Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III(NEW) EXAMINATION – SUMMER 2023

Subject Code:3130908 Date:24-07-2023

Subject Name:Applied Mathematics for Electrical Engineering

Time:02:30 PM TO 05:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1	**(a)**	State the iteration formula of Newton–Raphson method. Use it to find the approximate value of $\sqrt[3]{10}$ correct upto four decimal places.	**03**
	(b)	Evaluate $\int_0^5 \frac{1}{1+x^2} dx$ using the Trapezoidal rule taking $h = 1$. Also compare your answer with the exact integration.	**04**
	(c)	Write any two differences between Newton–Raphson method and Secant method. Using Secant method, find an approximate root of equation $f(x) = x^3 - 2x - 5 = 0$ taking $a = 2$ and $b = 3$ correct upto four decimal places.	**07**
Q.2	**(a)**	Using Gaussian two points quadrature formula, evaluate $\int_1^2 e^x \, dx$.	**03**
	(b)	The table below gives the temperature in $T^0\ C$ and resistance R in ohms of a circuit. If $= a + bT$, find the values of a and b by the least squares method. T: 10, 20, 30, 40, 50, 60 R: 20.1, 20.2, 20.4, 20.6, 20.8, 21.0	**04**
	(c)	Apply Runge–Kutta fourth order method to calculate approximate values of $y(0.1)$ and $y(0.2)$ for the differential equation $\frac{dy}{dx} = 2x + y\ ; y(0) = 1$ (Take h = 0.1).	**07**
		OR	
	(c)	Apply Euler's method to find an approximate value of $y(0.3)$ for the differential equation $\frac{dy}{dx} = 1 - y\ ;\ y(0) = 0$ (Take h = 0.1).	**07**
Q.3	**(a)**	Write normal equations to fit a parabola $y = ax^2 + bx + c$ by the least squares method.	**03**
	(b)	Using relation between finite difference operators, find the missing value in the following table. x: 16, 18, 20, 22, 24, 26 y: 43, 89, –, 155, 268, 388	**04**
	(c)	State Newton's divided difference interpolation formula. Apply it to find a polynomial satisfied by the following data and hence find y at $x = 4$. x: −4, −1, 0, 2, 5 y: 1245, 33, 5, 9, 1335	**07**
		OR	
Q.3	**(a)**	Write normal equations to fit the curve $y = ae^{bx}$ by the least squares method.	**03**

	(b)	The following are data from the steam table: T in C^0: 140, 150, 160, 170, 180 P in kgf/cm^2: 3.685, 4.854, 6.302, 8.076, 10.225 Using Newton's interpolation formula, find the pressure of the stream for temperature 175^0.	04
	(c)	Given that $f(-1) = -2, f(0) = -1, f(2) = 1, f(3) = 4$. Using Lagrange's interpolation formula, fit a polynomial of degree three in x.	07
Q.4	(a)	Let X denote the number of heads in a single toss of four fair coins. Determine $P(1 < X \leq 3)$	03
	(b)	A random variable has the following probability function: X: 0, 1, 2, 3, 4 $P(X)$: k, $3k$, $5k$, $7k$, $9k$ Find the value of k and the distribution function $F(x)$	04
	(c)	The probability of X, Y and Z becoming managers are $\frac{4}{9}, \frac{2}{9}$ and $\frac{1}{3}$ respectively. The probabilities that the bonus scheme will be introduced if X, Y and Z become managers are $\frac{3}{10}$, $\frac{1}{2}$ and $\frac{4}{5}$ respectively. (i) What is the probability that the bonus scheme will be introduced? (ii) If the bonus scheme has been introduced, what is the probability that the manager appointed was X ?	07
		OR	
Q.4	(a)	Define a continuous probability density function. Write its properties.	03
	(b)	The probability density function of a continuous random variable X is $f(x) = \frac{1}{2}e^{-\lvert x\rvert}$ Find the cumulative distribution function (cdf).	04
	(c)	Let X be a continuous random variable with the probability density function (pdf) $f(x) = kx(1 - x),\ 0 \leq X \leq 1$ Find k and a number b such that $P(X \leq b) = P(X \geq b)$	07
Q.5	(a)	A random variable X has the following distribution: X=x: 1, 2, 3, 4, 5, 6 $P(x)$: $\frac{1}{36}$, $\frac{3}{36}$, $\frac{5}{36}$, $\frac{7}{36}$, $\frac{9}{36}$, $\frac{11}{36}$ Find mean and variance.	03
	(b)	A continuous random variable X is distributed over the interval [0, 1] with the probability distribution function (pdf) $f(x) = ax^2 + bx$, where a and b are constants. If the mean of X is 0.5, find the values of a and b.	04
	(c)	Find the median, mode, quartile deviation and mean deviation from the following probability distribution. X=x: 0, 1, 2, 3, 4, 5, 6, 7, 8 $P(x)$: 0.004, 0.036, 0.10, 0.232, 0.28, 0.204, 0.112, 0.028, 0.004	07
		OR	
Q.5	(a)	Karl Pearson's coefficient of skewness of a distribution is 0.32. Its standard deviation is 6.5 and mean is 29.6. Find the mode of the distribution.	03
	(b)	Suppose X is a random variable such that $E(X) = 3$ and $E(X^2) = 13$. Calculate a lower bound for the probability that X lies between (−2) and 8 using Chebyshev's inequality.	04
	(c)	Define Kurtosis. The first four moments about the arbitrary mean "4" are −1.5, 17, −30, 108 respectively. Determine the shape of the distribution.	07

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