Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–III (NEW) EXAMINATION – WINTER 2020

Subject Code:3130005 Date:09/03/2021

Subject Name:Complex Variables and Partial Differential Equations

Time:10:30 AM TO 12:30 PM Total Marks:56

Instructions:

1. **Attempt any FOUR questions out of EIGHT questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			Marks
Q.1	**(a)**	Show that the function $u = x^2 - y^2 + x$ is harmonic.	**03**
	(b)	Find the fourth roots of -1.	**04**
	(c)	(i) Find and sketch the image of the region $\lvert z \rvert > 1$ under the transformation $w = 4z.$	**03**
		(ii) Find the real and imaginary parts of $f(z) = z^2 + 3z.$	**04**
Q.2	**(a)**	Evaluate $\int_0^{2+i} z^2 dz$ along the line $y = x/2.$	**03**
	(b)	Determine the Mobius transformation that maps $z = 0,1,\infty$ onto $w = -1,-i,1$ respectively.	**04**
	(c)	(i) Evaluate $\oint_C \frac{e^{-z}}{z+1} dz$, where C is the circle $\lvert z \rvert = 1/2.$	**03**
		(ii) For which values of z does the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ convergent?	**04**
Q.3	**(a)**	Evaluate $\oint_C \frac{dz}{z^2}$ where C is a unit circle.	**03**
	(b)	Find the residue *Res(f(z),2i)* of the function $f(z) = \frac{z^2 - 2z}{(z+1)^2 (z^2+4)}$.	**04**
	(c)	Expand $f(z) = \frac{1}{(z+1)(z-2)}$ in Laurent's series in the region $(i) \lvert z \rvert < 1,$ $(ii) 1 < \lvert z \rvert < 2$, $(iii) \lvert z \rvert > 2.$	**07**
Q.4	**(a)**	Evaluate $\oint_C \frac{5z+7}{z^2+2z-3} dz$, where $C : \lvert z-2 \rvert = 2.$	**03**
	(b)	Evaluate by using Cauchy's Residue Theorem $\int_C \frac{5z-2}{z(z-1)} dz$, $C : \lvert z \rvert = 2$.	**04**
	(c)	Find Laurent's series that represent $f(z) = \frac{1}{z(z-1)}$ in the region $(i) 0 < \lvert z \rvert < 1$, $(ii) 0 < \lvert z-1 \rvert < 1.$	**07**

Q.5	**(a)**	Solve $xp + yq = x - y$.	**03**
	(b)	Derive p.d.e. from $z = ax + by + ab$ by eliminating *a* and *b*.	**04**
	(c)	(i) Solve $(D^3 - 4D^2D' + 4DD'^2)z = 0$.	**03**
		(ii) Solve $pq = k$, where *k* is a constant.	**04**
Q.6	**(a)**	Solve $zp + yq = x$.	**03**
	(b)	Form the p.d.e. by eliminating ϕ from $x + y + z = \phi(x^2 + y^2 + z^2)$.	**04**
	(c)	(i) Solve $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$.	**03**
		(ii) Solve $zpq = p + q$ by Charpit's method.	**04**
Q.7	**(a)**	Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x}$.	**03**
	(b)	Solve the p.d.e. $u_x = 4u_y$, $u(0, y) = 8e^{-3y}$.	**04**
	(c)	A string of length l is initially at rest in its equilibrium position and motion is started by giving each of its points a velocity *kx* for $0 \le x \le l/2$ and *k(l-x)* for $l/2 \le x \le l$. Find the displacement *u(x,t)*.	**07**
Q.8	**(a)**	Solve $DD''(D - 2D' - 3)z = 0$.	**03**
	(b)	Solve the pde $u_{xx} = 16u_{xy}$.	**04**
	(c)	A bar of length 2 *m* is fully insulated along its sides. It is initially at a uniform temperature of $10^o C$ and at *t = 0* the ends are plunged into ice and maintained at a temperature of $0^o C$. Determine an expression for the temperature at a point P at a distance x from one end at any subsequent time t seconds after *t = 0*.	**07**

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