Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER– III(NEW) EXAMINATION – WINTER 2022**

**Subject Code: 3130005** **Date: 20-02-2023**

**Subject Name: Complex Variables and Partial Differential Equations**

**Time: 02:30 PM TO 05:00 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

| | | | **Marks** |
|---|---|---|---|
| **Q.1** | **(a)** | If $z = \frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$ then find its modulus value and the principal value of its argument. | **03** |
| | **(b)** | Find all the values of $(1-i)^{2/3}$ | **04** |
| | **(c)** | Solve $\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = \cos(x+2y)$ | **07** |
| **Q.2** | **(a)** | Give examples of functions $f(z)$ and $g(z)$ which are analytic everywhere in the complex plane C such that $f(z)$ is never zero and $g(z) = 0$ if and only if $z = i$. | **03** |
| | **(b)** | Discuss the continuity of $$f(z) = \frac{Re(z^2)}{\lvert z\rvert^2}, \qquad z \neq 0,$$ at $z = 0$ if $f(0) = 0$. | **04** |
| | **(c)** | Define Mobius transformation. Find the mobius transformation which maps the points $z = 0, -i, -1$ to $w = i, 1, 0$ respectively. | **07** |
| | | **OR** | |
| | **(c)** | Define Harmonic function and show that $u(x,y) = 2x - x^3 + 3xy^2$ is harmonic. State what are Cauchy Riemann equations and use it to find the harmonic conjugate of the given function $u(x,y)$. | **07** |
| **Q.3** | **(a)** | State: (i) Liouville Theorem and (ii) Cauchy-Goursat Theorem. | **03** |
| | **(b)** | Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n!}{n^n} z^n$ using the appropriate formula. | **04** |
| | **(c)** | Evaluate $\int_C \bar{z}dz$ where C is along the sides of the triangle having vertices $z = 0, 1, i$. | **07** |
| | | **OR** | |
| **Q.3** | **(a)** | Expand Ln $(1+z)$ as a Taylor series about $z = 0$ for $\lvert z\rvert < 1$. | **03** |
| | **(b)** | Without computing the integral, show that $$\left\lvert \int_C \frac{e^z}{z+1} dz \right\rvert \leq \frac{8\pi e^4}{3}$$ where C denotes the circle centered at origin and with radius 4. | **04** |
| | **(c)** | State the Generalized Cauchy integral formula and use it to compute $$\int_C \frac{z+e^z}{(z-1)^3} dz$$ where C is $\lvert z\rvert = 2$. | **07** |

**Q.4** **(a)** Identify the type of singularities of the function $f(z) = (z^2 - z^6)^{-1}$. **03**

**(b)** Using the Cauchy residue theorem, compute $\int_C \frac{1}{(z-1)^2(z-3)}\, dz$ where C is $|z| = 4$. **04**

**(c)** Solve $(x^2 + 2y^2)p - xyq = xz$. **07**

**OR**

**Q.4** **(a)** Form the partial differentiation equation by eliminating the arbitrary constants from $(x - a)(x - b) - z^2 = x^2 + y^2$. **03**

**(b)** Find the complete integral (complete solution) of $q^2 = z^2p^2(1 - p^2)$. **04**

**(c)** Find the Laurent's series of $f(z) = \frac{3}{(z-2)(z+1)}$ in the regions: **07**

(i) $|z| < 1$ and (ii) $1 < |z| < 2$.

**Q.5** **(a)** Solve $25\frac{\partial^2 z}{\partial x^2} - 40\frac{\partial^2 z}{\partial x \partial y} + 16\frac{\partial^2 z}{\partial y^2} = 0$. **03**

**(b)** Find the complete integral (complete solution) of $p^2 - q^2 = x - y$. **04**

**(c)** Solve $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2(x + y)u$ using the method of separation of variables. **07**

**OR**

**Q.5** **(a)** Solve $\frac{\partial^2 z}{\partial x^2} + 10\frac{\partial^2 z}{\partial x \partial y} + 25\frac{\partial^2 z}{\partial y^2} = e^x e^{-y}$. **03**

**(b)** Using the contour integration, show that the value of the improper integral $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx$ is equal to $\pi$. **04**

**(c)** A tightly stretched string with fixed end points $x = 0$ and $x = 1$ in the shape defined by $y = x(1 - x)$ is released from this position of rest. Find $y(x, t)$ using the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2\, \frac{\partial^2 y}{\partial x^2}$. **07**

****************