dot{x}$ and acceleration $\ddot{x}$ are also downwards)

As the sum of the inertia force and the external force on the body in any direction is to be zero (D'Alembert's principle),

$$m\ddot{x} + sx = 0 \tag{18.2}$$

If the mass is released, it will start oscillating above and below the equilibrium position. The oscillation will continue for ever if there is no frictional resistance to the motion.

The above equation can be written as

$$\ddot{x} + \left(\frac{s}{m}\right)x = 0 \tag{18.3}$$

This is the equation of a simple harmonic motion and is analogous to

$$\ddot{x} + \omega_n^2 x = 0 \tag{18.4}$$

The solution of which is given by

This shows that the inertia effect of the spring is equal to that of a mass one third of the mass of the spring, concentrated at its free end.

Thus

equivalent mass at the free end $= m + \frac{m_1}{3}$

where

m_1 = mass of the spring

$$f_n = \frac{1}{2}\sqrt{\frac{s}{m + \frac{m_1}{3}}} \tag{18.18}$$

It can be noted that the net force on the spring at any instant tending to restore the vibrating mass to the equilibrium position is sx which is proportional to the displacement of the mass. This is true for any vibration due to the elastic forces. Thus in a vibrating system in which the restoring force is proportional to the displacement from the equilibrium position, the frequency of the system will always be given by

$$f_n = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{mg/\Delta}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} \tag{18.19}$$

where Δ is the static deflection under the suspended mass m.

For example, consider a rod of length l suspended vertically. A mass m is suspended at the free end [Fig. 18.1(a)].

Then

Static deflection, $\Delta = \frac{mgl}{AE}$

where

A = cross-sectional area of the rod

l = length of the rod

E = Young's modulus of the rod material.

Frequency,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{gAE}{mgl}} = \frac{1}{2\pi}\sqrt{\frac{AE}{ml}}$$

However, if the mass of the suspended rod is also to be considered,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{AE}{\left(m + \frac{1}{3}m_1\right)l}}$$

where m_1 = mass of rod

and

$$\text{time period, } T_d = \frac{\omega_d}{2\pi}$$

let X_0 = displacement at the start of motion when $t = 0$

X_1 = displacement at the end of first oscillation when $t = T_d$

$$= Xe^{-\zeta\omega_n T_d} \sin(\omega_d T_d + \varphi)$$

$$= Xe^{-\zeta\omega_n T_d} \sin\left(\omega_d \frac{2\pi}{\omega_d} + \varphi\right)$$

$$= Xe^{-\zeta\omega_n T_d} \sin\varphi$$

X_2 = displacement at the end of second oscillation

$$= Xe^{-\zeta\omega_n \times 2T_d} \sin\varphi$$

Similarly,

$$X_3 = Xe^{-\zeta\omega_n \times 3T_d} \sin\varphi$$

....

$$X_n = Xe^{-\zeta\omega_n \times nT_d} \sin\varphi$$

$$X_{n+1} = Xe^{-\zeta\omega_n \times (n+1)T_d} \sin\varphi$$

Then

$$\frac{X_n}{X_{n+1}} = e^{\zeta\omega_n T_d} = \frac{X_0}{X_1} = \frac{X_1}{X_2} = \frac{X_2}{X_3} = \ldots \tag{18.31}$$

which shows that the ratio of amplitudes of two successive oscillations is constant (Fig. 18.16).

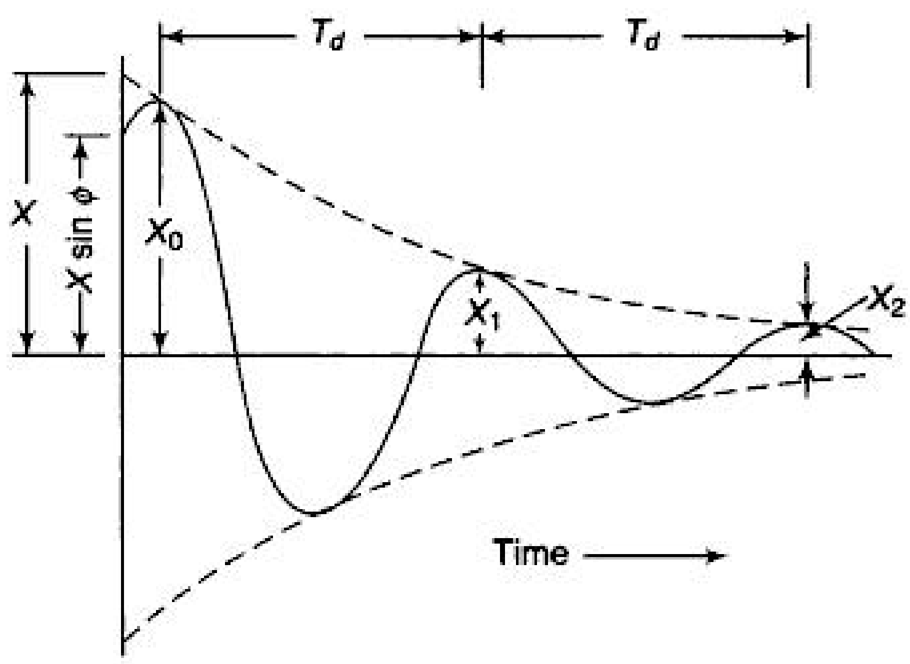

Fig. 18.16

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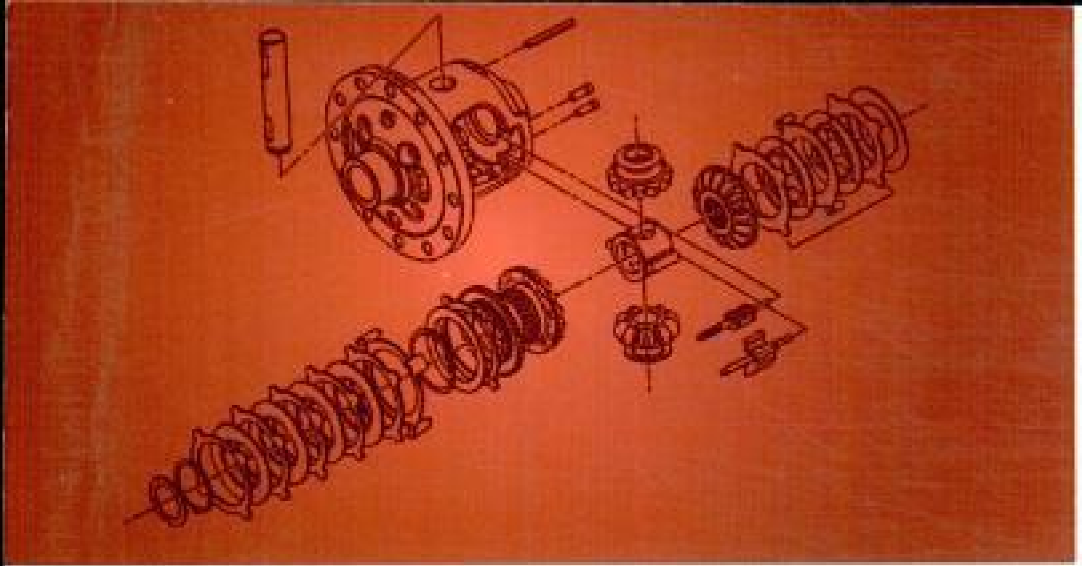

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