Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER–III(NEW) EXAMINATION – SUMMER 2023**

**Subject Code:3130005 Date:24-07-2023**

**Subject Name: Complex Variables and Partial Differential Equations**

**Time: 02:30 PM TO 05:00 PM Total Marks:70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

| | | | MARKS |
|---|---|---|---|
| **Q.1** | **(a)** | Determine an analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$ | **03** |
| | **(b)** | Solve the equation $z^2 + (2i - 3)z + 5 - i = 0$ | **04** |
| | **(c)** | Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic and Find a harmonic conjugate of u(x,y) | **07** |
| **Q.2** | **(a)** | Evaluate $\int_C |z|^2 dz$ around the square with vertices at (0,0), (1,0), (1,1), (0,1). | **03** |
| | **(b)** | Expand $f(z) = \frac{1}{(z+2)(z+4)}$ valid for the following regions (i) $|z| < 2$ (ii) $2 < |z| < 4$ | **04** |
| | **(c)** | (i) Evaluate $\int_C \frac{zdz}{(z-1)(z-2)}$ where C is the circle $|z| = \frac{1}{2}$ | **03** |
| | | (ii) Evaluate $\int_C \frac{dz}{z^2-7z+12}$ where C is the circle $|z| = 3.5$ | **04** |
| | | **OR** | |
| | **(c)** | Define mobius transformation. Determine the mobius transformation which maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ | **07** |
| **Q.3** | **(a)** | Find and plot the image of triangular region in the z-plane with vertices (0,0), (1,0), (0,1) under the transformation $w = (1 - i)z + 3$ | **03** |
| | **(b)** | Find the values of a and b such that the function $f(z) = x^2 + ay^2 - 2xy + i(bx^2 - y^2 + 2xy)$ is analytic. | **04** |
| | **(c)** | Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and Residue at each pole. Hence evaluate $\int_C f(z)dz$ where C is the circle $|z| = 3$ | **07** |
| | | **OR** | |
| **Q.3** | **(a)** | Expand $f(z) = \frac{1-e^z}{z}$ in Laurent's series about z = 0. | **03** |
| | **(b)** | Find modulus and argument of (i) $\frac{1+2i}{1-(1-i)^2}$ (ii) $\frac{(1+i)^2}{1-i}$ | **04** |
| | **(c)** | Evaluate (i) $\int_C \frac{3z^2+7z+1}{z+1} dz$ Where C is $|z| = \frac{1}{2}$ | **03** |
| | | (ii) $\int_C \frac{z^2+1}{z^2-1} dz$ Where C is $|z - 1| = 1$ | **04** |
| **Q.4** | **(a)** | Solve $yzp - xzq = xy$ | **03** |
| | **(b)** | Form partial differential equation by eliminating the arbitrary constants a and b from $z = axe^y + \frac{1}{2}a^2e^{2y} + b$ | **04** |
| | **(c)** | (i) Solve $25r - 40s + 16t = 0$ | **03** |
| | | (ii) Solve $p^2 + q^2 = x + y$ | **04** |
| | | **OR** | |
| **Q.4** | **(a)** | Solve $(mz - ny)p + (nx - lz)q = ly - mx$ | **03** |

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|---|---|---|---|
| | **(b)** | Form a partial differential equation by eliminating the arbitrary functions from $f(x^2 - y^2, xyz) = 0$ | **04** |
| | **(c)** | (i) Solve $(D^2 - DD' + D' - 1)z = \cos(x + 2y)$ | **03** |
| | | (ii) Solve using Charpit's Method $z^2 = pqxy$ | **04** |
| **Q.5** | **(a)** | Solve $\left(D^2 + 10DD' + 25D'^2\right)z = e^{3x+2y}$ | **03** |
| | **(b)** | Solve $x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 0$ using method of separation of variables. | **04** |
| | **(c)** | (i) Solve $\left(D^2 - D'^2\right)z = x - y$ | **03** |
| | | (ii) Solve $\left(2D^2 - 5DD' + 2D'^2\right) = \sin(2x + y)$ | **04** |

**OR**

| | | | |
|---|---|---|---|
| **Q.5** | **(a)** | Solve $(1 - x)p + (z - y)q = 3 - z$ | **03** |
| | **(b)** | Solve $2\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u$ using method of separation of variables subject to the condition $u(x, 0) = 4e^{-3x}$ | **04** |
| | **(c)** | Find the solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ such that $y = acospt\ when\ x = l\ and\ y = 0\ when\ x = 0$ | **07** |

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