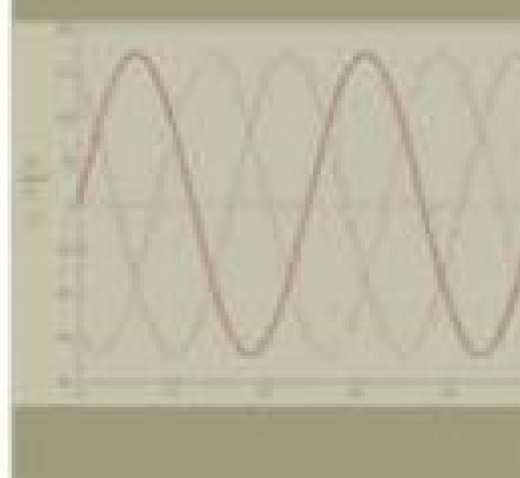
Revised Edition

Basic Electrical Engineering

(For B.E./B.Tech. and other Engineering Examinations)



V.K. MEHTA
ROHIT MEHTA

S. CHAND

BASIC ELECTRICAL ENGINEERING

For B.E./B.Tech. and Other Engineering Examinations

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Preface to Sixth Edition

The general response to the Fifth Edition of the book was very encouraging. Authors feel that their work has been amply rewarded and wish to express their deep sense of gratitude to the large number of readers who have used it and in particular to those of them who have sent helpful suggestions from time to time for the improvement of the book.

The popularity of the book is judged from the fact that authors frequently receive feedback from many quarters including teachers, students and serving engineers. This feedback helps the authors to make the book up-to-date. In the present edition, many new topics/numericals/illustrations have been added to make the book more useful.

Authors lay no claim to the original research in preparing the book. Liberal use of materials available in the works of eminent authors has been made. What they may claim, in all modesty, is that they have tried to fashion the vast amount of material available from primary and secondary sources into coherent body of description and analysis.

The authors wish to thank their colleagues and friends who have contributed many valuable suggestions regarding the scope and content sequence of the book. Authors are also indebted to S. Chand & Company Ltd., New Delhi for bringing out this revised edition in a short time and pricing the book moderately inspite of heavy cost of paper and printing.

Errors might have crept in despite utmost care to avoid them. Authors shall be grateful if these are pointed out along with other suggestions for the improvement of the book.

V.K. MEHTA

ROHIT MEHTA

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1
Basic Concepts

Introduction

Everybody is familiar with the functions that electricity can perform. It can be used for lighting, heating, traction and countless other purposes. The question always arises, "What is electricity" ? Several theories about electricity were developed through experiments and by observation of its behaviour. The only theory that has survived over the years to explain the nature of electricity is the *Modern Electron theory of matter*. This theory has been the result of research work conducted by scientists like Sir William Crooks, J.J. Thomson, Robert A. Millikan, Sir Earnest Rutherford and Neils Bohr. In this chapter, we shall deal with some basic concepts concerning electricity.

1.1. Nature of Electricity

We know that matter is electrical in nature *i.e.* it contains particles of electricity *viz*. protons and electrons. The positive charge on a proton is equal to the negative charge on an electron. Whether a given body exhibits electricity (*i.e.* charge) or not depends upon the relative number of these particles of electricity.

(*i*) If the number of protons is equal to the number of electrons in a body, the resultant charge is zero and the body will be electrically neutral. Thus, the paper of this book is electrically neutral (*i.e.* paper exhibits no charge) because it has the same number of protons and electrons.

(*ii*) If from a neutral body, some *electrons are removed, there occurs a deficit of electrons in the body. Consequently, the body attains a *positive charge*.

(*iii*) If a neutral body is supplied with electrons, there occurs an excess of electrons. Consequently, the body attains a *negative charge*.

1.2. Unit of Charge

The charge on an electron is so small that it is not convenient to select it as the unit of charge. In practice, *coulomb* is used as the unit of charge *i.e*. SI unit of charge is coulomb abbreviated as C. *One coulomb of charge is equal to the charge on 625 × 10^{16} electrons, i.e.*

$$1 \text{ coulomb} = \text{Charge on } 625 \times 10^{16} \text{ electrons}$$

Thus when we say that a body has a positive charge of one coulomb (*i.e.* +1 C), it means that the body has a deficit of 625 × 10^{16} electrons from normal due share. The charge on one electron is given by ;

$$\text{Charge on electron} = -\frac{1}{625\times10^{16}} = -1.6 \times 10^{-19} \text{ C}$$

1.3. The Electron

Since electrical engineering generally deals with tiny particles called electrons, these small particles require detailed study. We know that an electron is a negatively charged particle having negligible mass. Some of the important properties of an electron are :

(*i*) Charge on an electron, $e = 1.602 \times 10^{-19}$ coulomb

(*ii*) Mass of an electron, $m = 9.0 \times 10^{-31}$ kg

(*iii*) Radius of an electron, $r = 1.9 \times 10^{-15}$ metre

* Electrons have very small mass and, therefore, are much more mobile than protons. On the other hand, protons are powerfully held in the nucleus and cannot be removed or detached.

The ratio *e*/*m* of an electron is 1.77×10^{11} coulombs/kg. This means that mass of an electron is very small as compared to its charge. It is due to this property of an electron that it is very mobile and is greatly influenced by electric or magnetic fields.

1.4. Energy of an Electron

An electron moving around the nucleus possesses two types of energies *viz*. kinetic energy due to its motion and potential energy due to the charge on the nucleus. The total energy of the electron is the sum of these two energies. The energy of an electron increases as its distance from the nucleus increases. Thus, an electron in the second orbit possesses more energy than the electron in the first orbit ; electron in the third orbit has higher energy than in the second orbit. It is clear that electrons in the last orbit possess very high energy as compared to the electrons in the inner orbits. These last orbit electrons play an important role in determining the physical, chemical and electrical properties of a material.

1.5. Valence Electrons

The electrons in the outermost orbit of an atom are known as **valence electrons**.

The outermost orbit can have a maximum of 8 electrons *i.e.* the maximum number of valence electrons can be 8. The valence electrons determine the physical and chemical properties of a material. These electrons determine whether or not the material is chemically active; metal or non-metal or, a gas or solid. These electrons also determine the electrical properties of a material.

On the basis of electrical conductivity, materials are generally classified into *conductors, insulators* and *semi-conductors*. As a rough rule, one can determine the electrical behaviour of a material from the number of valence electrons as under :

(*i*) *When the number of valence electrons of an atom is less than 4* (*i.e.* half of the maximum eight electrons), the material is usually *a metal and a conductor*. Examples are sodium, magnesium and aluminium which have 1, 2 and 3 valence electrons respectively.

(*ii*) *When the number of valence electrons of an atom is more than 4*, the material is usually *a non-metal and an insulator*. Examples are nitrogen, sulphur and neon which have 5, 6 and 8 valence electrons respectively.

(*iii*) *When the number of valence electrons of an atom is 4* (*i.e.* exactly one-half of the maximum 8 electrons), the material has both metal and non-metal properties and is usually a *semi-conductor*. Examples are carbon, silicon and germanium.

1.6. Free Electrons

We know that electrons move around the nucleus of an atom in different orbits. The electrons in the inner orbits (*i.e.*, orbits close to the nucleus) are tightly bound to the nucleus. As we move away from the nucleus, this binding goes on decreasing so that electrons in the last orbit (called valence electrons) are quite loosely bound to the nucleus. In certain substances, especially metals (*e.g.* copper, aluminium etc.), the valence electrons are so weakly attached to their nuclei that they can be easily removed or detached. Such electrons are called free electrons.

Those valence electrons which are very loosely attached to the nucleus of an atom are called **free electrons.**

The free electrons move at random from one atom to another in the material. Infact, they are so loosely attached that they do not know the atom to which they belong. It may be noted here that all valence electrons in a metal are not free electrons. It has been found that one atom of a metal can provide at the most one free electron. Since a small piece of metal has billions of atoms, one can expect a very large number of free electrons in metals. For instance, one cubic centimetre of copper has about 8.5×10^{22} free electrons at room temperature.

(*i*) A substance which has a large number of free electrons at room temperature is called a **conductor** of electricity *e.g.* all metals. If a voltage source (*e.g.* a cell) is applied across the wire of a conductor material, free electrons readily flow through the wire, thus constituting electric current. The best conductors are silver, copper and gold in that order. Since copper is the least expensive out of these materials, it is widely used in electrical and electronic industries.

(*ii*) A substance which has very few free electrons is called an **insulator** of electricity. If a voltage source is applied across the wire of insulator material, practically no current flows through the wire. Most substances including plastics, ceramics, rubber, paper and most liquids and gases fall in this category. Of course, there are many practical uses for insulators in the electrical and electronic industries including wire coatings, safety enclosures and power-line insulators.

(*iii*) There is a third class of substances, called **semi-conductors**. As their name implies, they are neither conductors nor insulators. These substances have crystalline structure and contain very few free electrons at room temperature. Therefore, at room temperature, a semiconductor practically behaves as an insulator. However, if suitable controlled impurity is imparted to a semi-conductor, it is possible to provide controlled conductivity. Most common semi-conductors are silicon, germanium, carbon etc. However, *silicon* is the principal material and is widely used in the manufacture of electronic devices (*e.g.* crystal diodes, transistors etc.) and integrated circuits.

1.7. Electric Current

The directed flow of free electrons (*or charge*) *is called* **electric current**. The flow of electric current can be beautifully explained by referring to Fig. 1.1. The copper strip has a large number of free electrons. When electric pressure or voltage is applied, then free electrons, being negatively charged, will start moving towards the positive terminal around the circuit as shown in Fig. 1.1. This directed flow of electrons is called electric current.

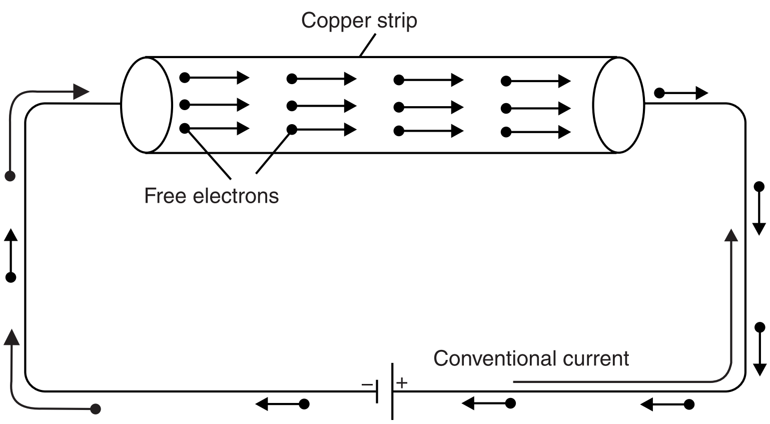


Fig. 1.1

The reader may note the following points :

(*i*) Current is flow of electrons and electrons are the constituents of matter. Therefore, electric current is matter (*i.e.* free electrons) in motion.

(*ii*) The actual direction of current (*i.e.* flow of electrons) is from negative terminal to the positive terminal through that part of the circuit external to the cell. However, prior to Electron theory, it was assumed that current flowed from positive terminal to the negative terminal of the cell *via* the circuit. This convention is so firmly established that it is still in use. This assumed direction of current is now called *conventional current*.

Unit of Current. The strength of electric current I is the rate of flow of electrons *i.e.* charge flowing per second.

$$\therefore \qquad \text{Current, } I = \frac{Q}{t}$$

The charge Q is measured in coulombs and time t in seconds. Therefore, the unit of electric current will be *coulombs/sec or ampere*. If $Q = 1$ coulomb, $t = 1$ sec, then $I = 1/1 = 1$ ampere.

One ampere *of current is said to flow through a wire if at any cross-section one coulomb of charge flows in one second.*

Thus, if 5 amperes current is flowing through a wire, it means that 5 coulombs per second flow past any cross-section of the wire.

Note. 1 C = charge on 625×10^{16} electrons. Thus when we say that current through a wire is 1 A, it means that 625×10^{16} electrons per second flow past any cross-section of the wire.

$$\therefore \qquad I = \frac{Q}{t} = \frac{ne}{t} \qquad \text{where } e = -1.6 \times 10^{-19}\text{ C ; } n = \text{number of electrons}$$

1.8. Electric Current is a Scalar Quantity

(*i*) Electric current, $I = \dfrac{Q}{t}$

As both charge and time are scalars, electric current is a scalar quantity.

(*ii*) We show electric current in a wire by an arrow to indicate the direction of flow of positive charge. But such arrows are not vectors because they do not obey the laws of vector algebra. This point can be explained by referring to Fig. 1.2. The wires *OA* and *OB* carry currents of 3 A and 4 A respectively. The total current in the wire *CO* is 3 + 4 = 7 A irrespective of the angle between the wires *OA* and *OB*. This is not surprising because the charge is conserved so that the magnitudes of currents in wires *OA* and *OB* must add to give the magnitude of current in the wire *CO*.

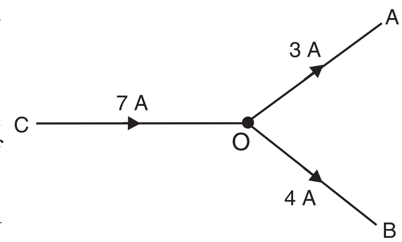


Fig. 1.2

1.9. Types of Electric Current

The electric current may be classified into three main classes: (*i*) steady current (*ii*) varying current and (*iii*) alternating current.

(*i*) **Steady current.** *When the magnitude of current does not change with time, it is called a steady current.* Fig. 1.3 (*i*) shows the graph between steady current and time. Note that value of current remains the same as the time changes. The current provided by a battery is almost a steady current (*d.c.*).

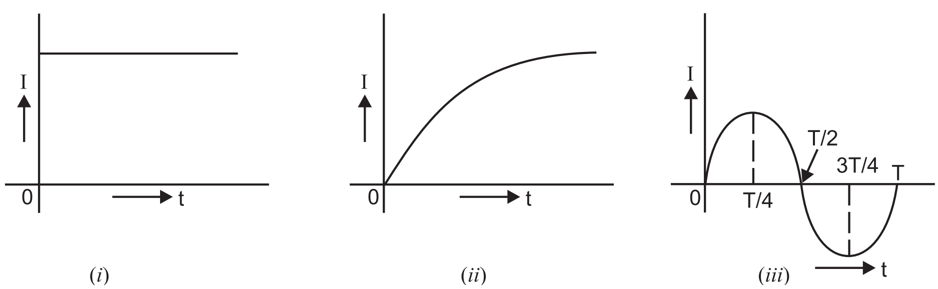


Fig. 1.3

(*ii*) Varying current. *When the magnitude of current changes with time, it is called a varying current*. Fig. 1.3 (*ii*) shows the graph between varying current and time. Note that value of current varies with time.

(*iii*) Alternating current. *An alternating current is one whose magnitude changes continuously with time and direction changes periodically.* Due to technical and economical reasons, we produce alternating currents that have sine waveform (or cosine waveform) as shown in Fig. 1.3 (*iii*). It is called *alternating current* because current flows in alternate directions in the circuit, *i.e.,* from 0 to $T/2$ second (T is the time period of the wave) in one direction and from $T/2$ to T second in the opposite direction. The current provided by an a.c. generator is alternating current that has sine (or cosine) waveform.

1.10. Mechanism of Current Conduction in Metals

Every metal has a large number of free electrons which wander randomly within the body of the conductor somewhat like the molecules in a gas. The average speed of free electrons is sufficiently high ($\simeq 10^5$ ms^{-1}) at room temperature. During random motion, the free electrons collide with positive ions (positive atoms of metal) again and again and after each collision, their direction of motion changes. When we consider all the free electrons, their random motions average to zero. In other words, there is no net flow of charge (electrons) in any particular direction. Consequently, no current is established in the conductor.

When potential difference is applied across the ends of a conductor (say copper wire) as shown in Fig. 1.4, electric field is applied at every point of the copper wire. The electric field exerts force on the free electrons which start accelerating towards the positive terminal (*i.e.,* opposite to the direction of the field). As the free electrons move, they *collide again and again with positive ions of the metal. Each collision destroys the extra velocity gained by the free electrons.

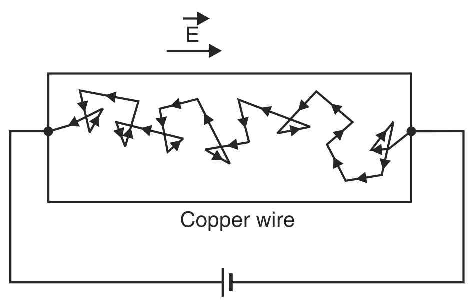


Fig. 1.4

The average time that an electron spends between two collisions is called the **relaxation time** (τ). Its value is of the order of 10^{-14} second.

Although the free electrons are continuously accelerated by the electric field, collisions prevent their velocity from becoming large. The result is that electric field provides a small constant velocity towards positive terminal which is superimposed on the random motion of the electrons. This constant velocity is called the drift velocity.

The average velocity with which free electrons get drifted in a metallic conductor under the influence of electric field is called **drift velocity** $(\vec{v}_d)$. The drift velocity of free electrons is of the order of 10^{-5} ms^{-1}.

Thus when a metallic conductor is subjected to electric field (or potential difference), free electrons move towards the positive terminal of the source with drift velocity $\vec{v}_d$. Small though it is, the drift velocity is entirely responsible for electric current in the metal.

Note. The reader may wonder that if electrons drift so slowly, how room light turns on quickly when switch is closed ? The answer is that propagation of electric field takes place with the speed of light. When we apply electric field (*i.e.,* potential difference) to a wire, the free electrons everywhere in the wire begin drifting almost at once.

* What happens to an electron after collision with an ion ? It moves off in some new and quite random direction. However, it still experiences the applied electric field, so it continues to accelerate again, gaining a velocity in the direction of the positive terminal. It again encounters an ion and loses its directed motion. This situation is repeated again and again for every free electron in a metal.

1.11. Relation Between Current and Drift Velocity

Consider a portion of a copper wire through which current I is flowing as shown in Fig. 1.5. Clearly, copper wire is under the influence of electric field.

Let A = area of X-section of the wire

n = electron density, *i.e.,* number of free electrons per unit volume

e = charge on each electron

v_d = drift velocity of free electrons

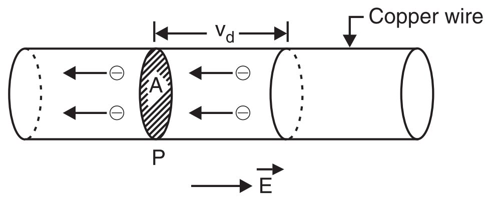


Fig. 1.5

In one second, all those free electrons within a distance v_d to the right of cross-section at P (*i.e.,* in a volume Av_d) will flow through the cross-section at P as shown in Fig. 1.5. This volume contains $n\,Av_d$ electrons and, hence, a charge $(nAv_d)e$. Therefore, a charge of $neAv_d$ per second passes the cross-section at P.

$$\therefore \qquad I = n\,e\,A\,v_d$$

Since A, n and e are constant, $I \propto v_d$.

Hence, current flowing through a conductor is directly proportional to the drift velocity of free electrons.

(*i*) The drift velocity of free electrons is very small. Since the number of free electrons in a metallic conductor is very large, even small drift velocity of free electrons gives rise to sufficient current.

(*ii*) The current density J is defined as current per unit area and is given by ;

Current density, $J = \dfrac{I}{A} = \dfrac{n\,e\,Av_d}{A} = n\,e\,v_d$

The SI unit of current density is amperes/m^2.

Note. Current density is a vector quantity and is denoted by the symbol $\vec{J}$. Therefore, in vector notation, the relation between I and $\vec{J}$ is $I = \vec{J}.\vec{A}$

where $\vec{A}$ = Area vector

Example 1.1. *A 60 W light bulb has a current of 0.5 A flowing through it. Calculate (i) the number of electrons passing through a cross-section of the filament (ii) the number of electrons that pass the cross-section in one hour.*

Solution. (*i*) $I = \dfrac{Q}{t} = \dfrac{n\,e}{t}$

$$\therefore \qquad n = \frac{I\,t}{e} = \frac{0.5 \times 1}{1.6 \times 10^{-19}} = \mathbf{3.1 \times 10^{18}\ electrons/s}$$

(*ii*) Charge passing the cross-section in one hour is

$$Q = I\,t = (0.5) \times (60 \times 60) = 1800\text{ C}$$

Now, $Q = n\,e$

$$\therefore \qquad n = \frac{Q}{e} = \frac{1800}{1.6 \times 10^{-19}} = \mathbf{1.1 \times 10^{22}\ electrons/hour}$$

Example 1.2. *A copper wire of area of X-section 4 mm^2 is 4 m long and carries a current of 10 A. The number density of free electrons is 8 × 10^{28} m^{-3}. How much time is required by an electron to travel the length of wire ?*

Solution. $I = n\,A\,e\,v_d$

Here $I = 10\text{ A}$; $A = 4\text{ mm}^2 = 4 \times 10^{-6}\text{ m}^2$; $e = 1.6 \times 10^{-19}\text{ C}$; $n = 8 \times 10^{28}\text{ m}^{-3}$

$$\therefore \quad \text{Drift velocity, } v_d = \frac{I}{n\,A\,e} = \frac{10}{8\times10^{28}\times(4\times10^{-6})\times1.6\times10^{-19}} = 1.95\times10^{-4}\text{ ms}^{-1}$$

∴ Time taken by the electron to travel the length of the wire is

$$t = \frac{l}{v_d} = \frac{4}{1.95\times10^{-4}} = 2.05\times10^4\text{s} = \mathbf{5.7\ hours}$$

Example 1.3. *The area of X-section of copper wire is 3×10^{-6} m^2. It carries a current of 4.2 A. Calculate (i) current density in the wire and (ii) the drift velocity of electrons. The number density of conduction electrons is 8.4×10^{28} m^{-3}.*

Solution. (*i*) Current density, $J = \frac{I}{A} = \frac{4.2}{3\times10^{-6}} = \mathbf{1.4 \times 10^6\ A/m^2}$

(*ii*) $I = n\,e\,A\,v_d$

$$\therefore \quad \text{Drift velocity, } v_d = \frac{I}{n\,A\,e} = \frac{4.2}{(8.4\times10^{28})\times(1.6\times10^{-19})\times3\times10^{-6}} = \mathbf{1.04 \times 10^{-4}\ ms^{-1}}$$

Tutorial Problems

1. How much current is flowing in a circuit where 1.27×10^{15} electrons move past a given point in 100 ms ? **[2.03 A]**
2. The current in a certain conductor is 40 mA.
   (*i*) Find the total charge in coulombs that passes through the conductor in 1.5 s.
   (*ii*) Find the total number of electrons that pass through the conductor in that time.
   [(*i*) 60 mC (*ii*) 3.745×10^{17} electrons]
3. The density of conduction electrons in a wire is 10^{22} m^{-3}. If the radius of the wire is 0.6 mm and it is carrying a current of 2 A, what will be the average drift velocity ? **[1.1×10^{-3} ms^{-1}]**
4. Find the velocity of charge leading to 1 A current which flows in a copper conductor of cross-section 1 cm^2 and length 10 km. Free electron density of copper = 8.5×10^{28} per m^3. How long will it take the electric charge to travel from one end of the conductor to the other ? **[0.735 μm/s; 431 years]**

1.12. Electric Potential

When a body is charged, work is done in charging it. This work done is stored in the body in the form of potential energy. The charged body has the capacity to do work by moving other charges either by attraction or repulsion. The ability of the charged body to do work is called electric potential.

The capacity of a charged body to do work is called its **electric potential.**

The greater the capacity of a charged body to do work, the greater is its electric potential. Obviously, the work done to charge a body to 1 coulomb will be a measure of its electric potential *i.e.*

$$\text{Electric potential, } V = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}$$

The work done is measured in joules and charge in coulombs. Therefore, the unit of electric potential will be *joules/coulomb* or *volt*. If $W = 1$ joule, $Q = 1$ coulomb, then $V = 1/1 = 1$ volt.

Hence a body is said to have an electric potential of **1 volt** *if 1 joule of work is done to give it a charge of 1 coulomb.*

Thus, when we say that a body has an electric potential of 5 volts, it means that 5 joules of work has been done to charge the body to 1 coulomb. In other words, every coulomb of charge possesses an energy of 5 joules. The greater the joules/coulomb on a charged body, the greater is its electric potential.

1.13. Potential Difference

The difference in the potentials of two charged bodies is called **potential difference.**

If two bodies have different electric potentials, a potential difference exists between the bodies. Consider two bodies *A* and *B* having potentials of 5 volts and 3 volts respectively as shown in Fig. 1.6 (*i*). Each coulomb of charge on body *A* has an energy of 5 joules while each coulomb of charge on body *B* has an energy of 3 joules. Clearly, body *A* is at higher potential than the body *B*.

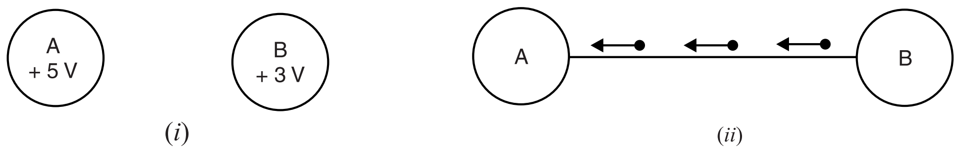


Fig. 1.6

If the two bodies are joined through a conductor [See Fig. 1.6 (*ii*)], then electrons will *flow from body *B* to body *A*. When the two bodies attain the same potential, the flow of current stops. Therefore, we arrive at a very important conclusion that current will flow in a circuit if potential difference exists. No potential difference, no current flow. It may be noted that potential difference is sometimes called voltage.

Unit. Since the unit of electric potential is volt, one can expect that unit of potential difference will also be *volt*. It is defined as under :

The potential difference between two points is **1 volt** *if one joule of work is **done or released in transferring 1 coulomb of charge from one point to the other.*

1.14. Maintaining Potential Difference

A device that maintains potential difference between two points is said to develop electromotive force (e.m.f.). A simple example is that of a cell. Fig. 1.7 shows the familiar voltaic cell. It consists of a copper plate (called anode) and a zinc rod (called cathode) immersed in dilute H_2SO_4.

The chemical action taking place in the cell removes electrons from copper plate and transfers them to the zinc rod. This transference of electrons takes place through the agency of dil. H_2SO_4 (called electrolyte). Consequently, the copper plate attains a positive charge of $+Q$ coulombs and zinc rod a charge of $-Q$ coulombs. The chemical action of the cell has done a certain amount of work (say W joules) to do so. Clearly, the potential difference between the two plates will be W/Q volts. If the two plates are joined through a wire, some electrons from zinc rod will be attracted through the wire to copper plate. The chemical action of the cell now transfers an equal amount of electrons from copper plate to zinc rod internally through the cell to maintain original potential difference (*i.e.* W/Q). This process continues so long as the

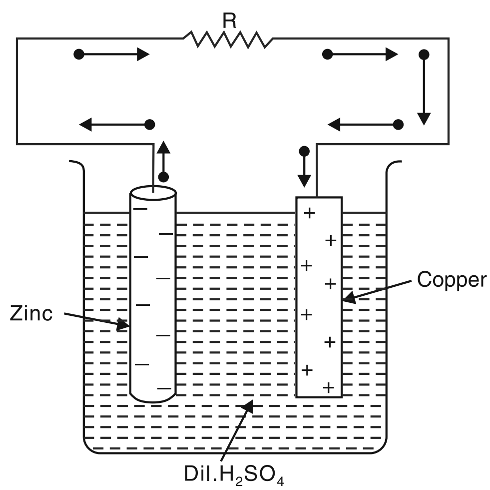


Fig. 1.7

* The conventional current flow will be in the opposite direction *i.e.* from body *A* to body *B*.

** 1 joule of work will be done in the case if 1 coulomb is transferred from point of lower potential to that of higher potential. However, 1 joule of work will be released (as heat) if 1 coulomb of charge moves from a point of higher potential to a point of lower potential.

circuit is complete or so long as there is chemical energy. The flow of electrons through the external wire from zinc rod to copper plate is the electric current.

Thus potential difference causes current to flow while an *e.m.f.* maintains the potential difference. Although both *e.m.f.* and *p.d.* are measured in volts, they do not mean exactly the same thing.

1.15. Concept of E.M.F. and Potential Difference

There is a distinct difference between *e.m.f.* and potential difference. The *e.m.f.* of a device, say a battery, is a measure of the energy the battery gives to each coulomb of charge. Thus if a battery supplies 4 joules of energy per coulomb, we say that it has an *e.m.f.* of 4 volts. The energy given to each coulomb in a battery is due to the chemical action.

The potential difference between two points, say *A* and *B*, is a measure of the energy used by one coulomb in moving from *A* to *B*. Thus if potential difference between points *A* and *B* is 2 volts, it means that each coulomb will give up an energy of 2 joules in moving from *A* to *B*.

Illustration. The difference between e.m.f. and p.d. can be made more illustrative by referring to Fig. 1.8. Here battery has an e.m.f. of 4 volts. It means battery supplies 4 joules of energy to each coulomb continuously. As each coulomb travels from the positive terminal of the battery, it gives up its most of energy to resistances (2 Ω and 2 Ω in this case) and remaining to connecting wires. When it returns to the negative terminal, it has lost all its energy originally supplied by the battery. The battery now supplies fresh energy to each coulomb (4 joules in the present case) to start the journey once again.

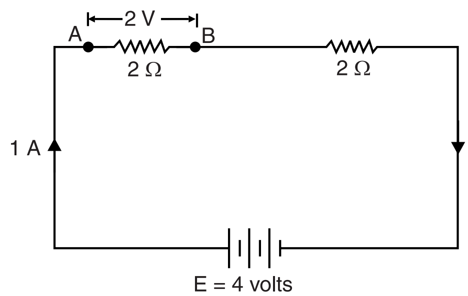


Fig. 1.8

The *p.d.* between any two points in the circuit is the energy used by one coulomb in moving from one point to another. Thus in Fig. 1.8, p.d. between *A* and *B* is 2 volts. It means that 1 coulomb will give up an energy of 2 joules in moving from *A* to *B*. This energy will be released as heat from the part *AB* of the circuit.

The following points may be noted carefully :

(*i*) The name e.m.f. at first sight implies that it is a force that causes current to flow. This is not correct because it is not a force but energy supplied to charge by some active device such as a battery.

(*ii*) *Electromotive force (e.m.f.) maintains potential difference while p.d. causes current to flow.*

1.16. Potential Rise and Potential Drop

Fig. 1.9 shows a circuit with a cell and a resistor. The cell provides a potential difference of 1.5 V. Since it is an energy source, there is a *rise* in potential associated with a cell. The cell's potential difference represents an e.m.f. so that symbol *E* could be used. The resistor is also associated with a potential difference. Since it is a consumer (converter) of energy, there is a *drop* in potential across the resistor. We can combine the idea of potential rise or drop with the popular term "voltage". It is customary to refer to the potential difference across the cell as a *voltage rise* and to the potential difference across the resistor as a *voltage drop*.

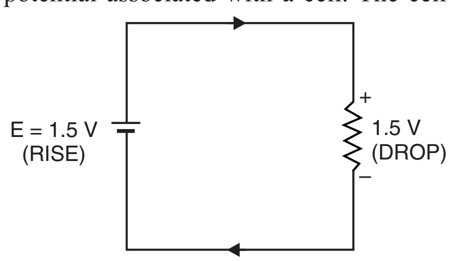


Fig. 1.9

Note. The term voltage refers to a potential difference across two points. There is no such thing as a voltage at one point. In cases where a single point is specified, some reference must be used as the other point. Unless stated otherwise, the ground or common point in any circuit is the reference when specifying a voltage at some other point.

Example 1.4. *A charge of 4 coulombs is flowing between points A and B of a circuit. If the potential difference between A and B is 2 volts, how many joules will be released by part AB of the circuit ?*

Solution. The p.d. of 2 volts between points *A* and *B* means that each coulomb of charge will give up an energy of 2 joules in moving from *A* to *B*. As the charge flowing is 4 coulombs, therefore, total energy released by part *AB* of the circuit is = 4 × 2 = **8 joules.**

Example 1.5. *How much work will be done by an electric energy source with a potential difference of 3 kV that delivers a current of 1 A for 1 minute ?*

Solution. We know that 1 A of current represents a charge transfer rate of 1 C/s. Therefore, the total charge for a period of 1 minute is $Q = It = 1 \times 60 = 60$ C.

$$\text{Total work done, } W = Q \times V = 60 \times (3 \times 10^3) = 180 \times 10^3 \text{ J} = \mathbf{180\ kJ}$$

Tutorial Problems

1. Calculate the potential difference of an energy source that provides 6.8 J for every milli-coulomb of charge that it delivers. **[6.8 kV]**
2. The potential difference across a battery is 9 V. How much charge must it deliver to do 50 J of work ? **[5.56 C]**
3. A 300 V energy source delivers 500 mA for 1 hour. How much energy does this represent ? **[540 kJ]**

1.17. Resistance

The opposition offered by a substance to the flow of electric current is called its **resistance.**

Since current is the flow of free electrons, resistance is the opposition offered by the substance to the flow of free electrons. This opposition occurs because atoms and molecules of the substance obstruct the flow of these electrons. Certain substances (*e.g.* metals such as silver, copper, aluminium etc.) offer very little opposition to the flow of electric current and are called conductors. On the other hand, those substances which offer high opposition to the flow of electric current (*i.e.* flow of free electrons) are called insulators *e.g.* glass, rubber, mica, dry wood etc.

It may be noted here that resistance is the electric friction offered by the substance and causes production of heat with the flow of electric current. The moving electrons collide with atoms or molecules of the substance ; each collision resulting in the liberation of minute quantity of heat.

Unit of resistance. The practical unit of resistance is ohm and is represented by the symbol Ω. It is defined as under :

A wire is said to have a resistance of **1 ohm** *if a p.d. of 1 volt across its ends causes 1 ampere to flow through it* (See Fig. 1.10).

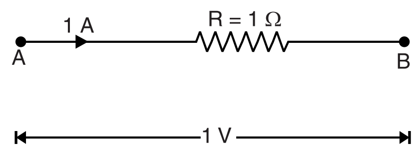


Fig. 1.10

There is another way of defining ohm.

A wire is said to have a resistance of **1 ohm** *if it releases 1 joule (or develops 0.24 calorie of heat) when a current of 1 A flows through it for 1 second.*

A little reflection shows that second definition leads to the first definition. Thus 1 A current flowing for 1 second means that total charge flowing is $Q = I \times t = 1 \times 1 = 1$ coulomb. Now the charge flowing between *A* and *B* (See Fig. 1.10) is 1 coulomb and energy released is 1 joule (or 0.24 calorie). Obviously, by definition, p.d. between *A* and *B* should be 1 volt.

1.18. Factors Upon Which Resistance Depends

The resistance R of a conductor

(*i*) is directly proportional to its length *i.e.*

$$R \propto l$$

(*ii*) is inversely proportional to its area of X-section *i.e.*

$$R \propto \frac{1}{a}$$

(*iii*) depends upon the nature of material.

(*iv*) depends upon temperature.

From the first three points (leaving temperature for the time being), we have,

$$R \propto \frac{l}{a} \quad \text{or} \quad R = \rho\frac{l}{a}$$

where ρ (Greek letter 'Rho') is a constant and is known as *resistivity* or *specific resistance* of the material. Its value depends upon the nature of the material.

1.19. Specific Resistance or Resistivity

We have seen above that $R = \rho\frac{l}{a}$

If $l = 1$ m, $a = 1$ m^2, then, $R = \rho$

Hence **specific resistance** *of a material is the resistance offered by 1 m length of wire of material having an area of cross-section of 1 m*^{2} [See Fig. 1.11 (*i*)].

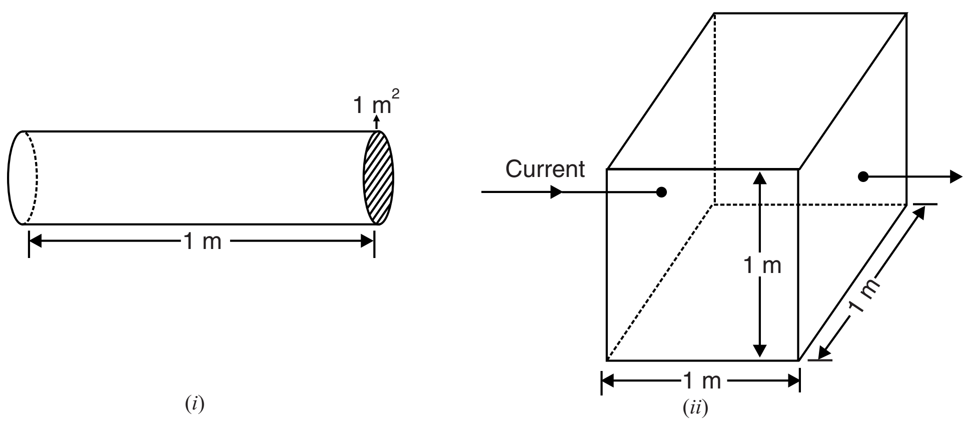


Fig. 1.11

Specific resistance can also be defined in another way. Take a cube of the material having each side 1 m. Considering any two opposite faces, the area of cross-section is 1 m^2 and length is 1 m [See Fig. 1.11 (*ii*)] *i.e.* $l = 1$ m, $a = 1$ m^2.

Hence **specific resistance** *of a material may be defined as the resistance between the opposite faces of a metre cube of the material.*

Unit of resistivity. We know $R = \frac{\rho l}{a}$ or $\rho = \frac{Ra}{l}$

Hence the unit of resistivity will depend upon the units of area of cross-section (a) and length (l).

(*i*) If the length is measured in metres and area of cross-section in m^2, then unit of resistivity will be ohm-metre (Ω m).

$$\rho = \frac{\text{ohm} \times \text{m}^2}{\text{m}} = \text{ohm-m}$$

(ii) If length is measured in cm and area of cross-section in cm^2, then unit of resistivity will be ohm-cm (Ω cm).

$$\rho = \frac{\text{ohm} \times \text{cm}^2}{\text{cm}} = \text{ohm-cm}$$

The resistivity of substances varies over a wide range. To give an idea to the reader, the following table may be referred :

S.No.	Material	Nature	Resistivity (Ω-m) at room temperature
1	Copper	metal	1.7×10^{-8}
2	Iron	metal	9.68×10^{-8}
3	Manganin	alloy	48×10^{-8}
4	Nichrome	alloy	100×10^{-8}
5	Pure silicon	semiconductor	2.5×10^{3}
6	Pure germanium	semiconductor	0.6
7	Glass	insulator	10^{10} to 10^{14}
8	Mica	insulator	10^{11} to 10^{15}

The reader may note that resistivity of metals and alloys is very small. Therefore, these materials are good conductors of electric current. On the other hand, resistivity of insulators is extremely large. As a result, these materials hardly conduct any current. There is also an intermediate class of semiconductors. The resistivity of these substances lies between conductors and insulators.

1.20. Conductance

The reciprocal of resistance of a conductor is called its **conductance** *(G).* If a conductor has resistance R, then its conductance G is given by ;

$$G = 1/R$$

Whereas resistance of a conductor is the opposition to current flow, the conductance of a conductor is the inducement to current flow.

The SI unit of conductance is mho (*i.e.*, ohm spelt backward). These days, it is a usual practice to use **siemen** as the unit of conductance. It is denoted by the symbol S.

Conductivity. *The reciprocal of resistivity of a conductor is called its* **conductivity.** It is denoted by the symbol σ. If a conductor has resistivity ρ, then its conductivity is given by ;

$$\text{Conductivity,} \quad \sigma = \frac{1}{\rho}$$

We know that $G = \frac{1}{R} = \frac{a}{\rho l} = \sigma \frac{a}{l}$. Clearly, the SI unit of conductivity is *Siemen metre*$^{-1}$ ($S\ m^{-1}$).

Example 1.6. *A coil consists of 2000 turns of copper wire having a cross-sectional area of 0.8 mm^2. The mean length per turn is 80 cm and the resistivity of copper is 0.02 μΩ m. Find the resistance of the coil and power absorbed by the coil when connected across 110 V d.c. supply.*

Solution. Length of coil, $l = 0.8 \times 2000 = 1600$ m; cross-sectional area of coil, $a = 0.8$ mm^2 $= 0.8 \times 10^{-6}$ m^2; Resistivity of copper, $\rho = 0.02 \times 10^{-6}$ Ωm

$$\therefore \quad \text{Resistance of coil,} \quad R = \rho \frac{l}{a} = 0.02 \times 10^{-6} \frac{1600}{0.8 \times 10^{-6}} = \mathbf{40\ \Omega}$$

$$\text{Power absorbed,} \quad P = \frac{V^2}{R} = \frac{(110)^2}{40} = \mathbf{302.5\ W}$$

Example 1.7. *Find the resistance of 1000 metres of a copper wire 25 sq. mm in cross-section. The resistance of copper is 1/58 ohm per metre length and 1 sq. mm cross-section. What will be the resistance of another wire of the same material, three times as long and one-half area of cross-section ?*

Solution. For the first case, $R_1 =$? ; $a_1 = 25\ \text{mm}^2$; $l_1 = 1000$ m

For the second case, $R_2 = 1/58\ \Omega$; $a_2 = 1\ \text{mm}^2$; $l_2 = 1$ m

$$R_1 = \rho\,(l_1/a_1)\ ;\quad R_2 = \rho\,(l_2/a_2)$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{a_2}{a_1} = \left(\frac{1000}{1}\right) \times \left(\frac{1}{25}\right) = 40$$

$$\text{or} \quad R_1 = 40\,R_2 = 40 \times \frac{1}{58} = \frac{\mathbf{20}}{\mathbf{29}}\ \Omega$$

For the third case, $R_3 =$? ; $a_3 = a_1/2$; $l_3 = 3l_1$

$$\therefore \quad \frac{R_3}{R_1} = \left(\frac{l_3}{l_1}\right) \times \left(\frac{a_1}{a_3}\right) = (3) \times (2) = 6$$

$$\text{or} \quad R_3 = 6R_1 = 6 \times \frac{20}{29} = \frac{\mathbf{120}}{\mathbf{29}}\ \Omega$$

Example 1.8. *A copper wire of diameter 1 cm had a resistance of 0.15 Ω. It was drawn under pressure so that its diameter was reduced to 50%. What is the new resistance of the wire ?*

Solution. Area of wire before drawing, $a_1 = \frac{\pi}{4}\,(1)^2 = 0.785\ \text{cm}^2$

Area of wire after drawing, $a_2 = \frac{\pi}{4}\,(0.5)^2 = 0.196\ \text{cm}^2$

As the volume of wire remains the same before and after drawing,

$$\therefore \quad a_1 l_1 = a_2 l_2$$

$$\text{or} \quad l_2/l_1 = a_1/a_2 = 0.785/0.196 = 4$$

For the first case, $R_1 = 0.15\ \Omega$; $a_1 = 0.785\ \text{cm}^2$; $l_1 = l$

For the second case, $R_2 =$? ; $a_2 = 0.196\ \text{cm}^2$; $l_2 = 4l$

$$\text{Now} \quad R_1 = \rho\frac{l_1}{a_1};\quad R_2 = \rho\frac{l_2}{a_2}$$

$$\therefore \quad \frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right) \times \left(\frac{a_1}{a_2}\right) = (4) \times (4) = 16$$

$$\text{or} \quad R_2 = 16R_1 = 16 \times 0.15 = \mathbf{2.4\ \Omega}$$

Example 1.9. *Two wires of aluminium and copper have the same resistance and same length. Which of the two is lighter? Density of copper is 8.9 × 10^3 kg/m^3 and that of aluminium is 2.7 × 10^3 kg/m^3. The resistivity of copper is 1.72 × 10^{-8} Ω m and that of aluminium is 2.6 × 10^{-8} Ω m.*

Solution. That wire will be lighter which has less mass. Let suffix 1 represent aluminium and suffix 2 represent copper.

$$R_1 = R_2 \quad \text{or} \quad \rho_1\frac{l_1}{A_1} = \rho_2\frac{l_2}{A_2}$$

$$\text{or} \quad \frac{\rho_1}{A_1} = \frac{\rho_2}{A_2} \qquad (\because\ l_1 = l_2)$$

$$\text{or} \quad \frac{A_1}{A_2} = \frac{\rho_1}{\rho_2} = \frac{2.6\times10^{-8}}{1.72\times10^{-8}} = 1.5$$

Now $$\frac{m_1}{m_2} = \frac{(A_1 l_1)d_1}{(A_2 l_2)d_2} = \frac{A_1 d_1}{A_2 d_2} \qquad (\because l_1 = l_2)$$

or $$\frac{m_1}{m_2} = \left(\frac{A_1}{A_2}\right) \times \left(\frac{d_1}{d_2}\right) = 1.5 \times \frac{2.7\times10^3}{8.9\times10^3} = 0.46$$

or $$m_1/m_2 = 0.46$$

It is clear that for the same length and same resistance, **aluminium wire is lighter than copper wire**. For this reason, aluminium wires are used for overhead power transmission lines.

Example 1.10. *A rectangular metal strip has the dimensions x = 10 cm, y = 0.5 cm and z = 0.2 cm. Determine the ratio of the resistances R_x, R_y and R_z between the respective pairs of opposite faces.*

Solution. $$R_x : R_y : R_z = \frac{\rho x}{yz} : \frac{\rho y}{zx} : \frac{\rho z}{xy} = \frac{10}{0.5\times0.2} : \frac{0.5}{0.2\times10} : \frac{0.2}{10\times0.5}$$

$$= \frac{10}{0.1} : \frac{1}{4} : 0.04 = \mathbf{2500 : 6.25 : 1}$$

Example 1.11. *Calculate the resistance of a copper tube 0.5 cm thick and 2 m long. The external diameter is 10 cm. Given that resistance of copper wire 1 m long and 1 mm² in cross-section is 1/58 Ω.*

Solution. External diameter, $D = 10$ cm

Internal diameter, $d = 10 - 2 \times 0.5 = 9$ cm

Area of cross-section, $$a = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}\left[(10)^2 - (9)^2\right] \text{cm}^2$$

$$= \frac{\pi}{4}\left[(10)^2 - (9)^2\right] \times 100 \text{ mm}^2$$

∴ Resistance of copper tube $$= \frac{\rho l}{a} = \frac{1}{58} \times \frac{\text{length in metres}}{\text{area of X-section in mm}^2}$$

$$= \frac{1}{58} \times \frac{2}{\frac{\pi}{4}\left[(10)^2 - (9)^2\right] \times 100} = 23.14 \times 10^{-6}\ \Omega = \mathbf{23.14\ \mu\Omega}$$

Example 1.12. *A copper wire is stretched so that its length is increased by 0.1%. What is the percentage change in its resistance ?*

Solution. $$R = \rho\frac{l}{a}; \quad R' = \rho\frac{l'}{a'}$$

Now $$l' = l + \frac{0.1}{100} \times l = 1.001\, l$$

As the volume remains the same, $al = a'l'$.

∴ $$a' = a\frac{l}{l'} = \frac{a}{1.001}$$

∴ $$\frac{R'}{R} = \left(\frac{l'}{l}\right) \times \left(\frac{a}{a'}\right) = (1.001) \times (1.001) = 1.002$$

or $$\frac{R' - R}{R} = 0.002$$

∴ Percentage increase $$= \frac{R' - R}{R} \times 100 = 0.002 \times 100 = \mathbf{0.2\%}$$

Example 1.13. *A lead wire and an iron wire are connected in parallel. Their respective specific resistances are in the ratio 49 : 24. The former carries 80% more current than the latter and the latter 47% longer than the former. Determine the ratio of their cross-sectional areas.*

Solution. Let us represent lead and iron by suffixes 1 and 2 respectively. Then as per the conditions of the problem, we have,

$$\frac{\rho_1}{\rho_2} = \frac{49}{24} \; ; \; I_1 = 1.8\, I_2 \; ; \; l_2 = 1.47\, l_1$$

Now $$R_1 = \rho_1 \frac{l_1}{a_1} \; ; \; R_2 = \rho_2 \frac{l_2}{a_2}$$

$$I_1 = \frac{V}{R_1} \quad \text{and} \quad I_2 = \frac{V}{R_2}$$

$\therefore$ $$\frac{I_2}{I_1} = \frac{R_1}{R_2} = \frac{\rho_1 l_1}{a_1} \times \frac{a_2}{\rho_2 l_2} = \left(\frac{\rho_1}{\rho_2}\right) \times \left(\frac{l_1}{l_2}\right) \times \left(\frac{a_2}{a_1}\right)$$

or $$\frac{1}{1.8} = \frac{49}{24} \times \frac{1}{1.47} \times \frac{a_2}{a_1}$$

$\therefore$ $$\frac{a_2}{a_1} = \frac{1}{1.8} \times \frac{24}{49} \times 1.47 = \mathbf{0.4}$$

Example 1.14. *An aluminium wire 7.5 m long is connected in parallel with a copper wire 6 m long. When a current of 5 A is passed through the combination, it is found that the current in the aluminium wire is 3 A. The diameter of the aluminium wire is 1 mm. Determine the diameter of the copper wire. Resistivity of copper is 0.017 μΩm ; that of the aluminium is 0.028 μΩ m.*

Solution. Let us assign subscripts *a* and *c* to aluminium and copper respectively.

Current through *Al* wire, $I_a = 3$ A

$\therefore$ Current through Cu wire, $I_c = 5 - 3 = 2$ A

Since R_a and R_c are in parallel, the voltage across them is the same [See Fig. 1.12] *i.e.*

$$I_a R_a = I_c R_c \quad \text{or} \quad \frac{R_a}{R_c} = \frac{I_c}{I_a} = \frac{2}{3}$$

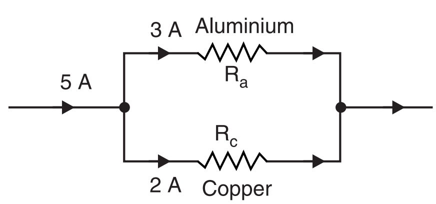


Fig. 1.12

Now $$R_a = \frac{\rho_a l_a}{a_a} \; ; \; R_c = \frac{\rho_c l_c}{a_c}$$

$\therefore$ $$\frac{R_c}{R_a} = \frac{\rho_c}{\rho_a} \times \frac{l_c}{l_a} \times \frac{a_a}{a_c}$$

Here $$\frac{R_c}{R_a} = \frac{3}{2} \; ; \; \frac{\rho_c}{\rho_a} = \frac{0.017}{0.028} \; ; \; \frac{l_c}{l_a} = \frac{6}{7.5} \; ;$$

$$a_a = \frac{\pi}{4} d^2 = \frac{\pi \times (1)^2}{4} = \frac{\pi}{4} \text{ mm}^2$$

$\therefore$ $$\frac{3}{2} = \frac{0.017}{0.028} \times \frac{6}{7.5} \times \frac{\pi / 4}{a_c}$$

or $$a_c = \frac{2}{3} \times \frac{0.017}{0.028} \times \frac{6}{7.5} \times \frac{\pi}{4} = 0.2544 \text{ mm}^2$$

or $$\frac{\pi}{4} d_c^2 = 0.2544 \qquad \therefore \quad d_c = \mathbf{0.569 \ mm}$$

Example 1.15. *A transmission line cable consists of 19 strands of identical copper conductors, each 1.5 mm in diameter. The length of the cable is 2 km but because of the twist of the strands, the actual length of each conductor is increased by 5 percent. What is resistance of the cable ? Take the resistivity of the copper to be 1.78 × 10^{-8} Ω m.*

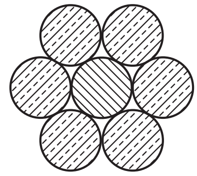


Fig. 1.13

Solution. Fig. 1.13 shows the general shape of a stranded conductor. Allowing for twist, the length of the strands is

$$l = 2000 \text{ m} + 5\% \text{ of } 2000 \text{ m} = 2100 \text{ m}$$

Area of X-section of 19 strands, $a = (19)\left(\frac{\pi}{4}\right)\times(1.5\times10^{-3})^2 = 33.576\times10^{-6} \text{ m}^2$

∴ Resistance of line, $R = \rho\frac{l}{a} = 1.72\times10^{-8}\times\frac{2100}{33.576\times10^{-6}}$ = **1.076 Ω**

Example 1.16. *The resistance of the wire used for telephone is 35 Ω per kilometre when the weight of the wire is 5 kg per kilometre. If the specific resistance of the material is 1.95 × 10^{-8} Ω m, what is the cross-sectional area of the wire ? What will be the resistance of a loop to a subscriber 8 km from the exchange if wire of the same material but weighing 20 kg per kilometre is used?*

Solution. For the first case, R = 35 Ω ; l = 1000 m ; $\rho = 1.95 \times 10^{-8}$ Ω m

Now $R = \rho\frac{l}{a}$ ∴ $a = \frac{\rho l}{R} = \frac{1.95\times10^{-8}\times1000}{35}$ = **55.7 × 10^{-8} m²**

Since weight of conductor is directly proportional to the area of cross-section, for the second case, we have,

$$a = \frac{20}{5}\times 55.7\times10^{-8} = 222.8\times10^{-8} \text{ m}^2 \text{ ; } l = 2\times 8 = 16 \text{ km} = 16000 \text{ m}$$

∴ $R = \rho\frac{l}{a} = 1.95\times10^{-8}\times\frac{16000}{222.8\times10^{-8}}$ = **140.1 Ω**

Example 1.17. *Find the resistance of a cubic centimetre of copper (i) when it is drawn into a wire of diameter 0.32 mm and (ii) when it is hammered into a flat sheet of 1.2 mm thickness, the current flowing through the sheet from one face to another, specific resistance of copper is 1.6 × 10^{-8} Ω-m.*

Solution. Volume of copper wire, $v = 1 \text{ cm}^3 = 1\times10^{-6} \text{ m}^3$

(*i*) Resistance when drawn into wire.

Area of X-section, $a = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.32\times10^{-3})^2 = 0.804\times10^{-7} \text{ m}^2$

Length of wire, $l = \frac{v}{a} = \frac{1\times10^{-6}}{0.804\times10^{-7}} = 12.43 \text{ m}$

∴ Resistance of wire, $R = \rho\frac{l}{a} = 1.6\times10^{-8}\frac{12.43}{0.804\times10^{-7}}$ = **2.473 Ω**

(*ii*) Resistance when hammered into flat sheet.

Length of flat sheet, $l = 1.2\times10^{-3}$ m ; Area of cross-section of flat sheet is

$$a = \frac{v}{l} = \frac{1\times10^{-6}}{1.2\times10^{-3}} = \frac{10^{-3}}{1.2} \text{ m}^2$$

∴ Resistance of copper flat sheet is $R = \rho\frac{l}{a} = 1.6\times10^{-8}\frac{1.2\times10^{-3}}{10^{-3}/1.2}$ = **2.3 × 10^{-8} Ω**

Tutorial Problems

1. Calculate the resistance of 915 metres length of a wire having a uniform cross-sectional area of 0.77 cm^2 if the wire is made of copper having a resistivity of $1.7 \times 10^{-6}\ \Omega$ cm. **[0.08 Ω]**
2. A wire of length 1 m has a resistance of 2 ohms. What is the resistance of second wire, whose specific resistance is double the first, if the length of wire is 3 metres and the diameter is double of the first? **[3 Ω]**
3. A rectangular copper strip is 20 cm long, 0.1 cm wide and 0.4 cm thick. Determine the resistance between (*i*) opposite ends (*ii*) opposite sides. The resistivity of copper is $1.7 \times 10^{-6}\ \Omega$ cm. **[(*i*) $0.85 \times 10^{-4}\ \Omega$ (*ii*) $0.212 \times 10^{-6}\ \Omega$]**
4. A cube of a material of side 1 cm has a resistance of 0.001 Ω between its opposite faces. If the same material has a length of 9 cm and a uniform cross-sectional area 1 cm^2, what will be the resistance of this length ? **[0.009 Ω]**
5. An aluminium wire 10 metres long and 2 mm in diameter is connected in parallel with a copper wire 6 metres long. A total current of 2 A is passed through the combination and it is found that current through the aluminium wire is 1.25 A. Calculate the diameter of copper wire. Specific resistance of copper is $1.6 \times 10^{-6}\ \Omega$ cm and that of aluminium is $2.6 \times 10^{-6}\ \Omega$ cm. **[0.94 mm]**
6. A copper wire is stretched so that its length is increased by 0.1%. What is the percentage change in its resistance ? **[0.2%]**

1.21. Types of Resistors

A component whose function in a circuit is to provide a specified value of resistance is called a **resistor**. The principal applications of resistors are to limit current, divide voltage and in certain cases, generate heat. Although there are a variety of different types of resistors, the following are the commonly used resistors in electrical and electronic circuits :

(*i*) Carbon composition types (*ii*) Film resistors
(*iii*) Wire-wound resistors (*iv*) Cermet resistors

(*i*) Carbon composition type. This type of resistor is made with a mixture of finely ground carbon, insulating filler and a resin binder. The ratio of carbon and insulating filler decides the resistance value [See Fig. 1.14]. The mixture is formed into a rod and lead connections are made. The entire resistor is then enclosed in a plastic case to prevent the entry of moisture and other harmful elements from outside.

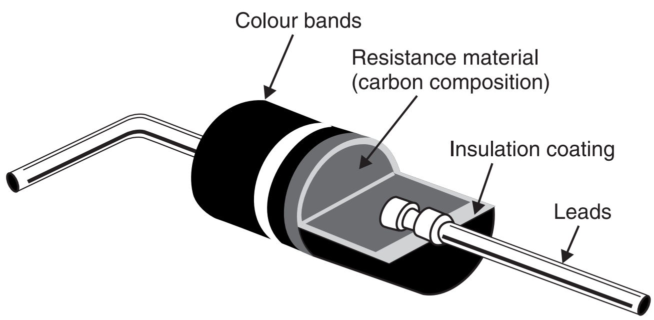


Fig. 1.14

Carbon resistors are relatively inexpensive to build. However, they are highly sensitive to temperature variations. The carbon resistors are available in power ratings ranging from 1/8 to 2 W.

(*ii*) Film resistors. In a film resistor, a resistive material is deposited uniformly onto a high-grade ceramic rod. The resistive film may be carbon (carbon film resistor) or nickel-chromium (metal film resistor). In these types of resistors, the desired resistance value is obtained by removing a part of the resistive material in a helical pattern along the rod as shown in Fig. 1.15.

Metal film resistors have better characteristics as compared to carbon film resistors.

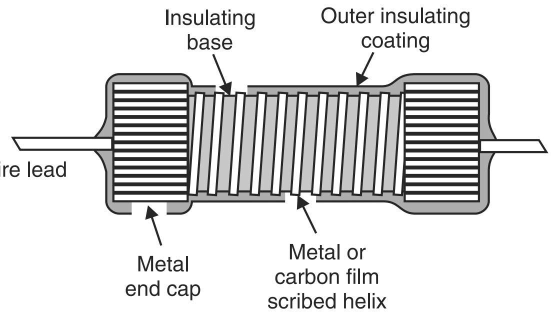


Fig. 1.15

(*iii*) Wire-wound resistors. A wire-wound resistor is constructed by winding a resistive wire of some alloy around an insulating rod. It is then enclosed in an insulating cover. Generally, nickle-chromium alloy is used because of its very small temperature coefficient of resistance. Wire-wound resistors can safely operate at higher temperatures than carbon types. These resistors have high power ratings ranging from 12 to 225 W.

(*iv*) Cermet resistors. A cermet resistor is made by depositing a thin film of metal such as nichrome or chromium cobalt on a ceramic substrate. They are cermet which is a contraction for ceramic and metal. These resistors have very accurate values.

1.22. Effect of Temperature on Resistance

In general, the resistance of a material changes with the change in temperature. The effect of temperature upon resistance varies according to the type of material as discussed below :

(*i*) The resistance of pure metals (*e.g.* copper, aluminium) increases with the increase of temperature. The change in resistance is fairly regular for normal range of temperatures so that temperature/resistance graph is a straight line as shown in Fig. 1.16 (for copper). Since the resistance of metals increases with the rise in temperature, they have *positive temperature co-efficient of resistance.*

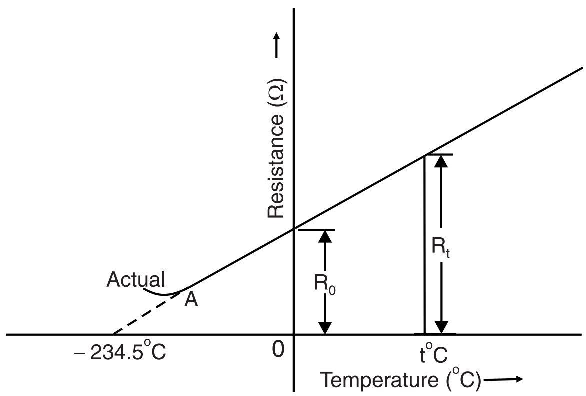


Fig. 1.16

(*ii*) The resistance of electrolytes, insulators (*e.g.* glass, mica, rubber etc.) and semiconductors (*e.g.* germanium, silicon etc.) decreases with the increase in temperature. Hence these materials have *negative temperature co-efficient of resistance.*

(*iii*) The resistance of alloys increases with the rise in temperature but this increase is very small and irregular. For some high resistance alloys (*e.g.* Eureka, manganin, constantan etc.), the change in resistance is practically negligible over a wide range of temperatures.

Fig. 1.16 shows temperature/resistance graph for copper which is a straight line. If this line is extended backward, it would cut the temperature axis at −234.5°C. It means that theoretically, the resistance of copper wire is zero at −234.5°C. However, in actual practice, the curve departs (point *A*) from the straight line path at very low temperatures.

1.23. Temperature Co-efficient of Resistance

Consider a conductor having resistance R_0 at 0°C and R_t at t °C. It has been found that in the normal range of temperatures, the increase in resistance (*i.e.* $R_t - R_0$)

(*i*) is directly proportional to the initial resistance *i.e.*

$$R_t - R_0 \propto R_0$$

(*ii*) is directly proportional to the rise in temperature *i.e.*

$$R_t - R_0 \propto t$$

(*iii*) depends upon the nature of material.

Combining the first two, we get,

$$R_t - R_0 \propto R_0\, t$$

or
$$R_t - R_0 = {}^*\alpha_0\, R_0\, t \qquad ...(i)$$

where α_0 is a constant and is called temperature co-efficient of resistance at 0°C. Its value depends upon the nature of material and temperature.

Rearranging eq. (*i*), we get,

$$R_t = R_0\,(1 + \alpha_0\, t) \qquad ...(ii)$$

Definition of α_0. From eq. (*i*), we get,

$$\alpha_0 = \frac{R_t - R_0}{R_0 \times t}$$

= Increase in resistance/ohm original resistance/°C rise in temperature

Hence **temperature co-efficient of resistance** *of a conductor is the increase in resistance per ohm original resistance per °C rise in temperature.*

A little reflection shows that unit of α will be ohm/ohm°C *i.e.*/°C. Thus, copper has a temperature co-efficient of resistance of 0.00426/°C. It means that if a copper wire has a resistance of 1 Ω at 0°C, then it will increase by 0.00426 Ω for 1°C rise in temperature *i.e.* it will become 1.00426 Ω at 1°C. Similarly, if temperature is raised to 10°C, then resistance will become 1 + 10 × 0.00426 = 1.0426 ohms.

The following points may be noted carefully :

(*i*) Those substances (*e.g.* pure metals) whose resistance increases with rise in temperature are said to have *positive* temperature co-efficient of resistance. On the other hand, those substances whose resistance decreases with increase in temperature are said to have *negative* temperature co-efficient of resistance.

(*ii*) If a conductor has a resistance R_0, R_1 and R_2 at 0°C, t_1°C and t_2°C respectively, then,

$$R_1 = R_0\,(1 + \alpha_0\, t_1)$$

$$R_2 = R_0\,(1 + \alpha_0\, t_2)$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{1+\alpha_0\, t_2}{1+\alpha_0 t_1} \qquad ...(iii)$$

This relation is often utilised in determining the rise of temperature of the winding of an electrical machine. The resistance of the winding is measured both before and after the test run. Let R_1 and t_1 be the resistance and temperature before the commencement of the test. After the operation of the machine for a given period, let these values be R_2 and t_2. Since R_1 and R_2 can be measured and t_1 (ambient temperature) and α_0 are known, the value of t_2 can be calculated from eq. (*iii*). The average rise in temperature of the winding will be $(t_2 - t_1)$°C.

Note. The life expectancy of electrical apparatus is limited by the temperature of its insulation; the higher the temperature, the shorter the life. The useful life of electrical apparatus reduces approximately by half every time the temperature increases by 10°C. This means that if a motor has a normal life expectancy of eight years

* It will be shown in Art. 1.25 that value of α depends upon temperature. Therefore, it is referred to the original temperature *i.e.* 0°C in this case. Hence the symbol α_0.

at a temperature of 100°C, it will have a life expectancy of only four years at a temperature of 110°C, of two years at a temperature of 120°C and of only one year at 130°C.

1.24. Graphical Determination of α

The value of temperature co-efficient of resistance can also be determined graphically from temperature/resistance graph of the material. Fig. 1.17 shows the temperature/resistance graph for a conductor. The graph is a straight line *AX* as is the case with all conductors. The resistance of the conductor is R_0 (represented by *OA*) at 0°C and it becomes R_t at *t*°C. By definition,

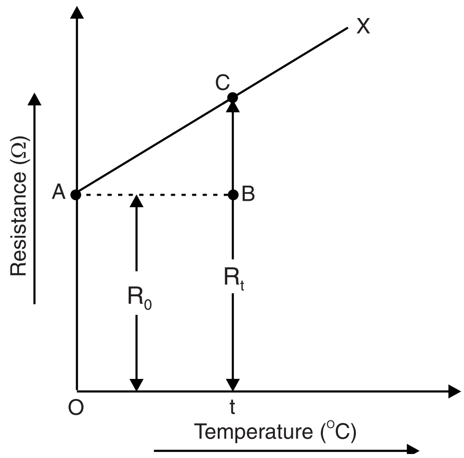


Fig. 1.17

$$\alpha_0 = \frac{R_t - R_0}{R_0 \times t}$$

But $R_t - R_0 = BC$

and t = rise in temperature = AB

$$\therefore \quad \alpha_0 = \frac{BC}{R_0 \times AB}$$

But *BC*/*AB* is the slope of temperature/resistance graph.

$$\therefore \quad \alpha_0 = \frac{\text{Slope of temp./resistance graph}}{\text{Original resistance}} \quad ...(i)$$

Hence, **temperature co-efficient** *of resistance of a conductor at 0°C is the slope of temp./resistance graph divided by resistance at 0°C (i.e. R_0).*

The following points may be particularly noted :

(*i*) The value of α depends upon temperature. At any temperature, α can be calculated by using eq. (*i*).

Thus, $$\alpha_0 = \frac{\text{Slope* of temperature/resistance graph}}{R_0}$$

and $$\alpha_t = \frac{\text{Slope of temperature/resistance graph}}{R_t}$$

(*ii*) The value of α_0 is maximum and it decreases as the temperature is increased. This is clear from the fact that the slope of temperature/resistance graph is constant and R_0 has the minimum value.

1.25. Temperature Co-efficient at Various Temperatures

Consider a conductor having resistances R_0 and R_1 at temperatures 0°C and t_1°C respectively. Let α_0 and α_1 be the temperature co-efficients of resistance of the conductor at 0°C and t_1°C respectively. It is desired to establish the relationship between α_1 and α_0. Fig. 1.18 shows the temperature/resistance graph of the conductor. As proved in Art. 1.24,

$$\alpha_0 = \frac{\text{Slope of graph}}{R_0}$$

$$\therefore \quad \text{Slope of graph} = \alpha_0 R_0$$

* The slope of temp./resistance graph of a conductor is always constant (being a straight line).

Similarly, $\alpha_1 = \dfrac{\text{Slope of graph}}{R_1}$

or Slope of graph $= \alpha_1 R_1$

Since the slope of temperature/resistance graph is constant,

$\therefore \quad \alpha_0 R_0 = \alpha_1 R_1$

or $\alpha_1 = \dfrac{\alpha_0 R_0}{R_1} = \dfrac{\alpha_0 R_0}{R_0(1+\alpha_0 t_1)}$ $[\because R_1 = R_0(1+\alpha_0 t_1)]$

$\therefore \quad \alpha_1 = \dfrac{\alpha_0}{1+\alpha_0 t_1}$ ...(*i*)

Similarly,* $\alpha_2 = \dfrac{\alpha_0}{1+\alpha_0 t_2}$ ...(*ii*)

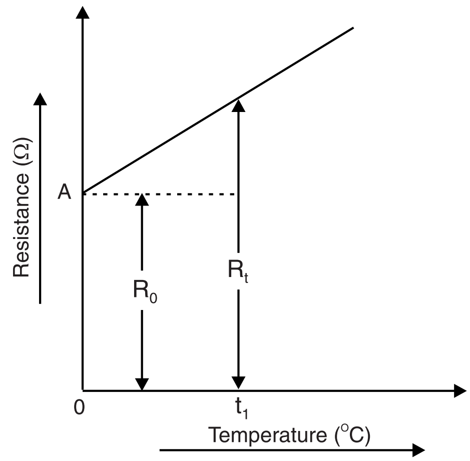


Fig. 1.18

Subtracting the reciprocal of eq. (*i*) from the reciprocal of eq. (*ii*),

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_1} = \frac{1+\alpha_0 t_2}{\alpha_0} - \frac{1+\alpha_0 t_1}{\alpha_0} = t_2 - t_1$$

$$\therefore \quad \alpha_2 = \frac{1}{\dfrac{1}{\alpha_1} + (t_2 - t_1)} \qquad ...(iii)$$

Eq. (*i*) gives the relation between α_1 and α_0 while Eq. (*iii*) gives the relation between α_2 and α_1.

1.26. Summary of Temperature Co-efficient Relations

(*i*) If R_0 and α_0 are the resistance and temperature co-efficient of resistance of a conductor at 0°C, then its resistance R_t at t°C is given by ;

$$R_t = R_0(1 + \alpha_0 t)$$

(*ii*) If α_0, α_1 and α_2 are the temperature co-efficients of resistance at 0°C, t_1°C and t_2°C respectively, then,

$$\alpha_1 = \frac{\alpha_0}{1+\alpha_0 t_1} \;;\quad \alpha_2 = \frac{\alpha_0}{1+\alpha_0 t_2} \;;\quad \alpha_2 = \frac{1}{\dfrac{1}{\alpha_1} + (t_2 - t_1)}$$

(*iii*) Suppose R_1 and R_2 are the resistances of a conductor at t_1°C and t_2°C respectively. If α_1 is the temperature co-efficient of resistance at t_1°C, then,

$$R_2^{**} = R_1[1 + \alpha_1(t_2 - t_1)]$$

1.27. Variation of Resistivity With Temperature

Not only resistance but resistivity or specific resistance of a material also changes with temperature. The change in resistivity per °C change in temperature is called *temperature*

* $\alpha_0 R_0 = \alpha_2 R_2$ where R_2 is the resistance at t_2°C

or $\alpha_2 = \dfrac{\alpha_0 R_0}{R_2} = \dfrac{\alpha_0 R_0}{R_0(1+\alpha_0 t_2)} = \dfrac{\alpha_0}{1+\alpha_0 t_2}$

** Slope of graph, $\tan\theta = R_0\,\alpha_0 = R_1\,\alpha_1 = R_2\,\alpha_2$

Increase in resistance as temperature is raised from t_1°C to t_2°C

$= \tan\theta(t_2 - t_1) = R_1\alpha_1(t_2 - t_1)$

$\therefore$ Resistance at t_2°C, $R_2 = R_1 + R_1\alpha_1(t_2 - t_1) = R_1[1 + \alpha_1(t_2 - t_1)]$

coefficient of resistivity. In case of metals, the resistivity increases with increase in temperature and vice-versa. It is found that resistivity of a metallic conductor increases linearly over a wide range of temperatures and is given by ;

$$\rho_t = \rho_0(1 + \alpha_0 t)$$

where ρ_0 = resistivity of metallic conductor at 0°C

ρ_t = resistivity of metallic conductor at temperature t°C

Note that temperature coefficient of resistivity is equal to temperature coefficient of resistance α_0.

Example 1.18. *A coil has a resistance of 18 Ω when its mean temperature is 20°C and of 20 Ω when its mean temperature is 50°C. Find its mean temperature rise when its resistance is 21Ω and the surrounding temperature is 15°C.*

Solution. Let R_0 be the resistance of the coil at 0°C and α_0 be its temperature coefficient of resistance at 0°C. Then,

$$18 = R_0(1 + \alpha_0 \times 20) \quad \text{and} \quad 20 = R_0(1 + \alpha_0 \times 50)$$

$$\therefore \quad \frac{20}{18} = \frac{1+50\alpha_0}{1+20\alpha_0} \quad \text{or} \quad \alpha_0 = \frac{1}{250} = 0.004/°C$$

If t°C is the temperature of the coil when its resistance is 21Ω, then,

$$21 = R_0(1 + 0.004\,t)$$

$$\therefore \quad \frac{21}{18} = \frac{R_0(1+0.004\,t)}{R_0(1+0.004\times 20)} \quad \text{or} \quad t = 65°C$$

$\therefore$ Temperature rise = $t - 15 = 65° - 15°$ = **50°C**

Example 1.19. *The resistance of the field coils of a dynamo is 173 Ω at 16°C. After working for 6 hours on full-load, the resistance of the coils increases to 212 Ω. Calculate (i) the temperature of the coils (ii) mean rise of temperature of the coils. Assume temperature co-efficient of resistance of copper is 0.00426/°C at 0°C.*

Solution. ***(i)*** Let t°C be the final temperature.

$$\frac{R_{16}}{R_t} = \frac{R_0(1+\alpha_0 \times 16)}{R_0(1+\alpha_0 \times t)}$$

or
$$\frac{173}{212} = \frac{1+0.00426\times 16}{1+0.00426\times t}$$

or
$$0.816 = \frac{1.068}{1+0.00426\,t} \qquad \therefore \quad t = \mathbf{72.5°C}$$

(ii) Rise in temperature = $t - 16 = 72.5 - 16$ = **56.5°C**

Example 1.20. *The resistance of a transformer winding is 460 Ω at room temperature of 25°C. When the transformer is running and the final temperature is reached, the resistance of the winding increases to 520 Ω. Find the average temperature rise of winding, assuming that α_{20} = 1/250 per °C.*

Solution.
$$\alpha_{25} = \frac{1}{1/\alpha_{20} + (25-20)} = \frac{1}{250+5} = \frac{1}{255}/°C$$

Let t°C be the final temperature of the winding. Then, the rise in temperature is $t - 25$.

Now, $R_{25} = 460\ \Omega$; $R_t = 520\ \Omega$

$$R_t = R_{25}[1 + \alpha_{25}(t - 25)]$$

or
$$t - 25 = \frac{1}{\alpha_{25}}\left(\frac{R_t}{R_{25}} - 1\right) = 255(520/460 - 1) = 33.26°C$$

$\therefore$ Temperature rise = $t - 25$ = **32.26°C**

Example 1.21. *The filament of a 60 watt, 230 V lamp has a normal working temperature of 2000°C. Find the current flowing in the filament at the instant of switching, when the lamp is cold. Assume the temperature of cold lamp to be 15°C and* $\alpha_{15} = 0.005/°C$.

Solution. Resistance of lamp at 2000°C is

$$R_{2000} = V^2/P = (230)^2/60 = 881.67\ \Omega$$

$$R_{2000} = R_{15}[1 + \alpha_{15}(2000 - 15)]$$

$$\therefore \quad R_{15} = \frac{R_{2000}}{1+0.005(1985)} = \frac{881.67}{10.925} = 80.7\,\Omega$$

∴ Current taken by cold lamp (*i.e.* at the time of switching) is

$$I = V/R_{15} = 230/80.7 = \mathbf{2.85\ A}$$

Example 1.22. *Two coils connected in series have resistances of 600 Ω and 300 Ω and temperature coefficients of 0.1% and 0.4% per °C at 20°C respectively. Find the resistance of combination at a temperature of 50°C. What is the effective temperature coefficient of the combination at 50°C ?*

Solution. Resistance of 600 Ω coil at 50°C

$$= 600\,[1 + 0.001(50 - 20)] = 618\ \Omega$$

Resistance of 300 Ω coil at 50°C

$$= 300\,[1 + 0.004\,(50 - 20)] = 336\ \Omega$$

Resistance of series combination at 50°C is

$$R_{50} = 618 + 336 = \mathbf{954}\ \Omega$$

Resistance of series combination at 20°C is

$$R_{20} = 600 + 300 = 900\ \Omega$$

Now $$R_{50} = R_{20}\,[1 + \alpha_{20}\,(t_2 - t_1)]$$

$$\therefore \quad \alpha_{20} = \frac{\frac{R_{50}}{R_{20}} - 1}{t_2 - t_1} = \frac{\frac{954}{900} - 1}{50 - 20} = 0.002$$

Now $$\alpha_{50} = \frac{1}{1/\alpha_{20} + (t_2 - t_1)} = \frac{1}{1/0.002 + (50-20)} = \mathbf{\frac{1}{530}\ /°C}$$

Example 1.23. *The coil of a relay takes a current of 0.12 A when it is at the room temperature of 15°C and connected across a 60 V supply. If the minimum operating current of the relay is 0.1 A, calculate the temperature above which the relay will fail to operate when connected to the same supply. Resistance temperature coefficient of the coil material is 0.0043 per °C at 0°C.*

Solution. Resistance of relay coil at 15°C, $R_{15} = 60/0.12 = 500\ \Omega$

If the temperature increases, the resistance of relay coil increases and current in relay coil decreases. Let t°C be the temperature at which the current in relay coil becomes 0.1 A (= the minimum relay coil current for its operation). Clearly, $R_t = 60/0.1 = 600\ \Omega$.

Now, $$R_{15} = R_0\,(1 + 15\,\alpha_0) = R_0\,(1 + 15 \times 0.0043)$$

$$R_t = R_0\,(1 + \alpha_0 t) = R_0\,(1 + 0.0043\,t)$$

$$\therefore \quad \frac{R_t}{R_{15}} = \frac{1+0.0043\,t}{1.0645}$$

or $$\frac{600}{500} = \frac{1+0.0043\,t}{1.0645}$$

On solving, $t = \mathbf{64.5°C}$

If the temperature of relay coil increases above 64.5°C, the resistance of relay coil will increase and the relay coil current will be less than 0.1 A. As a result, the relay will fail to operate.

Example 1.24. *Two materials, A and B, have resistance temperature coefficients of 0.004 and 0.0004 respectively at a given temperature. In what proportion must A and B be joined in series to produce a circuit having a temperature coefficient of 0.001 ?*

Solution. Let the resistance of A be 1 Ω and that of B be x Ω *i.e.* $R_A = 1\ \Omega$ and $R_B = x\ \Omega$.

Resistance of series combination $= R_A + R_B = (1 + x)\ \Omega$

Suppose the temperature rises by t°C.

Resistance of series combination at the raised temperature $= (1 + x)(1 + 0.001\ t)$ ...(*i*)

Resistance of A at the raised temperature $= 1\ (1 + 0.004\ t)$ ...(*ii*)

Resistance of B at the raised temperature $= x\ (1 + 0.0004\ t)$ ...(*iii*)

As per the conditions of the problem, we have, (*ii*) + (*iii*) = (*i*)

or $1\ (1 + 0.004\ t) + x\ (1 + 0.0004\ t) = (1 + x)(1 + 0.001\ t)$

or $0.004\ t + 0.0004\ t\,x = (1 + x) \times 0.001\ t$

Dividing by t and multiplying throughout by 10^4, we have,

$$40 + 4x = 10\ (1 + x) \quad \therefore\ x = 5$$

$\therefore$ $R_A : R_B =$ **1 : 5** *i.e.* R_B should be 5 times R_A.

Example 1.25. *A resistor of 80 Ω resistance, having a temperature coefficient of 0.0021/°C is to be constructed. Wires of two materials of suitable cross-sectional areas are available. For material A, the resistance is 80 Ω per 100 m and the temperature coefficient is 0.003/°C. For material B, the corresponding figures are 60 Ω per 100 m and 0.0015/°C. Calculate suitable lengths of wires of materials A and B to be connected in series to construct the required resistor. All data are referred to the same temperature.*

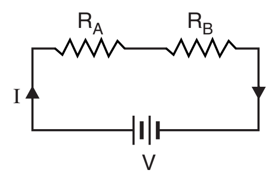


Fig. 1.19

Solution. Let R_A and R_B be the required resistances of materials A and B respectively which when joined in series have a combined temperature coefficient of 0.0021 [See Fig. 1.19].

Resistance of series combination $= R_A + R_B$

Resistance of series combination at raised temperature $= (R_A + R_B)(1 + 0.0021\ t)$ ...(*i*)

Resistance of A at raised temperature $= R_A\ (1 + 0.003\ t)$ ...(*ii*)

Resistance of B at raised temperature $= R_B\ (1 + 0.0015\ t)$ ...(*iii*)

As per conditions of the problem, (*ii*) + (*iii*) = (*i*).

$\therefore$ $R_A\ (1 + 0.003\ t) + R_B\ (1 + 0.0015\ t) = (R_A + R_B)(1 + 0.0021\ t)$

On solving, $\dfrac{R_B}{R_A} = \dfrac{3}{2}$ ...(*iv*)

Now, $R_A + R_B = 80$ ...(*v*)

From eqs. (*iv*) and (*v*), $R_A = 32\ \Omega$ and $R_B = 48\ \Omega$

$\therefore$ Length of wire A, $L_A = (100/80) \times 32 =$ **40 m**

Length of wire B, $L_B = (100/60) \times 48 =$ **80 m**

Example 1.26. *Two wires A and B are connected in series at 0°C and resistance of B is 3.5 times that of A. The resistance temperature coefficient of A is 0.4% and that of combination is 0.1%. Find the resistance temperature coefficient of B.*

Solution. Let the temperature coefficient of resistance of wire B be α_B. If R is the resistance of wire A, then,

$$R_A = R \;;\; R_B = 3.5\,R$$

Total resistance of two wires at 0°C = $R_A + R_B = R + 3.5\,R = 4.5\,R$

Increase in resistance of wire A per °C rise = $\alpha_A\,R = 0.004\,R$

Increase in resistance of wire B per °C rise = $\alpha_B \times 3.5\,R = 3.5\,R\,\alpha_B$

Total increase in the resistance of combination per °C rise = $0.004\text{ R} + 3.5\,R\,\alpha_B$ ... (*i*)

Also, total increase in the resistance of combination per °C rise = α_C × Total resistance of combination = $0.001 \times 4.5\text{ R} = 0.0045\text{ R}$ ... (*ii*)

From eqs. (*i*) and (*ii*), $0.004\,R + 3.5\,R\alpha_B = 0.0045\,R$

$$\therefore \qquad \alpha_B = \frac{0.0045\,R - 0.004\,R}{3.5\,R} = \mathbf{0.000143/°C} \text{ or } \mathbf{0.0143\%}$$

Example 1.27. *Two conductors, one of copper and the other of iron, are connected in parallel and carry equal currents at 25°C. What proportion of current will pass through each if the temperature is raised to 100°C? The temperature co-efficients of resistance at 0°C are 0.0043/°C and 0.0063/°C for copper and iron respectively.*

Solution. Since copper and iron conductors carry equal currents at 25°C, their resistances are the same at this temperature. Let their resistance be R ohms at 25°C. If R_1 and R_2 are the resistances of copper and iron conductors respectively at 100°C, then,

$$R_1 = R\,[1 + 0.0043\,(100 - 25)] = 1.3225\,R$$

$$R_2 = R\,[1 + 0.0063\,(100 - 25)] = 1.4725\,R$$

If I is the total current at 100°C, then,

$$\text{Current in copper conductor} = I \times \frac{R_2}{R_1 + R_2} = I \times \frac{1.4725\,R}{1.3225\,R + 1.4725\,R} = 0.5268\,I$$

$$\text{Current in iron conductor} = I \times \frac{R_1}{R_1 + R_2} = I \times \frac{1.3225\,R}{1.3225R + 1.4725\,R} = 0.4732\,I$$

Therefore, at 100°C, the copper conductor will carry **52.68%** of total current and the remaining **47.32%** will be carried by iron conductor.

Example 1.28. *A semi-circular ring of copper has an inner radius 6 cm, radial thickness 3 cm and an axial thickness 4 cm. Find the resistance of the ring at 50°C between its two end-faces. Assume specific resistance of copper at 20°C = 1.724×10^{-6} Ω-cm and resistance temperature coefficient of copper at 0°C = 0.0043/°C.*

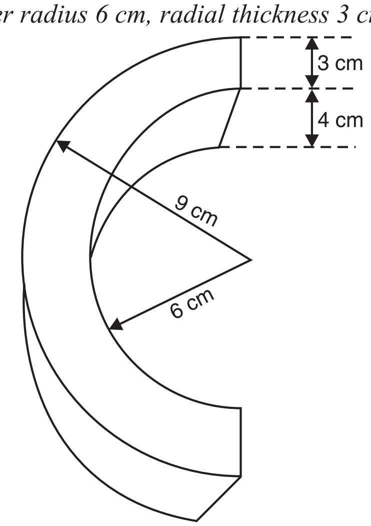


Fig. 1.20

Solution. Fig. 1.20 shows the semi-circular ring.

Mean radius of the ring, $r_m = (6 + 9)/2 = 7.5$ cm

Mean length between end faces is

$$l_m = \pi r_m = \pi \times 7.5 = 23.56 \text{ cm}$$

Cross-sectional area of the ring is

$$a = 3 \times 4 = 12 \text{ cm}^2$$

$$\text{Now} \qquad \alpha_{20} = \frac{\alpha_0}{1 + \alpha_0\,t} = \frac{0.0043}{1 + 0.0043 \times 20}$$

$$= 0.00396/°C$$

Also $\rho_{50} = \rho_{20}\,[1 + \alpha_{20}\,(t - 20)]$

$= 1.724 \times 10^{-6}\,[1 + 0.00396 \times (50 - 20)]$

$= 1.93 \times 10^{-6}\ \Omega\ \text{cm}$

$$\therefore \quad R_{50} = \frac{\rho_{50}\, l_m}{a} = \frac{1.93 \times 10^{-6} \times 23.56}{12} = \mathbf{3.79 \times 10^{-6}\ \Omega}$$

This example shows that resistivity of a conductor increases with the increase in temperature and vice-versa.

Example 1.29. *A copper conductor has its specific resistance of 1.6 ×10*$^{-6}$ *Ω cm at 0°C and a resistance temperature coefficient of 1/254.5 per °C at 20°C. Find (i) specific resistance and (ii) the resistance temperature coefficient at 60°C.*

Solution. $$\alpha_{20} = \frac{\alpha_0}{1 + \alpha_0 \times 20} \quad \text{or} \quad \frac{1}{254.5} = \frac{\alpha_0}{1 + \alpha_0 \times 20} \quad \therefore\ \alpha_0 = \frac{1}{234.5}\ /°\text{C}$$

(*i*) $$\rho_{60} = \rho_0\,(1 + \alpha_0 \times 60) = 1.6 \times 10^{-6}\,(1 + 60/234.5) = \mathbf{2.01 \times 10^{-6}\ \Omega\ cm}$$

(*ii*) $$\alpha_{60} = \frac{1}{\dfrac{1}{\alpha_{20}} + (t_2 - t_1)} = \frac{1}{254.5 + (60 - 20)} = \mathbf{\frac{1}{294.5}\ /°C}$$

Example 1.30. *The filament of a 240 V metal-filament lamp is to be constructed from a wire having a diameter of 0.02 mm and a resistivity at 20° C of 4.3 μΩ cm. If* α_{20} *= 0.005/°C, what length of filament is necessary if the lamp is to dissipate 60 W at a filament temperature of 2420°C ?*

Solution. Power to be dissipated by the lamp at 2420°C is

$$\frac{V^2}{R_{2420}} = 60 \qquad \therefore \quad R_{2420} = \frac{V^2}{60} = \frac{(240)^2}{60} = 960\,\Omega$$

Now $R_{2420} = R_{20}\,[1 + \alpha_{20}\,(2420 - 20)]$

or $960 = R_{20}\,[1 + 0.005\,(2420 - 20)]$

$\therefore$ $R_{20} = 960/13\ \Omega$

Now $\rho_{20} = 4.3 \times 10^{-6}\ \Omega\ \text{cm}\ ; \quad a = \dfrac{\pi}{4} d^2 = \dfrac{\pi}{4}(0.02 \times 10^{-1})^2\ \text{cm}^2$

$$\therefore \quad \text{Length of filament is } l = \frac{a \times R_{20}}{\rho_{20}} = \frac{\pi}{4} \times \frac{(0.02 \times 10^{-1})^2 \times 960}{4.3 \times 10^{-6} \times 13} = \mathbf{54\ cm}$$

Tutorial Problems

1. The shunt winding of a motor has a resistance of 35.1 Ω at 20°C. Find its resistance at 32.6°C. The temperature co-efficient of copper is 0.00427/°C at 0°C. **[39.6 Ω]**
2. The resistance of a coil of wire increases from 40 Ω at 10°C to 48.25 Ω at 60°C. Find the temperature coefficient at 0°C of the conductor material. **[0.0043/°C]**
3. The coil of an electromagnet, made of copper wire, has resistance of 4 Ω at a temperature of 22°C. After operating for 2 days, the coil current is 42 A at a terminal voltage of 210 V. Calculate the average temperature of the coil at that time. **[86.1°C]**
4. The filament of a 60 watt incandescent lamp possesses a cold resistance of 17.6 Ω at 20°C. The lamp draws a current of 0.25 A when connected to a 240 V source. Calculate the temperature of hot filament. Take temperature co-efficient at 0°C as 0.0055/°C. **[2571°C]**

5. A nichrome heater is operated at 1500°C. What is the percentage increase in its resistance over that at room temperature (20°C) ? Temperature co-efficient of nichrome is 0.00016/°C at 0°C. **[23.6%]**
6. Two wires *A* and *B* are connected in series at 0°C and resistance of *B* is 3.5 times that of *A*. The resistance temperature coefficient of *A* is 0.4% and that of the combination is 0.1%. Find the resistance temperature coefficient of *B*. **[0.0143%]**
7. A d.c. shunt motor after running for several hours on constant voltage mains of 400 V takes a field current of 1.6 A. If the temperature rise is known to be 40°C, what value of extra circuit resistance is required to adjust the field current to 1.6 A when starting from cold at 20°C ? Temperature coefficient = 0.0043/°C at 20°C. **[36.69 Ω]**
8. A potential difference of 250 V is applied to a copper field coil at a temperature of 15°C and the current is 5 A. What will be the mean temperature of the coil when the current has fallen to 3.91 A, the applied voltage being the same as before ? **[85°C]**
9. An insulating material has an insulation resistance of 100% at 0°C. For each rise in temperature of 5°C its resistance is reduced by 10%. At what temperature is the insulation resistance halved ? **[33°C]**
10. A carbon electrode has a resistance of 0.125 Ω at 20°C. The temperature coefficient of carbon is –0.0005 at 20°C. What will the resistance of the electrode be at 85°C ? **[0.121 Ω]**

1.28. Ohm's Law

The relationship between voltage (*V*), the current (*I*) and resistance (*R*) in a d.c. circuit was first discovered by German scientist George Simon *Ohm. This relationship is called Ohm's law and may be stated as under :

The ratio of potential difference (V) between the ends of a conductor to the current (I) flowing between them is constant, provided the physical conditions (e.g. temperature etc.) do not change i.e.

$$\frac{V}{I} = \text{Constant} = R$$

where *R* is the resistance of the conductor between the two points considered.

For example, if in Fig. 1.21 (*i*), the voltage between points *A* and *B* is *V* volts and current flowing is *I* amperes, then *V*/*I* will be constant and equal to *R*, the resistance between points *A* and *B*. If the voltage is doubled up, the current will also be doubled up so that the ratio *V*/*I* remains constant. If we draw a graph between *V* and *I*, it will be a straight line passing through the origin as shown in Fig. 1.21 (*ii*). The resistance *R* between points *A* and *B* is given by slope of the graph *i.e.*

$$R = \tan\theta = V/I = \text{Constant}$$

Ohm's law can be expressed in three forms *viz*.

$$I = V/R\ ;\ V = IR\ ;\ R = V/I$$

These formulae can be applied to any part of a d.c. circuit or to a complete circuit. It may be noted that if voltage is measured in volts and current in amperes, then resistance will be in ohms.

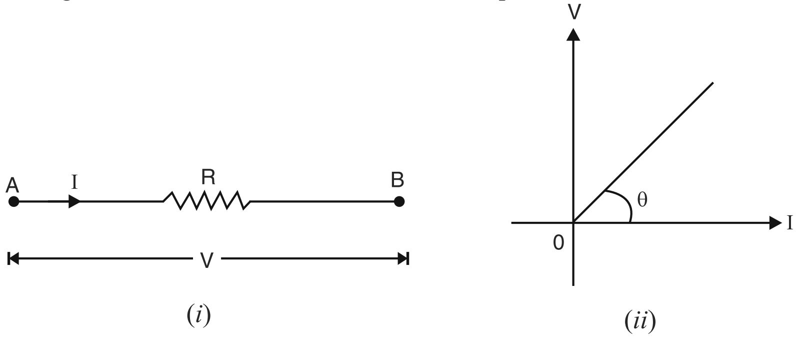


Fig. 1.21

* The unit of resistance (*i.e.* ohm) was named in his honour.

1.29. Non-ohmic Conductors

Those conductors which do not obey Ohm's law (I ∝ V) are called non-ohmic conductors e.g., vacuum tubes, transistors, electrolytes, etc. A non-ohmic conductor may have one or more of the following properties :

(*i*) The *V-I* graph is non-linear *i.e.* *V/I* is variable.

(*ii*) The *V-I* graph may not pass through the origin as in case of an ohmic conductor.

(*iii*) A non-ohmic conductor may conduct poorly or not at all when the p.d. is reversed.

The non-linear circuit problems are generally solved by graphical methods.

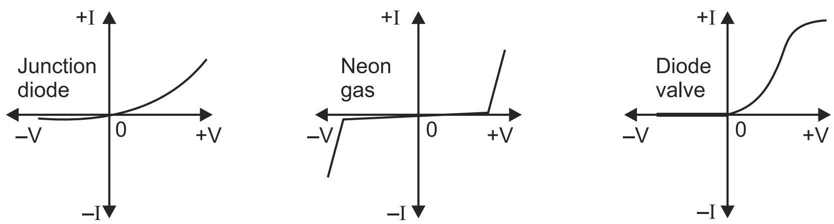


Fig. 1.22

Fig. 1.22 illustrates the graphs of non-ohmic conductors. Note that *V-I* graphs for these non-ohmic conductors are not a straight line.

Example 1.31. *What is the value of the unknown resistor R in Fig. 1.23 (i) if the voltage drop across the 500 Ω resistor is 2.5 volts ? All resistances are in ohm.*

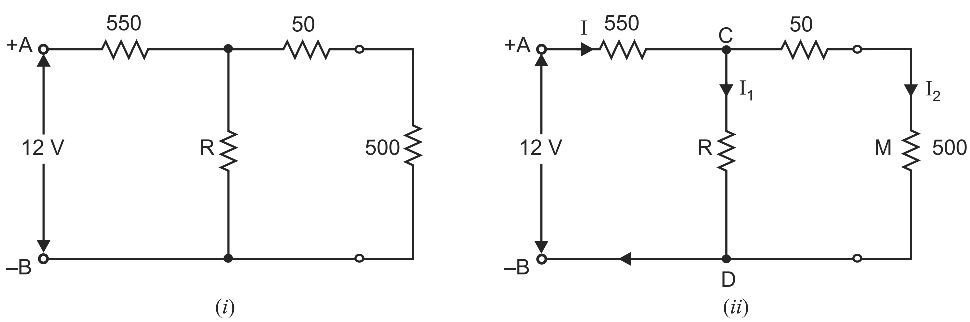


Fig. 1.23

Solution. Fig. 1.23 (*ii*) shows the various currents in the circuit.

$$I_2 = \frac{\text{Voltage drop across 500 } \Omega}{500\ \Omega} = \frac{2.5}{500} = 0.005 \text{ A}$$

Voltage across *CMD* or *CD* is given by ;

$$V_{CMD} = V_{CD} = I_2\,(50 + 500) = 0.005 \times 550 = 2.75 \text{ V}$$

Now $$I = \frac{12 - V_{CD}}{550} = \frac{12 - 2.75}{550} = 0.0168 \text{ A}$$

∴ $$I_1 = I - I_2 = 0.0168 - 0.005 = 0.0118 \text{ A}$$

Now $$V_{CD} = I_1 R \quad \therefore \quad R = \frac{V_{CD}}{I_1} = \frac{2.75}{0.0118} = \mathbf{233\ \Omega}$$

Example 1.32. *A metal filament lamp takes 0.3 A at 230 V. If the voltage is reduced to 115 V, will the current be halved ? Explain your answer.*

Solution. No. It is because Ohm's law is applicable only if the resistance of the circuit does not change. In the present case, when voltage is reduced from 230 V to 115 V, the temperature of the lamp will decrease too much, resulting in an enormous decrease of lamp resistance. Consequently, Ohm's law ($I = V/R$) cannot be applied. To give an idea to the reader, the hot resistance (*i.e.* at normal operating temperature) of an incandescent lamp is more than 10 times its cold resistance.

Example 1.33. *A coil of copper wire has resistance of 90 Ω at 20°C and is connected to a 230 V supply. By how much must the voltage be increased in order to maintain the current constant if the temperature of the coil rises to 60°C? Take α_0 for copper = 0.00428/°C.*

Solution. $R_{20} = R_0(1 + \alpha_0 \times 20)$; $R_{60} = R_0(1 + \alpha_0 \times 60)$

$$\therefore \quad \frac{R_{60}}{R_{20}} = \frac{1+0.00428\times 60}{1+0.00428\times 20} = \frac{1.2568}{1.0856}$$

or $$R_{60} = R_{20} \times \frac{1.2568}{1.0856} = 90\times\frac{1.2568}{1.0856} = 104.2\,\Omega$$

$$\text{Now, current at } 20°\text{C} = \frac{230}{90} = \frac{23}{9}\text{ A}$$

The wire resistance has become 104.2 Ω at 60°C. Therefore, in order to keep the current constant at the previous value, the new voltage required = (23/9) × 104.2 = 266.3 V.

∴ Required voltage increase = 266.3 – 230 = **36.3 V**

Tutorial Problems

1. A battery has an e.m.f. of 12.8 V and supplies a current of 3.2 A. What is the resistance of the circuit ? How many coulombs leave the battery in 5 minutes ? **[4 Ω ; 960 C]**
2. In a discharge tube, the number of hydrogen ions (*i.e.* protons) drifting across a cross-section per second is 1.2×10^{18} while the number of electrons drifting in the opposite direction is 2.8×10^{18} per second. If the supply voltage is 220 V, what is the effective resistance of the tube ? **[344 Ω]**
3. An electromagnet of resistance 12.4 Ω requires a current of 1.5 A to operate it. Find the required voltage. **[18.6 V]**
4. The cold resistance of a certain gas-filled tungsten lamp is 18.2 Ω and its hot resistance at the operating voltage of 220 V is 202 Ω. Find the current (*i*) at the instant of switching (*ii*) under normal operating conditions. **[(*i*) 12.08 A (*ii*) 1.09 A]**

1.30. Electric Power

The rate at which work is done in an electric circuit is called its **electric power** *i.e.*

$$\text{Electric power} = \frac{\text{Work done in electric circuit}}{\text{Time}}$$

When voltage is applied to a circuit, it causes current (*i.e.* electrons) to flow through it. Clearly, work is being done in moving the electrons in the circuit. This work done in moving the electrons in a unit time is called the electric power. Thus referring to the part *AB* of the circuit (See Fig. 1.24),

V = P.D. across *AB* in volts
I = Current in amperes
R = Resistance of *AB* in Ω
t = Time in sec. for which current flows

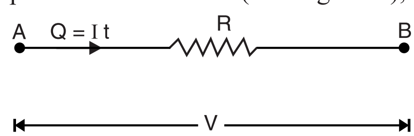


Fig. 1.24

The total charge that flows in t seconds is $Q = I \times t$ coulombs and by definition (See Art. 1.12),

$$V = \frac{\text{Work}}{Q}$$

or $$\text{Work} = VQ = VIt \qquad (\because Q = It)$$

$$\therefore \quad \text{Electric power, } P = \frac{\text{Work}}{t} = \frac{VIt}{t} = VI \text{ joules/sec or watts}$$

$$\therefore \quad P = VI = I^2R = \frac{V^2}{R} \qquad [\because V = IR \text{ and } I = V/R]$$

The above three formulae are equally valid for calculation of electric power in a d.c. circuit. Which one is to be used depends simply on which quantities are known or most easily determined.

Unit of electric power. The basic unit of electric power is *joules/sec* or *watt*. The power consumed in a circuit is 1 watt if a p.d. of 1 V causes 1 A current to flow through the circuit.

$$\text{Power in watts} = \text{Voltage in volts} \times \text{Current in amperes}$$

The bigger units of electric power are kilowatts (kW) and megawatts (MW).

$$1 \text{ kW} = 1000 \text{ watts} \quad ; \quad 1 \text{ MW} = 10^6 \text{ watts or } 10^3 \text{ kW}$$

1.31. Electrical Energy

The total work done in an electric circuit is called **electrical energy** *i.e.*

$$\text{Electrical energy} = \text{Electrical power} \times \text{Time}$$
$$= V\,I\,t = I^2R\,t = \frac{V^2}{R}t$$

The reader may note that formulae for electrical energy can be readily derived by multiplying the electric power by '*t*', the time for which the current flows. The unit of electrical energy will depend upon the units of electric power and time.

(*i*) If power is taken in watts and time in seconds, then the unit of electrical energy will be *watt-sec. i.e.*

$$\text{Energy in watt-sec.} = \text{Power in watts} \times \text{Time in sec.}$$

(*ii*) If power is expressed in watts and time in hours, then unit of electrical energy will be *watt-hour i.e.*

$$\text{Energy in watt-hours} = \text{Power in watts} \times \text{Time in hours}$$

(*iii*) If power is expressed in kilowatts and time in hours, then unit of electrical energy will be *kilowatt-hour* (kWh) *i.e.*

$$\text{Energy in kWh} = \text{Power in kW} \times \text{Time in hours}$$

It may be pointed out here that in practice, electrical energy is measured in kilowatt-hours (kWh). Therefore, it is profitable to define it.

One kilowatt-hour (kWh) *of electrical energy is expended in a circuit if 1 kW (1000 watts) of power is supplied for 1 hour.*

The electricity bills are made on the basis of total electrical energy consumed by the consumer. The unit for charge of electricity is 1 kWh. One kWh is also called Board of Trade (B.O.T.) unit or simply unit. Thus when we say that a consumer has consumed 100 units of electricity, it means that electrical energy consumption is 100 kWh.

1.32. Use of Power and Energy Formulas

It has already been discussed that electric power as well as electrical energy consumed can be expressed by three formulas. While using these formulas, the following points may be kept in mind:

(*i*) $$\text{Electric power, } P = I^2 R = \frac{V^2}{R} \text{ watts}$$

$$\text{Electrical energy consumed, } W = I^2 Rt = \frac{V^2}{R}t \text{ joules}$$

The above formulas apply *only* to resistors and to devices (*e.g.* electric bulb, heater, electric kettle etc) where all electrical energy consumed is converted into heat.

(*ii*) Electric power, $P = VI$ watts

Electrical energy consumed, $W = VIt$ joules

These formulas apply to any type of load including the one mentioned in point (*i*).

Example 1.34. *A 100 V lamp has a hot resistance of 250 Ω. Find the current taken by the lamp and its power rating in watts. Calculate also the energy it will consume in 24 hours.*

Solution. Current taken by lamp, $I = V/R = 100/250 =$ **0.4 A**

Power rating of lamp, $P = VI = 100 \times 0.4 =$ **40 W**

Energy consumption in 24 hrs. = Power × time = 40 × 24 = **960 watt-hours**

Example 1.35. *A heating element supplies 300 kilojoules in 50 minutes. Find the p.d. across the element when current is 2 amperes.*

Solution. Total charge, $Q = I \times t = 2 \times 50 \times 60 = 6000$ C

$$\text{P.D., } V = \frac{\text{Work}}{\text{Charge}} = \frac{300 \times 10^3}{6000} = \mathbf{50\ V}$$

Example 1.36. *A 10 watt resistor has a value of 120 Ω. What is the rated current through the resistor ?*

Solution. Rated power, $P = I^2R$

$$\therefore \quad \text{Rated current, } I = \sqrt{\frac{P}{R}} = \sqrt{\frac{10}{120}} = \mathbf{0.2887\ A}$$

If current through the resistor exceeds this value, the resistor will be burnt due to excessive heat.

Note. Every electrical equipment has power and current ratings marked on its body. While the equipment is in operation, care should be taken that neither of these limits is exceeded, otherwise the equipment may be damaged/burnt due to excessive heat.

Example 1.37. *The following are the details of load on a circuit connected through a supply metre :*

(*i*) *Six lamps of 40 watts each working for 4 hours per day*

(*ii*) *Two flourescent tubes 125 watts each working for 2 hours per day*

(*iii*) *One 1000 watt heater working for 3 hours per day*

If each unit of energy costs 70 P, what will be the electricity bill for the month of June ?

Solution. Total wattage of lamps = 40 × 6 = 240 watts

Total wattage of tubes = 125 × 2 = 250 watts

Wattage of heater = 1000 watts

Energy consumed by the appliances per day

= (240 × 4) + (250 × 2) + (1000 × 3)

= 4460 watt-hours = 4.46 kWh

Total energy consumed in the month of June (*i.e.* in 30 days)

= 4.46 × 30 = 133.8 kWh

Bill for the month of June = Rs. 0.7 × 133.8 = **Rs. 93.66**

Tutorial Problems

1. A resistor of 50 Ω has a p.d. of 100 volts d.c. across it for 1 hour. Calculate (*i*) power and (*ii*) energy. **[(*i*) 200 watts (*ii*) 7.2 × 10^5 J]**
2. A current of 10 A flows through a resistor for 10 minutes and the power dissipated by the resistor is 100 watts. Find the p.d. across the resistor and the energy supplied to the circuit. **[10 V ; 6 × 10^4 J]**

3. A factory is supplied with power at 210 volts through a pair of feeders of total resistance 0.0225 Ω. The load consists of 354, 250 V, 60 watt lamps and 4 motors each taking 40 amperes. Find :
   (*i*) total current required
   (*ii*) voltage at the station end of feeders
   (*iii*) power wasted in feeders. **[(*i*) 231.4 A (*ii*) 215.78 V (*iii*) 1.4 kW]**
4. How many kilowatts will be required to light a factory in which 250 lamps each taking 1.3 A at 230 V are used ? **[74.75 kW]**

1.33. Power Rating of a Resistor

The ability of a resistor to dissipate power as heat without destructive temperature build-up is called **power rating** *of the resistor*.

Power rating of resistor = I^2R or V^2/R [See Fig. 1.25]

Suppose the power rating of a resistor is 2 W. It means that I^2R or V^2/R should not exceed 2 W. Suppose the quantity I^2R (or V^2/R) for this resistor becomes 4 W. The resistor is able to dissipate 2 W as heat and the remaining 2 W will start building up the temperature. In a matter of seconds, the resistor will burn out.

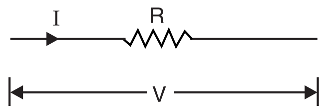


Fig. 1.25

The physical size of a resistor is not necessarily related to its resistance value but rather to its *power rating*. A large resistor is able to dissipate (throw off) more heat because of its large physical size. In general, the greater the physical size of a resistor, the greater is its power rating and vice-versa.

Example 1.38. *A 0.1 Ω resistor has a power rating of 5 W. Is this resistor safe when conducting a current of 10 A ?*

Solution. Power developed in the resistor is

$$P = I^2R = (10)^2 \times 0.1 = 10 \text{ W}$$

The resistor is **not safe** since the power developed in the resistor exceeds its dissipation rating.

Example 1.39. *What is the maximum safe current flow in a 47 Ω, 2 W resistor ?*

Solution. Power rating $= I^2R$

or $2 = I^2 \times 47$ ∴ Maximum safe current, $I = \sqrt{\frac{2}{47}}$ = **0.21 A**

Example 1.40. *What is the maximum voltage that can be applied across a 100 Ω, 10 W resistor in order to keep within the resistor's power rating ?*

Solution. Power rating $= V^2/R$

or $10 = V^2/100$ ∴ Max. safe voltage, $V = \sqrt{10 \times 100}$ = **31.6 volts**

Tutorial Problems

1. A 200 Ω resistor has a 2 W power rating. What is the maximum current that can flow in the resistor without exceeding the power rating ? **[100 mA]**
2. A 6.8 kΩ, 0.25 W resistor shows a potential difference of 40 V. Is the resistor safe ? **[Yes]**
3. A 1.5 kΩ resistor has 1 W power rating. What maximum voltage can be applied across the resistor without exceeding the power rating ? **[38.73 V]**

1.34. Nonlinear Resistors

A device or circuit element whose V/I characteristic is not a straight line is said to exhibit **nonlinear resistance.**

The examples of nonlinear resistors are thermistors, varistors, diodes, filaments of incandescent lamps etc.

1. Thermistors. *A* **thermistor** *is a heat sensitive device usually made of a semiconductor material whose resistance changes very rapidly with change of temperature.* A thermistor has the following important properties :

(*i*) The resistance of a thermistor changes very rapidly with change of temperature.

(*ii*) The temperature coefficient of a thermistor is very high.

(*iii*) The temperature co-efficient of a thermistor can be both positive and negative.

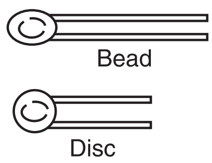


Fig. 1.26

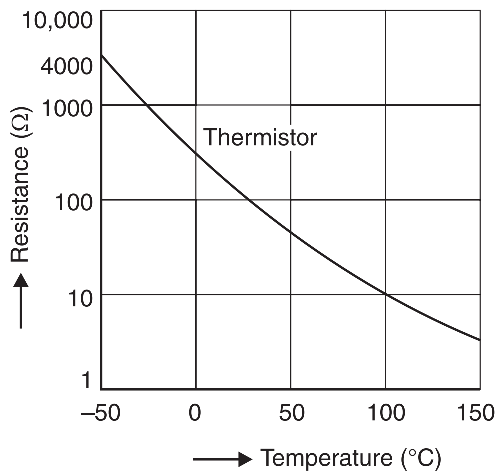


Fig. 1.27

Construction. Thermistors are made from semiconductor oxides of iron, nickel and cobalt. They are generally in the form of beads, discs or rods (See Fig. 1.26). A pair of platinum leads are attached at the two ends for electrical connections. The arrangement is enclosed in a very small glass bulb and sealed.

Fig. 1.27 shows the resistance/temperature characteristic of a typical thermistor with negative temperature coefficient. The resistance decreases progressively from 4000 Ω to 3 Ω as its temperature varies from – 50°C to +150°C.

Applications

(*a*) A thermistor with negative temperature coefficient of resistance may be used to safeguard against current surges in a circuit where this could be harmful *e.g.* in a circuit where the heaters of the radio valves are in series (See Fig. 1.28).

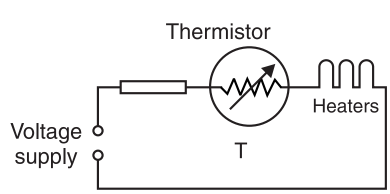


Fig. 1.28

A thermistor *T* is included in the circuit. When the supply voltage is switched on, the thermistor has a high resistance at first because it is cold. It thus limits the current to a moderate value. As it warms up, the thermistor resistance drops appreciably and an increased current then flows through the heaters.

(*b*) A thermistor with a negative temperature coefficient can be used to issue an alarm for excessive temperature of winding of motors, transformers and generators [See Fig. 1.29].

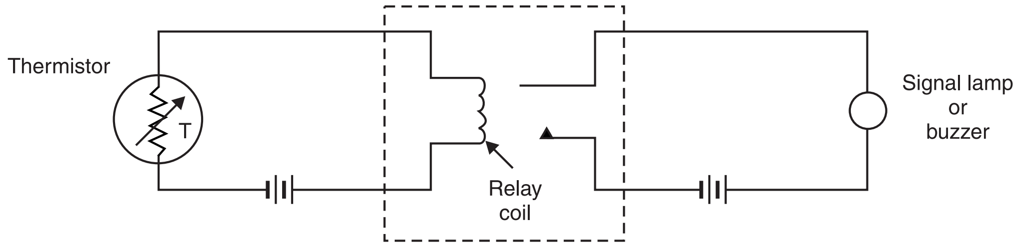


Fig. 1.29

When the temperature of windings is low, the thermistor is cool and its resistance is high. Therefore, only a small current flows through the thermistor and the relay coil. When the temperature of the windings is high, the thermistor is hot and its resistance is low. Therefore, a large current flows in the relay coil to close the contacts. This completes the circuit for the signal lamp or buzzer.

2. Varistor (Thyrite). A varistor is a nonlinear resistor whose resistance decreases as the voltage increases. Therefore, a varistor is a voltage-dependent resistor. It is made of silicon-carbide powder and is built in the shape of a disc. The *V-I* characteristic of a typical varistor is shown in Fig. 1.30. The curve shows that the current increases dramatically with increasing voltage. Thus when the voltage increases from 1.5 kV to 10 kV, the current rises from 1 mA to 100 A. Varistors are placed in parallel with critical components which might be damaged by high transient voltages. Under normal conditions, the varistor remains in high-resistance state and draws very little current. On the application of surge, the varistor is driven to its low-resistance state. The varistor then conducts a relatively large amount of current and dissipates much of the surge as heat. Thus the component is saved from damage.

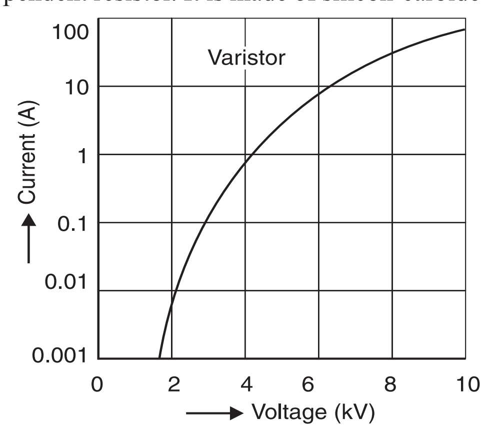


Fig. 1.30

OBJECTIVE QUESTIONS

1. The resistance of a wire is R ohms. It is stretched to double its length. The new resistance of the wire in ohms is
   (*i*) $R/2$ (*ii*) $2R$
   (*iii*) $4R$ (*iv*) $R/4$
2. The example of non-ohmic resistance is
   (*i*) copper wire
   (*ii*) carbon resistance
   (*iii*) tungsten wire
   (*iv*) diode
3. In which of the following substances, the resistance decreases with the increase of temperature
   (*i*) carbon (*ii*) constantan
   (*iii*) copper (*iv*) silver
4. The resistance of a wire of uniform diameter d and length l is R. The resistance of another wire of the same material but diameter $2d$ and length $4l$ will be
   (*i*) $2R$ (*ii*) R
   (*iii*) $R/2$ (*iv*) $R/4$
5. The temperature coefficient of resistance of a wire is 0.00125 $°C^{-1}$. At 300 K, its resistance is one ohms. The resistance of the wire will be 2 ohms at
   (*i*) 1154 K (*ii*) 1100 K
   (*iii*) 1400 K (*iv*) 1127 K
6. The resistance of 20 cm long wire is 5 ohms. The wire is stretched to a uniform wire of 40 cm length. The resistance now will be (in ohms)
   (*i*) 5 (*ii*) 10
   (*iii*) 20 (*iv*) 200

7. A current of 4.8 A is flowing in a conductor. The number of electrons flowing per second through the X-section of conductor will be

(*i*) 3×10^{19} electrons
(*ii*) 76.8×10^{20} electrons
(*iii*) 7.68×10^{20} electrons
(*iv*) 3×10^{20} electrons

8. A carbon resistor has coloured strips as brown, green, orange and silver respectively. The resistance is

(*i*) 15 k Ω ± 10% (*ii*) 10 k Ω ± 10%
(*iii*) 15 k Ω ± 5% (*iv*) 10 k Ω ± 5%

9. A wire has a resistance of 10 Ω. It is stretched by one-tenth of its original length. Then its resistance will be

(*i*) 10 Ω (*ii*) 12.1 Ω
(*iii*) 9 Ω (*iv*) 11 Ω

10. A 10 m long wire of resistance 20 Ω is connected in series with a battery of e.m.f. 3 V (negligible internal resistance) and a resistance of 10 Ω. The potential gradient along the wire in volt per metre is

(*i*) 0.02 (*ii*) 0.1
(*iii*) 0.2 (*iv*) 1.2

11. The diameter of an atom is about

(*i*) 10^{-10} m (*ii*) 10^{-8} m
(*iii*) 10^{-2} m (*iv*) 10^{-15} m

12. 1 cm^3 of copper at room temperature has about

(*i*) 200 free electrons
(*ii*) 20×10^{10} free electrons
(*iii*) 8.5×10^{22} free electrons
(*iv*) 3×10^{5} free electrons

13. The electric current is due to the flow of

(*i*) positive charges only
(*ii*) negative charges only
(*iii*) both positive and negative charges
(*iv*) neutral particles only

14. The quantity of charge that will be transferred by a current flow of 10 A over 1 hour period is

(*i*) 10 C (*ii*) 3.6×10^4 C
(*iii*) 2.4×10^3 C (*iv*) 1.6×10^2 C

15. The drift velocity of electrons is of the order of

(*i*) 1 ms^{-1} (*ii*) 10^{-3} ms^{-1}
(*iii*) 10^6 ms^{-1} (*iv*) 3×10^8 ms^{-1}

16. Insulators have .... temperature co-efficient of resistance.

(*i*) zero (*ii*) positive
(*iii*) negative (*iv*) none of the above

17. Eureka has .... temperature co-efficient of resistance.

(*i*) almost zero (*ii*) negative
(*iii*) positive (*iv*) none of the above

18. Constantan wire is used for making standard resistances because it has ....

(*i*) low specific resistance
(*ii*) high specific resistance
(*iii*) negligibly small temperature co-efficient of resistance
(*iv*) high melting point

19. Two resistors *A* and *B* have resistances R_A and R_B respectively with $R_A < R_B$. The resistivities of their materials are ρ_A and ρ_B. Then,

(*i*) $\rho_A > \rho_B$ (*ii*) $\rho_A = \rho_B$
(*iii*) $\rho_A < \rho_B$
(*iv*) Information insufficient

20. In case of liquids, Ohm's law is ....

(*i*) fully obeyed
(*ii*) partially obeyed
(*iii*) there is no relation between current and p.d.
(*iv*) none of the above.

ANSWERS

1. (*iii*)	**2.** (*ii*) and (*iii*)	**3.** (*i*)	**4.** (*ii*)	**5.** (*ii*)
6. (*iii*)	**7.** (*i*)	**8.** (*i*)	**9.** (*ii*)	**10.** (*iii*)
11. (*i*)	**12.** (*iii*)	**13.** (*iii*)	**14.** (*ii*)	**15.** (*ii*)
16. (*iii*)	**17.** (*i*)	**18.** (*iii*)	**19.** (*iv*)	**20.** (*i*)

2
D.C. Circuits

Introduction

It is well known that electric current flows in a closed path. The closed path followed by electric current is called an electric circuit. The essential parts of an electric circuit are (*i*) the source of power (*e.g.* battery, generator etc.), (*ii*) the conductors used to carry current and (*iii*) the load* (*e.g.* lamp, heater, motor etc.). The source supplies electrical energy to the load which converts it into heat or other forms of energy. Thus, conversion of electrical energy into other forms of energy is possible only with suitable circuits. For instance, conversion of electrical energy into mechanical energy is achieved by devising a suitable motor circuit. In fact, the innumerable uses of electricity have been possible only due to the proper use and application of electric circuits. In this chapter, we shall confine our discussion to d.c. circuits only *i.e.* circuits carrying direct current.

2.1. D.C. Circuit

The closed path followed by direct current (d.c.) is called a **d.c. circuit.**

A d.c. circuit essentially consists of a source of d.c. power (*e.g.* battery, d.c. generator etc.), the conductors used to carry current and the load. Fig. 2.1 shows a torch bulb connected to a battery through conducting wires. The direct current **starts from the positive terminal of the battery and comes back to the starting point *via* the load. The direct current follows the path *ABCDA* and *ABCDA* is a d.c. circuit. The load for a d.c. circuit is usually a *** resistance. In a d.c. circuit, loads (*i.e.* resistances) may be connected in series or parallel or series-parallel. Accordingly, d.c. circuits can be classified as :

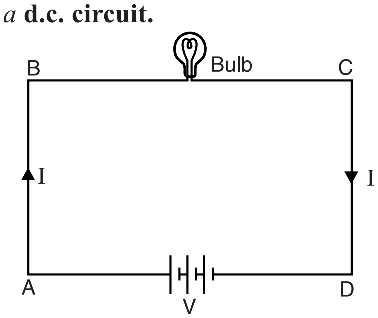


Fig. 2.1

(*i*) Series circuits

(*ii*) Parallel circuits

(*iii*) Series-parallel circuits.

2.2. D.C. Series Circuit

The d.c. circuit in which resistances are connected end to end so that there is only one path for current to flow is called a **d.c. series circuit.**

Consider three resistances R_1, R_2 and R_3 ohms connected in series across a battery of V volts as shown in Fig. 2.2 (*i*). Obviously, there is only one path for current I *i.e.* current is same throughout the circuit. By Ohm's law, voltage across the various resistances is

$$V_1 = I R_1 \; ; \; V_2 = I R_2 \; ; V_3 = I R_3$$

Now $$V = V_1 + V_2 + V_3$$
$$= I R_1 + I R_2 + IR_3$$

---

* The device which utilises electrical energy is called load. For instance, heater converts electrical energy supplied to it into heat. Therefore, heater is the load.

** This is the direction of conventional current. However, the electron flow will be in the opposite direction.

*** Other passive elements *viz.* inductance and capacitance are relevant only in a.c. circuits.

$$= I(R_1 + R_2 + R_3)$$

or $$\frac{V}{I} = R_1 + R_2 + R_3$$

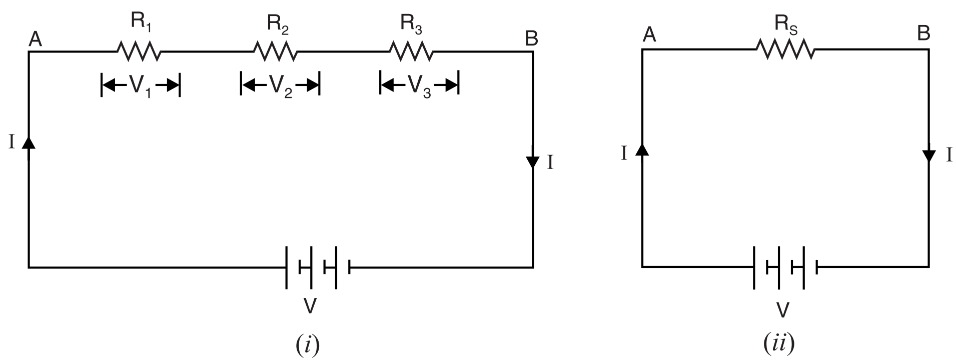


Fig. 2.2

But V/I is the total resistance R_S between points A and B. Note that R_S is called the *total or equivalent resistance of the three resistances.

$\therefore$ $$R_S = R_1 + R_2 + R_3$$

Hence when a number of resistances are connected in series, the total resistance is equal to the sum of the individual resistances.

The total conductance G_S of the circuit is given by ;

$$G_S = \frac{1}{R_S} = \frac{1}{R_1 + R_2 + R_3}$$

Also $$\frac{1}{G_S} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$$

The main characteristics of a series circuit are :

(*i*) The current in each resistor is the same.

(*ii*) The total resistance in the circuit is equal to the sum of individual resistances.

(*iii*) The total power dissipated in the circuit is equal to the sum of powers dissipated in individual resistances. Thus referring to Fig. 2.2 (*i*),

$$R_S = R_1 + R_2 + R_3$$

or $$I^2R_S = I^2R_1 + I^2R_2 + I^2R_3$$

or $$P_S = P_1 + P_2 + P_3$$

Thus total power dissipated in a series circuit is equal to the sum of powers dissipated in individual resistances. As we shall see, this is also true for parallel and series-parallel d.c. circuits.

Note. A series resistor circuit [See Fig. 2.2 (*i*)] can be considered to be a *voltage divider circuit* because the potential difference across any one resistor is a fraction of the total voltage applied across the series combination; the fraction being determined by the values of the resistances.

Example 2.1. *Two filament lamps A and B take 0.8 A and 0.9 A respectively when connected across 110 V supply. Calculate the value of current when they are connected in series across a 220-V supply, assuming the filament resistances to remain unaltered. Also find the voltage across each lamp.*

* Total or equivalent resistance is the single resistance, which if substituted for the series resistances, would provide the same current in the circuit.

Solution. For lamp A, $R_A = 110/0.8 = 137.5\ \Omega$

For lamp B, $R_B = 110/0.9 = 122.2\ \Omega$

When the lamps are connected in series, total resistance is

$$R_S = 137.5 + 122.2 = 259.7\ \Omega$$

∴ Circuit current, $I = V/R_S = 220/259.7 =$ **0.847 A**

Voltage across lamp A $= I\,R_A = 0.847 \times 137.5 =$ **116.5 V**

Voltage across lamp B $= I\,R_B = 0.847 \times 122.2 =$ **103.5 V**

Example 2.2. *A 100 watt, 250 V lamp is connected in series with a 100 watt, 200 V lamp across 250 V supply. Calculate (i) circuit current and (ii) voltage across each lamp. Assume the lamp resistances to remain unaltered.*

Solution. **(*i*)** Resistance, $R = \dfrac{V^2}{P}$

Resistance of 100 watt, 250 V lamp, $R_1 = (250)^2/100 = 625\ \Omega$

Resistance of 100 watt, 200 V lamp, $R_2 = (200)^2/100 = 400\ \Omega$

When the lamps are connected in series, total resistance is

$$R_S = 625 + 400 = 1025\ \Omega$$

∴ Circuit current, $I = V/R_S = 250/1025 =$ **0.244 A**

(*ii*) Voltage across 100 W, 250 V lamp $= I\,R_1 = 0.244 \times 625 =$ **152.5 V**

Voltage across 100 W, 200 V lamp $= I\,R_2 = 0.244 \times 400 =$ **97.6 V**

Example 2.3. *The element of 500 watt electric iron is designed for use on a 200 V supply. What value of resistance is needed to be connected in series in order that the iron can be operated from 240 V supply?*

Solution. Current rating of iron, $I = \dfrac{\text{Wattage}}{\text{Voltage}} = \dfrac{500}{200} = 2.5$ A

If R ohms is the required value of resistance to be connected in series, then voltage to be dropped across this resistance = 240 − 200 = 40 V.

∴ $R = 40 / 2.5 =$ **16 Ω**

Example 2.4. *Determine the resistance and the power dissipation of a resistor that must be placed in series with a 75 - ohm resistor across 120 V source in order to limit the power dissipation in the 75 - ohm resistor to 90 watts.*

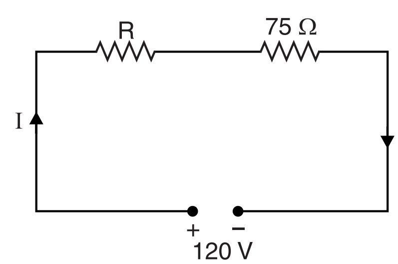


Fig. 2.3

Solution. Fig. 2.3 represents the conditions of the problem.

$$I^2 \times 75 = 90$$

∴ $I = \sqrt{90/75} = 1.095$ A

Now, $I = \dfrac{120}{R + 75}$

or $1.095 = \dfrac{120}{R + 75}$

∴ $R =$ **34.6 Ω**

Power dissipation in $R = I^2 R = (1.095)^2 \times 34.6 =$ **41.5 watts**

Example 2.5. *A generator of e.m.f. E volts and internal resistance r ohms supplies current to a water heater. Calculate the resistance R of the heater so that three-quarter of the total energy developed by the generator is absorbed by the water.*

Solution. Current supplied by generator, $I = \dfrac{E}{R+r}$

Power developed by generator $= E\,I = \dfrac{E^2}{R+r}$

Power dissipated by heater $= I^2R = R \times \dfrac{E^2}{(R+r)^2} = \dfrac{E^2 R}{(R+r)^2}$

As per the conditions of the problem, we have,

$$\frac{E^2 R}{(R+r)^2} = \frac{3}{4} \times \frac{E^2}{R+r} \quad \text{or} \quad \frac{R}{R+r} = \frac{4}{3} \quad \therefore \quad R = \mathbf{3\,r}$$

Example 2.6. *A direct current arc has a voltage/current relation expressed as :*

$$V = 44 + \frac{30}{I} \text{ volts}$$

It is connected in series with a resistor across 100 V supply. If voltages across the arc and resistor are equal, find the ohmic value of the resistor.

Solution. Let R ohms be resistance of the resistor. The voltage across the arc as well as resistor = 50 volts.

Now $\quad 50 = 44 + \dfrac{30}{I} \quad \therefore \quad I = 5$ A

$\therefore \quad R = \dfrac{V}{I} = \dfrac{50}{5} = \mathbf{10\ \Omega}$

Tutorial Problems

1. If the resistance of a circuit having 12 V source is increased by 4 Ω, the current drops by 0.5 A. What is the original resistance of the circuit ? **[8 Ω]**
2. A searchlight takes 100 A at 80 V. It is to be operated from a 220 V supply. Find the value of the resistor to be connected in series. **[1.4 Ω]**
3. The maximum resistance of a rheostat is 4.8 Ω and the minimum resistance is 0.5 Ω. Find for each condition the voltage across the rheostat when current is 1.2 A. **[5.76V ; 0.6V]**
4. What is the drop across the 150 Ω resistor in Fig. 2.4 ? **[5.33 V]**

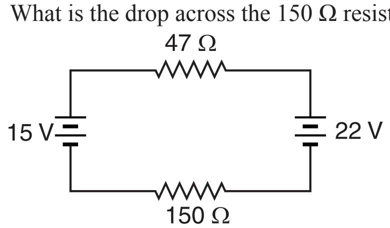


Fig. 2.4

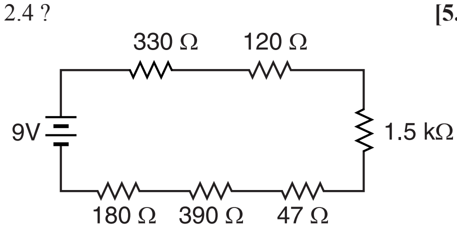


Fig. 2.5

5. Calculate the current flow for Fig. 2.5. **[3.51 mA]**

2.3. D.C. Parallel Circuit

When one end of each resistance is joined to a common point and the other end of each resistance is joined to another common point so that there are as many paths for current flow as the number of resistances, it is called a **parallel circuit.**

Consider three resistances R_1, R_2 and R_3 ohms connected in parallel across a battery of V volts as shown in Fig. 2.6 (*i*). The total current I divides into three parts : I_1 flowing through R_1, I_2 flowing through R_2 and I_3 flowing through R_3. Obviously, the voltage across each resistance is the same (*i.e.* V volts in this case) and there are as many current paths as the number of resistances. By Ohm's law, current through each resistance is

$$I_1 = V/R_1 \;;\; I_2 = V/R_2 \;;\; I_3 = V/R_3$$

Now,
$$I = I_1 + I_2 + I_3$$
$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$
$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

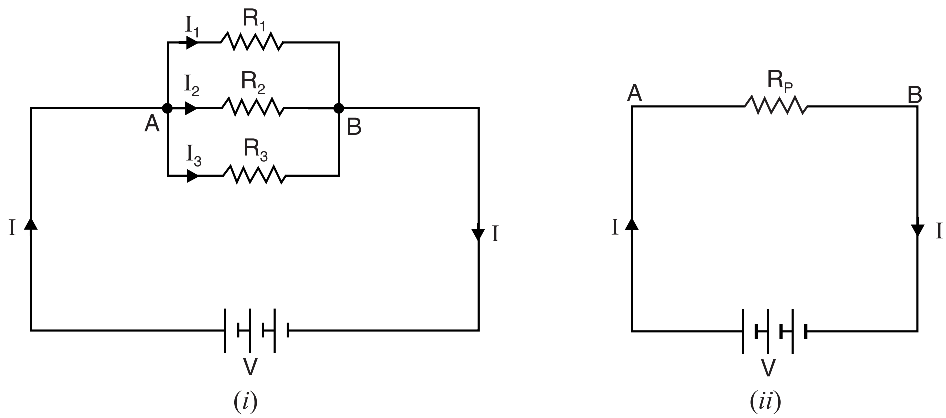


Fig. 2.6

or
$$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

But V/I is equivalent resistance R_P of the parallel resistances [See Fig. 2.6 (*ii*)] so that $I/V = 1/R_P$.

∴
$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Hence when a number of resistances are connected in parallel, the reciprocal of total resistance is equal to the sum of the reciprocals of the individual resistances.

Also
$$G_P = G_1 + G_2 + G_3$$

Hence total conductance G_P of resistors in parallel is equal to the sum of their individual conductances.

We can also express currents I_1, I_2 and I_3 in terms of conductances.

$$I_1 = \frac{V}{R_1} = VG_1 = \frac{I}{G_P}G_1 = I \times \frac{G_1}{G_P} = I \times \frac{G_1}{G_1 + G_2 + G_3}$$

Similarly,
$$I_2 = I \times \frac{G_2}{G_1 + G_2 + G_3} \;;\; I_3 = I \times \frac{G_3}{G_1 + G_2 + G_3}$$

2.4. Main Features of Parallel Circuits

The following are the characteristics of a parallel circuit :

(*i*) The voltage across each resistor is the same.

(*ii*) The current through any resistor is inversely proportional to its resistance.

(*iii*) The total current in the circuit is equal to the sum of currents in its parallel branches.

(*iv*) The reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.

(*v*) As the number of parallel branches is increased, the total resistance of the circuit is decreased.

(*vi*) The total resistance of the circuit is always less than the smallest of the resistances.

(*vii*) If n resistors, each of resistance R, are connected in parallel, then total resistance $R_P = R/n$.

(*viii*) The conductances are additive.

(*ix*) The total power dissipated in the circuit is equal to the sum of powers dissipated in the individual resistances. Thus referring to Fig. 2.6 (*i*),

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

or $$\frac{V^2}{R_P} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3}$$

or $$P_P = P_1 + P_2 + P_3$$

Like a series circuit, *the total power dissipated in a parallel circuit is equal to the sum of powers dissipated in the individual resistances.*

Note. A parallel resistor circuit [See Fig. 2.6 (*i*)] can be considered to be a *current divider circuit* because the current through any one resistor is a fraction of the total circuit current; the fraction depending on the values of the resistors.

2.5. Two Resistances in Parallel

A frequent special case of parallel resistors is a circuit that contains two resistances in parallel. Fig. 2.7 shows two resistances R_1 and R_2 connected in parallel across a battery of V volts. The total current I divides into two parts ; I_1 flowing through R_1 and I_2 flowing through R_2.

(*i*) Total resistance R_P. $$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

$\therefore$ $$R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{\text{Product}}{\text{Sum}}$$

Hence the total value of two resistors connected in parallel is equal to the product divided by the sum of the two resistors.

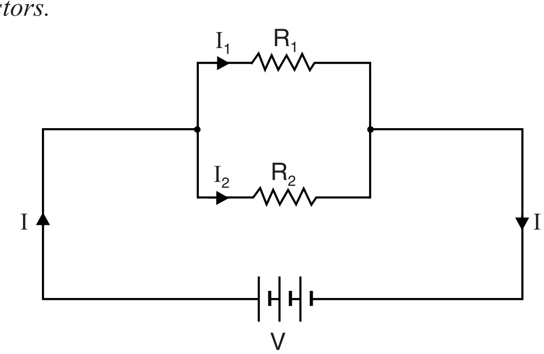


Fig. 2.7

(*ii*) Branch Currents. $$R_P = \frac{R_1 R_2}{R_1 + R_2}$$

$$V = I R_P = I \frac{R_1 R_2}{R_1 + R_2}$$

Current through R_1, $$I_1 = \frac{V}{R_1} = I \frac{R_2}{R_1 + R_2} \qquad \left[\text{Putting } V = I \frac{R_1 R_2}{R_1 + R_2}\right]$$

Current through R_2, $I_2 = \dfrac{V}{R_2} = I\dfrac{R_1}{R_1 + R_2}$

Hence in a parallel circuit of two resistors, the current in one resistor is the line current (i.e. total current) times the opposite resistor divided by the sum of the two resistors.

We can also express currents in terms of conductances.

$$G_P = G_1 + G_2$$

$$I_1 = \frac{V}{R_1} = VG_1 = \frac{I}{G_P} \times G_1 = I \times \frac{G_1}{G_P} = I \times \frac{G_1}{G_1 + G_2}$$

$$I_2 = \frac{V}{R_2} = VG_2 = \frac{I}{G_P} \times G_2 = I \times \frac{G_2}{G_P} = I \times \frac{G_2}{G_1 + G_2}$$

Note. When two resistances are connected in parallel and one resistance is much greater than the other, then the total resistance of the combination is very nearly equal to the smaller of the two resistances. For example, if $R_1 = 10\ \Omega$ and $R_2 = 10\ \text{k}\Omega$ and they are connected in parallel, then total resistance R_P of the combination is given by ;

$$R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 10^4}{10 + 10^4} = \frac{10^5}{10{,}010} = 9.99\ \Omega \simeq R_1$$

In general, if R_2 is 10 times (or more) greater than R_1, then their combined resistance in parallel is nearly equal to R_1.

2.6. Advantages of Parallel Circuits

The most useful property of a parallel circuit is the fact that potential difference has the same value between the terminals of each branch of parallel circuit. This feature of the parallel circuit offers the following advantages :

(*i*) The appliances rated for the same voltage but different powers can be connected in parallel without disturbing each other's performance. Thus a 230 V, 230 W TV receiver can be operated independently in parallel with a 230 V, 40 W lamp.

(*ii*) If a break occurs in any one of the branch circuits, it will have no effect on other branch circuits.

Due to above advantages, electrical appliances in homes are connected in parallel. We can switch on or off any light or appliance without affecting other lights or appliances.

2.7. Applications of Parallel Circuits

Parallel circuits find many applications in electrical and electronic circuits. We shall give two applications by way of illustration.

(*i*) Identical voltage sources may be connected in parallel to provide a greater current capacity. Fig. 2.8 shows two 12 V automobile storage batteries in parallel. If the starter motor draws 400 A at starting, then each battery will supply half the current *i.e.* 200 A. A single battery might not be able to provide a load current of 400 A. Another benefit is that two batteries in parallel will supply a given load current for twice the time when compared to a single battery before discharge is reached.

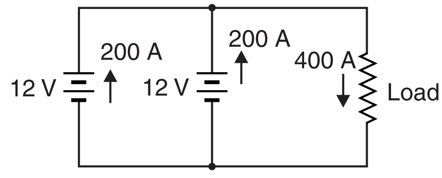


Fig. 2.8

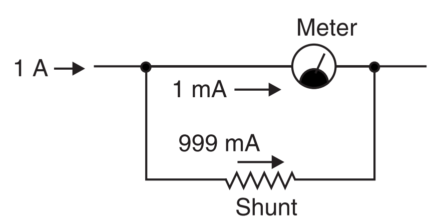


Fig. 2.9

(*ii*) Fig. 2.9 shows another application for parallel connection. A low resistor, called a *shunt*, is connected in parallel with an ammeter to increase the current range of the meter. If shunt is not used, the ammeter is able to measure currents up to 1 mA. However, the use of shunt permits to measure currents up to 1 A. Thus shunt increases the range of the ammeter.

Example 2.7. *Two coils connected in series have a resistance of 18 Ω and when connected in parallel have a resistance of 4 Ω. Find the value of resistances.*

Solution. Let R_1 and R_2 be the resistances of the coils. When resistances are connected in series, $R_S = 18\ \Omega$.

$$\therefore \quad R_1 + R_2 = 18 \quad \text{...}(i)$$

When resistances are connected in parallel, $R_P = 4\ \Omega$.

$$\therefore \quad 4 = \frac{R_1 R_2}{R_1 + R_2} \quad \text{....}(ii)$$

Multiplying Eqns. (*i*) and (*ii*), we get, $R_1 R_2 = 18 \times 4 = 72$

$$\text{Now} \quad R_1 - R_2 = \sqrt{(R_1 + R_2)^2 - 4R_1R_2} = \sqrt{(18)^2 - 4 \times 72}$$

$$\therefore \quad R_1 - R_2 = \pm 6 \quad \text{...}(iii)$$

Solving Eqns. (*i*) and (*iii*), we get, R_1 = **12 Ω** or **6 Ω** ; R_2 = **6 Ω** or **12 Ω**

Example 2.8. *A 100 watt, 250 V lamp is connected in parallel with an unknown resistance R across a 250 V supply. The total power dissipated in the circuit is 1100 watts. Find the value of unknown resistance. Assume the resistance of lamp remains unaltered.*

Solution. The total power dissipated in the circuit is equal to the sum of the powers consumed by the lamp and unknown resistance *R*.

$$\therefore \quad \text{Power consumed by } R = 1100 - 100 = 1000 \text{ watts}$$

$$\therefore \quad \text{Value of resistance, } R = \frac{V^2}{\text{Power consumed}} = \frac{(250)^2}{1000} = \mathbf{62.5\ \Omega}$$

Example 2.9. *A coil has a resistance of 5.2 ohms; the resistance has to be reduced to 5 Ω by connecting a shunt across the coil. If this shunt is made of manganin wire of diameter 0.025 cm, find the length of wire required. Specific resistance for manganin is 47 × 10^{-8} Ω m.*

Solution. Let *R* ohms be the required resistance of the shunt.

$$R_P = \frac{R \times 5.2}{R + 5.2} \quad \text{or} \quad 5 = \frac{5.2R}{R + 5.2} \quad \therefore \quad R = 130\ \Omega$$

$$a = \frac{\pi}{4}\left(0.025 \times 10^{-2}\right)^2 = 490 \times 10^{-10}\ \text{m}^2\ ; \ \rho = 47 \times 10^{-8}\ \Omega\text{-}m$$

$$\text{Now} \quad R = \rho\frac{l}{a}$$

$$\therefore \quad l = \frac{Ra}{\rho} = \frac{130 \times (490 \times 10^{-10})}{47 \times 10^{-8}} = \mathbf{13.55\ m}$$

Example 2.10. *Three equal resistors are connected as shown in Fig 2.10. Find the equivalent resistance between points A and B.*

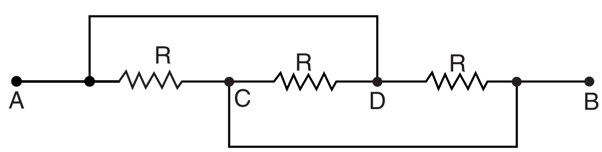


Fig. 2.10

Solution. The reader may observe that one end of each resistor is connected to point *A* and the other end of each resistor is connected to point *B*. Hence the three resistors are in parallel.

$$\therefore \quad \frac{1}{R_{AB}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R} \quad \text{or} \quad R_{AB} = \mathbf{\frac{R}{3}}$$

Example 2.11. *Find the branch currents for Fig. 2.11 using the current divider rule for parallel conductances.*

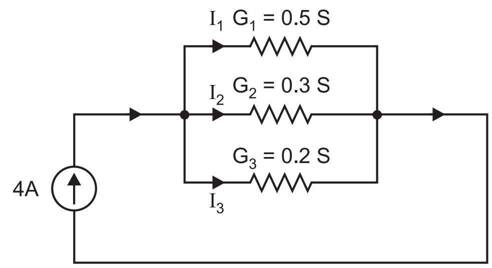


Fig. 2.11

Solution.

$$G_P = G_1 + G_2 + G_3 = 0.5 + 0.3 + 0.2 = 1 \text{ S}$$

$\therefore$

$$I_1 = I\frac{G_1}{G_P} = 4 \times \frac{0.5}{1} = \mathbf{2\ A}$$

$$I_2 = I\frac{G_2}{G_P} = 4 \times \frac{0.3}{1} = \mathbf{1.2\ A}$$

$$I_3 = I\frac{G_3}{G_P} = 4 \times \frac{0.2}{1} = \mathbf{0.8\ A}$$

Example 2.12. *Find the three branch currents in the circuit shown in Fig. 2.12. What is the potential difference between points A and B?*

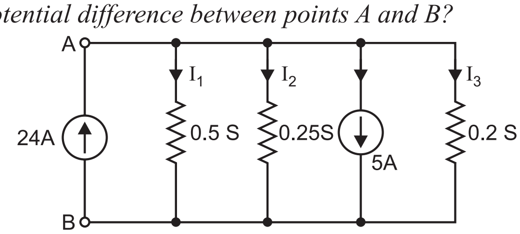


Fig. 2.12

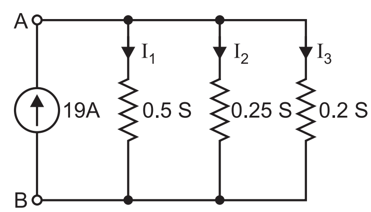


Fig. 2.13

Solution. Current sources in parallel add *algebraically*. Therefore, the two current sources can be combined to give the resultant current source of current $I = 24 - 5 = 19$ A as shown in Fig. 2.13. Referring to Fig. 2.13,

$$G_P = G_1 + G_2 + G_3 = 0.5 + 0.25 + 0.2 = 0.95 \text{ S}$$

$\therefore$

$$I_1 = I \times \frac{G_1}{G_P} = 19 \times \frac{0.5}{0.95} = \mathbf{10\ A}$$

$$I_2 = I \times \frac{G_2}{G_P} = 19 \times \frac{0.25}{0.95} = \mathbf{5\ A}$$

$$I_3 = I \times \frac{G_3}{G_P} = 19 \times \frac{0.2}{0.95} = \mathbf{4\ A}$$

The voltage across each conductance is the same.

$\therefore$

$$V_{AB} = \frac{I_1}{G_1} = \frac{I_2}{G_2} = \frac{I_3}{G_3}$$

or

$$V_{AB} = \frac{I_1}{G_1} = \frac{10 \text{ A}}{0.5 \text{ S}} = \mathbf{20\ V}$$

Example 2.13. *A current of 90 A is shared by three resistances in parallel. The wires are of the same material and have their lengths in the ratio 2 : 3 : 4 and their cross-sectional areas in the ratio 1 : 2 : 3. Determine current in each resistance.*

Solution. Conductance, $G = \sigma\frac{a}{l}$ so that $G \propto \frac{a}{l}$ $(\because \sigma$ is same)

$$\therefore \quad G_1 : G_2 : G_3 :: \frac{a_1}{l_1} : \frac{a_2}{l_2} : \frac{a_3}{l_3}$$

$$\text{or} \quad G_1 : G_2 : G_3 :: \frac{1}{2} : \frac{2}{3} : \frac{3}{4}$$

$$\text{or} \quad G_1 : G_2 : G_3 :: 6 : 8 : 9$$

$$\therefore \quad I_1 = I \times \frac{G_1}{G_1 + G_2 + G_3} = 90 \times \frac{6}{6+8+9} = \mathbf{23.48\ A}$$

$$I_2 = I \times \frac{G_2}{G_1 + G_2 + G_3} = 90 \times \frac{8}{6+8+9} = \mathbf{31.30\ A}$$

$$I_3 = I \times \frac{G_3}{G_1 + G_2 + G_3} = 90 \times \frac{9}{6+8+9} = \mathbf{35.22\ A}$$

Example 2.14. *An aluminium wire 7.5 m long is connected in parallel with a copper wire 6 m long. When a current of 5 A is passed through the combination, it is found that current in the aluminium wire is 3 A. The diameter of aluminium wire is 1 mm. Determine the diameter of copper wire. Resistivity of copper is 0.017* μΩ *m and that of aluminium is 0.028* μΩ *m.*

Solution. Assign suffix A to aluminium and C to copper. Then,

$$I_A = 3\text{ A} \quad \text{and} \quad I_C = 5 - I_A = 5 - 3 = 2\text{ A}$$

In a parallel circuit, the current in any branch is directly proportional to conductance (G) of that branch ($\because I = VG$).

$$\therefore \quad I_A \propto G_A \quad \text{and} \quad I_C \propto G_C$$

$$\therefore \quad \frac{G_C}{G_A} = \frac{I_C}{I_A} = \frac{2}{3}$$

$$\text{Now,} \quad G_C = \frac{a_C}{\rho_C l_C} \quad \text{and} \quad G_A = \frac{a_A}{\rho_A l_A}$$

$$\therefore \quad \frac{G_C}{G_A} = \frac{a_C}{\rho_C l_C} \times \frac{\rho_A l_A}{a_A}$$

$$\text{or} \quad \frac{2}{3} = \frac{a_C}{0.017 \times 6} \times \frac{0.028 \times 7.5}{a_A}$$

$$\text{or} \quad \frac{a_C}{a_A} = \frac{2}{3} \times \frac{0.017 \times 6}{0.028 \times 7.5} = 0.3238$$

$$\therefore \quad a_C = 0.3238 \times a_A = 0.3238 \times \frac{\pi}{4}(1\text{ mm})^2$$

$$\text{or} \quad \frac{\pi}{4}(d_C)^2 = 0.3238 \times \frac{\pi}{4}$$

$$\therefore \quad d_C = \sqrt{0.3238} = \mathbf{0.57\ mm}$$

Example 2.15. *A voltage of 200 V is applied to a tapped resistor of 500* Ω. *Find the resistance between two tapping points connected to a circuit needing 0.1 A at 25 V. Calculate the total power consumed.*

Solution. Fig. 2.14 shows the conditions of the problem.

$$\text{Current in } AB = 0.1 + \frac{25}{R}$$

$$\text{Also current in } AB = \frac{200 - 25}{500 - R} = \frac{175}{500 - R}$$

$$\therefore \quad 0.1+\frac{25}{R} = \frac{175}{500-R}$$

$$\text{or} \quad \frac{0.1R+25}{R} = \frac{175}{500-R}$$

$$\text{or} \quad (500-R)(0.1R+25) = 175\,R$$

$$\text{or} \quad 0.1\,R^2 + 150\,R - 12500 = 0$$

On solving and taking the positive value, R = **79 Ω.**

$$\text{Total current, } I = \text{Current in } AB$$

$$= 0.1+\frac{25}{79} = 0.4165 \text{ A}$$

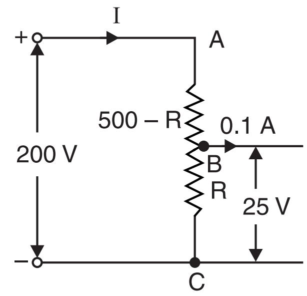


Fig. 2.14

$$\therefore \quad \text{Total power} = 200 \times I = 200 \times 0.4165 = \mathbf{83.5\ W}$$

Example 2.16. *A heater has two similar elements controlled by a 3-heat switch. Draw a connection diagram of each position of the switch. What is the ratio of heat developed for each position of the switch?*

Solution. Fig. 2.15 shows the connections of 3-heat switch controlling two similar elements. Suppose the supply voltage is V.

With points 1 and 3 linked and supply connected across 1 and 3, the two elements will be in parallel.

$$\therefore \quad \text{Power dissipated, } P_1 = \frac{V^2}{R/2} = \frac{2V^2}{R}$$

With voltage across 1 and 2 or 2 and 3, only one element is in the circuit.

$$\therefore \quad \text{Power dissipated, } P_2 = \frac{V^2}{R}$$

With voltage across 1 and 3, the two elements are in series.

$$\therefore \quad \text{Power dissipated, } P_3 = \frac{V^2}{2R}$$

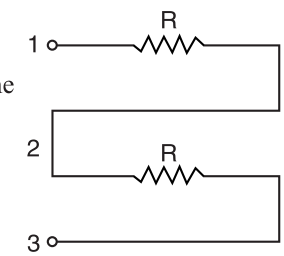


Fig. 2.15

$$\therefore \quad P_1 : P_2 : P_3 = \frac{2V^2}{R} : \frac{V^2}{R} : \frac{V^2}{2R} = 2:1:\frac{1}{2} = \mathbf{4 : 2 : 1}$$

Example 2.17. *The frame of an electric motor is connected to three earthing plates having resistance to earth of 10 Ω, 20 Ω and 30 Ω respectively. Due to a fault, the frame becomes live. What proportion of total fault energy is dissipated at each earth connection ?*

Solution. The three resistances are in parallel. During the fault, suppose voltage to ground is V. Then ratios of energy dissipated are :

$$\frac{V^2}{10} : \frac{V^2}{20} : \frac{V^2}{30} = \frac{1}{10} : \frac{1}{20} : \frac{1}{30} = 6 : 3 : 2$$

$$\text{\% of fault energy dissipated in } 10\ \Omega = \frac{6}{6+3+2} \times 100 = \mathbf{54.5\%}$$

$$\text{\% of fault energy dissipated in } 20\ \Omega = \frac{3}{6+3+2} \times 100 = \mathbf{27.3\%}$$

$$\text{\% of fault energy dissipated in } 30\ \Omega = \frac{2}{6+3+2} \times 100 = \mathbf{18.2\%}$$

Example 2.18. *A 50 Ω resistor is in parallel with 100 Ω resistor. Current in 50 Ω resistor is 7.2 A. How will you add a third resistor and what will be its value if the line current is to be 12.1 A?*

Solution. Source voltage = 50 × 7.2 = 360 V

$$\therefore \quad \text{Current in } 100\ \Omega \text{ resistor} = \frac{360}{100} = 3.6 \text{ A}$$

Total current drawn by 50 Ω and 100 Ω resistors = 7.2 + 3.6 = 10.8 A

In order that line current is 12.1 A, **some resistance *R* must be added in parallel**. The current in R is to be = 12.1 – 10.8 = 1.3 A.

$$\therefore \qquad \text{Value of } R = \frac{360}{1.3} = \mathbf{277\ \Omega}$$

Tutorial Problems

1. Two resistors of 4 Ω and 6 Ω are connected in parallel. If the total current is 30 A, find the current through each resistor. **[18 A ; 12 A]**
2. Four resistors are in parallel. The currents in the first three resistors are 4 mA, 5 mA and 6 mA respectively. The voltage drop across the fourth resistor is 200 volts. The total power dissipated is 5 watts. Determine the values of the resistances of the branches and the total resistance. **[50 k Ω, 40 k Ω, 33.33 k Ω, 8 k Ω, 5 k Ω]**
3. Four resistors of 2 Ω, 3 Ω, 4 Ω and 5 Ω respectively are connected in parallel. What potential difference must be applied to the group in order that total power of 100 watts may be absorbed ? **[8.826 volts]**
4. Three resistors are in parallel. The current in the first resistor is 0.1 A. The power dissipated in the second is 3 watts. The voltage drop across the third is 100 volts. Determine the ohmic values of resistors and the total resistance if total current is 0.2 A. **[1000 Ω, 3333.3 Ω, 1428.5 Ω, 500 Ω]**
5. Two coils each of 250 Ω resistance are connected in series across a constant voltage mains. Calculate the value of resistance to be connected in parallel with one of the coils to reduce the p.d. across its terminals by 1%. **[12,375 Ω]**
6. When a resistor is placed across a 230 volt supply, the current is 12 A. What is the value of resistor that must be placed in parallel to increase the load to16 A ? **[57.5 Ω]**
7. A 50-ohm resistor is in parallel with a 100-ohm resistor. The current in 50 Ω resistor is 7.2 A. What is the value of third resistance to be added in parallel to make the line current 12.1 A ? **[276.9 Ω]**
8. Five equal resistors each of 2 Ω are connected in a network as shown in Fig. 2.16. Find the equivalent resistance between points *A* and *B*. **[2 Ω]**

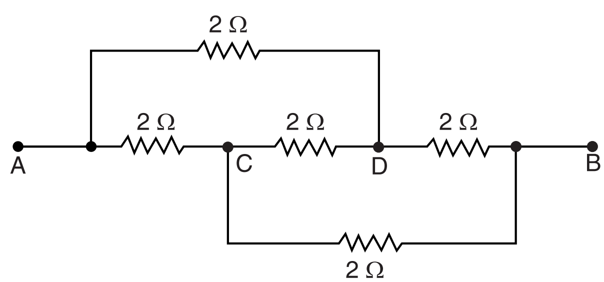


Fig. 2.16

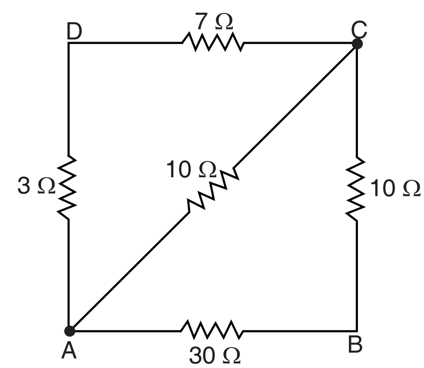


Fig. 2.17

9. Find the equivalent resistance between points *A* and *B* in the circuit shown in Fig. 2.17. **[10 Ω]**
10. Fig. 2.18 shows a 50 V source connected to three resistances : R_1 = 5 kΩ; R_2 = 25 kΩ and R_3 = 10 kΩ. Calculate (*i*) branch currents (*ii*) total current for the given figure.
[(*i*) I_1 = 10 mA ; I_2 = 2 mA; I_3 = 5 mA (*ii*) I = 17 mA]
11. A parallel circuit consists of four parallel-connected 480 Ω resistors in parallel with six 360 Ω resistors. What is the total resistance and total conductance of the circuit? **[40 Ω ; 0.025 S]**

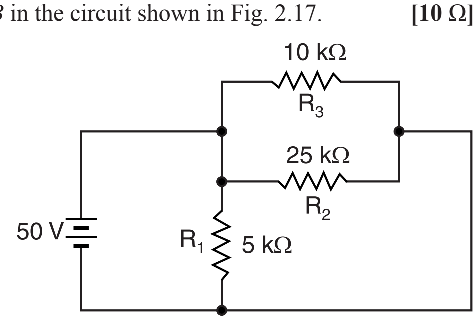


Fig. 2.18

2.8. D.C. Series-Parallel Circuit

As the name suggests, this circuit is a combination of series and parallel circuits. A simple example of such a circuit is illustrated in Fig. 2.19. Note that R_2 and R_3 are connected in parallel with each other and that both together are connected in series with R_1. One simple rule to solve such circuits is to first reduce the parallel branches to an equivalent series branch and then solve the circuit as a simple series circuit.

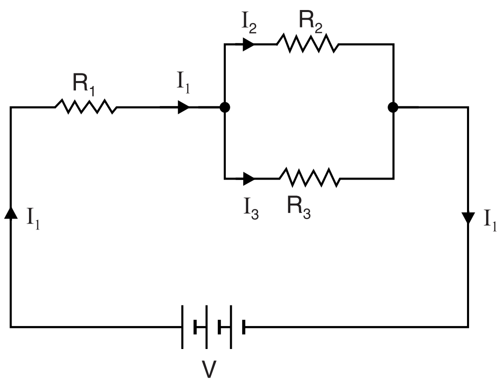


Fig. 2.19

Referring to the series-parallel circuit shown in Fig. 2.19,

$$R_P \text{ for parallel combination } = \frac{R_2 R_3}{R_2 + R_3}$$

$$\text{Total circuit resistance } = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

$$\text{Voltage across parallel combination } = I_1 \times \frac{R_2 R_3}{R_2 + R_3}$$

The reader can now readily find the values of I_1, I_2, I_3.

Like series and parallel circuits, the total power dissipated in the circuit is equal to the sum of powers dissipated in the individual resistances *i.e.*,

$$\text{Total power dissipated, } P = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3$$

2.9. Applications of Series-Parallel Circuits

Series-parallel circuits combine the advantages of both series and parallel circuits and minimise their disadvantages. Generally, less copper is required and a smaller size wire can be used. Such circuits are used whenever various types of circuits must be fed from the same power supply. A few common applications of series-parallel circuits are given below :

(*i*) In an automobile, the starting, lighting and ignition circuits are all individual circuits joined to make a series-parallel circuit drawing its power from one battery.

(*ii*) Radio and television receivers contain a number of separate circuits such as tuning circuits, r.f. amplifiers, oscillator, detector and picture tube circuits. Individually, they may be simple series or parallel circuits. However, when the receiver is considered as a whole, the result is a series-parallel circuit.

(*iii*) Power supplies are connected in series to get a higher voltage and in parallel to get a higher current.

2.10. Internal Resistance of a Supply

All supplies (*e.g.* a cell) must have some internal resistance, however small. This is shown as a series resistor external to the supply. Fig 2.20 shows a cell of *e.m.f.* E volts and internal resistance r. When the cell is delivering no current (*i.e.* on no load), the p.d. across the terminals will be equal to *e.m.f.* E of the cell as shown in Fig. 2.20 (*i*).

When some load resistance R is connected across the terminals of the cell, the current I starts flowing in the circuit. This current causes a voltage drop across internal resistance r of the cell so that terminal voltage V available will be less than E. The relationship between E and V can be easily established [See Fig. 2.20 (*ii*)].

$$I = \frac{E}{R+r}$$

or $$IR = E - Ir$$

But $$IR = V, \text{ the terminal voltage of the cell.}$$

$\therefore$ $$V = E - Ir$$

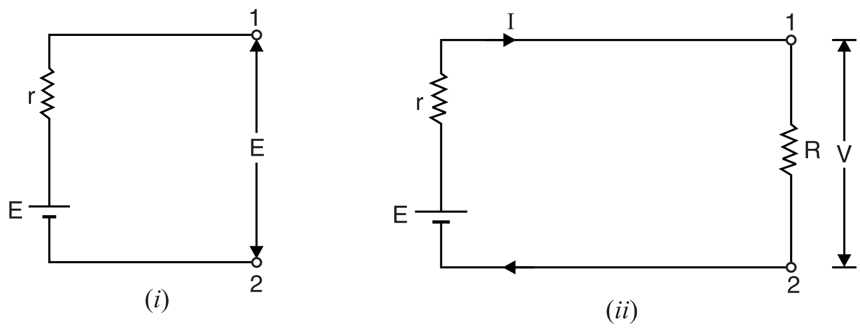


Fig. 2.20

Internal resistance of cell, $r = \dfrac{E-V}{I} = \dfrac{(E-V)}{V}R$ $\qquad \left(\because I = \dfrac{V}{R}\right)$

2.11. Equivalent Resistance

Sometimes we come across a complicated circuit consisting of many resistances. The resistance between the two desired points (or terminals) of such a circuit can be replaced by a single resistance between these points using laws of series and parallel resistances. Then this single resistance is called equivalent resistance of the circuit between these points.

The **equivalent resistance** *of a circuit or network between its any two points* (*or terminals*) *is that single resistance which can replace the entire circuit between these points* (*or terminals*).

Once equivalent resistance is found, we can use Ohm's law to solve the circuit. It is important to note that resistance between two points of a circuit is different for different point-pairs. This is illustrated in Fig. 2.21.

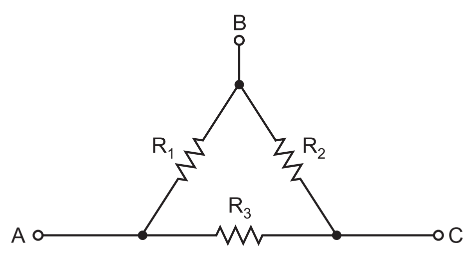


Fig. 2.21

(*i*) Between points A and B, R_1 is in parallel with the series combination of R_2 and R_3 *i.e.*

$$R_{AB} = R_1 \parallel (R_2 + R_3) = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$$

(*ii*) Between points *A* and *C*, R_3 is in parallel with the series combination of R_1 and R_2 *i.e.*

$$R_{AC} = R_3 \parallel (R_1 + R_2) = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

(*iii*) Between points *B* and *C*, R_2 is in parallel with the series combination of R_1 and R_3 *i.e.*

$$R_{BC} = R_2 \parallel (R_1 + R_3) = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3}$$

Example 2.19. *A battery having an e.m.f. of E volts and internal resistance 0.1 Ω is connected across terminals A and B of the circuit shown in Fig. 2.22. Calculate the value of E in order that power dissipated in 2 Ω resistor shall be 2 W.*

Solution. Resistance between *E* and *F* is given by ;

$$\frac{1}{R_{EF}} = \frac{1}{3} + \frac{1}{2} + \frac{1}{6} = \frac{6}{6}$$

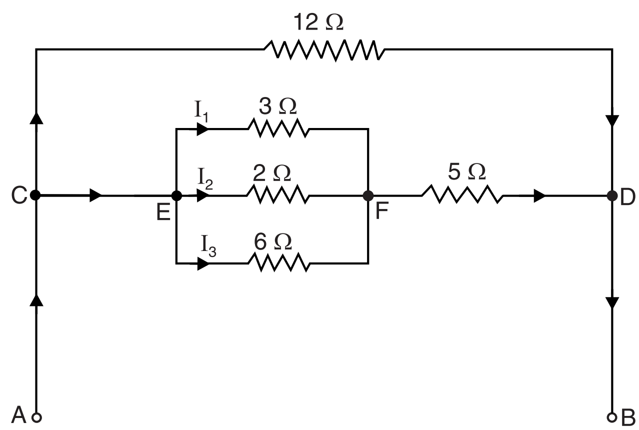


Fig. 2.22

$$\therefore \quad R_{EF} = 6/6 = 1\ \Omega$$

$$\text{Resistance of branch } CEFD = 1 + 5 = 6\ \Omega$$

$$\text{Current through } 2\ \Omega = \sqrt{\frac{\text{Power loss}}{\text{Resistance}}} = \sqrt{\frac{2}{2}} = 1\text{A}$$

$$\text{P.D. across } EF = 1 \times 2 = 2\text{V}$$

$$\text{Current through } 3\ \Omega = 2/3 = 0.67\ \text{A}$$

$$\text{Current through } 6\ \Omega = 2/6 = 0.33\ \text{A}$$

$$\text{Current in branch } CED = 1 + 0.67 + 0.33 = 2\ \text{A}$$

$$\text{P.D. across } CD = 6 \times 2 = 12\ \text{V}$$

$$\text{Current through } 12\ \Omega = 12/12 = 1\ \text{A}$$

$$\text{Current supplied by battery} = 2 + 1 = 3\ \text{A}$$

$$\therefore \quad E = \text{P.D. across } AB \text{ or } CD + \text{Drop in battery resistance}$$

$$= 12 + 0.1 \times 3 = \mathbf{12.3\ V}$$

Example 2.20. *Calculate the values of various currents in the circuit shown in Fig. 2.23. What is total circuit conductance and total resistance?*

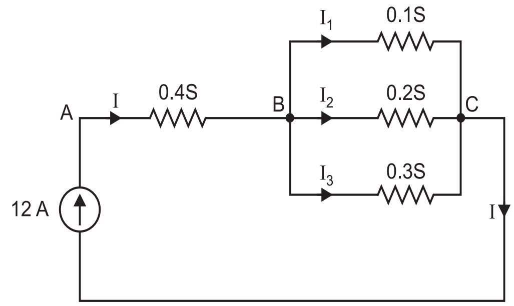


Fig. 2.23

Solution. $I = 12$ A ; $G_{BC} = 0.1 + 0.2 + 0.3 = 0.6$ S

$$\therefore \quad I_1 = I \times \frac{0.1}{G_{BC}} = 12 \times \frac{0.1}{0.6} = \mathbf{2\ A} \ ; \ I_2 = I \times \frac{0.2}{0.6} = 12 \times \frac{0.2}{0.6} = \mathbf{4\ A} \ ;$$

$$I_3 = I \times \frac{0.3}{0.6} = \mathbf{6\ A} \ ; \ I = \mathbf{12\ A}$$

Now, $G_{AB} = 0.4$ S and $G_{BC} = 0.6$ S are in series.

$$\therefore \quad \frac{1}{G_{AC}} = \frac{1}{G_{AB}} + \frac{1}{G_{BC}} = \frac{1}{0.4} + \frac{1}{0.6} = \frac{25}{6} \quad \therefore \quad G_{AC} = \mathbf{\frac{6}{25}\ S}$$

$$\text{Total circuit resistance, } R_{AC} = \frac{1}{G_{AC}} = \frac{1}{6/25} = \mathbf{\frac{25}{6}\ \Omega}$$

Example 2.21. *Six resistors are connected as shown in Fig. 2.24. If a battery having an e.m.f. of 24 volts and internal resistance of 1 Ω is connected to the terminals A and B, find (i) the current from the battery, (ii) p.d. across 8 Ω and 4 Ω resistors and (iii) the current taken from the battery if a conductor of negligible resistance is connected in parallel with 8 Ω resistor.*

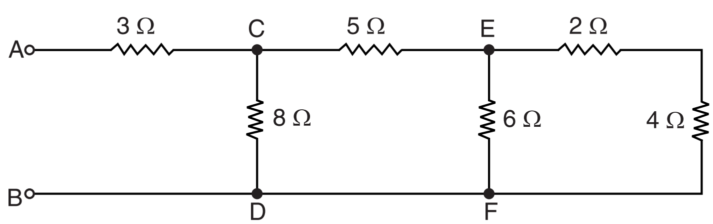


Fig. 2.24

Solution.

Resistance between E and F, $R_{EF} = \dfrac{(4+2)\times 6}{(4+2)+6} = 3\ \Omega$

Resistance between C and D, $R_{CD} = \dfrac{(5+3)\times 8}{(5+3)+8} = 4\ \Omega$

Resistance between A and B, $R_{AB} = 3 + 4 = 7\ \Omega$

Total circuit resistance, $R_T = R_{AB}$ + Supply resistance $= 7 + 1 = 8\ \Omega$

(*i*) Current from battery, $I = E/R_T = 24/8 = \mathbf{3\ A}$

(*ii*) P.D. across 8 Ω $= E - I(3+1) = 24 - 3(4) = \mathbf{12\ V}$

Current through 8 Ω $= 12/8 = 1.5$ A

Current through 5 Ω $= 3 - 1.5 = 1.5$A

P.D. across $EF = 12 - 1.5 \times 5 = 4.5$ V

Current through 6Ω $= 4.5/6 = 0.75$A

$\therefore$ Current through 4 Ω = 1.5 − 0.75 = 0.75 A

$\therefore$ Voltage across 4Ω = 0.75 × 4 = **3 V**

(iii) When a conductor of negligible resistance is connected across 8 Ω, then resistance between *C* and *D* is zero. Therefore, total resistance in the circuit is now 3 Ω resistor in series with 1 Ω internal resistance of battery.

$$\therefore \quad \text{Current from battery} = \frac{24}{3+1} = \mathbf{6\ A}$$

Example 2.22. *Two resistors R_1 = 2500 Ω and R_2 = 4000 Ω are joined in series and connected to a 100 V supply. The voltage drops across R_1 and R_2 are measured successively by a voltmeter having a resistance of 50000 Ω. Find the sum of two readings.*

Solution. When voltmeter is connected across resistor R_1 [See Fig. 2.25 (*i*)], it becomes a series-parallel circuit and total circuit resistance decreases.

$$\text{Total circuit resistance} = 4000 + \frac{2500 \times 50000}{2500 + 50000} = 4000 + 2381 = 6381\ \Omega$$

$$\text{Circuit current, } I = \frac{100}{6381}\text{ A}$$

$$\text{Voltmeter reading, } V_1 = I \times 2381 = \frac{100}{6381} \times 2381 = 37.3\text{ V}$$

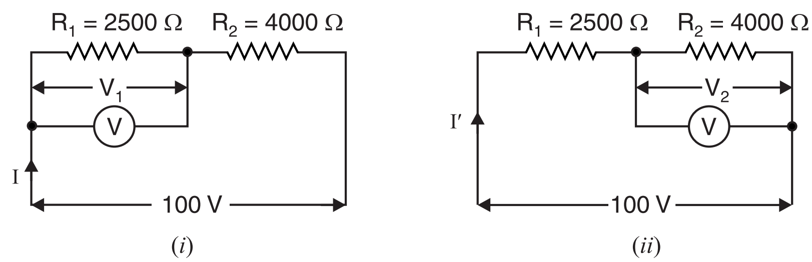


Fig. 2.25

When voltmeter is connected across R_2 [See Fig. 2.25 (*ii*)], it becomes a series-parallel circuit.

$$\text{Total circuit resistance} = 2500 + \frac{4000 \times 50000}{4000 + 50000} = 2500 + 3703.7 = 6203.7\ \Omega$$

$$\text{Circuit current, } I' = \frac{100}{6203.7}\text{ A}$$

$$\text{Voltmeter reading, } V_2 = I' \times 3703.7 = \frac{100}{6203.7} \times 3703.7 = 59.7\text{ V}$$

$$\therefore \quad \text{Sum of two readings} = V_1 + V_2 = 37.3 + 59.7 = \mathbf{97\ V}$$

Example 2.23. *A battery of unknown e.m.f. is connected across resistances as shown in Fig. 2.26. The voltage drop across the 8 Ω resistor is 20 V. What will be the current reading in the ammeter? What is the e.m.f. of the battery?*

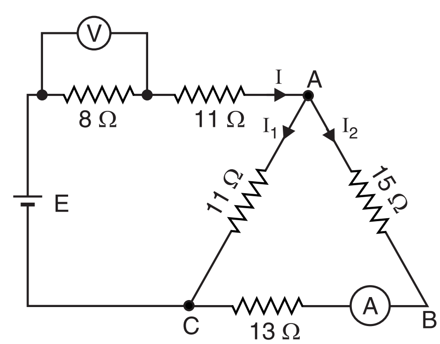


Fig. 2.26

Solution. The current through 8 Ω resistance is I = 20/8 = 2.5 A. At point *A* in Fig. 2.26, the current I is divided into two paths *viz* I_2 flowing in path *ABC* of 15 + 13 = 28 Ω resistance and current I_1 flowing in path *AC* of 11 Ω resistor. By current divider rule, the value of I_2 is given by ;

$$I_2 = I \times \frac{11}{11+28} = 2.5 \times \frac{11}{39} = 0.7 \text{ A}$$

Therefore, ammeter reads **0.7 A**.

Resistance between A and C = (28 × 11)/39 = 308/39 Ω

Total circuit resistance, R_T = 8 + 11 + (308/39) = 1049/39 Ω

$\therefore \quad E = I \times R_T = 2.5 \times (1049/39) =$ **67.3 V**

Example 2.24. *Find the voltage V_{AB} in the circuit shown in Fig. 2.27.*

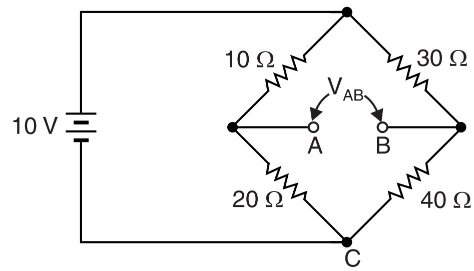


Fig. 2.27

Solution. The resistors 10 Ω and 20 Ω are in series and voltage across this combination is 10 V.

$$\therefore \quad V_{AC} = \frac{20}{10+20} \times 10 = 6.667 \text{ V}$$

The resistors 30 Ω and 40 Ω are in series and voltage across this combination is 10 V.

$$\therefore \quad V_{BC} = \frac{40}{30+40} \times 10 = 5.714 \text{ V}$$

The point A is positive *w.r.t.* point B.

$\therefore \quad V_{AB} = V_{AC} - V_{BC} = 6.667 - 5.714 =$ **0.953 V**

Example 2.25. *A circuit consists of four 100 W lamps connected in parallel across a 230 V supply. Inadvertently, a voltmeter has been connected in series with the lamps. The resistance of the voltmeter is 1500 Ω and that of the lamps under the conditions stated is six times their value then burning normally. What will be the reading of the voltmeter ?*

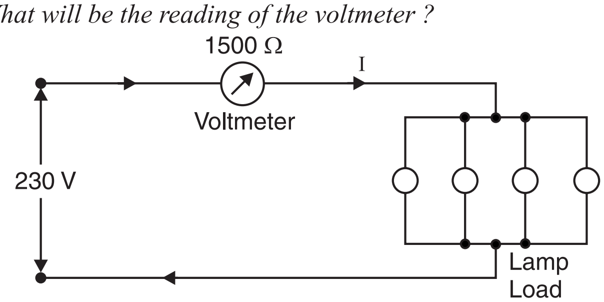


Fig. 2.28

Solution. Fig. 2.28 shows the conditions of the problem. When burning normally, the resistance of each lamp is $R = V^2/P = (230)^2/100 = 529$ Ω. Under the conditions shown in Fig. 2.28, resistance of each lamp = 6 × 529 = 3174 Ω.

∴ Equivalent resistance of 4 lamps under stated conditions is $R_P = 3174/4 = 793$ Ω

Total circuit resistance = $1500 + R_P$

= 1500 + 793.5 = 2293.5 Ω

$\therefore$ Circuit current, $I = \dfrac{230}{2293.5}$ A

$\therefore$ Voltage drop across voltmeter $= I \times 1500 = \dfrac{230}{2293.5} \times 1500 \simeq$ **150 V**

Example 2.26. *Find the current supplied by the d.c. source in the circuit shown in Fig. 2.29.*

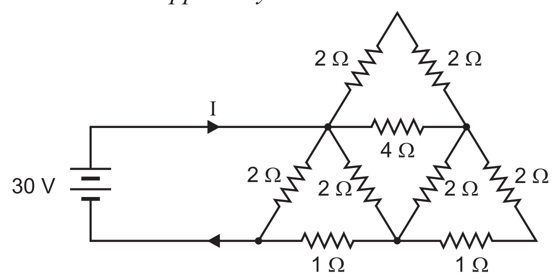


Fig. 2.29

Solution. In the circuit shown in Fig. 2.29, the resistances in series can be combined and the circuit reduces to the one shown in Fig. 2.30 (*i*). In Fig. 2.30 (*i*), the resistances in parallel can be combined using the formula product divided by sum and the circuit reduces to the one shown in Fig. 2.30 (*ii*).

In Fig. 2.30 (*ii*), the resistances in series can be combined and the circuit reduces to the one shown in Fig. 2.30 (*iii*). In Fig. 2.30 (*iii*), 3.2 Ω and 2 Ω are in parallel and their combined resistance is 16/13 Ω. Now 16/13 Ω and 1 Ω are in series and this series combination is in parallel with 2 Ω.

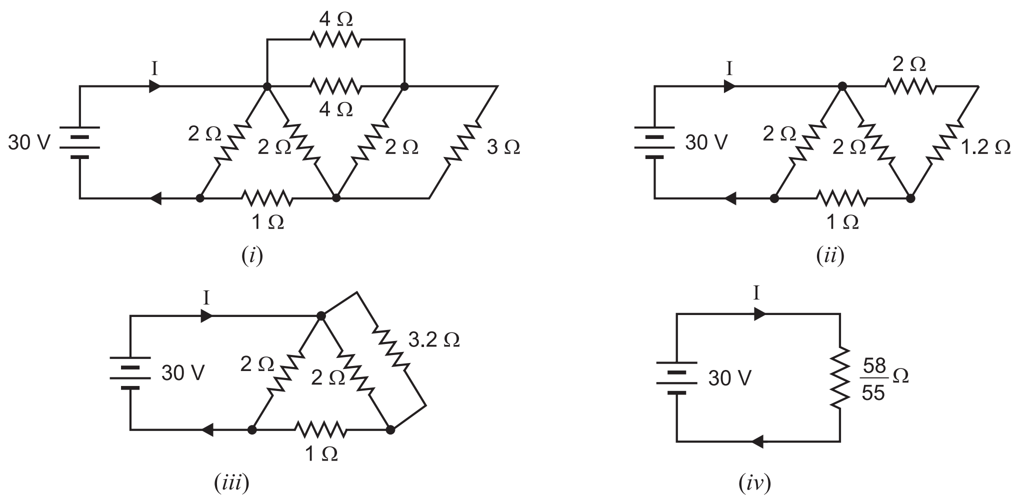


Fig. 2.30

$\therefore$ Effective resistance of the circuit is

$$R_{eff} = \left(\frac{16}{13}+1\right)\Omega \,||\, 2\,\Omega = \frac{58}{55}\,\Omega \qquad \text{[See Fig. 2.30 (iv)]}$$

$\therefore$ Current supplied by source $= \dfrac{30}{R_{eff}} = \dfrac{30}{58/55} =$ **28.45 A**

Example 2.27. *Determine the current drawn by a 12 V battery with internal resistance 0.5 Ω by the following infinite network (See Fig. 2.31).*

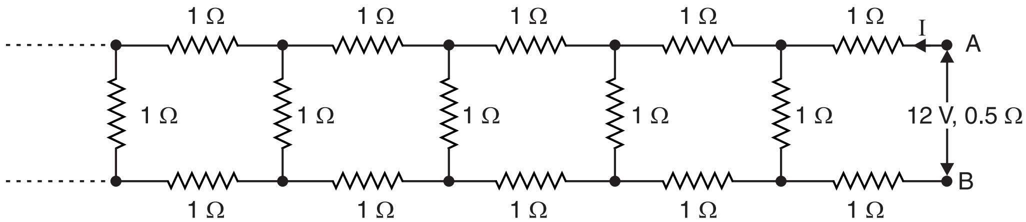


Fig. 2.31

Solution. Let x be the equivalent resistance of the network. Since the network is infinite, the addition of one set of three resistances, each of 1 Ω, will not change the total resistance, *i.e.*, it will remain x. The network would then become as shown in Fig. 2.32. The resistances x and 1 Ω are in parallel and their total resistance is R_P given by ;

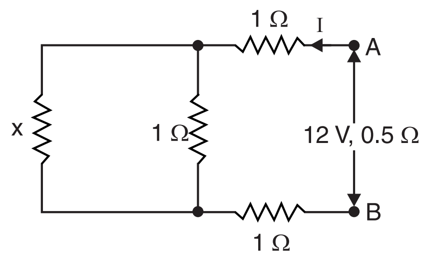


Fig. 2.32

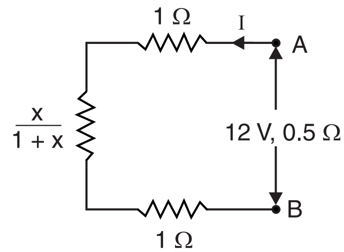


Fig. 2.33

$$R_P = \frac{x \times 1}{x+1} = \frac{x}{1+x}$$

The circuit then reduces to the one shown in Fig. 2.33. Referring to Fig. 2.33,

$$\text{Total resistance of the network} = 1 + 1 + \frac{x}{1+x} = 2 + \frac{x}{1+x}$$

But total resistance of the network is x as mentioned above.

$$\therefore \quad x = 2 + \frac{x}{1+x}$$

$$\text{or} \quad x + x^2 = 2 + 2x + x$$

$$\text{or} \quad x^2 - 2x - 2 = 0$$

$$\therefore \quad x = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$$

$$\text{or} \quad x = 1 \pm \sqrt{3}$$

As the value of the resistance cannot be negative,

$$\therefore \quad x = 1 + \sqrt{3} = 1 + 1.732 = 2.732\ \Omega$$

$$\text{Total circuit resistance, } R_T = x + \text{internal resistance of the supply}$$

$$= 2.732 + 0.5 = 3.232\ \Omega$$

∴ Current drawn by the network is

$$I = \frac{E}{R_T} = \frac{12}{3.232} = \mathbf{3.71\ A}$$

Example 2.28. *Find R_{AB} in the circuit shown in Fig. 2.34.*

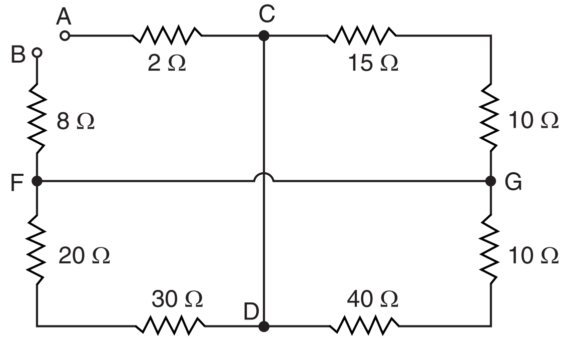


Fig. 2.34

Solution. The circuit shown in Fig. 2.34 reduces to the one shown in Fig. 2.35 (*i*). This circuit further reduces to the circuit shown in Fig. 2.35 (*ii*).

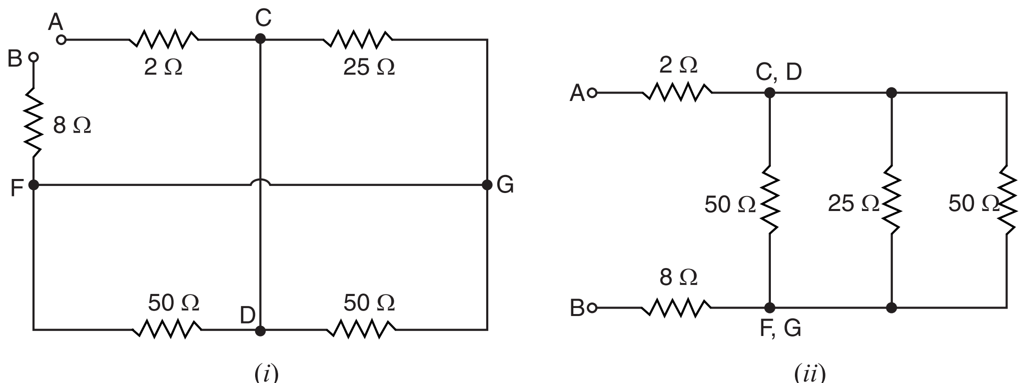


Fig. 2.35

Referring to Fig. 2.35 (*ii*), we have,

$$
\begin{aligned}
R_{AB} &= 2 + (50 \parallel 25 \parallel 50) + 8 \\
&= 2 + (25 \parallel 25) + 8 \\
&= 2 + 12.5 + 8 = \mathbf{22.5\ \Omega}
\end{aligned}
$$

Example 2.29. *What is the equivalent resistance between the terminals A and B in Fig. 2.36?*

Solution. The network shown in Fig. 2.36 can be redrawn as shown in Fig. 2.37 (*i*). It is a balanced Wheatstone bridge. Therefore, points *C* and *D* are at the same potential. Since no current flows in the branch *CD*, this branch is ineffective in determining the equivalent resistance between terminals *A* and *B* and can be removed. The circuit then reduces to that shown in Fig. 2.37 (*ii*).

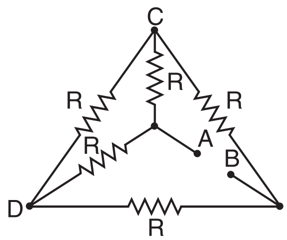


Fig. 2.36

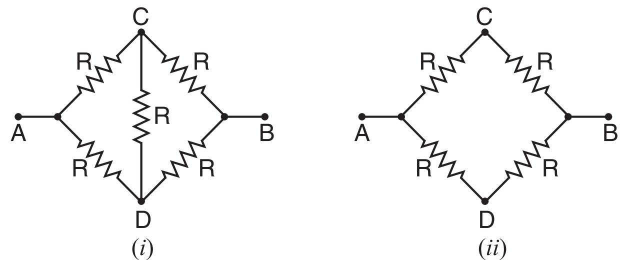


Fig. 2.37

The branch *ACB* ($= R + R = 2R$) is in parallel with branch *ADB* ($= R + R = 2R$).

$$\therefore \qquad R_{AB} = \frac{(2R)\times(2R)}{2R+2R} = \mathbf{R}$$

Example 2.30. *An electrical network is arranged as shown in Fig. 2.38. Find the value of current in the branch AF.*

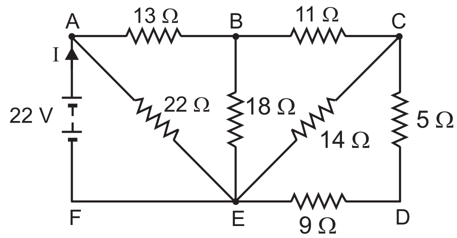


Fig. 2.38

Solution. Resistance between E and C,

$$R_{EC} = \frac{(5+9)\times 14}{(5+9)+14} = 7\ \Omega$$

Resistance between B and E,

$$R_{BE} = \frac{(11+7)\times 18}{(11+7)+18} = 9\ \Omega$$

Resistance between A and E,

$$R_{AE} = \frac{(13+9)\times 22}{(13+9)+22} = 11\ \Omega$$

i.e.,Total circuit resistance, $R_T = 11\ \Omega$

$\therefore$ Current in branch AF, $I = V/R_T = 22/11 =$ **2 A**

Example 2.31. *A resistor of 5 Ω is connected in series with a parallel combination of a number of resistors each of 5 Ω. If the total resistance of the combination is 6 Ω, how many resistors are in parallel?*

Solution. Let n be the required number of 5 Ω resistors to be connected in parallel. The equivalent resistance of this parallel combination is

$$\frac{1}{R_P} = \frac{1}{5}+\frac{1}{5}+\frac{1}{5}\ldots n \text{ times} = \frac{n}{5}$$

Therefore, $R_P = 5/n$

Now R_P (= $5/n$) in series with 5 Ω is equal to 6 Ω *i.e.*,

$$\frac{5}{n}+5 = 6 \quad \therefore \quad n = \mathbf{5}$$

Example 2.32. *A letter A consists of a uniform wire of resistance 1 Ω per cm. The sides of the letter are each 20 cm long and the cross-piece in the middle is 10 cm long while the apex angle is 60°. Find the resistance of the letter between the two ends of the legs.*

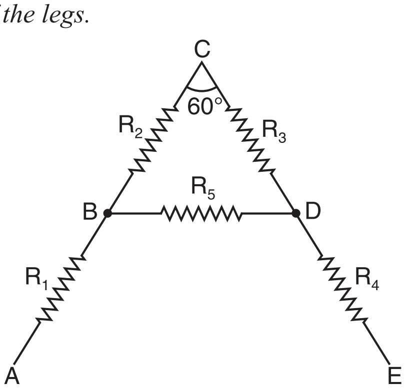


Fig. 2.39

Solution. Fig. 2.39 shows the conditions of the problem. Point B is the mid-point of AC, point D is the mid-point of EC and BD = 10 cm.

$\therefore \quad AB = BC = CD = DE = BD = 10$ cm

or $\quad R_1 = R_2 = R_3 = R_4 = R_5 = 10\ \Omega (\because 1 \text{ cm} = 1\ \Omega)$

Now R_2 and R_3 are in series and their total resistance = 10 + 10 = 20 Ω. This 20 Ω resistance is in parallel with R_5.

$$\therefore \quad R_{BD} = 20\ \Omega \parallel R_5 = 20\ \Omega \parallel 10\ \Omega$$

$$= \frac{20\times 10}{20+10} = \frac{20}{3}\ \Omega$$

Now, R_1, R_{BD} and R_4 are in series so that :

$$R_{AE} = R_1 + R_{BD} + R_4$$

$$= 10 + \frac{20}{3} + 10 = \mathbf{26.67\ \Omega}$$

Example 2.33. *All the resistances in Fig. 2.40 are in ohms. Find the effective resistance between the points A and B.*

Solution. Resistance between points A and D is

$$R_{AD} = (3 + 3)\ \Omega \parallel 6\ \Omega = \frac{6\times 6}{6+6} = 3\ \Omega$$

$$R_{AE} = (R_{AD} + 3)\ \Omega \parallel 6\ \Omega = \frac{6\times 6}{6+6} = 3\ \Omega$$

$$R_{AF} = (R_{AE} + 3)\ \Omega \parallel 6\ \Omega = \frac{6\times 6}{6+6} = 3\ \Omega$$

$\therefore$ Resistance between points A and B is

$$R_{AB} = (R_{AF} + 3)\ \Omega \parallel 3\ \Omega$$

$$= \frac{6\times 3}{6+3} = \frac{18}{9} = \mathbf{2\ \Omega}$$

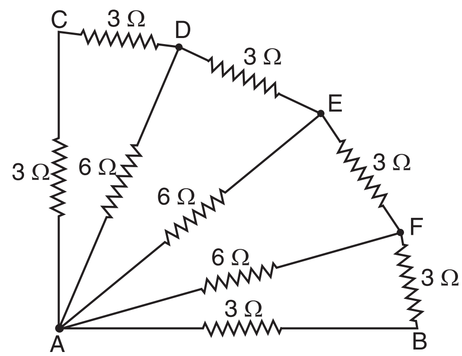


Fig. 2.40

Example 2.34. *What is the equivalent resistance of the ladder network shown in Fig. 2.41?*

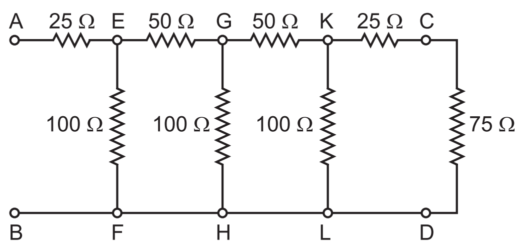


Fig. 2.41

Solution. Referring to Fig. 2.41, the resistance between points K and L is

$$R_{KL} = (25 + 75)\ \Omega \parallel 100\ \Omega = \frac{100\times 100}{100+100} = 50\ \Omega$$

The circuit of Fig. 2.41 then reduces to the one shown in Fig. 2.42 (*i*). Referring to Fig. 2.42 (*i*),

$$R_{GH} = (50 + 50)\ \Omega \parallel 100\ \Omega = \frac{100\times 100}{100+100} = 50\ \Omega$$

The circuit of Fig. 2.42 (*i*) then reduces to the one shown in Fig. 2.42 (*ii*). Referring to Fig. 2.42 (*ii*),

$$R_{EF} = (50 + 50)\ \Omega \parallel 100\ \Omega = \frac{100\times 100}{100+100} = 50\ \Omega$$

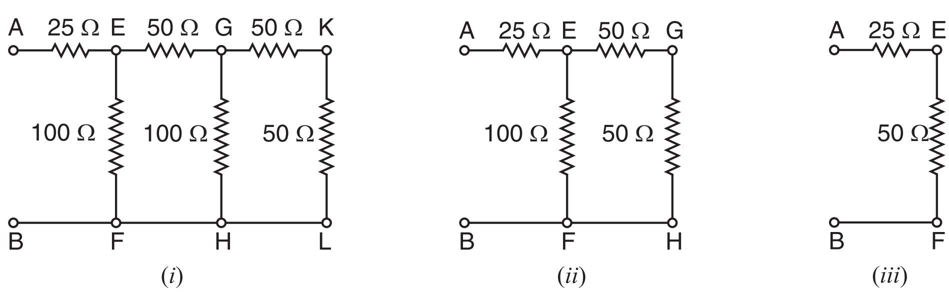


Fig. 2.42

The circuit of Fig. 2.42 (*ii*) then reduces to the one shown in Fig. 2.42 (*iii*). Referring to Fig. 2.42 (*iii*),

Equivalent resistance of the ladder network

$$= 25 + 50 = \mathbf{75\ \Omega}$$

Tutorial Problems

1. A resistor of 3.6 Ω is connected in series with another of 4.56 Ω. What resistance must be placed across 3.6 Ω so that the total resistance of the circuit shall be 6 Ω ? **[2.4 Ω]**
2. A circuit consists of three resistors of 3 Ω, 4 Ω and 6 Ω in parallel and a fourth resistor of 4 Ω in series. A battery of e.m.f. 12 V and internal resistance 6 Ω is connected across the circuit. Find the total current in the circuit and terminal voltage across the battery. **[1.059 A, 5.65 V]**
3. A resistance *R* is connected in series with a parallel circuit comprising two resistors of 12 Ω and 8 Ω respectively. The total power dissipated in the circuit is 70 W when the applied voltage is 22 volts. Calculate the value of *R*. **[0.91 Ω]**
4. Two resistors R_1 and R_2 of 12 Ω and 6 Ω are connected in parallel and this combination is connected in series with a 6.25 Ω resistance R_3 and a battery which has an internal resistance of 0.25 Ω. Determine the e.m.f. of the battery. **[12.6 V]**
5. Find the voltage across and current through 4 kΩ resistor in the circuit shown in Fig. 2.43. **[4 V ; 1 mA]**

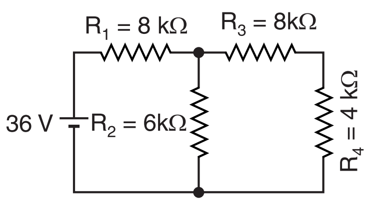


Fig. 2.43

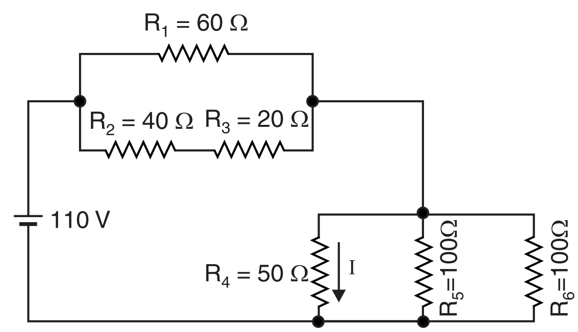


Fig. 2.44

6. Find the current *I* in the 50 Ω resistor in the circuit shown in Fig. 2.44. **[1 A]**
7. Find the current in the 1 kΩ resistor in Fig. 2.45. **[6.72 mA]**

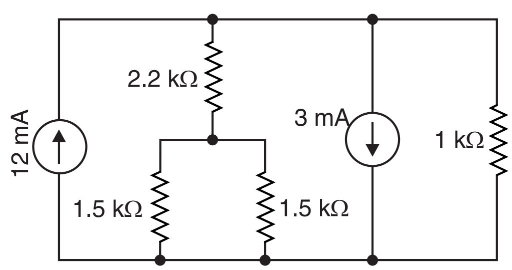


Fig. 2.45

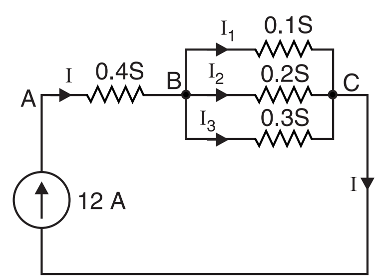


Fig. 2.46

8. Calculate the value of different currents for the circuit shown in Fig. 2.46. What is the total circuit conductance and resistance ? **[$I = 12$ A ; $I_1 = 2$ A ; $I_2 = 4$ A ; $I_3 = 6$ A ; $G_{AC} = 6/25$ S ; $R_{AC} = 25/6$ Ω]**
9. For the parallel circuit of Fig. 2.47, calculate (*i*) *V* (*ii*) I_1 (*iii*) I_2. **[(*i*) 20 V (*ii*) 5 A (*iii*) –5 A]**

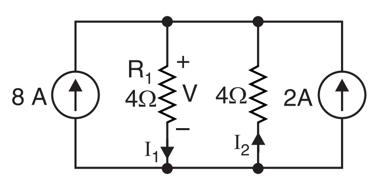


Fig. 2.47

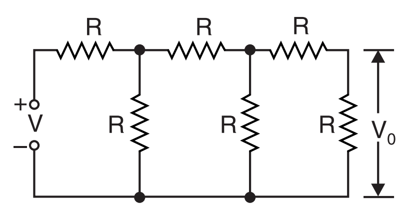


Fig. 2.48

10. Prove that output voltage V_0 in the circuit of Fig. 2.48 is $V/13$.

11. Find the current I supplied by the 50 V source in Fig. 2.49. **[I = 13.7 A]**

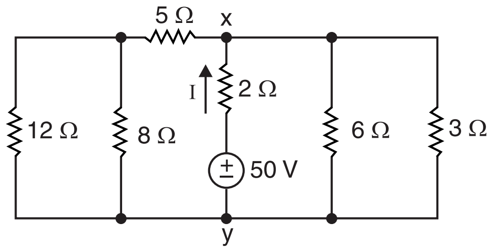


Fig. 2.49

12. An electric heating pad rated at 110 V and 55 W is to be used at a 220 V source. It is proposed to connect the heating pad in series with a series-parallel combination of light bulbs, each rated at 100 V ; bulbs are having ratings of 25 W, 60 W, 75 W and 100 W. Obtain a possible scheme of the pad-bulbs combination. At what rate will heat be produced by the pad with this modification ?

[100 W bulb in series with a parallel combination of two 60 W bulbs ; 54.54 W]

2.12. Open Circuits

As the name implies, an *open* is a gap or break or interruption in a circuit path.

When there is a break in any part of a circuit, that part is said to be **open-circuited.**

No current can flow through an open. Since no current can flow through an open, according to Ohm's law, an open has infinite resistance ($R = V/I = V/0 = \infty$). An open circuit may be as a result of component failure or disintegration of a conducting path such as the breaking of a wire.

1. Open circuit in a series circuit. Fig. 2.50 shows an open circuit fault in a series circuit. Here resistor R_4 is burnt out and an open develops. Because of the open, no current can flow in the circuit.

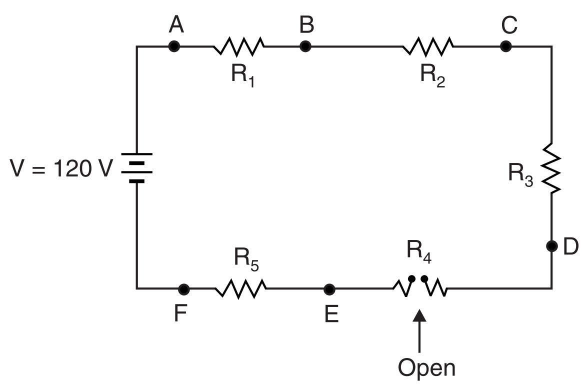


Fig. 2.50

When an open occurs in a series circuit, the following symptoms can be observed :

(*i*) The circuit current becomes zero.

(*ii*) There will be no voltage drop across the resistors that are normal.

(*iii*) *The entire voltage drop appears across the open.* This can be readily proved. Applying Kirchhoff's voltage law to the loop *ABCDEFA*, we have,

$$-0 \times R_1 - 0 \times R_2 - 0 \times R_3 - V_{DE} - 0 \times R_5 + 120 = 0$$

$$\therefore \quad V_{DE} = 120 \text{ V}$$

(*iv*) Since the circuit current is zero, there is no voltage drop in the internal resistance of the source. Therefore, terminal voltage may appear higher than the normal.

2. Open circuit in a parallel circuit. One or more branches of a parallel circuit may develop an open. Fig. 2.51 shows a parallel circuit with an open. Here resistor R_3 is burnt out and now has infinite resistance.

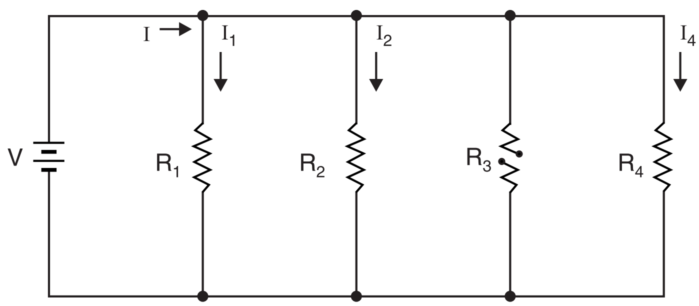


Fig. 2.51

The following symptoms can be observed :

(*i*) Branch current I_3 will be zero because R_3 is open.

(*ii*) The total current I will be less than the normal.

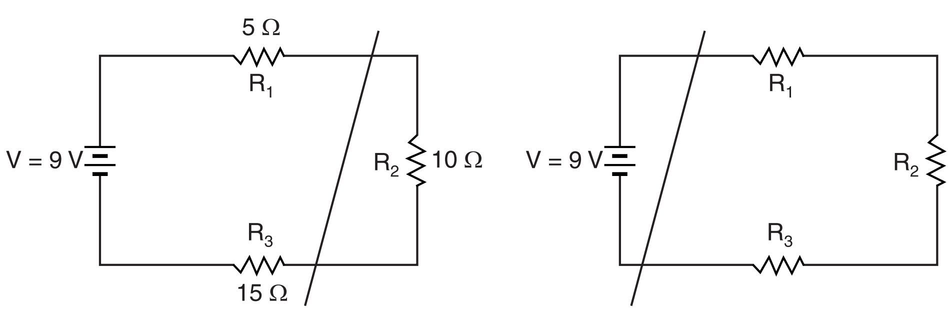
(*iii*) The operation of the branches without opens will be normal.

(*iv*) The open device will not operate. If R_3 is a lamp, it will be out. If it is a motor, it will not run.

2.13. Short Circuits

A short circuit or short is a path of low resistance. *A* **short circuit** *is an unwanted path of low resistance.* When a short circuit occurs, the resistance of the circuit becomes low. As a result, current greater than the normal flows which can cause damage to circuit components. The short circuit may be due to insulation failure, components get shorted etc.

1. Partial short in a series circuit. Fig. 2.52 (*i*) shows a **series circuit** with a **partial short.** An unwanted path has connected R_1 to R_3 and has eliminated R_2 from the circuit. Therefore, the circuit resistance decreases and the circuit current becomes greater than normal. The voltage drop across components that are not shorted will be higher than normal. Since current is increased, the power dissipation in the components that are not shorted will be greater than the normal. A partial short may cause healthy component to burn out due to abnormally high dissipation.



Partial short
(*i*)

Dead short
(*ii*)

Fig. 2.52

2. Dead short in a series circuit. Fig. 2.52 (*ii*) shows a **series circuit** with a **dead short.** Here all the loads (*i.e.* resistors in this case) have been removed by the unwanted path. Therefore, the circuit resistance is almost zero and the circuit current becomes extremely high. If there are no protective devices (fuse, circuit breaker etc.) in the circuit, drastic results (smoke, fire, explosion etc.) may occur.

3. Partial short in a parallel circuit. Fig. 2.53 (*i*) shows a **parallel circuit** with a **partial short.** The circuit resistance will decrease and total current becomes greater than the normal. Further, the current flow in the healthy branches will be less than the normal. Therefore, healthy branches may operate but not as they are supposed to.

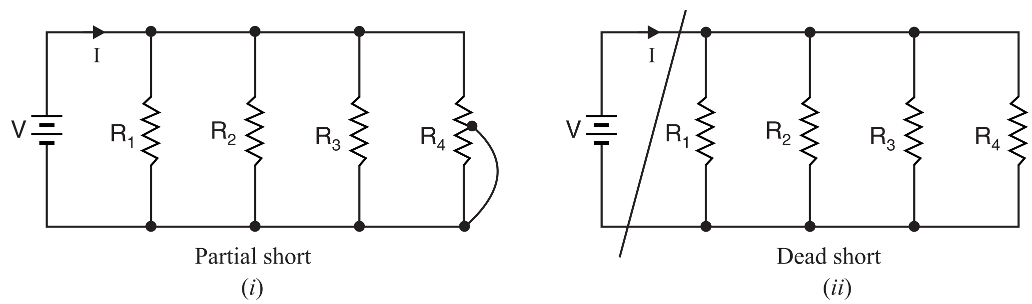


Fig. 2.53

4. Dead short in a parallel circuit. Fig. 2.53 (*ii*) shows a **parallel circuit** with a **dead short.** Note that all the loads are eliminated by the short circuit so that the circuit resistance is almost zero. As a result, the circuit current becomes abnormally high and may cause extensive damage unless it has protective devices (*e.g.* fuse, circuit breaker etc.).

2.14. Duality Between Series and Parallel Circuits

Two physical systems or circuits are called dual if they are described by equations of the same mathematical form.

This peculiar pattern of relationship exists between series and parallel circuits. For example, consider the following table for d.c. series circuit and d.c. parallel circuit.

D.C. series circuit	**D.C. parallel circuit**
$I_1 = I_2 = I_3 = \ldots$	$V_1 = V_2 = V_3 = \ldots$
$V = V_1 + V_2 + V_3 + \ldots$	$I = I_1 + I_2 + I_3 + \ldots$
$R_S = R_1 + R_2 + R_3 + \ldots$	$G_P = G_1 + G_2 + G_3 + \ldots$
$I = \dfrac{V_1}{R_1} = \dfrac{V_2}{R_2} = \dfrac{V_3}{R_3} = \ldots$	$V = \dfrac{I_1}{G_1} = \dfrac{I_2}{G_2} = \dfrac{I_3}{G_3} = \ldots$
$V_1 = V\dfrac{R_1}{R_S}$; $V_2 = V\dfrac{R_2}{R_S}$	$I_1 = I\dfrac{G_1}{G_P}$; $I_2 = I\dfrac{G_2}{G_P}$

Note that the relations for parallel circuit can be obtained from the series circuit by replacing voltage by current, current by voltage and resistance by conductance. In like manner, relations for series circuit can be obtained from the parallel circuit by replacing current by voltage, voltage by current and conductance by resistance. Such a pattern is known as *duality* and the two circuits are said to be dual of each other. Thus series and parallel circuits are dual of each other. Other examples of duals are : short circuits and open circuits are duals and nodes and meshes are duals.

2.15. Wheatstone Bridge

This bridge was first proposed by Wheatstone (an English telegraph engineer) for measuring accurately the value of an unknown resistance. It consists of four resistors (two fixed known resistances P and Q, a known variable resistance R and the unknown resistance X whose value is to be found) connected to form a diamond-shaped circuit $ABCDA$ as shown in Fig.2.54 (*i*). Across one pair of opposite junctions (A and C), battery is connected and across the other opposite pair of

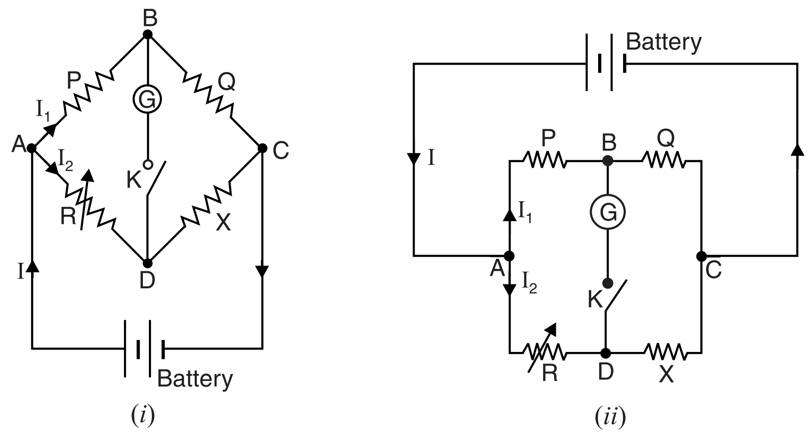


Fig. 2.54

junctions (*B* and *D*), a galvanometer is connected through the key *K*. The circuit is called a bridge because galvanometer bridges the opposite junctions *B* and *D*. Fig. 2.54 (*ii*) shows another* way of drawing the Wheatstone bridge.

Working. The values of *P* and *Q* are properly fixed. The value of *R* is varied such that on closing the key *K*, there is no current through the galvanometer. Under such conditions, the bridge is said to be *balanced*. The point at which the bridge is balanced is called the *null point*. Let I_1 and I_2 be the currents through *P* and *R* respectively when the bridge is balanced. Since there is no current through the galvanometer, the currents in *Q* and *X* are also I_1 and I_2 respectively. As the galvanometer reads zero, points *B* and *D* are at the same potential. This means that voltage drops from *A* to *B* and *A* to *D* must be equal. Also voltage drops from *B* to *C* and *D* to *C* must be equal. Hence,

$$I_1 P = I_2 R \quad ...(i)$$

and

$$I_1 Q = I_2 X \quad ...(ii)$$

Dividing exp. (*i*) by (*ii*), we get,

$$P/Q = R/X$$

or

$$P X = Q R$$

i.e. *Product of opposite arms = Product of opposite arms*

$$\text{Unknown resistance, } X = \frac{Q}{P} \times R \quad ...(iii)$$

Since the **values of *Q*, *P* and *R* are known, the value of unknown resistance *X* can be calculated. It should be noted that exp. (*iii*) is true only under the balanced conditions of Wheatstone bridge.

Note. When the bridge is balanced, $V_B = V_D$ so the voltage across galvanometer is zero *i.e.* $V_{BD} = V_B - V_D = 0$. When there is zero voltage across the galvanometer, there is also zero current though the galvanometer. Consequently, **in a balanced Wheatstone bridge, galvanometer can be replaced by either a short circuit or an open circuit without affecting the voltages and currents anywhere else in the circuit.**

Example 2.35. *Verify that the Wheatstone bridge shown in Fig. 2.55 is balanced. Then find the voltage V_T across the 0.2 A current source by* (*i*) *replacing the 200 Ω resistor with a short.* (*ii*) *replacing the 200 Ω resistor with an open.*

* Note the four points *A*,*B*, *C* and *D*, each lying at the junction between two resistors. A galvanometer should bridge a pair of opposite points such as *B* and *D* and the battery the other pair *A* and *C*.

** Resistances *P* and *Q* are called the ratio arms of bridge and are usually made equal to definite ratio such as 1 to 1, 10 to 1 or 100 to 1. The resistance *R* is ca]lled the rheostat arm and is made continuously variable from 1 to 1000 ohms or from 1 to 10,000 ohms.

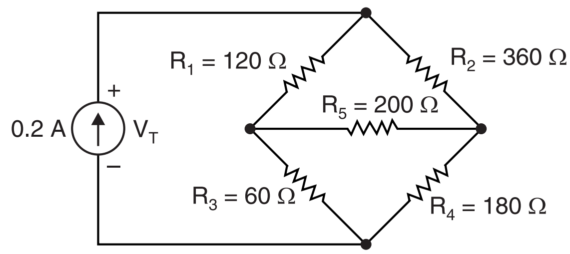


Fig. 2.55

Solution. The Wheatstone bridge is balanced if the products of the resistances of the opposite arms of the bridge are equal. An inspection of Fig. 2.55 shows that $R_1R_4 = R_2R_3$. Therefore, the bridge is balanced.

(*i*) When 200 Ω resistor is shorted. Fig. 2.56 (*i*) shows the bridge when the 200 Ω resistor (R_5) is replaced by a short. In this case, the circuit is equivalent to a series-parallel circuit as shown in Fig. 2.56 (*ii*). Referring to Fig. 2.56 (*ii*), the circuit is equivalent to parallel combination of R_1 and R_2 in series with the parallel combination of R_3 and R_4.

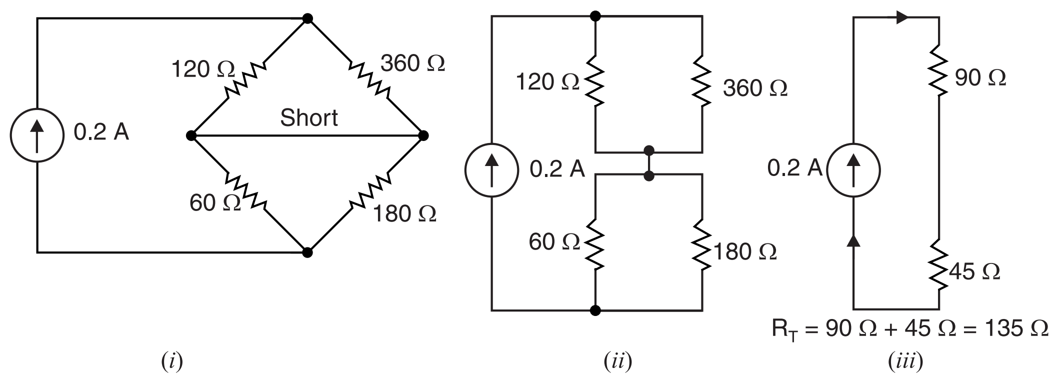


Fig. 2.56

The circuit shown in Fig. 2.56 (*ii*) further reduces to the one shown in Fig. 2.57 (*iii*). Therefore, total circuit resistance, $R_T = 90 + 45 = 135\ \Omega$.

∴ Voltage across 0.2 A current source is

$$V_T = I\,R_T = 0.2 \times 135 = \mathbf{27\ V}$$

(*ii*) When 200 Ω resistor is open-circuited. Fig. 2.57 (*i*) shows the bridge when 200 Ω resistor is replaced by an open. In this case, the circuit is equivalent to a series-parallel circuit in which series combination of R_1 and R_3 is in parallel with the series combination of R_2 and R_4. This is shown in Fig. 2.57 (*ii*).

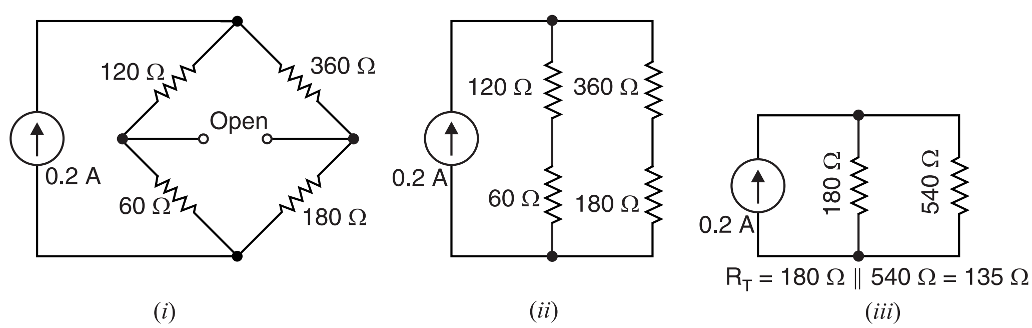


Fig. 2.57

The circuit shown in Fig. 2.57 (*ii*) further reduces to the one shown in Fig. 2.57 (*iii*). Referring to Fig. 2.57 (*iii*), the total circuit resistance R_T is given by ;

$$R_T = \frac{180 \times 540}{180 + 540} = 135\ \Omega$$

∴ Voltage across 0.2 A current source, $V_T = I\,R_T = 0.2 \times 135 =$ **27 V**

Note that the voltage across current source is unaffected whether 200 Ω resistor is replaced by a short or an open.

2.16. Complex Circuits

Sometimes we encounter circuits where simplification by series and parallel combinations is impossible. Consequently, Ohm's law cannot be applied to solve such circuits. This happens when there is more than one e.m.f. in the circuit or when resistors are connected in a complicated manner. Such circuits are called *complex circuits*. We shall discuss two such circuits by way of illustration.

(*i*) Fig. 2.58 shows a circuit containing two sources of *e.m.f.* E_1 and E_2 and three resistors. This circuit cannot be solved by series-parallel combinations. Are resistors R_1 and R_3 in parallel? Not quite, because there is an *e.m.f.* source E_1 between them. Are they in series? Not quite, because same current does not flow between them.

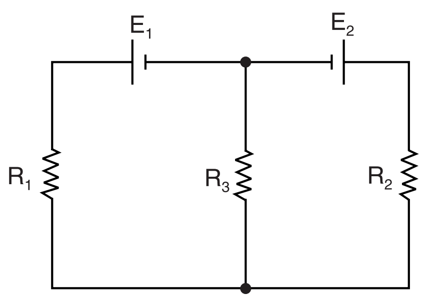


Fig. 2.58

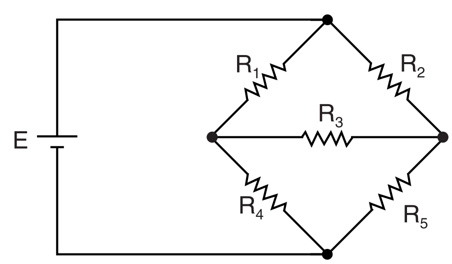


Fig. 2.59

(*ii*) Fig. 2.59 shows another circuit where we cannot solve the circuit by series-parallel combinations. Though this circuit has one source of *e.m.f.* (*E*), it cannot be solved by using series and parallel combinations. Thus resistors R_1 and R_2 are neither in series nor in parallel; the same is true for other pair of resistors.

In order to solve such complex circuits, Gustav Kirchhoff gave two laws, known as Kirchhoff's laws.

2.17. Kirchhoff's Laws

Kirchhoff gave two laws to solve complex circuits, namely ;

1. Kirchhoff's Current Law (KCL)
2. Kirchhoff's Voltage Law (KVL)

1. KIRCHHOFF'S CURRENT LAW (KCL)

This law relates to the currents at the *junctions of an electric circuit and may be stated as under :

The algebraic sum of the currents meeting at a junction in an electrical circuit is zero.

An algebraic sum is one in which the sign of the quantity is taken into account. For example, consider four conductors carrying currents I_1, I_2, I_3 and I_4 and meeting at point O as shown in Fig. 2.60.

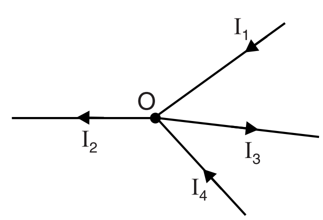


Fig. 2.60

* A junction is that point in an electrical circuit where *three or more* circuit elements meet.

If we take the signs of currents flowing towards point O as positive, then currents flowing away from point O will be assigned negative sign. Thus, applying Kirchhoff's current law to the junction O in Fig. 2.60, we have,

$$(I_1) + (I_4) + (-I_2) + (-I_3) = 0$$

or $$I_1 + I_4 = I_2 + I_3$$

i.e., Sum of incoming currents = Sum of outgoing currents

Hence, Kirchhoff's current law may also be stated as under :

The sum of currents flowing towards any junction in an electrical circuit is equal to the sum of currents flowing away from that junction. Kirchhoff's current law is also called junction rule.

Kirchhoff's current law is true because electric current is merely the flow of free electrons and they cannot accumulate at any point in the circuit. This is in accordance with the law of conservation of charge. Hence, Kirchhoff's current law is based on the law of conservation of charge.

2. KIRCHHOFF'S VOLTAGE LAW (KVL)

This law relates to *e.m.fs* and voltage drops in a closed circuit or loop and may be stated as under :

In any closed electrical circuit or mesh, the algebraic sum of all the electromotive forces (e.m.fs) and voltage drops in resistors is equal to zero, i.e.,

In any closed circuit or mesh,

Algebraic sum of e.m.fs + Algebraic sum of voltage drops = 0

The validity of Kirchhoff's voltage law can be easily established by referring to the closed loop *ABCDA* shown in Fig. 2.61. If we start from any point (say point *A*) in this closed circuit and go back to this point (*i.e.,* point *A*) after going around the circuit, then there is no increase or decrease in potential. This means that algebraic sum of the *e.m.fs* of all the sources (here only one *e.m.f.* source is considered) met on the way *plus* the algebraic sum of the voltage drops in the resistances must be zero. Kirchhoff's voltage law is based on the law of *conservation of energy, *i.e.*, net change in the energy of a charge after completing the closed path is zero.

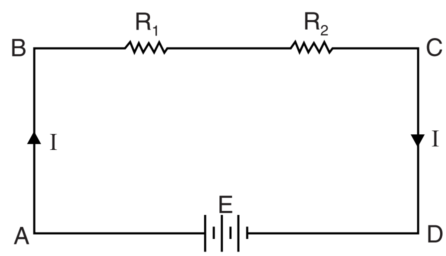


Fig. 2.61

Note. Kirchhoff's voltage law is also called *loop rule*.

2.18. Sign Convention

While applying Kirchhoff's voltage law to a closed circuit, algebraic sums are considered. Therefore, it is very important to assign proper signs to *e.m.fs* and voltage drops in the closed circuit. The following convention may be followed :

A **rise in potential should be considered positive and fall in potential should be considered negative.

(*i*) Thus if we go from the positive terminal of the battery to the negative terminal, there is fall of potential and the *e.m.f.* should be assigned negative sign. Thus in Fig. 2.62 (*i*), as we go from *A* to *B*, there is a fall in potential and the *e.m.f.* of the cell will be assigned negative

* As a charge traverses a loop and returns to the starting point, the sum of rises of potential energy associated with *e.m.fs* in the loop must be equal to the sum of the drops of potential energy associated with resistors.

** The reverse convention is equally valid *i.e.* rise in potential may be considered negative and fall in potential as positive.

sign. On the other hand, if we go from the negative terminal to the positive terminal of the battery or source, there is a rise in potential and the *e.m.f* should be assigned positive sign. Thus in Fig. 2.62 (*ii*) as we go from *A* to *B*, there is a rise in potential and the *e.m.f.* of the cell will be assigned positive sign. *It may be noted that the sign of e.m.f. is independent of the direction of current through the branch under consideration.*

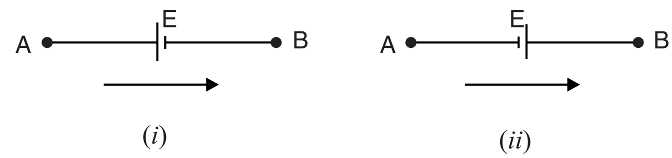


Fig. 2.62

(*ii*) When current flows through a resistor, there is a voltage drop across it. If we go through the resistor in the same direction as the current, there is a fall in potential because current flows from higher potential to lower potential. Hence this voltage drop should be assigned negative sign. In Fig. 2.63 (*i*), as we go from *A* to *B*, there is a fall in potential and the voltage drop across the resistor will be assigned negative sign. On the other hand, if we go through the resistor against the current flow, there is a rise in potential and the voltage drop should be given positive sign. Thus referring to Fig. 2.63 (*ii*), as we go from *A* to *B*, there is a rise in potential and this voltage drop will be given positive sign. *It may be noted that sign of voltage drop depends on the direction of current and is independent of the polarity of the e.m.f. of source in the circuit under consideration.*

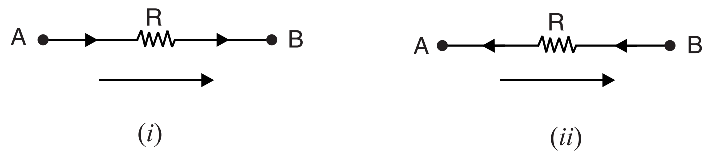


Fig. 2.63

2.19. Illustration of Kirchhoff's Laws

Kirchhoff's Laws can be beautifully explained by referring to Fig. 2.64. Mark the directions of currents as indicated. The direction in which currents are assumed to flow is unimportant, since if wrong direction is chosen, it will be indicated by a negative sign in the result.

(*i*) The magnitude of current in any branch of the circuit can be found by applying Kirchhoff's current law. Thus at junction *C* in Fig. 2.64, the incoming currents to the junction are I_1 and I_2. Obviously, the current in branch *CF* will be I_1+I_2.

(*ii*) There are three closed circuits in Fig 2.64 *viz*. *ABCFA, CDEFC* and *ABCDEFA*. Kirchhoff's voltage law can be applied to these closed circuits to get the desired equations.

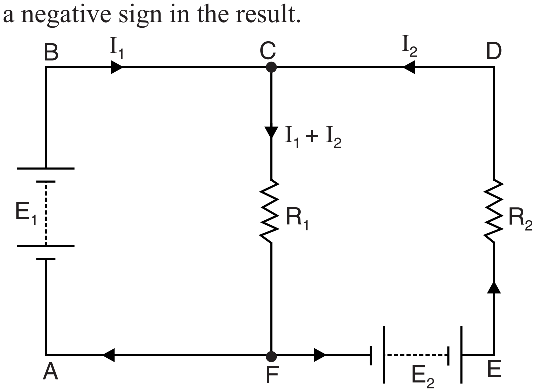


Fig. 2.64

Loop ABCFA. In this loop, *e.m.f.* E_1 will be given *positive* sign. It is because as we consider the loop in the order *ABCFA*, we go from −ve terminal to the positive terminal of the battery in the branch *AB* and hence there is a rise in potential. The voltage drop in branch *CF* is $(I_1 + I_2)R_1$ and shall bear *negative* sign. It is because as we consider the loop in the order *ABCFA*, we go with current in branch *CF* and there is a fall in potential. Applying Kirchhoff's voltage law to the loop *ABCFA*,

$$-(I_1 + I_2)R_1 + E_1 = 0$$

or $$E_1 = (I_1 + I_2) R_1 \quad ...(i)$$

Loop CDEFC. As we go around the loop in the order *CDEFC*, drop I_2R_2 is *positive,* e.m.f. E_2 is *negative* and drop $(I_1 + I_2) R_1$ is *positive*. Therefore, applying Kirchhoff's voltage law to this loop, we get,

$$I_2 R_2 + (I_1 + I_2) R_1 - E_2 = 0$$

or $$I_2 R_2 + (I_1 + I_2) R_1 = E_2 \quad ...(ii)$$

Since E_1, E_2, R_1 and R_2 are known, we can find the values of I_1 and I_2 from the above two equations. Hence currents in all branches can be determined.

2.20. Method to Solve Circuits by Kirchhoff's Laws

(*i*) Assume unknown currents in the given circuit and show their direction by arrows.

(*ii*) Choose any closed circuit and find the algebraic sum of voltage drops *plus* the algebraic sum of e.m.fs in that loop.

(*iii*) Put the algebraic sum of voltage drops plus the algebraic sum of e.m.fs equal to zero.

(*iv*) Write equations for as many closed circuits as the number of unknown quantities. Solve equations to find unknown currents.

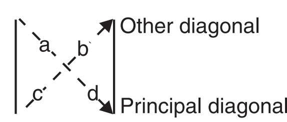
(*v*) If the value of the assumed current comes out to be negative, it means that actual direction of current is opposite to that of assumed direction.

Note. It may be noted that Kirchhoff's laws are also applicable to a.c. circuits. The only thing to be done is that **I**, **V** and **Z** are substituted for *I*, *V* and *R*. Here **I**, **V** and **Z** are phasor quantities.

2.21. Matrix Algebra

The solution of two or three simultaneous equations can be achieved by a method that uses *determinants.* A determinant is a numerical value assigned to a square arrangement of numbers called a *matrix.* The advantage of determinant method is that it is less difficult for three unknowns and there is less chance of error. The theory behind this method is not presented here but is available in any number of mathematics books.

Second-order determinant. A 2 × 2 matrix has four numbers arranged in two rows and two columns. The value of such a matrix is called a *second-order determinant* and is *equal to the product of the principal diagonal minus the product of the other diagonal.* For example, value of the matrix = *ad* – *cb*.



Second-order determinant can be used to solve simultaneous equations with two unknowns. Consider the following equations :

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

The unknowns are *x* and *y* in these equations. The numbers associated with the unknowns are called *coefficients.* The coefficients in these equations are a_1, a_2, b_1 and b_2. The right hand number (c_1 or c_2) of each equation is called a *constant.* The coefficients and constants can be arranged as a *numerator matrix* and as a *denominator matrix.* The matrix for the numerator is formed by replacing the coefficients of the unknown by the constants. The denominator matrix is called *characteristic matrix* and is the same for each fraction. It is formed by the coefficients of the simultaneous equations.

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad ; \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Note that the characteristic determinant (denominator) is the same in both cases and needs to be evaluated only once. Also note that the coefficients for x are replaced by the constants when solving for x and that the coefficients for y are replaced by the constants when solving for y.

Third-order determinant. A third-order determinant has 9 numbers arranged in 3 rows and 3 columns. Simultaneous equations with three unknowns can be solved with third-order determinants. Consider the following equations :

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

The characteristic matrix forms the denominator and is the same for each fraction. It is formed by the coefficients of the simultaneous equations.

$$\text{Denominator} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

The matrix for each numerator is formed by replacing the coefficient of the unknown with the constant.

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\text{Denominator}} \quad ; \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\text{Denominator}} \quad ; \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\text{Denominator}}$$

Example 2.36. *In the network shown in Fig. 2.65, the different currents and voltages are as under :*

$i_2 = 5e^{-2t}$; $i_4 = 3 \sin t$; $v_3 = 4e^{-2t}$

Using KCL, find voltage v_1.

Solution. Current through capacitor is

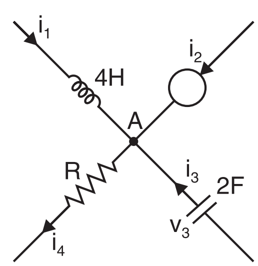


Fig. 2.65

$$i_3 = C\frac{dv_3}{dt} = C\frac{d}{dt}(v_3) = \frac{2d}{dt}(4e^{-2t})$$
$$= -16e^{-2t}$$

Applying *KCL* to junction *A* in Fig. 2.65,

$$i_1 + i_2 + i_3 + (-i_4) = 0$$

or $\quad i_1 + 5e^{-2t} - 16e^{-2t} - 3 \sin t = 0$

or $\quad i_1 = 3 \sin t + 11e^{-2t}$

∴ Voltage developed across 4H coil is

$$v_1 = L\frac{di_1}{dt} = L\frac{d}{dt}(i_1) = 4\frac{d}{dt}(3 \sin t + 11e^{-2t})$$
$$= 4(3 \cos t - 22e^{-2t}) = \mathbf{12 \cos t - 88e^{-2t}}$$

Example 2.37. *For the circuit shown in Fig. 2.66, find the currents flowing in all branches.*

Solution. Mark the currents in various branches as shown in Fig. 2.66. Since there are two unknown quantities I_1 and I_2, two loops will be considered.

Loop *ABCFA*. Applying *KVL*,

$$30 - 2I_1 - 10 + 5I_2 = 0$$

or $$2I_1 - 5I_2 = 20 \qquad ...(i)$$

Loop *FCDEF*. Applying *KVL*,

$$-5I_2 + 10 - 3(I_1 + I_2) - 5 - 4(I_1 + I_2) = 0$$

or $$7I_1 + 12I_2 = 5 \qquad ...(ii)$$

Multiplying eq. *(i)* by 7 and eq. *(ii)* by 2, we get,

$$14I_1 - 35I_2 = 140 \qquad ...(iii)$$

$$14I_1 + 24I_2 = 10 \qquad ...(iv)$$

Subtracting eq. (*iv*) from eq. (*iii*), we get,

$$-59I_2 = 130$$

$\therefore$ $I_2 = -130/59 = -2.2\text{A} =$ **2.2 A** *from C to F*

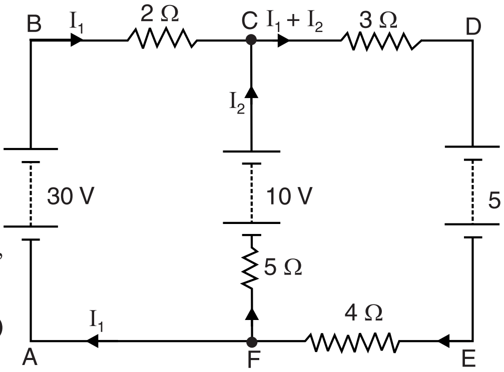


Fig. 2.66

Substituting the value of $I_2 = -2.2$ A in eq. (*i*), we get, $I_1 =$ **4.5 A**

Current in branch *CDEF* $= I_1 + I_2 = (4.5) + (-2.2) =$ **2.3 A**

Example 2.38. *A Wheatstone bridge ABCD has the following details ; AB = 1000 Ω ; BC = 100 Ω; CD = 450 Ω ; DA = 5000 Ω.*

A galvanometer of resistance 500 Ω is connected between B and D. A 4.5-volt battery of negligible resistance is connected between A and C with A positive. Find the magnitude and direction of galvanometer current.

Solution. Fig. 2.67 shows the Wheatstone bridge *ABCD*. Mark the currents in the various sections as shown. Since there are three unknown quantities (*viz.* I_1, I_2 and I_g), three loops will be considered.

Loop *ABDA*. Applying *KVL*,

$$-1000I_1 - 500I_g + 5000I_2 = 0$$

or $$2I_1 + I_g - 10I_2 = 0 \qquad ...(i)$$

Loop *BCDB*. Applying *KVL*,

$$-100(I_1 - I_g) + 450(I_2 + I_g) + 500I_g = 0$$

or $$2I_1 - 21I_g - 9I_2 = 0 \qquad ...(ii)$$

Loop *EABCFE*. Applying *KVL*,

$$-1000I_1 - 100(I_1 - I_g) + 4.5 = 0$$

or $$1100I_1 - 100I_g = 4.5 \qquad ...(iii)$$

Subtracting eq. (*ii*) from eq. (*i*), we get,

$$22I_g - I_2 = 0 \qquad ...(iv)$$

Multiplying eq. (*i*) by 550 and subtracting eq. (*iii*) from it, we get,

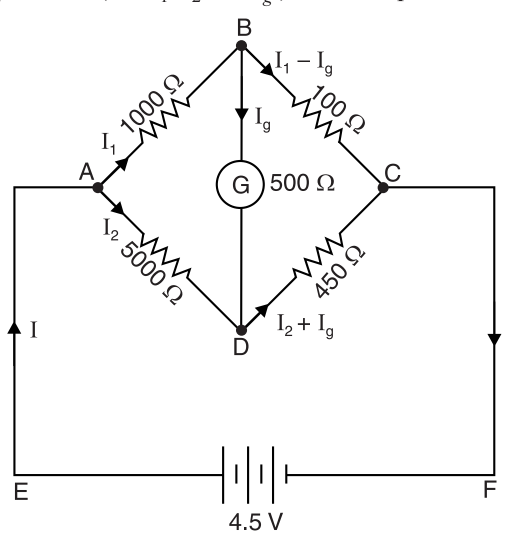


Fig. 2.67

$$650I_g - 5500I_2 = -4.5 \qquad ...(v)$$

Multiplying eq. (*iv*) by 5500 and subtracting eq. (*v*) from it, we get,

$$120350I_g = 4.5$$

$\therefore$ $I_g = \dfrac{4.5}{120350} = 37.4 \times 10^{-6}\text{ A} =$ **37.4 μA** *from B to D*

Example 2.39. *A Wheatstone bridge ABCD is arranged as follows : AB = 1 Ω ; BC = 2 Ω ; CD = 3 Ω ; DA = 4 Ω. A resistance of 5 Ω is connected between B and D. A 4-volt battery of internal resistance 1 Ω is connected between A and C. Calculate (i) the magnitude and direction of current in 5 Ω resistor and (ii) the resistance between A and C.*

Solution. **(*i*)** Fig. 2.68 shows the Wheatstone bridge *ABCD*. Mark the currents in the various branches as shown. Since there are three unknown quantities (*viz.* I_1, I_2 and I_3), three loops will be considered.

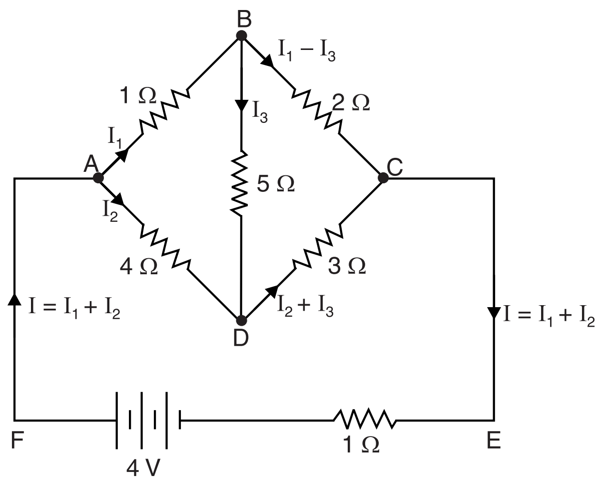


Fig. 2.68

Loop *ABDA*. Applying *KVL*,

$$-1 \times I_1 - 5\,I_3 + 4\,I_2 = 0$$

or $$I_1 + 5\,I_3 - 4\,I_2 = 0 \quad ...(i)$$

Loop *BCDB*. Applying *KVL*,

$$-2\,(I_1 - I_3) + 3\,(I_2 + I_3) + 5I_3 = 0$$

or $$2\,I_1 - 10\,I_3 - 3\,I_2 = 0 \quad ...(ii)$$

Loop *FABCEF*. Applying *KVL*,

$$-I_1 \times 1 - 2\,(I_1 - I_3) - 1\,(I_1 + I_2) + 4 = 0$$

or $$4\,I_1 - 2\,I_3 + I_2 = 4 \quad ...(iii)$$

Multiplying eq.(*i*) by 2 and subtracting *eq.* (*ii*) from it, we get,

$$20\,I_3 - 5\,I_2 = 0 \quad ...(iv)$$

Multiplying *eq.* (*i*) by 4 and subtracting *eq.* (*iii*) from it, we get,

$$22\,I_3 - 17\,I_2 = -4 \quad ...(v)$$

Multiplying *eq.* (*iv*) by 17 and eq. (*v*) by 5, we get,

$$340\,I_3 - 85\,I_2 = 0 \quad ...(vi)$$

$$110\,I_3 - 85\,I_2 = -20 \quad ...(vii)$$

Subtracting *eq.* (*vii*) from *eq.* (*vi*), we get,

$$230\,I_3 = 20$$

$$\therefore \quad I_3 = 20/230 = 0.087 \text{ A}$$

i.e Current in 5 Ω, I_3 = **0.087 A** *from B* to *D*

(*ii*) Substituting the value of I_3 = 0.087 A in eq. (*iv*), we get, I_2 = 0.348 A.

Substituting values of I_3 = 0.087 A and I_2 = 0.348 A in *eq. (ii)*, I_1 = 0.957 A.

Current supplied by battery, $I = I_1 + I_2 = 0.957 + 0.348 = 1.305$ A

P.D. between *A* and *C* = E.M.F. of battery − Drop in battery = 4 − 1.305 × 1 = 2.695 V

$$\therefore \quad \text{Resistance between } A \text{ and } C = \frac{P.D. \text{ across } AC}{\text{Battery current}} = \frac{2.695}{1.305} = \mathbf{2.065\ \Omega}$$

Example 2.40. *Determine the current in 4 Ω resistance of the circuit shown in Fig. 2.69.*

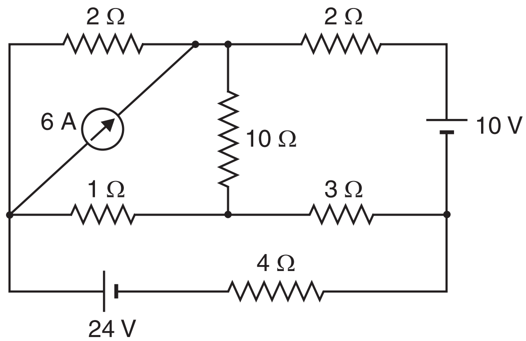


Fig. 2.69

Solution. The given circuit is redrawn as shown in Fig. 2.70. Mark the currents in the various branches of the circuit using *KCL*. Since there are three unknown quantities (*viz.* I_1, I_2 and I_3), three loops will be considered. While applying *KVL* to any loop, rise in potential is considered positive while fall in potential is considered negative. This convention is followed throughout the book.

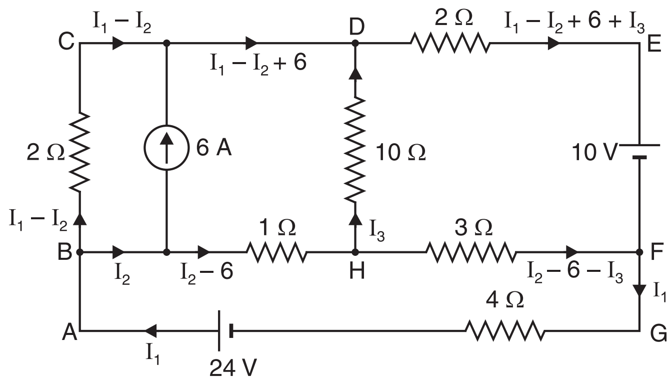


Fig. 2.70

Loop *BCDHB*. Applying *KVL*, we have,

$$-2(I_1 - I_2) + 10I_3 + 1 \times (I_2 - 6) = 0$$

or $$2I_1 - 3I_2 - 10I_3 = -6 \quad ...(i)$$

Loop *DEFHD*. Applying K*VL*, we have,

$$-2(I_1 - I_2 + 6 + I_3) - 10 + 3(I_2 - 6 - I_3) - 10I_3 = 0$$

or $$2I_1 - 5I_2 + 15I_3 = -40 \quad ...(ii)$$

Loop *BHFGAB*. Applying *KVL*, we have,

$$-1(I_2 - 6) - 3(I_2 - 6 - I_3) - 4I_1 + 24 = 0$$

or $$4I_1 + 4I_2 - 3I_3 = 48 \quad ...(iii)$$

Solving eqs. (*i*), (*ii*) and (*iii*), we get, $I_1 = 4.1$ A.

∴ Current in 4 Ω resistance = I_1 = **4.1 A**

Example 2.41. *Two batteries E_1 and E_2 having e.m.fs of 6V and 2V respectively and internal resistances of 2* **Ω** *and 3* **Ω** *respectively are connected in parallel across a 5* **Ω** *resistor. Calculate* (*i*) *current through each battery and* (*ii*) *terminal voltage.*

Solution. Fig. 2.71 shows the conditions of the problem. Mark the currents in the various branches. Since there are two unknown quantities I_1 and I_2, two loops will be considered.

(*i*) **Loop *HBCDEFH*.** Applying Kirchhoff's voltage law to loop *HBCDEFH*, we get,

$$2I_1 - 6 + 2 - 3I_2 = 0$$

or $$2I_1 - 3I_2 = 4 \quad ...(i)$$

Loop *ABHFEGA*. Applying Kirchhoff's voltage law to loop *ABHFEGA*, we get,

$$3I_2 - 2 + 5(I_1 + I_2) = 0$$

or $$5I_1 + 8I_2 = 2 \quad ...(ii)$$

Multiplying eq. (*i*) by 8 and eq. (*ii*) by 3 and then adding them, we get,

$$31\,I_1 = 38$$

or $\quad I_1 = \frac{38}{31} = \mathbf{1.23\ A}$

i.e. battery E_1 is being discharged at 1.23 A. Substituting I_1 = 1.23 A in eq. (*i*), we get, I_2 = **− 0.52A** *i.e.* battery E_2 is being charged.

(ii) Terminal voltage = $(I_1 + I_2)\,5$

= (1.23 − 0.52) 5 = **3.55 V**

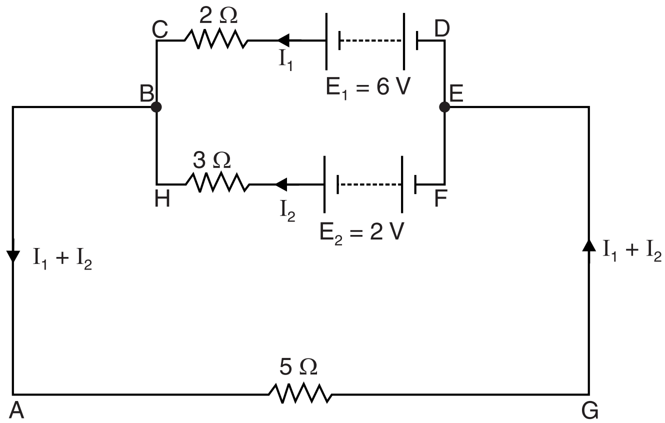


Fig. 2.71

Example 2.42. *Twelve wires, each of resistance r, are connected to form a skeleton cube. Find the equivalent resistance between the two diagonally opposite corners of the cube.*

Solution. Let *ABCDEFGH* be the skeleton cube formed by joining 12 wires, each of resistance *r* as shown in Fig. 2.72. Suppose a current of 6*I* enters the cube at the corner *A*. Since the resistance of each wire is the same, the current at corner *A* is divided into three equal parts: 2*I* flowing in *AE*, 2*I* flowing in *AB* and 2*I* flowing in *AD*. At points *B, D* and *E*, these currents are divided into equal parts, each part being equal to *I*. Applying Kirchhoff's current law, 2*I* current flows in each of the wires *CG, HG* and *FG*. These three currents add up at the corner *G* so that current flowing out of this corner is 6*I*.

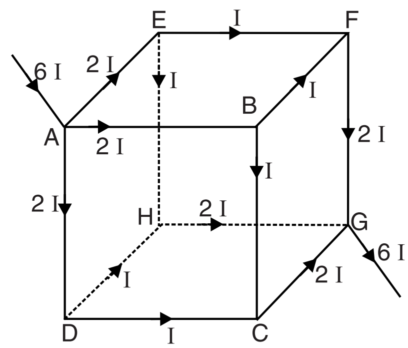


Fig. 2.72

Let *E* = e.m.f. of the battery connected to corners *A* and *G* of the cube ; corner *A* being connected to the +ve terminal. Now consider any closed circuit between corners *A* and *G*, say the closed circuit *AEFGA*. Applying Kirchhoff's voltage law to the closed circuit *AEFGA*, we have,

$$-2\,I\,r - I\,r - 2\,I\,r = -E \quad \text{or} \quad 5\,I\,r = E \qquad ...(i)$$

Let *R* be the equivalent resistance between the diagonally opposite corners *A* and *G*.

Then, $\qquad E = 6\,I\,R \qquad ...(ii)$

From eqs. (*i*) and (*ii*), we get, $6IR = 5I\,r$ or $R = \mathbf{(5/6)\ r}$

Example 2.43. *Determine the current supplied by the battery in the circuit shown in Fig. 2.73.*

Solution. Mark the currents in the various branches as shown in Fig. 2.73. Since there are three unknown quantities *x*, *y* and *z*, three equations must be formed by considering three loops.

Loop *ABCA.* Applying *KVL*, we have,

$$-100x - 300z + 500y = 0$$

or $\qquad x - 5y + 3z = 0 \qquad ...(i)$

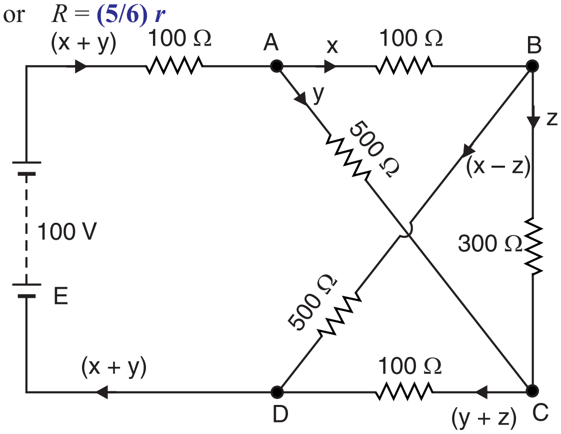


Fig. 2.73

Loop *BCDB*. Applying *KVL*, we have,

$-300x - 100(y + z) + 500(x - z) = 0$

or $\quad 5x - y - 9z = 0 \quad$ ...(*ii*)

Loop *ABDEA*. Applying *KVL*, we have,

$-100x - 500(x - z) + 100 - 100(x + y) = 0$

or $\quad 7x + y - 5z = 1 \quad$ ...(*iii*)

From eqs. (*i*), (*ii*) and (*iii*), $x = \frac{1}{5}\text{A} \quad ; \quad y = \frac{1}{10}\text{A} \quad ; \quad z = \frac{1}{10}\text{A}$

By Determinant Method. We shall now find the values of *x*, *y* and *z* by determinant method.

$$x - 5y + 3z = 0 \quad \text{...}(i)$$

$$5x - y - 9z = 0 \quad \text{...}(ii)$$

$$7x + y - 5z = 1 \quad \text{...}(iii)$$

$$\begin{bmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore \quad x = \frac{\begin{vmatrix} 0 & -5 & 3 \\ 0 & -1 & -9 \\ 1 & 1 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{vmatrix}} = \frac{0\begin{vmatrix} -1 & -9 \\ 1 & -5 \end{vmatrix} + 5\begin{vmatrix} 0 & -9 \\ 1 & -5 \end{vmatrix} + 3\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}}{1\begin{vmatrix} -1 & -9 \\ 1 & -5 \end{vmatrix} + 5\begin{vmatrix} 5 & -9 \\ 7 & -5 \end{vmatrix} + 3\begin{vmatrix} 5 & -1 \\ 7 & 1 \end{vmatrix}}$$

$$= \frac{0[(-1\times-5)-(1\times-9)]+5[(0\times-5)-(1\times-9)]+3[(0\times1)-(1\times-1)]}{1[(-1\times-5)-(1\times-9)]+5[(5\times-5)-(7\times-9)+3[(5\times1)-(7\times-1)]}$$

$$= \frac{0+45+3}{14+190+36} = \frac{48}{240} = \frac{1}{5}\text{A}$$

$$y = \frac{\begin{vmatrix} 1 & 0 & 3 \\ 5 & 0 & -9 \\ 7 & 1 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{vmatrix}} = \frac{24}{240} = \frac{1}{10}\text{A}$$

$$z = \frac{\begin{vmatrix} 1 & -5 & 0 \\ 5 & -1 & 0 \\ 7 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{vmatrix}} = \frac{24}{240} = \frac{1}{10}\text{A}$$

$\therefore$ Current supplied by battery $= x + y = \frac{1}{5} + \frac{1}{10} = \mathbf{\frac{3}{10}\,A}$

Example 2.44. *Use Kirchhoff's voltage law to find the voltage V_{ab} in Fig. 2.74.*

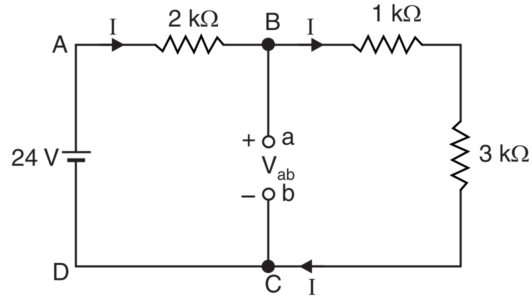


Fig. 2.74

Solution. We shall use Kirchhoff's voltage law to solve this problem, although other methods can be used.

Total circuit resistance, $R_T = 2 + 1 + 3 = 6\ \text{k}\Omega$

Circuit current, $I = \dfrac{V}{R_T} = \dfrac{24\ \text{V}}{6\ \text{k}\Omega} = 4\ \text{mA}$

Applying Kirchhoff's voltage law to loop *ABCDA*, we have,

$$24 - 4\ \text{mA} \times 2\ \text{k}\Omega - {}^{*}V_{ab} = 0$$

or $\quad 24 - 8 - V_{ab} = 0 \qquad \therefore \quad V_{ab} = 24 - 8 =$ **16 V**

Example 2.45. *For the ladder network shown in Fig. 2.75, find the source voltage V_s which results in a current of 7.5 mA in the 3 Ω resistor.*

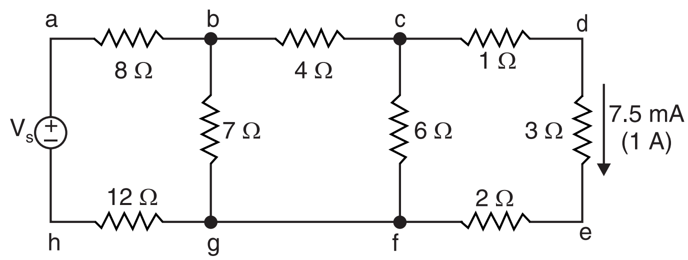


Fig. 2.75

Solution. Let us assume that current in branch *de* is 1 A.

Since the circuit is linear, the voltage necessary to produce 1 A is in the same ratio to 1 A as V_s to 7.5 mA.

Voltage between *c* and *f*, $\quad V_{cf} = 1\,(1 + 3 + 2) = 6\ \text{V}$

$\therefore$ Current in branch *cf*, $I_{cf} = 6/6 = 1\ \text{A}$

Applying *KCL* at junction *c*,

$$I_{bc} = 1 + 1 = 2\ \text{A}$$

Applying *KVL* to loop *bcfgb,* we have,

$$-4 \times 2 - 6 \times 1 + V_{bg} = 0 \quad \therefore \quad V_{bg} = 8 + 6 = 14\ \text{V}$$

$\therefore$ Current in branch *bg*, $I_{bg} = \dfrac{V_{bg}}{7} = \dfrac{14}{7} = 2\ \text{A}$

Applying *KCL* to junction *b*, we have, $I_{ab} = 2 + 2 = 4\ \text{A}$

Applying *KVL* to loop *abgha,* we have,

$$-8 \times 4 - 7 \times 2 - 12 \times 4 + V_{ah} = 0 \quad \therefore \quad V_{ah} = 94\ \text{V}$$

Now $\quad \dfrac{V_{ah}}{1\ \text{A}} = \dfrac{V_s}{7.5\ \text{mA}} \quad$ or $\quad \dfrac{94}{1\ \text{A}} = \dfrac{V_s}{7.5 \times 10^{-3}\ \text{A}} \qquad \therefore \quad V_s =$ **0.705 V**

Example 2.46. *Determine the readings of an ideal voltmeter connected in Fig. 2.76 to (i) terminals a and b, (ii) terminals c and g. The average power dissipated in the 5 Ω resistor is equal to 20 W.*

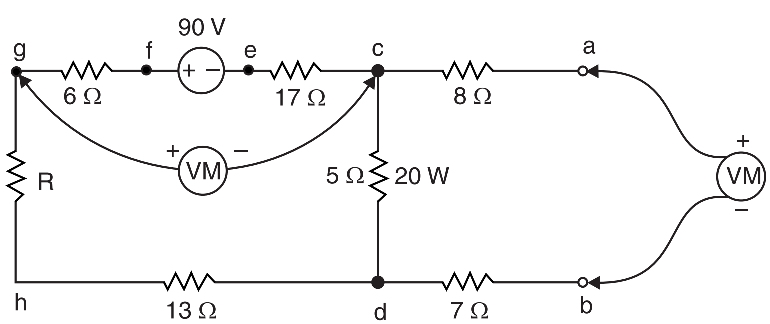


Fig. 2.76

* Note that point *a* is positive w.r.t. point *b*.

Solution. The polarity of 90 V source suggests that point *d* is positive w.r.t. *c*. Therefore, current flows from point *d* to *c*. The average power in 5 Ω resistor is 20 W so that $V_{dc}^2/5 = 20$. Therefore, $V_{dc} = 10$ V. An *ideal* voltmeter has an infinite resistance and indicates the voltage without drawing any current.

(*i*) Applying *KVL* to loop *acdba,* we have,

$$V_{ac} + V_{cd} + V_{db} + V_{ba} = 0$$

or $$0 + 10 + 0 + V_{ba} = 0 \qquad \therefore \qquad V_{ba} = \mathbf{-10\ V}$$

If the meter is of digital type, it will indicate – 10 V. For moving-coil galvanometer, the leads of voltmeter will be reversed to obtain the reading.

(*ii*) Applying *KVL* to loop *cefgc*, we have,

$$-V_{ce} + V_{ef} - V_{fg} - V_{gc} = 0$$

or $$-17 \times 2 + 90 - 6 \times 2 - V_{gc} = 0 \qquad \therefore \qquad V_{gc} = \mathbf{44\ V}$$

Example 2.47. *Using Kirchhoff's current law and Ohm's law, find the magnitude and polarity of voltage V in Fig. 2.77. Directions of the two current sources are shown.*

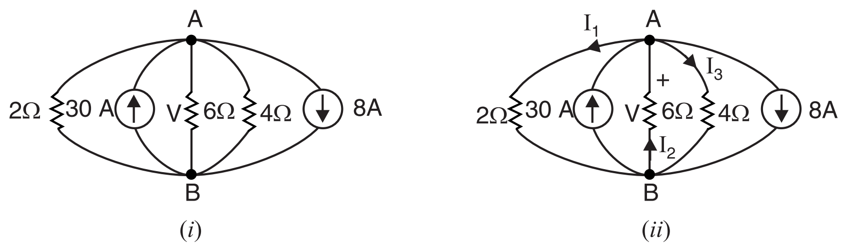


Fig. 2.77

Solution. Let us assign the directions of I_1, I_2 and I_3 and polarity of *V* as shown in Fig. 2.77 (*ii*). We shall see in the final result whether our assumptions are correct or not. Referring to Fig. 2.77 (*ii*) and applying *KCL* to junction *A*, we have,

$$\text{Incoming currents} = \text{Outgoing currents}$$

or $$I_2 + 30 = I_1 + I_3 + 8$$

$$\therefore \qquad I_1 - I_2 + I_3 = 22 \qquad \text{...(i)}$$

Applying Ohm's law to Fig. 2.77 (*ii*), we have,

$$I_1 = \frac{V}{2} \quad ; \quad I_3 = \frac{V}{4} \quad ; \quad I_2 = -\frac{V}{6}$$

Putting these values of I_1, I_2 and I_3 in eq. (*i*), we have,

$$\frac{V}{2} - \left(-\frac{V}{6}\right) + \frac{V}{4} = 22 \quad \text{or} \quad V = \mathbf{24\ V}$$

Now $$I_1 = V/2 = 24/2 = 12\text{ A} \quad ; \quad I_2 = -24/6 = -4\text{ A} \quad ; \quad I_3 = 24/4 = 6\text{ A}$$

The negative sign of I_2 indicates that the direction of its flow is opposite to that shown in Fig. 2.77 (*ii*).

Example 2.48. *In the network shown in Fig. 2.78,* $v_1 = 4$ *volts ;* $v_4 = 4 \cos 2t$ *and* $i_3 = 2e^{-t/3}$. *Determine* i_2.

Solution. Voltage across 6 H coil is

$$v_3 = L\frac{di_3}{dt} = L\frac{d}{dt}(i_3)$$

$$= 6\frac{d}{dt}(2e^{-t/3}) = -4e^{-t/3}$$

Applying *KVL* to loop *ABCDA*, we have,

$$-v_1 - v_2 + v_3 + v_4 = 0$$

or $$-4 - v_2 - 4e^{-t/3} + 4\cos 2t = 0$$

$\therefore$ $$v_2 = 4\cos 2t - 4e^{-t/3} - 4$$

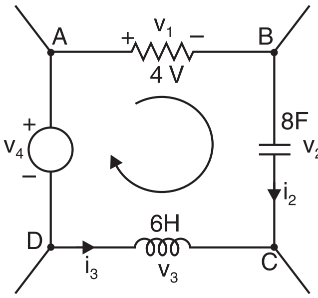


Fig. 2.78

Current through 8 F capacitor is

$$i_2 = C\frac{dv_2}{dt} = C\frac{d}{dt}(v_2)$$

$$= 8\frac{d}{dt}(4\cos 2t - 4e^{-t/3} - 4)$$

$$= 8\left(-8\sin 2t + \frac{4}{3}e^{-t/3}\right)$$

$$= \mathbf{-64\sin 2t + \frac{32}{3}e^{-t/3}}$$

Tutorial Problems

1. Using Kirchhoff's laws, find the current in various resistors in the circuit shown in Fig. 2.79.

[6.574 A, 3.611 A ,10.185 A]

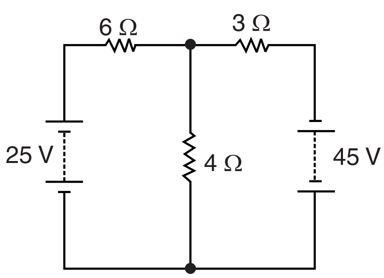


Fig. 2.79

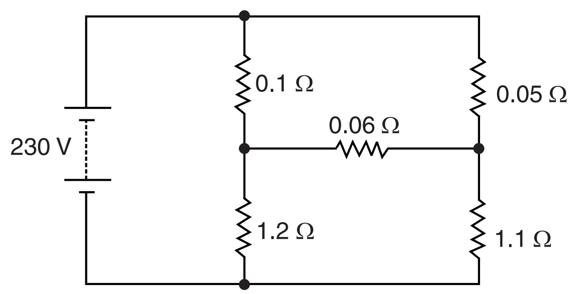


Fig. 2.80

2. For the circuit shown in Fig. 2.80, determine the branch currents using Kirchhoff's laws.

[151.35A, 224.55A, 27.7A , 179.05 A, 196.84 A]

3. Two batteries *A* and *B* having e.m.fs. 12 V and 8 V respectively and internal resistances of 2 Ω and 1 Ω respectively, are connected in parallel across 10 Ω resistor. Calculate (*i*) the current in each of the batteries and the external resistor and (*ii*) p.d. across external resistor.

[(*i*) I_A = 1.625 A *discharge* ; I_B = 0.75 A *charge*; 0.875 A (*ii*) 8.75 V]

4. A Wheatstone bridge *ABCD* is arranged as follows : *AB* = 20 Ω, *BC* = 5 Ω, *CD* = 4 Ω and *DA* = 10 Ω. A galvanometer of resistance 6Ω is connected between *B* and *D*. A 100-volt supply of negligible resistance is connected between *A* and *C* with *A* positive. Find the magnitude and direction of galvanometer current.

[0.667 A *from D to B*]

5. A network *ABCD* consists of the following resistors : *AB* = 5 kΩ, *BC* = 10 kΩ, *CD* = 15 kΩ and *DA* = 20 kΩ. A fifth resistor of 10 kΩ is connected between *A* and *C*. A dry battery of e.m.f. 120 V and internal resistance 500Ω is connected across the resistor *AD*. Calculate (*i*) the total current supplied by the battery, (*ii*) the p.d. across points *C* and *D* and (*iii*) the magnitude and direction of current through branch *AC*. **[(*i*) 11.17 mA (*ii*) 81.72 V (*iii*) 3.27 mA *from A to C*]**

6. A Wheatstone bridge *ABCD* is arranged as follows : *AB* = 10 Ω, *BC* = 30 Ω, *CD* = 15Ω and *DA* = 20Ω. A 2 volt battery of internal resistance 2Ω is connected between *A* and *C* with *A* positive. A galvanometer of resistance 40Ω is connected between *B* and *D*. Find the magnitude and direction of galvanometer current. **[11.5 mA *from B to D*]**

7. Two batteries E_1 and E_2 having *e.m.fs* 6 V and 2 V respectively and internal resistances of 2 Ω and 3 Ω respectively are connected in parallel across a 5 Ω resistor. Calculate (*i*) current through each battery and (*ii*) terminal voltage. **[(*i*) 1.23A; –0.52A (*ii*) 3.55V]**

8. Calculate the current in 20 Ω resistor in Fig. 2.81. **[26.67 mA]**

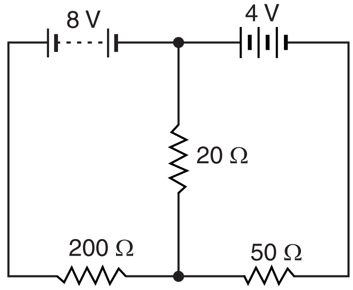


Fig. 2.81

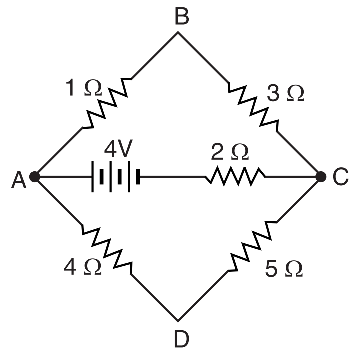


Fig. 2.82

9. In the circuit shown in Fig. 2.82, find the current in each branch and the current in the battery. What is the p.d. between *A* and *C* ?

[Branch *ABC* = 0.581 A ; Branch *ADC* = 0.258 A ; Branch AC = 0.839 A ; V_{AC} = 2.32 V]

10. Two batteries *A* and *B* having e.m.f.s of 20 V and 21 V respectively and internal resistances of 0.8 Ω and 0.2 Ω respectively, are connected in parallel across 50 Ω resistor. Calculate (*i*) the current through each battery and (*ii*) the terminal voltage. **[(*i*) Battery *A* = 0.4725 A ; Battery *B* = 0.0714 A (*ii*) 20 V]**

11. A battery having an e.m.f. of 10 V and internal resistance 0.01 Ω is connected in parallel with a second battery of e.m.f. 10 V and internal resistance 0.008 Ω. The two batteries in parallel are properly connected for charging from a d.c. supply of 20 V through a 0.9 Ω resistor. Calculate the current taken by each battery and the current from the supply. **[4.91 A, 6.14 A, 10.05 A]**

12. Find i_x and v_x in the network shown in Fig. 2.83. **[i_x = – 5 A; v_x = – 15 V]**

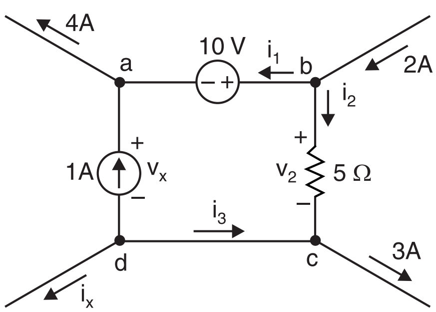


Fig. 2.83

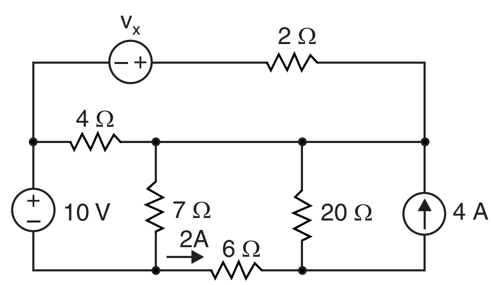


Fig. 2.84

13. Find v_x for the network shown in Fig. 2.84. **[31 V]**

14. Find *i* and v_{ab} for the network shown in Fig. 2.85. **[3 A ; 19 V]**

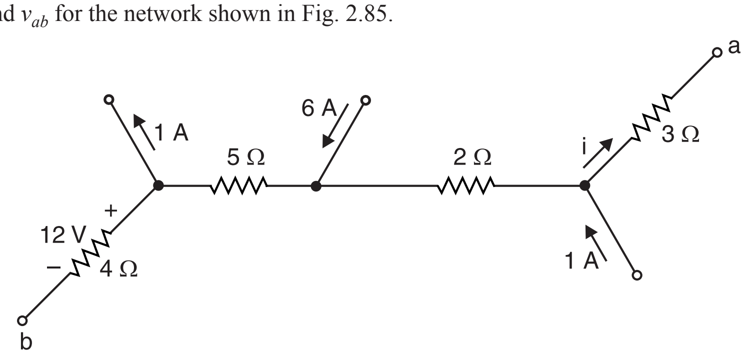


Fig. 2.85

2.22. Voltage and Current Sources

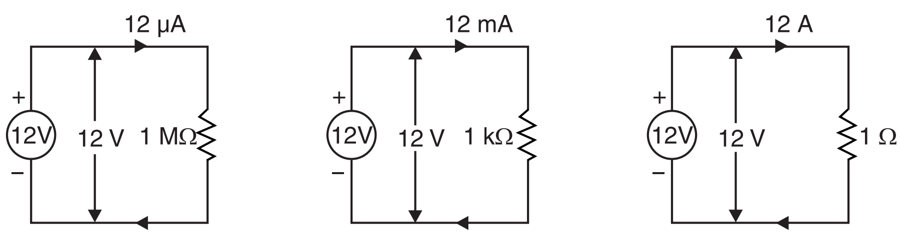
The term *voltage source* is used to describe a source of energy which establishes a potential difference across its terminals. Most of the sources encountered in everyday life are voltage sources *e.g.*, batteries, d.c. generators, alternators etc. The term *current source* is used to describe a source of energy that provides a current *e.g.*, collector circuits of transistors. Voltage and current sources are called active elements because they provide electrical energy to a circuit.

Unlike a voltage source, which we can imagine as two oppositely charged electrodes, it is difficult to visualise the structure of a current source. However, as we will learn in later sections, a real current source can always be converted into a real voltage source. In other words, we can regard a current source as a convenient fiction that aids in solving circuit problems, yet we feel secure in the knowledge that the current source can be replaced by the equivalent voltage source, if so desired.

2.23. Ideal Voltage Source or Constant-Voltage Source

An **ideal voltage source** (*also called constant-voltage source*) *is one that maintains a constant terminal voltage, no matter how much current is drawn from it.*

An ideal voltage source has zero internal resistance. Therefore, it would provide constant terminal voltage regardless of the value of load connected across its terminals. For example, an ideal 12V source would maintain 12V across its terminals when a 1 MΩ resistor is connected (so I = 12 V/1 MΩ = 12A) as well as when a 1 kΩ resistor is connected (I = 12 mA) or when a 1 Ω resistor is connected (I = 12A). This is illustrated in Fig. 2.86.



(*i*) (*ii*) (*iii*)

Fig. 2.86

It is not possible to construct an ideal voltage source because every voltage source has some internal resistance that causes the terminal voltage to fall due to the flow of current. *However, if the internal resistance of voltage source is very small, it can be considered as a constant voltage source.* This is illustrated in Fig. 2.87. It has a d.c. source of 6 V with an internal resistance R_i = 0.005 Ω. If the load current varies over a wide range of 1 to 10 A, for any of these values, the internal drop across R_i (= 0.005 Ω) is less than 0.05 volt. Therefore, the voltage output of the source is between 5.995 and 5.95 volts. This can be considered constant voltage compared with wide variations in load current. The practical example of a constant voltage source is the lead-acid cell. The internal resistance of lead-acid cell is very small (about 0.01 Ω) so that it can be regarded as a constant voltage source for all practical purposes. A constant voltage source is represented by the symbol shown in Fig. 2.88.

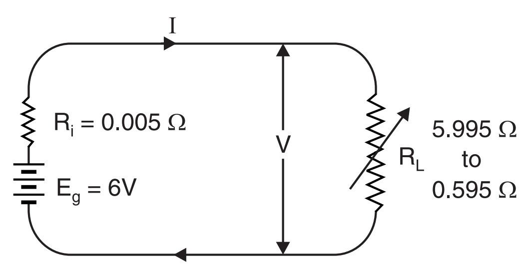


Fig. 2.87

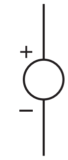


Fig. 2.88

2.24. Real Voltage Source

A real or non-ideal voltage source has low but finite internal resistance (R_{int}) that causes its terminal voltage to decrease when load current is increased and *vice-versa. A* **real voltage source** *can be represented as an ideal voltage source in series with a resistance equal to its internal resistance* (R_{int}) *as shown in Fig. 2.89.*

When load R_L is connected across the terminals of a real voltage source, a load current I_L flows through the circuit so that output voltage V_o is given by ;

$$V_o = E - I_L R_{int}$$

Here E is the voltage of the ideal voltage source *i.e.*, it is the potential difference between the terminals of the source when no current (*i.e.*, $I_L = 0$) is drawn. Fig. 2.90 shows the graph of output voltage V_o versus load current I_L of a real or non-ideal voltage source.

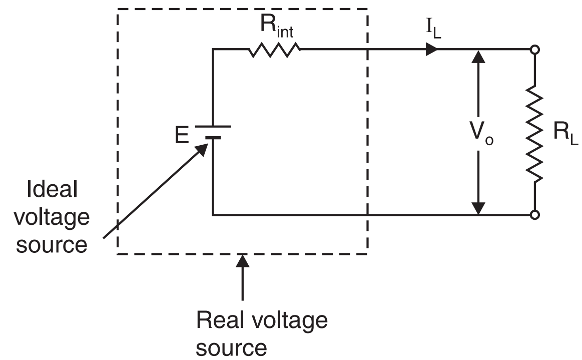


Fig. 2.89

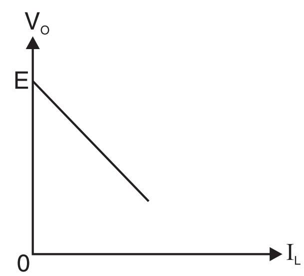


Fig. 2.90

As R_{int} becomes smaller, the real voltage source more closely approaches the ideal voltage source. Sometimes it is convenient when analysing electric circuits to assume that a real voltage source behaves like an ideal voltage source. This assumption is justified by the fact that in circuit analysis, we are not usually concerned with changing currents over a wide range of values.

2.25. Ideal Current Source

An **ideal current source** *or* **constant current source** *is one which will supply the same current to any resistance (load) connected across its terminals.*

An ideal current source has infinite internal resistance. Therefore, it supplies the same current to any resistance connected across its terminals. This is illustrated in Fig. 2.91. The symbol for ideal current source is shown in Fig. 2.92. The arrow shows the direction of current (conventional) produced by the current source.

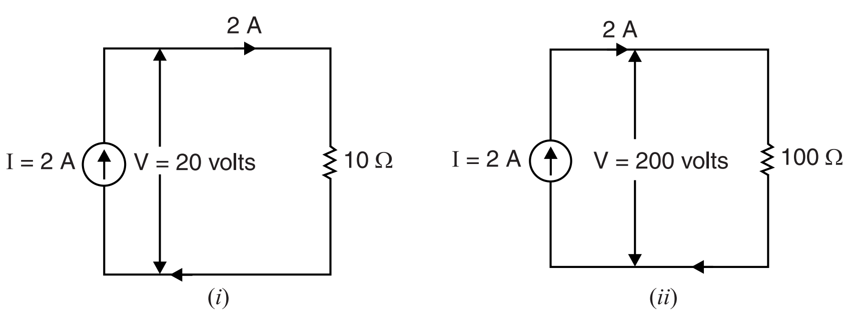


Fig. 2.91

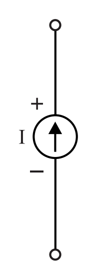


Fig. 2.92

Since an ideal current source supplies the same current regardless of the value of resistance connected across its terminals, it is clear that the terminal voltage V of the current source will depend on the value of load resistance. For example, if a 2 A current source has 10 Ω across its terminals, then terminal voltage of the source is $V = 2\text{ A} \times 10\ \Omega = 20$ volts. If load resistance is changed to 100 Ω, then terminal voltage of the current source becomes $V = 2\text{ A} \times 100\ \Omega = 200$ volts. This is illustrated in Fig. 2.91.

2.26. Real Current Source

A real or non-ideal current source has high but finite internal resistance (R_{int}). Therefore, the load current (I_L) will change as the value of load resistance (R_L) changes. *A* **real current source** *can be represented by an ideal current source* (I) *in parallel with its internal resistance* (R_{int}) *as shown in Fig. 2.93*. When load resistance R_L is connected across the terminals of the real current source, the load current I_L is equal to the current I from the ideal current source *minus* that part of the current that passes through the parallel internal resistance (R_{int}) *i.e.*,

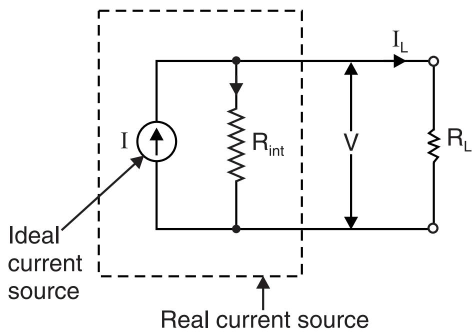


Fig. 2.93

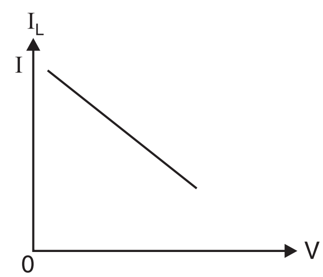


Fig. 2.94

$$I_L = I - \frac{V}{R_{int}}$$

where V = P.D. between output terminals

Fig. 2.94 shows the graph of load current I_L versus output voltage V of a real current source.

Note that load current I_L is less than it would be if the source were ideal. As the internal resistance of real current source becomes greater, the current source more closely approaches the ideal current source.

Note. Current sources in parallel add *algebraically*. If two current sources are supplying currents in the same direction, their equivalent current source supplies current equal to the sum of the individual currents. If two current sources are supplying currents in the opposite directions, their equivalent current source supplies a current equal to the difference of the currents of the two sources.

2.27. Source Conversion

A real voltage source can be converted to an *equivalent* real current source and *vice-versa*. When the conversion is made, the sources are equivalent in every sense of the word; it is impossible to make any measurement or perform any test at the external terminals that would reveal whether the source is a voltage source or its equivalent current source.

(*i*) Voltage to current source conversion. Let us see how a real voltage source can be converted to an equivalent current source. We know that a real voltage source can be represented by constant voltage E in series with its internal resistance R_{int} as shown in Fig. 2.95 (*i*).

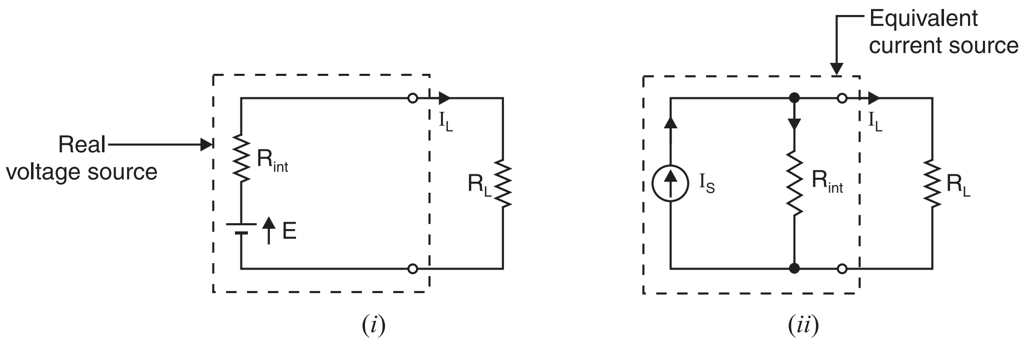


Fig. 2.95

It is clear from Fig. 2.95 (i) that load current I_L is given by ;

$$I_L = \frac{E}{R_{int} + R_L} = \frac{\dfrac{E}{R_{int}}}{\dfrac{R_{int} + R_L}{R_{int}}} = \frac{E}{R_{int}} \times \frac{R_{int}}{R_{int} + R_L}$$

$$\therefore \quad I_L = I_S \times \frac{R_{int}}{R_{int} + R_L} \quad \text{...(}i\text{)}$$

where $I_S = \dfrac{E^*}{R_{int}}$

= Current which would flow in a short circuit across the output terminals of voltage source in Fig. 2.95 (i)

From eq. (i), the voltage source appears as a current source of current I_S which is dividing between the internal resistance R_{int} and load resistance R_L connected in parallel as shown in Fig. 2.95 (ii). Thus the current source shown in Fig. 2.95 (ii) (dotted box) is equivalent to the real voltage source shown in Fig. 2.95 (i) (dotted box).

Thus a real voltage source of constant voltage E and internal resistance R_{int} is equivalent to a current source of current $I_S = E/R_{int}$ and R_{int} in parallel with current source.

Note that internal resistance of the equivalent current source has the same value as the internal resistance of the original voltage source but is in parallel with current source. The two circuits shown in Fig. 2.95 are equivalent and either can be used for circuit analysis.

(*ii*) Current to voltage source conversion. Fig. 2.96 (i) shows a real current source whereas Fig. 2.96 (ii) shows its equivalent voltage source. Note that series resistance R_{int} of the voltage source

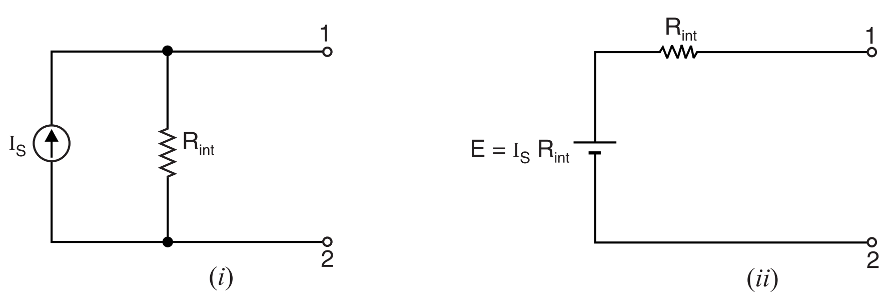


Fig. 2.96

* The source voltage is E and its internal resistance is R_{int}. Therefore, E/R_{int} is the current that would flow when source terminals in Fig. 2.95 (i) are shorted.

has the same value as the parallel resistance of the original current source. The value of voltage of the equivalent voltage source is $E = I_S R_{int}$ where I_S is the magnitude of current of the current source.

Note that the two circuits shown in Fig. 2.96 are equivalent and either can be used for circuit analysis.

Note. The source conversion (voltage source into equivalent current source and vice-versa) often simplifies the analysis of many circuits. Any resistance that is in series with a voltage source, whether it be internal or external resistance, can be included in its conversion to an equivalent current source. Similarly, any resistance in parallel with current source can be included when it is converted to an equivalent voltage source.

Example 2.49. *Show that the equivalent sources shown in Fig. 2.97 have exactly the same terminal voltage and produce exactly the same external current when the terminals (i) are shorted, (ii) are open and (iii) have a 500 Ω load connected.*

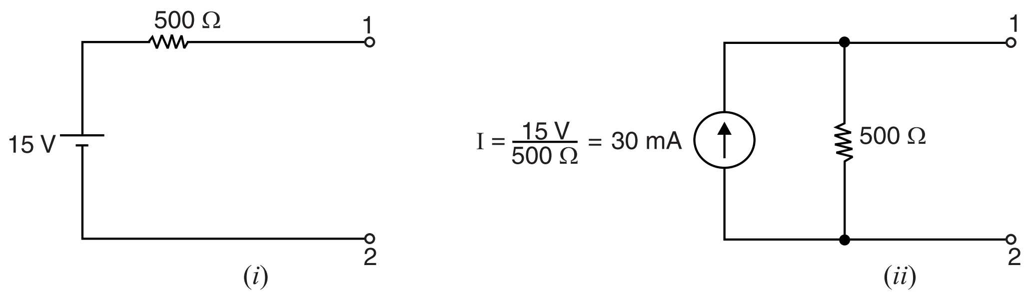


Fig. 2.97

Solution. Fig 2.97 (*i*) shows a voltage source whereas Fig. 2.97 (*ii*) shows its equivalent current source.

(*i*) When terminals are shorted. Referring to Fig. 2.98, the terminal voltage is 0 V in both circuits because the terminals are shorted.

$$I_L = \frac{15\text{ V}}{500\ \Omega} = 30\text{ mA ... voltage source}$$

$$I_L = 30\text{ mA ...current source}$$

Note that in case of current source, 30 mA flows in the shorted terminals because the short diverts all of the source current around the 500 Ω resistor.

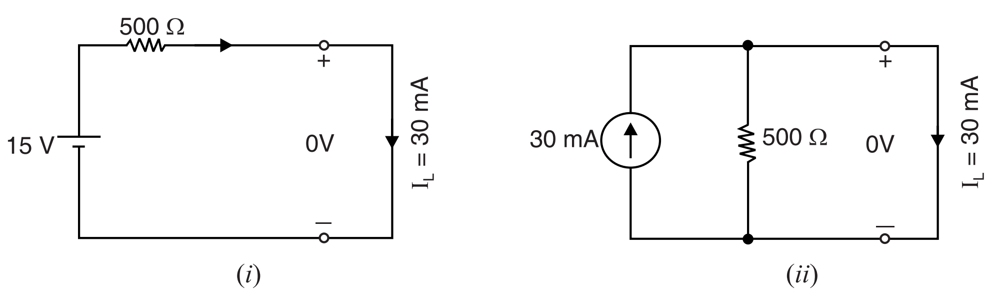


Fig. 2.98

(*ii*) When the terminals are open. Referring to Fig. 2.99 (*i*), the voltage across the open terminals of voltage source is 15 V because no current flows and there is no voltage drop across 500 Ω resistor. Referring to Fig. 2.99 (*ii*), the voltage across the open terminals of the current source is also 15 V ; $V = 30\text{ mA} \times 500\ \Omega = 15\text{ V}$. The current flowing from one terminal into the other is zero in both cases because the terminals are open.

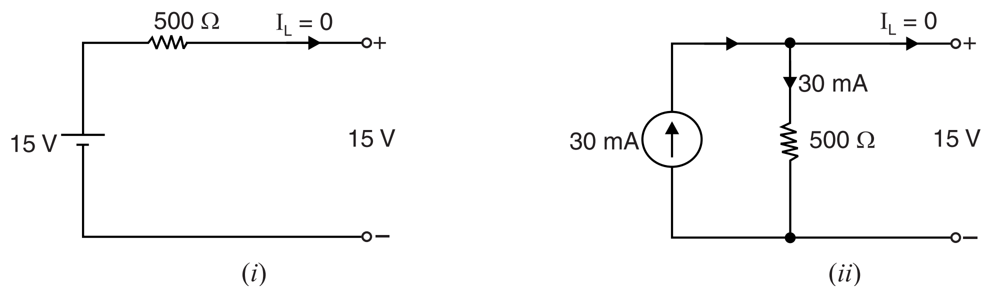


Fig. 2.99

***(iii)* Terminals have a 500 Ω load connected.**

***(a)* Voltage source.** Referring to Fig. 2.100 (*i*),

$$\text{Current in } R_L,\ I_L = \frac{15\ \text{V}}{(500 + 500)\ \Omega} = 15\ \text{mA}$$

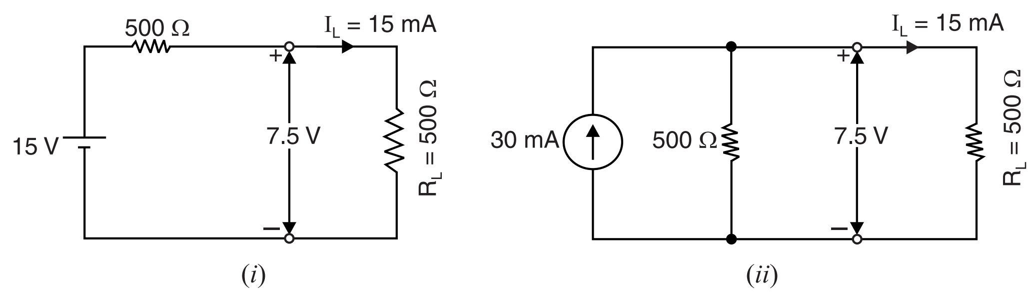


Fig. 2.100

Terminal voltage of source, $V = I_L R_L = 15\ \text{mA} \times 500\ \Omega = 7.5\ \text{V}$

***(b)* Current source.** Referring to Fig. 2.100 (*ii*),

$$\text{Current in } R_L,\ I_L = 30 \times \frac{500}{500 + 500} = 15\ \text{mA}$$

$$\text{Terminal voltage of source} = I_L R_L = 15\ \text{mA} \times 500\ \Omega = 7.5\ \text{V}$$

We conclude that equivalent sources produce exactly the same voltages and currents at their external terminals, no matter what the load and that they are therefore indistinguishable.

Example 2.50. *Find the current in 6 kΩ resistor in Fig. 2.101 (i) by converting the current source to a voltage source.*

Solution. Since we want to find the current in 6 kΩ resistor, we use 3 kΩ resistor to convert the current source to an equivalent voltage source. Referring to Fig. 2.101 (*ii*), the equivalent voltage is

$$E = 15\ \text{mA} \times 3\ \text{k}\Omega = 45\ \text{V}$$

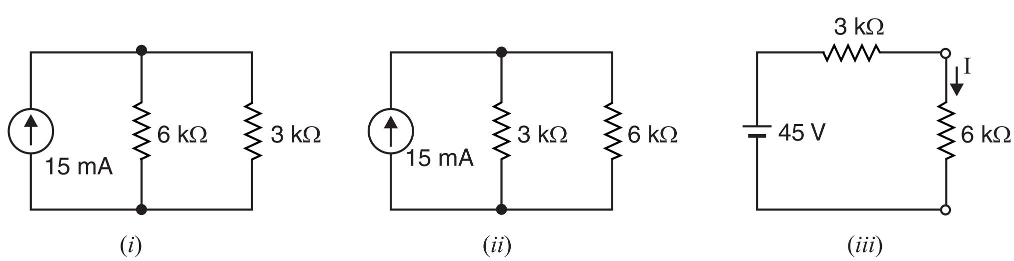


Fig. 2.101

The circuit then becomes as shown in Fig. 2.101 (*iii*). Note that polarity of the equivalent voltage source is such that it produces current in the same direction as the original current source.

Referring to Fig. 2.101 (*iii*), the current in 6 kΩ is

$$I = \frac{45\text{ V}}{(3+6)\text{ k}\Omega} = \mathbf{5\text{ mA}}$$

In the series circuit shown in Fig. 2.101 (*iii*), it would appear that current in 3 kΩ resistor is also 5 mA. However, 3 kΩ resistor was involved in source conversion, so we *cannot* conclude that there is 5 mA in the 3 kΩ resistor of the original circuit [See Fig. 2.101 (*i*)]. Verify that the current in the 3 kΩ resistor in that circuit is, in fact, 10 mA.

Example 2.51. *Find the current in the 3 kΩ resistor in Fig. 2.101* (*i*) *above by converting the current source to a voltage source.*

Solution. The circuit shown in Fig. 2.101 (*i*) is redrawn in Fig. 2.102 (*i*). Since we want to find the current in 3 kΩ resistor, we use 6 kΩ resistor to convert the current source to an equivalent voltage source. Referring to Fig. 2.102 (*i*), the equivalent voltage is

$$E = 15\text{ mA} \times 6\text{ k}\Omega = 90\text{ V}$$

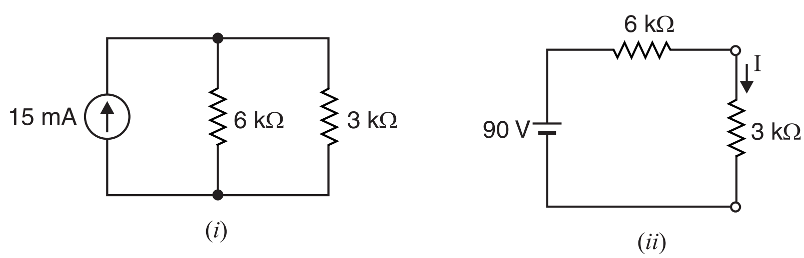


Fig. 2.102

The circuit then reduces to that shown in Fig. 2.102 (*ii*). The current in 3 kΩ resistor is

$$I = \frac{90\text{ V}}{(6+3)\text{k}\Omega} = \frac{90\text{ V}}{9\text{ k}\Omega} = \mathbf{10\text{ mA}}$$

Example 2.52. *Find the current in various resistors in the circuit shown in Fig. 2.103* (*i*) *by converting voltage sources into current sources.*

Solution. Referring to Fig. 2.103 (*i*), the 100 Ω resistor can be considered as the internal resistance of 15 V battery. The equivalent current is

$$I = \frac{15\text{ V}}{100\ \Omega} = 0.15\text{ A}$$

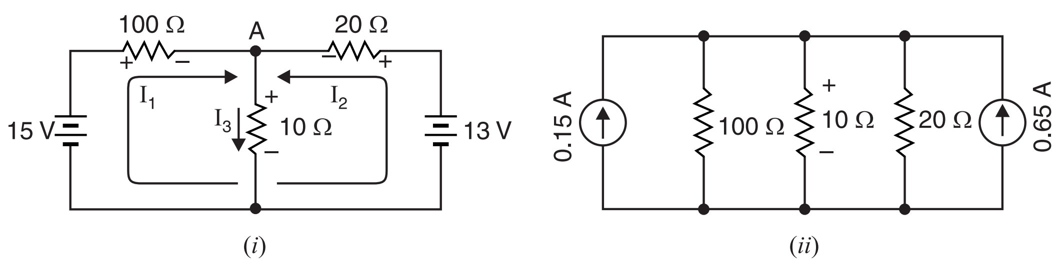


Fig. 2.103

Similarly, 20 Ω resistor can be considered as the internal resistance of 13 V battery. The equivalent current is

$$I = \frac{13\text{ V}}{20\ \Omega} = 0.65\text{ A}$$

Replacing the voltage sources with current sources, the circuit becomes as shown in Fig. 2.103 (*ii*). The current sources are parallel-aiding for a total flow = 0.15 + 0.65 = 0.8 A. The parallel resistors can be combined.

$$100\ \Omega \parallel 10\ \Omega \parallel 20\ \Omega = 6.25\ \Omega$$

The total current flowing through this resistance produces the drop :

$$0.8\text{ A} \times 6.25\ \Omega = 5\text{ V}$$

This 5 V drop can now be "transported" back to the original circuit. It appears across 10 Ω resistor [See Fig. 2.104]. Its polarity is negative at the bottom and positive at the top. Applying Kirchhoff's voltage law (KVL), the voltage drop across 100 Ω resistor = 15 – 5 = 10 V and drop across 20 Ω resistor = 13 – 5 = 8 V.

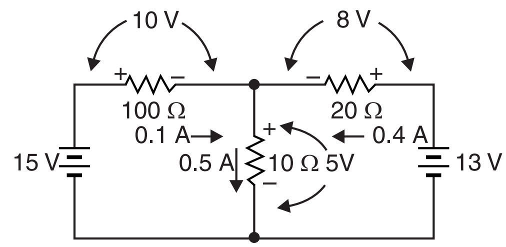


Fig. 2.104

$$\therefore \quad \text{Current in 100 } \Omega \text{ resistor} = \frac{10}{100} = \mathbf{0.1\ A}$$

$$\text{Current in 10 } \Omega \text{ resistor} = \frac{5}{10} = \mathbf{0.5\ A}$$

$$\text{Current in 20 } \Omega \text{ resistor} = \frac{8}{20} = \mathbf{0.4\ A}$$

Example 2.53. *Find the current in and voltage across 2 Ω resistor in Fig. 2.105.*

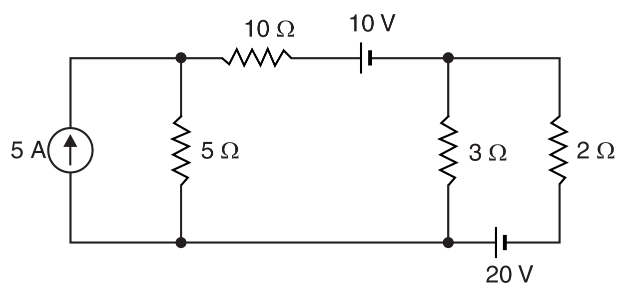


Fig. 2.105

Solution. We use 5 Ω resistor to convert the current source to an equivalent voltage source. The equivalent voltage is

$$E = 5\text{ A} \times 5\ \Omega = 25\text{ V}$$

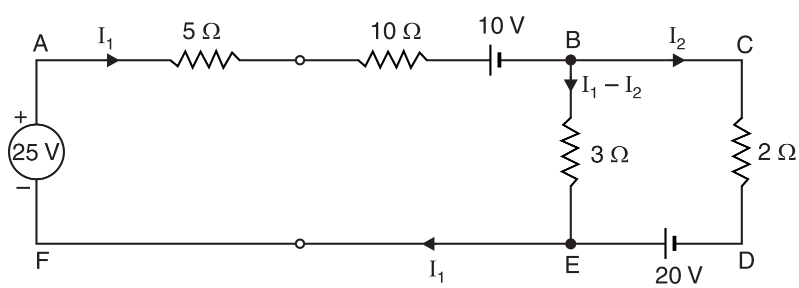


Fig. 2.106

The circuit shown in Fig. 2.105 then becomes as shown in Fig. 2.106.

Loop *ABEFA*. Applying Kirchhoff's voltage law to loop *ABEFA*, we have,

$$-5\,I_1 - 10\,I_1 - 10 - 3\,(I_1 - I_2) + 25 = 0$$

or $$-18\,I_1 + 3\,I_2 = -15 \qquad ...(i)$$

Loop *BCDEB*. Applying Kirchhoff's voltage law to loop *BCDEB*, we have,

$$-2\,I_2 + 20 + 3\,(I_1 - I_2) = 0$$

or $$3\,I_1 - 5\,I_2 = -20 \qquad ...(ii)$$

Solving equations (*i*) and (*ii*), we get, $I_2 = 5$ A.

∴ Current through 2 Ω resistor = I_2 = **5 A**

Voltage across 2 Ω resistor = $I_2 \times 2 = 5 \times 2$ = **10 V**

Example 2.54. *Find the current in 28 Ω resistor in the circuit shown in Fig. 2.107.*

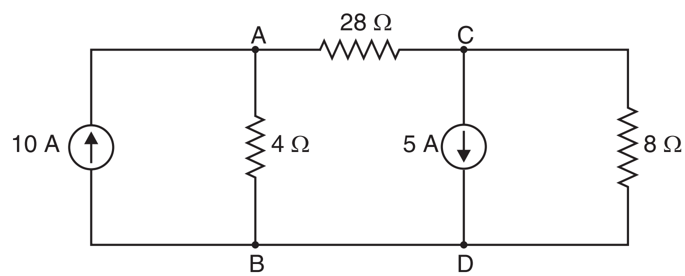
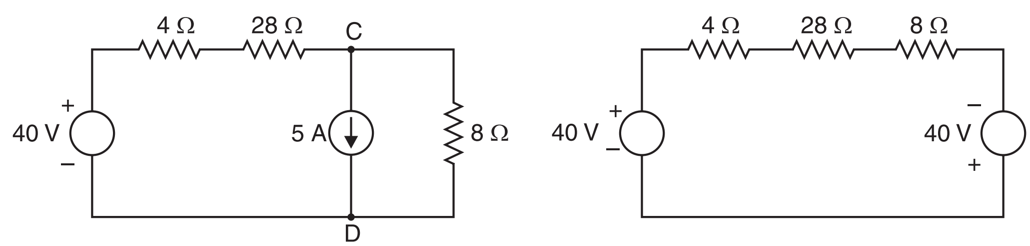


Fig. 2.107

Solution. The two current sources cannot be combined together because 28 Ω resistor is present between points *A* and *C*. However, this difficulty is overcome by converting current sources into equivalent voltage sources. Now 10 A current source in parallel with 4 Ω resistor can be converted into equivalent voltage source of voltage = 10 A × 4 Ω = 40 V in series with 4 Ω resistor as shown in Fig. 2.108 (*i*). Note that polarity of the equivalent voltage source is such that it provides current in the same direction as the original current source.



(*i*) (*ii*)

Fig. 2.108

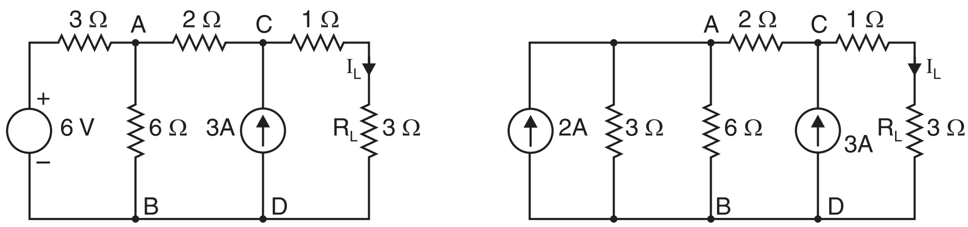
Similarly, 5 A current source in parallel with 8 Ω resistor can be converted into equivalent voltage source of voltage = 5 A × 8 Ω = 40 V in series with 8 Ω resistor. The circuit then becomes as shown in Fig. 2.108 (*ii*). Note that polarity of the voltage source is such that it provides current in the same direction as the original current source. Referring to Fig. 2.108 (*ii*),

Total circuit resistance = 4 + 28 + 8 = 40 Ω

Total voltage = 40 + 40 = 80 V

∴ Current in 28 Ω resistor = $\dfrac{80}{40}$ = **2 A**

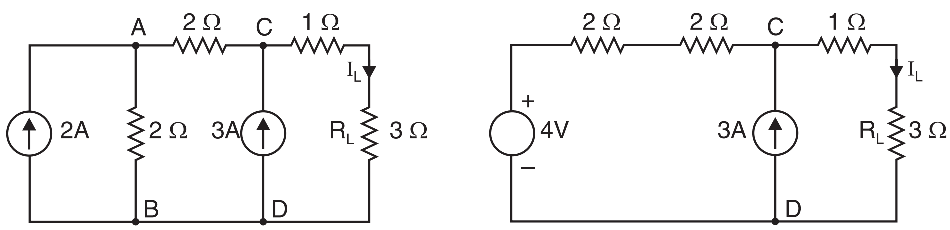
Example 2.55. *Using source conversion technique, find the load current I_L in the circuit shown in Fig. 2.109* (*i*).



(*i*) (*ii*)

Fig. 2.109

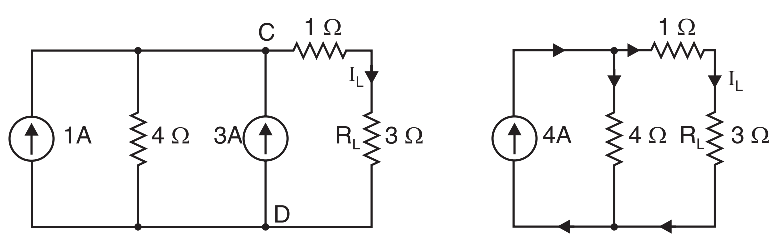
Solution. We first convert 6 V source in series with 3 Ω resistor into equivalent current source of current = 6 V/3 Ω = 2 A in parallel with 3 Ω resistor. The circuit then becomes as shown in Fig. 2.109 (*ii*). Note that polarity of current source is such that it provides current in the same direction as the original voltage source. In Fig. 2.109 (*ii*), 3 Ω and 6 Ω resistors are in parallel and their equivalent resistance = (3 × 6)/3 + 6 = 2 Ω. Therefore, circuit of Fig. 2.109 (*ii*) reduces to the one shown in Fig. 2.109 (*iii*).



(*iii*) (*iv*)

Fig. 2.109

In Fig. 2.109 (*iii*), we now convert 2 A current source in parallel with 2 Ω resistor into equivalent voltage source of voltage = 2 A × 2 Ω = 4 V in series with 2 Ω resistor. The circuit then becomes as shown in Fig. 2.109 (*iv*). The polarity of voltage source is marked correctly. In Fig. 2.109 (*iv*), we convert 4 V source in series with 2 + 2 = 4 Ω resistor into equivalent current source of current = 4 V/4 Ω = 1 A in parallel with 4 Ω resistor as shown in Fig. 2.109 (*v*). Note that direction of current of current source is shown correctly.



(*v*) (*vi*)

Fig. 2.109

In Fig. 2.109 (*v*), the two current sources can be combined together to give resultant current source of 3 + 1 = 4 A. The circuit then becomes as shown in Fig. 2.109 (*vi*). Referring to Fig. 2.109 (*vi*) and applying current-divider rule,

$$\text{Load current, } I_L = 4 \times \frac{4}{(3+1)+4} = \mathbf{2\ A}$$

Tutorial Problems

1. By performing an appropriate source conversion, find the voltage across 120 Ω resistor in the circuit shown in Fig. 2.110. **[20 V]**

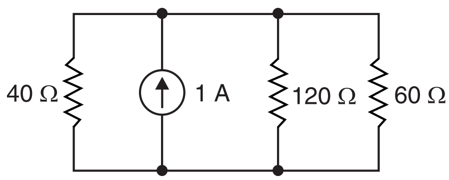


Fig. 2.110

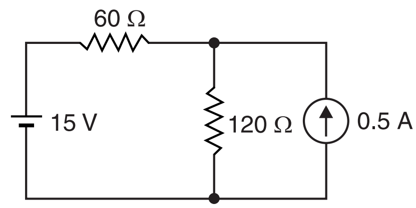


Fig. 2.111

2. By performing an appropriate source conversion, find the voltage across 120 Ω resistor in the circuit shown in Fig. 2.111. **[30 V]**

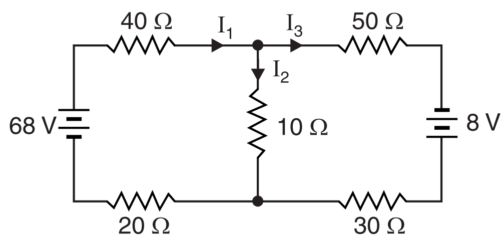


Fig. 2.112

3. By performing an appropriate source conversion, find the currents I_1, I_2 and I_3 in the circuit shown in Fig. 2.112. **[I_1 = 1 A; I_2 = 0.2 A; I_3 = 0.8 A]**

2.28. Independent Voltage and Current Sources

So far we have been dealing with independent voltage and current sources. We now give brief description about these two active elements.

(*i*) **Independent voltage source.** *An independent voltage source is a two-terminal element (e.g. a battery, a generator etc.) that maintains a specified voltage between its terminals.*

An independent voltage source provides a voltage independent of any other voltage or current. The symbol for independent voltage source having v volts across its terminals is shown in Fig. 2.113. (*i*). As shown, the terminal a is v volts above terminal b. If v is greater than zero, then terminal a is at a higher potential than terminal b. In Fig. 2.113 (*i*), the voltage v may be time varying or it may be constant in which case we label it V.

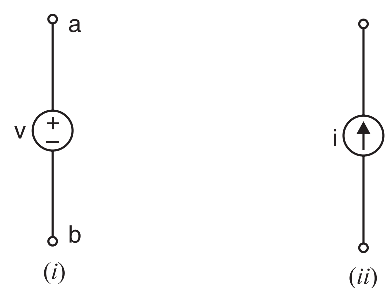


Fig. 2.113

(*ii*) **Independent current source.** *An independent current source is a two-terminal element through which a specified current flows.*

An independent current source provides a current that is completely independent of the voltage across the source. The symbol for an independent current source is shown in Fig. 2.113 (*ii*) where i is the specified current. The direction of the current is indicated by the arrow. In Fig. 2.113 (*ii*), the current i may be time varying or it may be constant in which case we label it I.

2.29. Dependent Voltage and Current Sources

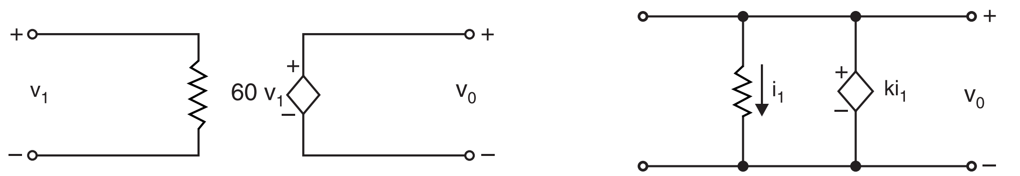
A dependent source provides a voltage or current between its output terminals which depends upon another variable such as voltage or current.

For example, a voltage amplifier can be considered to be a dependent voltage source. It is because the output voltage of the amplifier depends upon another voltage *i.e.* the input voltage to the amplifier. A dependent source is represented by a *diamond-shaped symbol as shown in the figures below. There are four possible dependent sources :

(*i*) Voltage-dependent voltage source (*ii*) Current-dependent voltage source

(*iii*) Voltage-dependent current source (*iv*) Current-dependent current source

(*i*) Voltage-dependent voltage source. *A voltage-dependent voltage source is one whose output voltage (v_0) depends upon or is controlled by an input voltage (v_1).* Fig. 2.114 (*i*) shows a voltage-dependent voltage source. Thus if in Fig. 2.114 (*i*), v_1 = 20 mV, then v_0 = 60 × 20 mV = 1.2 V. If v_1 changes to 30 mV, then v_0 changes to 60 × 30 mV = 1.8 V. Note that the constant (60) that multiplies v_1 is dimensionless.

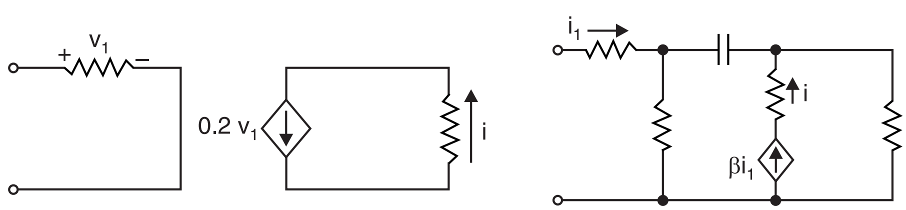


A voltage-dependent voltage source (*i*) A current-dependent voltage source (*ii*)

Fig. 2.114

(*ii*) Current-dependent voltage source. *A current-dependent voltage source is one whose output voltage (v_0) depends on or is controlled by an input current (i_1).* Fig. 2.114 (*ii*) shows a current-dependent voltage source. Note that the controlling current i_1 is in the same circuit as the controlled source itself. The constant that multiplies the value of voltage produced by the controlled source is sometimes designated by a letter k or β. Note that the constant k has the dimensions of V/A or ohm. Thus if i_1 = 50 µA and constant k is 0.5 V/A, then $v_0 = 50 \times 10^{-6} \times 0.5$ = 25 µV.

(*iii*) Voltage-dependent current source. *A voltage-dependent current source is one whose output current (i) depends upon or is controlled by an input voltage (v_1).* Fig. 2.115 (*i*) shows a voltage-dependent current source. The constant that multiplies the value of voltage v_1 has the dimensions of *A/V i.e.* mho or siemen. For example, in Fig. 2.115. (*i*), if the constant is 0.2 siemen and if input voltage v_1 is 10 mV, then the output current i = 0.2 S × 10 mV = 2 mA.



A voltage-dependent current source (*i*) A current-dependent current source (*ii*)

Fig. 2.115

* So as not to confuse with the symbol of independent source.

(*iv*) Current-dependent current source. *A current-dependent current source is one whose output current* (*i*) *depends upon or is controlled by an input current* (i_1). Fig. 2.115 (*ii*) shows a current-dependent current source. Note that controlling current i_1 is in the same circuit as the controlled source itself. The constant (β) that multiplies the value of current produced by the controlled source is dimensionless. Thus in Fig. 2.115 (*ii*), if $i_1 = 50$ μA and if constant β equals 100, then the current produced by the controlled current source is $i = 100 \times 50$ μA = 5 mA. If i_1 changes to 20 μA, then i changes to $i = 100 \times 20$ μA = 2 mA.

2.30. Circuits With Dependent-Sources

Fig. 2.116 shows the circuit that has an independent source, a dependent-source and two resistors. The dependent-source is a voltage source controlled by the current i_1. The constant for the dependent-source is 0.5 V/A. Dependent sources are essential components in amplifier circuits. Circuits containing dependent-sources are analysed in the same manner as those without dependent-sources. That is, Ohm's law for resistors and Kirchhoff's voltage and current laws apply, as well as the concepts of equivalent resistance and voltage and current division. We shall solve a few examples by way of illustration.

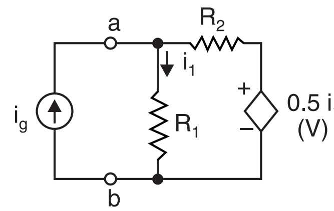


Fig. 2.116

Example 2.56. *Find the value of v in the circuit shown in Fig. 2.117. What is the value of dependent-current source ?*

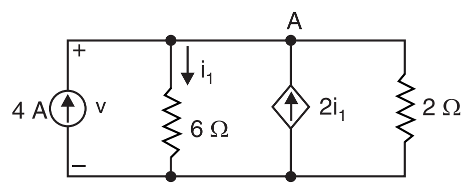


Fig. 2.117

Solution. By applying *KCL* to node* *A* in Fig. 2.117, we get,

$$4 - i_1 + 2i_1 = \frac{v}{2} \qquad ...(i)$$

By Ohm's law, $\quad i_1 = \dfrac{v}{6}$

Putting $i_1 = v/6$ in eq. (*i*), we get,

$$4 - \frac{v}{6} + \frac{2v}{6} = \frac{v}{2} \quad \therefore \quad v = \mathbf{12\ V}$$

Value of dependent-current source = =2 $i_1 = \dfrac{2v}{6} = \dfrac{2 \times 12}{6} = \mathbf{4\ A}$

Example 2.57. *Find the values of v, i_1 and i_2 in the circuit shown in Fig. 2.118 (i) which contains a voltage-dependent current source. Resistance values are in ohms.*

Solution. Applying *KCL* to node *A* in Fig. 2.118 (*i*), we get,

$$2 - i_1 + 4v = i_2 \qquad ...(i)$$

Now $\quad$ By Ohm's law, $i_1 = \dfrac{v}{3}$ and $i_2 = \dfrac{v}{6}$

Putting $\quad i_1 = \dfrac{v}{3}$ and $i_2 = \dfrac{v}{6}$ in eq. (*i*), we get,

$$2 - \frac{v}{3} + 4v = \frac{v}{6} \quad \therefore \quad v = \mathbf{\frac{-4}{7}\ V}$$

$$\therefore \quad i_1 = \frac{v}{3} = \frac{1}{3} \times v = \frac{1}{3} \times \frac{-4}{7} = \mathbf{\frac{-4}{21}\ A}$$

* A node of a network is an equipotential surface at which *two or more* circuit elements are joined.

$$\therefore \qquad i_2 = \frac{v}{6} = \frac{1}{6} \times v = \frac{1}{6} \times \frac{-4}{7} = \mathbf{\frac{-2}{21}\ A}$$

Value of dependent current source $= 4v = 4 \times \frac{-4}{7} = \frac{-16}{7}$ A

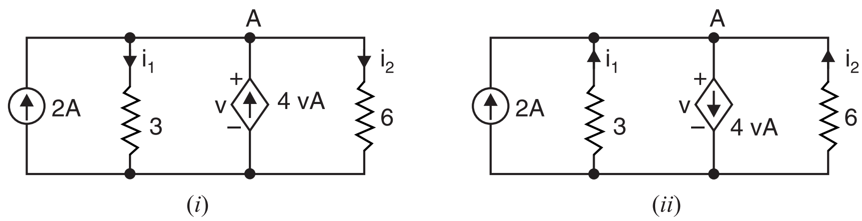


Fig. 2.118

Since the value of i_1, i_2 comes out to be negative, it means that directions of flow of currents are opposite to that assigned in Fig. 2.118. (*i*). The same is the case for current source. The actual directions are shown in Fig. 2.118 (*ii*).

Example 2.58. *Find the value of i in the circuit shown in Fig. 2.119 if R = 10 Ω.*

Solution. Applying *KVL* to the loop *ABEFA*, we have,

$$5 - 10\, i_1 + 5\, i_1 = 0 \qquad \therefore \quad i_1 = 1 \text{ A}$$

Applying *KVL* to the loop *BCDEB*, we have,

$$10\, i - 25 - 5\, i_1 = 0$$

or $$10\, i - 25 - 5 = 0 \qquad \therefore \quad i = \mathbf{3\ A}$$

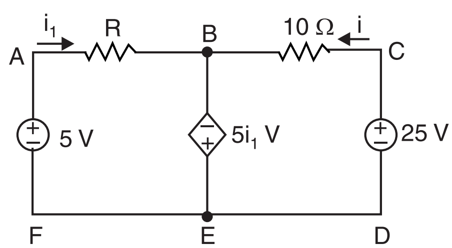


Fig. 2.119

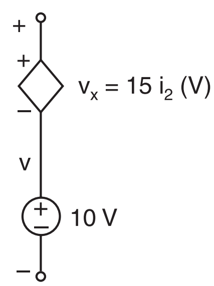


Fig. 2.120

Example 2.59. *Find the voltage v in the branch shown in Fig. 2.120. for (i) i_2 = 1 A, (ii) i_2 = – 2 A and (iii) i_2 = 0A.*

Solution. The voltage v is the sum of the current-independent 10 V source and the current-dependent voltage source v_x. Note the factor 15 multiplying the control current carries the units of ohm.

(*i*) $v = 10 + v_x = 10 + 15\,(1) = \mathbf{25\ V}$

(*ii*) $v = 10 + v_x = 10 + 15\,(-2) = \mathbf{-20\ V}$

(*iii*) $v = 10 + v_x = 10 + 15\,(0) = \mathbf{10\ V}$

Example 2.60. *Find the values of current i and voltage drops v_1 and v_2 in the circuit of Fig. 2.121 which contains a current-dependent voltage source. What is the voltage of the dependent-source? All resistance values are in ohms.*

Solution. Note that the factor 4 multiplying the control current carries the units of ohms. Applying *KVL* to the loop *ABCDA* in Fig. 2.121, we have,

$$-v_1 + 4\,i - v_2 + 6 = 0$$

or $$v_1 - 4\,i + v_2 = 6 \qquad ...(i)$$

By Ohm's law, $v_1 = 2\,i$ and $v_2 = 4\,i$.

Putting the values of $v_1 = 2\,i$ and $v_2 = 4\,i$ in eq. (*i*), we have,

$$2\,i - 4\,i + 4\,i = 6 \qquad \therefore \quad i = \mathbf{3\ A}$$

$$\therefore \qquad v_1 = 2i = 2 \times 3 = \mathbf{6\ V}\ ;\ v_2 = 4\,i = 4 \times 3 = \mathbf{12\ V}$$

Voltage of the dependent source = $4\,i = 4 \times 3 =$ **12 V**

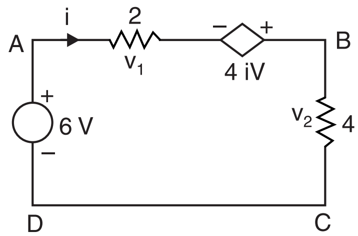


Fig. 2.121

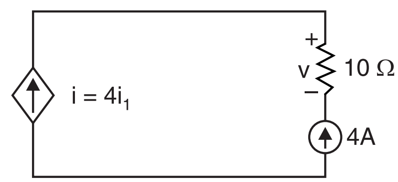


Fig. 2.122

Example 2.61. *Find the voltage v across the 10 Ω resistor in Fig. 2.122, if the control current i_1 in the dependent current-source is (i) 2A (ii) – 1A.*

Solution.

(*i*) $v = (i - 4)10 = [4\,(2) - 4]10 =$ **40 V**

(*ii*) $v = (i - 4)10 = [4(-1) - 4]\ 10 =$ **– 80 V**

Example 2.62. *Calculate the power delivered by the dependent-source in Fig. 2.123.*

Solution. Applying *KVL* to the loop *ABCDA*, we have,

$$-2\,I - 4\,I - 3\,I + 10 = 0$$

$$\therefore \qquad I = 10/9 = 1.11\ \text{A}$$

The current *I* enters the positive terminal of dependent-source. Therefore, power absorbed = 1.11 × 4 (1.11) = 4.93 watts. Hence power delivered is **– 4.93 W.**

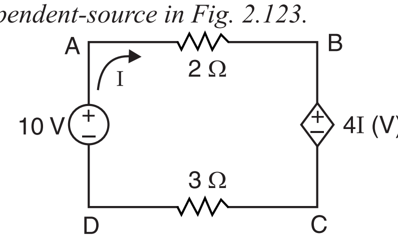


Fig. 2.123

Example 2.63. *In the circuit of Fig. 2.124, find the values of i and v. All resistances are in ohms.*

Solution. Referring to Fig. 2.124, it is clear that $v_a = 12 + v$.

Therefore, $\qquad v = v_a - 12$

Voltage drop across left 2 Ω resistor = $0 - v_a$

Voltage drop across top 2 Ω resistor = $v_a - 12$

Applying *KCL* to the node *a*, we have,

$$\frac{0 - v_a}{2} + \frac{v}{4} - \frac{v_a - 12}{2} = 0 \qquad \text{or} \quad v_a = 4\ \text{V}$$

$$\therefore \qquad v = v_a - 12 = 4 - 12 = \mathbf{-\,8V}$$

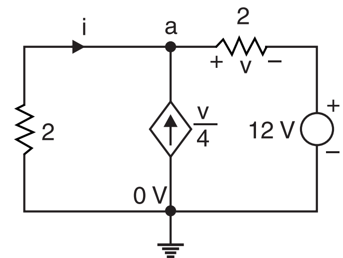


Fig. 2.124

The negative sign shows that the polarity of *v* is opposite to that shown in Fig. 2.124. The current that flows from point *a* to ground = 4/2 = 2 A. Hence *i* = **– 2 A**.

Example 2.64. *In Fig. 2.125, both independent and dependent-current sources drive current through resistor R. Is the value of R uniquely determined ?*

Solution. By definition of an independent source, the current *I* must be 10 A.

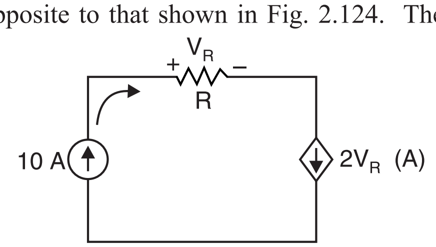


Fig. 2.125

$\therefore \qquad I = 10 \text{ A} = 2 V_R$

or $\qquad V_R = 10/2 = 5 \text{ V}$

Now $\qquad 5 \text{ V} = (10)(R) \quad \therefore R = 5/10 =$ **0.5 Ω**

No other value of R is possible.

Example 2.65. *Find the value of current i_2 supplied by the voltage-controlled current source (VCCS) shown in Fig. 2.126.*

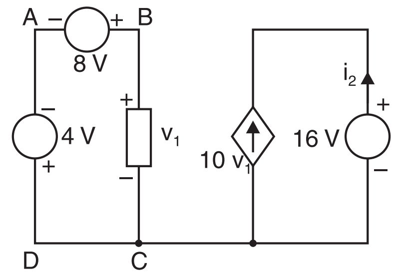


Fig. 2.126

Solution. Applying *KVL* to the loop *ABCDA*, we have,

$$8 - v_1 - 4 = 0 \quad \therefore v_1 = 4\text{V}$$

The current supplied by VCCS = 10 v_1 = 10 × 4 = 40A

As i_2 flows in opposite direction to this current, therefore, i_2 = **– 40A.**

Example 2.66. *By using voltage divider rule, calculate the voltages v_x and v_y in the circuit shown in Fig. 2.127.*

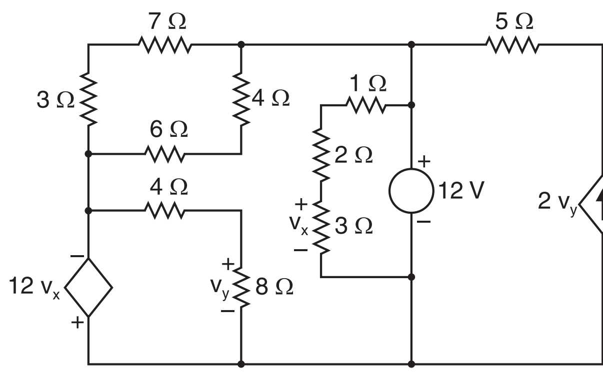


Fig. 2.127

Solution. As can be seen from Fig. 2.127, 12 V drop is over the series combination of 1Ω, 2Ω and 3Ω resistors. Therefore, by voltage divider rule,

Voltage drop over 3Ω, $v_x = 12 \times \dfrac{3}{1+2+3}$ = **6V**

$\therefore$ Voltage of dependent source = $12v_x$ = 12 × 6 = 72 V

As seen 72 V drop is over series combination of 4Ω and 8Ω resistors. Therefore, by voltage divider rule,

Voltage drop over 8Ω, $v_y = 72 \times \dfrac{8}{4+8}$ = 48 V

The actual sign of polarities of v_y is opposite to that shown in Fig. 2.127. Hence v_y = **– 48 V.**

Example 2.67. *Find the values of i_1, v_1, v_x and v_{ab} in the network shown in Fig. 2.128 with its terminals a and b open.*

Solution. It is clear from the circuit that i_1 = **4A.**

Applying *KVL* to the left-hand loop, we have,

$$20 - v_1 - 40 = 0 \quad \therefore v_1 = -\textbf{20 V}$$

Applying *KVL* to the second loop from left, we have,

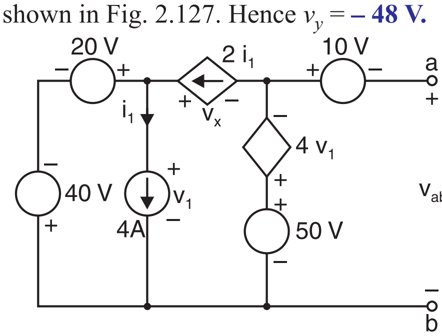


Fig. 2.128

$$-v_x + 4v_1 - 50 + v_1 = 0$$

$$\therefore \quad v_x = 5v_1 - 50 = 5(-20) - 50 = \mathbf{-150\ V}$$

Applying *KVL* to the third loop containing v_{ab}, we have,

$$-10 - v_{ab} + 50 - 4v_1 = 0$$

$$\therefore \quad v_{ab} = -10 + 50 - 4v_1 = -10 + 50 - 4(-20) = \mathbf{120\ V}$$

Tutorial Problems

1. The circuit of Fig. 2.129 contains a voltage-dependent voltage source. Find the current supplied by the battery and power supplied by the voltage source. **[8A; 1920 W]**

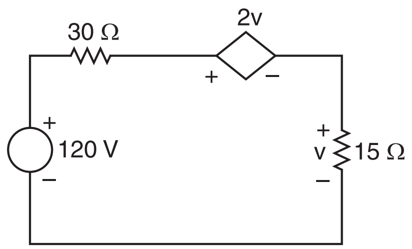


Fig. 2.129

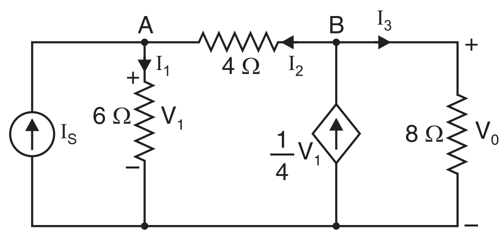


Fig. 2.130

2. Applying Kirchhoff's current law, determine current I_S in the electric circuit of Fig. 2.130. Take V_0 = 16V. **[1A]**

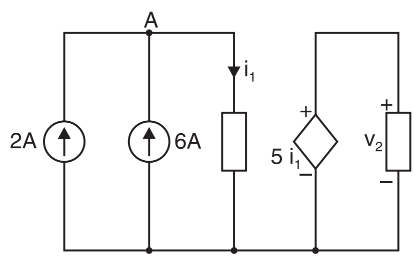


Fig. 2.131

3. Find the voltage drop v_2 across the current-controlled voltage source shown in Fig. 2.131. **[40 V]**

2.31. Ground

Voltage is relative. That is, the voltage at one point in a circuit is always measured relative to another point in the circuit. For example, if we say that voltage at a point in a circuit is + 100V, we mean that the point is 100V more positive than some reference point in the circuit. This reference point in a circuit is usually called the *ground point*. Thus ground is used as reference point for specifying voltages. The ground may be used as common connection (*common ground*) or as a zero reference point (*earth ground*). There are different symbols for chassis ground, common ground and earth ground as shown in Fig. 2.132. *However, earth ground symbol is often used in place of chassis ground or common ground.*

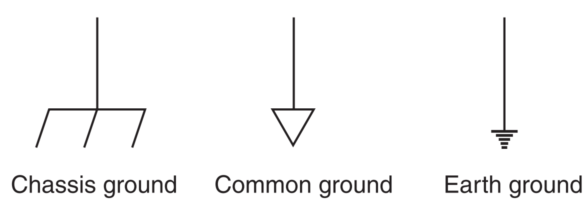


Fig. 2.132

(*i*) Ground as a common connection. It is a usual practice to mount the electronic and electrical components on a metal base called *chassis* (See Fig. 2.133). Since chassis is good conductor, it provides a conducting return path as shown in Fig. 2.134. It may be seen that all points connected to chassis are shown as grounded and represent the same potential. The adoption of this scheme (*i.e.* showing points of same potential as grounded) often simplifies the electrical and electronic circuits.

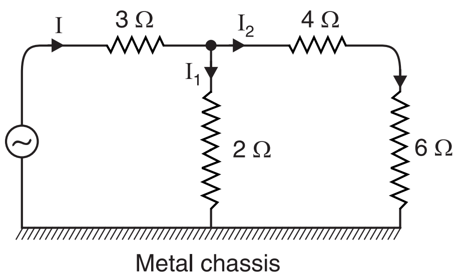


Fig. 2.133

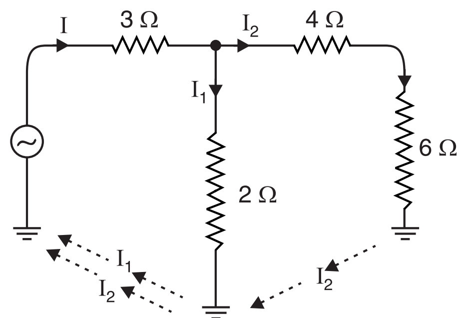
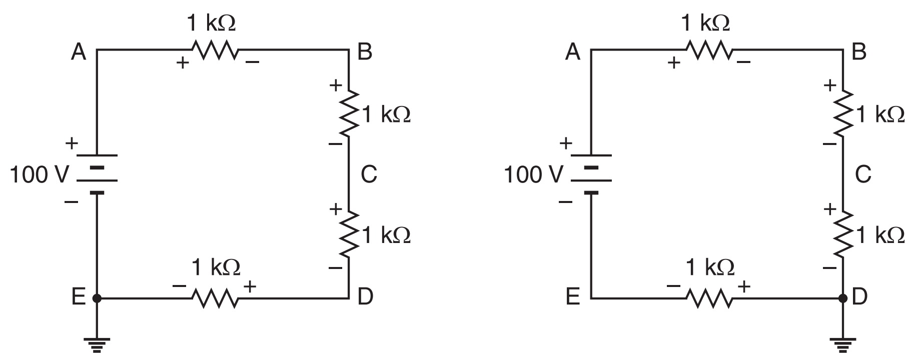


Fig. 2.134

(*ii*) Ground as a zero reference point. Many times connection is made to earth which acts as a reference point. The earth (ground) has a potential of zero volt (0V) with respect to all other points in the circuit. Thus in Fig. 2.135(*i*), point *E* is grounded (*i.e.,* point *E* is connected to earth) and has zero potential. The voltage across each resistor is 25 volts. The voltages of the various points with respect to ground or earth (*i.e.,* point *E*) are :

$$V_E = 0\text{V} \;;\; V_D = +25\text{ V} \;;\; V_C = +50\text{ V} \;;\; V_B = +75\text{ V} \;;\; V_A = +100\text{V}$$



(*i*) (*ii*)

Fig. 2.135

If instead of point *E*, the point *D* is grounded as shown in Fig. 2.135 (*ii*), then potentials of various points with respect to ground (*i.e.,* point *D*) will be :

$$V_E = -25\text{ V} \;;\; V_D = 0\text{ V} \;;\; V_C = +25\text{ V} \;;\; V_B = +50\text{ V} \;;\; V_A = +75\text{ V}$$

Example 2.68. *In Fig. 2.136, find the relative potentials of points A, B, C, D and E when point A is grounded.*

Solution. Net circuit voltage, $V = 34 - 10 = 24$ V

Total circuit resistance, $R_T = 6 + 4 + 2 = 12\ \Omega$

Circuit current, $I = V/R_T = 24/12 = 2$ A

Drop across 2 Ω resistor $= 2 \times 2 = 4$ V

Drop across 4 Ω resistor $= 2 \times 4 = 8$ V

Drop across 6 Ω resistor $= 2 \times 6 = 12$ V

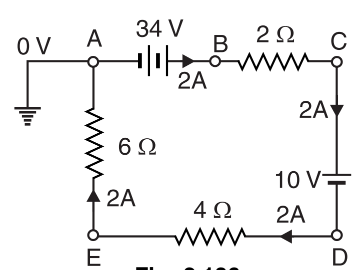


Fig. 2.136

$\therefore$ Potential at point B, V_B = 34 – 0 = **34 V**

Potential at point C, V_C = 34 – drop in 2 Ω

= 34 – 2 × 2 = **30 V**

Potential at point D, V_D = V_C – 10 = 30 – 10 = **20 V**

Potential at point E, V_E = V_D – drop in 4Ω = 20 – 2 × 4 = **12 V**

Potential at point A, V_A = V_E – drop in 6 Ω

= 12 – 6 × 2 = **0 V**

Example 2.69. *Fig. 2.137 shows the circuit with common ground symbols. Find the total current I drawn from the 25 V source.*

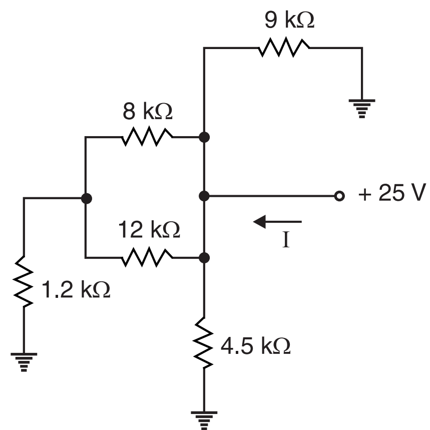
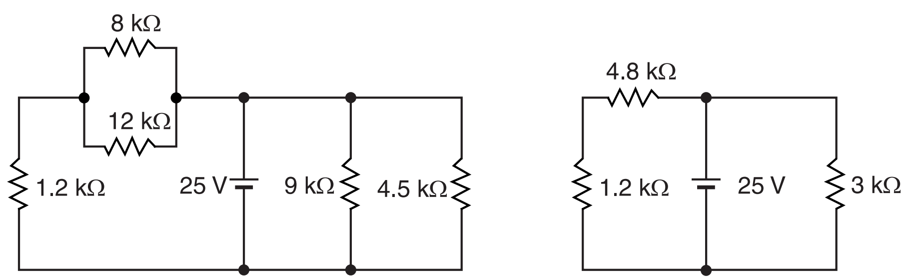


Fig. 2.137

Solution. The circuit shown in Fig. 2.137 is redrawn by eliminating the common ground symbols. The equivalent circuit then becomes as shown in Fig. 2.138. (*i*). We see that 8 kΩ and 12 kΩ resistors are in parallel as are the 9 kΩ and 4.5 kΩ resistors. Fig. 2.138 (*ii*) shows the circuit when these parallel combinations are replaced by their equivalent resistances :

$$\frac{8 \times 12}{8+12} = 4.8 \text{ k}\Omega \quad \text{and} \quad \frac{9 \times 4.5}{9+4.5} = 3 \text{ k}\Omega$$

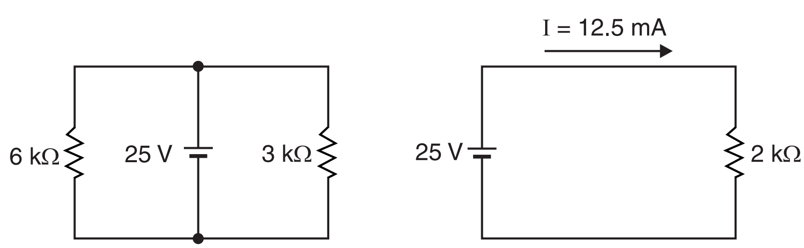


(*i*) (*ii*)

Fig. 2.138

Referring to Fig. 2.138 (*ii*), it is clear that 4.8 kΩ resistance is in series with 1.2 kΩ resistance, giving an equivalent resistance of 4.8 + 1.2 = 6 kΩ.

The circuit then becomes as shown in Fig. 2.139 (*i*).



(*i*) (*ii*)

Fig. 2.139

Referring to Fig. 2.139 (*i*), 6 kΩ is in parallel with 3 kΩ giving the total resistance R_T as :

$$R_T = \frac{6 \times 3}{6 + 3} = 2\ k\Omega$$

The circuit then reduces to the one shown in Fig. 2.139 (*ii*).

∴ Total current *I* drawn from 25 V source is

$$I = \frac{25\ V}{R_T} = \frac{25\ V}{2\ k\Omega} = \mathbf{12.5\ mA}$$

Example 2.70. *What is the potential difference between X and Y in the network shown in Fig. 2.140 ?*

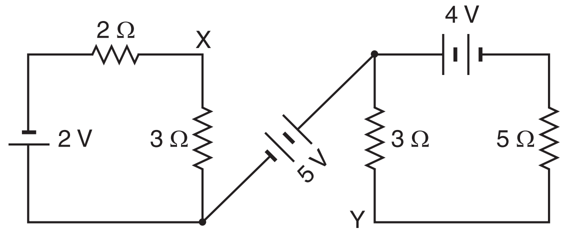


Fig. 2.140

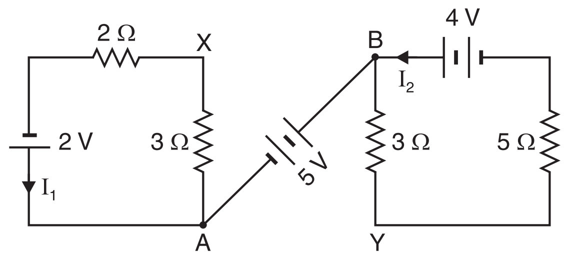


Fig. 2.141

Solution. Fig. 2.140 is reproduced as Fig 2.141 with required labeling. Consider the two battery circuits separately. Referring to Fig. 2.141,

Current flowing in 2Ω and 3Ω resistors is

$$I_1 = \frac{2}{2 + 3} = 0.4\text{A}$$

Current flowing in 3Ω and 5Ω resistors is

$$I_2 = \frac{4}{3 + 5} = 0.5\ \text{A}$$

∴ Potential difference between *X* and *Y* is

$$V_{XY} = V_{XA} + V_{AB} - V_{BY} \qquad \text{[See Fig. 2.141]}$$

$$= 3I_1 + 5 - 3I_2$$

$$= 3 \times 0.4 + 5 - 3 \times 0.5 = \mathbf{4.7\ V}$$

2.32. Voltage Divider Circuit

A **voltage divider** *(or potential divider) is a series circuit that is used to provide two or more reduced voltages from a single input voltage source.*

Fig. 2.142 shows a simple voltage divider circuit which provides two reduced voltages V_1 and V_2 from a single input voltage *V*. Since no load is connected to the circuit, it is called **unloaded voltage divider**. The values of V_1 and V_2 can be found as under :

$$\text{Circuit current, } I = \frac{V}{R_1 + R_2} = \frac{V}{R_T}$$

where R_T = Total resistance of the voltage divider

$$\therefore \quad V_1 = IR_1 = V \times \frac{R_1}{R_T}$$

and $$V_2 = IR_2 = V \times \frac{R_2}{R_T}$$

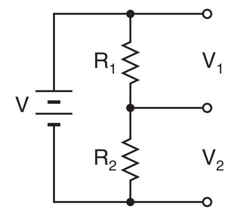


Fig. 2.142

Therefore, voltage drop across any resistor in an unloaded voltage divider is equal to the ratio of that resistance value to the total resistance multiplied by the source voltage.

Loaded voltage divider. When load R_L is connected to the output terminals of the voltage divider as shown in Fig. 2.143, the output voltage (V_2) is reduced by an amount depending on the value of R_L. It is because load resistor R_L is in parallel with R_2 and reduces the resistance from point *A* to point *B*. As a result, the output voltage is reduced. The larger the value of R_L, the less the output voltage is reduced from the unloaded value. Loading a voltage divider has the following effects :

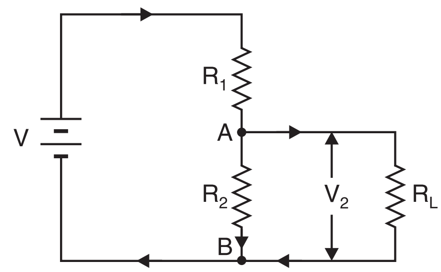


Fig. 2.143

(*i*) The output voltage is reduced depending upon the value of load resistance R_L.

(*ii*) The current drawn from the source is increased because total resistance of the circuit is reduced. The decrease in total resistance is due to the fact that loaded voltage divider becomes series-parallel circuit.

Example 2.71. *Design a voltage divider circuit that will operate the following loads from a 20 V source :*

5 V at 5 mA ; 12 V at 10 mA ; 15 V at 5 mA

The bleeder current is 4 mA.

Solution. A voltage divider that produces a *bleeder current requires $N + 1$ resistors where N is the number of loads. In this example, the number of loads is three. Therefore, four resistors are required for this voltage divider. The required circuit is shown in Fig. 2.144. Here R_1 is the bleeder resistor. The loads are arranged in ascending order of their voltage requirements, starting at the bottom of the divider network.

Voltage across bleeder resistor R_1 = 5 V ; Current through R_1, I_B = 4 mA .

$$\therefore \quad \text{Value of } R_1 = \frac{5\text{V}}{4\text{mA}} = \mathbf{1.25\ k\Omega}$$

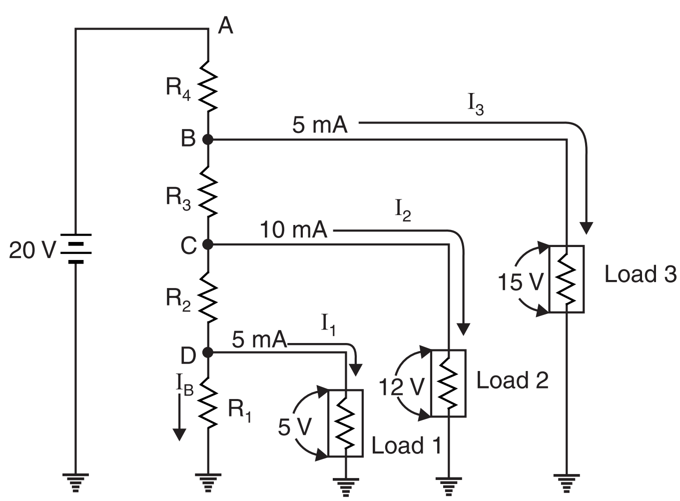


Fig. 2.144

Next we shall find the value of resistor R_2. For this purpose, we find the current through R_2 and voltage across R_2.

* The current drawn continuously from a power supply by the resistive voltage divider circuit is called bleeder current. Without a bleeder current, the voltage divider outputs go up to full value of supply voltage if all the loads are disconnected.

Current through $R_2 = I_B + 5$ mA = 4 mA + 5 mA = 9 mA

Voltage across $R_2 = V_C - V_D = 12 - 5 = 7$ V

$$\therefore \quad \text{Value of } R_2 = \frac{7\text{ V}}{9\text{ mA}} = \mathbf{778\ \Omega}$$

Now we shall find the value of resistor R_3.

Current through R_3 = Current in R_2 + 10 mA = 9 mA + 10 mA = 19 mA

Voltage across $R_3 = V_B - V_C = 15 - 12 = 3$ V

$$\therefore \quad \text{Value of } R_3 = \frac{3\text{ V}}{19\text{ mA}} = \mathbf{158\ \Omega}$$

Finally, we shall determine the value of resistor R_4.

Current through R_4 = Current through R_3 + 5 mA = 19 mA + 5 mA = 24 mA

Voltage across $R_4 = V_A - V_B = 20 - 15 = 5$ V

$$\therefore \quad \text{Value of } R_4 = \frac{5\text{ V}}{24\text{ mA}} = \mathbf{208\ \Omega}$$

The design of voltage divider circuit means finding the values of R_1, R_2, R_3 and R_4. Therefore, the design of voltage divider circuit stands completed.

Example 2.72. *Fig. 2.145 shows the voltage divider circuit. Find (i) the unloaded output voltage, (ii) the loaded output voltage for R_L = 10 kΩ and R_L = 100 kΩ.*

Solution. **(*i*)** When load R_L is removed, the voltage across R_2 is the unloaded output voltage of the voltage divider.

$$\therefore \quad \text{Unloaded output voltage} = \frac{R_2}{R_1 + R_2} \times V_S = \frac{10}{4.7 + 10} \times 5 = \mathbf{3.4\ V}$$

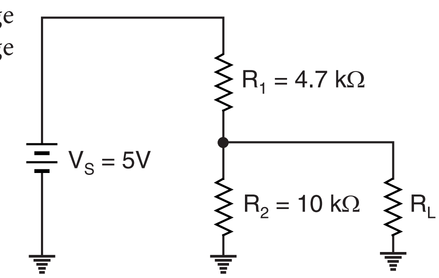


Fig. 2.145

(*ii*) When R_L = 10 kΩ is connected in parallel with R_2, then equivalent resistance of this parallel combination is

$$R_T = \frac{R_2 R_L}{R_2 + R_L} = \frac{10 \times 10}{10 + 10} = 5\text{ k}\Omega$$

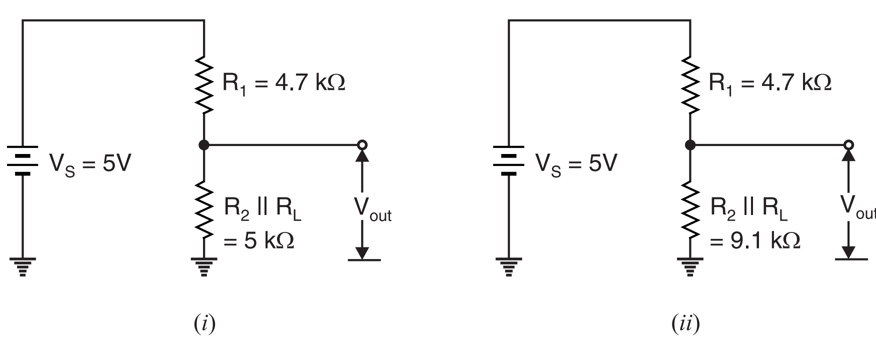


Fig. 2.146

The circuit then becomes as shown in Fig. 2.146 (*i*).

$$\therefore \quad \text{Loaded output voltage} = \frac{R_T}{R_1 + R_T} \times V_S = \frac{5}{4.7 + 5} \times 5 = \mathbf{2.58\ V}$$

When R_L = 100 kΩ is connected in parallel with R_2, then equivalent resistance of this parallel combination is given by ; $R'_T = \dfrac{R_2 R_L}{R_2 + R_L} = \dfrac{10 \times 100}{10 + 100}$ = 9.1 kΩ

The circuit then becomes as shown in Fig. 2.146 (*ii*).

$$\therefore \quad \text{Loaded output voltage} = \frac{R'_T}{R_1 + R'_T} \times V_S = \frac{9.1}{4.7 + 9.1} \times 5 = \mathbf{3.3\ V}$$

Example 2.73. *Find the values of different voltages that can be obtained from 25V source with the help of voltage divider circuit of Fig. 2.147.*

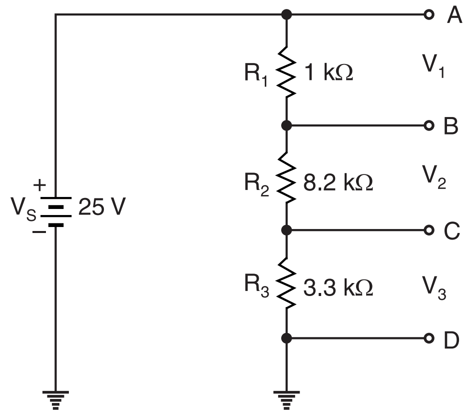


Fig. 2.147

Solution. Total circuit resistance, $R_T = R_1 + R_2 + R_3 = 1 + 8.2 + 3.3 = 12.5$ kΩ

$$\text{Voltage drop across } R_1,\ V_1 = \frac{R_1}{R_T} \times V_S = \frac{1}{12.5} \times 25 = 2\ \text{V}$$

$$\therefore \quad \text{Voltage at point } B,\ V_B = 25 - 2 = 23\ \text{V}$$

$$\text{Voltage drop across } R_2,\ V_2 = \frac{R_2}{R_T} \times V_S = \frac{8.2}{12.5} \times 25 = 16.4\ \text{V}$$

$$\therefore \quad \text{Voltage at point } C,\ V_C = V_B - V_2 = 23 - 16.4 = 6.6\ \text{V}$$

The different available load voltages are :

$V_{AB} = V_A - V_B = 25 - 23 =$ **2 V** ; $V_{AC} = V_A - V_C = 25 - 6.6 =$ **18.4 V**

$V_{BC} = V_B - V_C = 23 - 6.6 =$ **16.4 V** ; $V_{AD} =$ **25 V** ; $V_{CD} = V_C - V_D = 6.6 - 0 =$ **6.6 V**

$V_{BD} = V_B - V_D = 23 - 0 =$ **23 V**

Example 2.74. *Fig. 2.148 shows a 10 kΩ potentiometer connected in a series circuit as an adjustable voltage divider. What total range of voltage V_1 can be obtained by adjusting the potentiometer through its entire range ?*

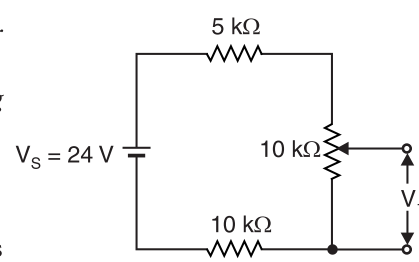


Fig. 2.148

Solution. Total circuit resistance is

$$R_T = 5 + 10 + 10 = 25\ \text{k}\Omega$$

The total voltage E that appears across the end terminals of potentiometer is

$$E = \frac{10}{R_T} \times V_S = \frac{10}{25} \times 24 = 9.6\ \text{V}$$

When the wiper arm is at the top of the potentiometer,

$$V_1 = \frac{10}{10} \times E = \frac{10}{10} \times 9.6 = 9.6\ \text{V}$$

When the wiper arm is at the bottom of the potentiometer,

$$V_1 = \frac{0}{10} \times E = \frac{0}{10} \times 9.6 = 0 \text{ V}$$

Therefore, V_1 can be adjusted between **0 and 9.6 V.**

Example 2.75. *Fig. 2.149 shows the voltage divider circuit. Find (i) the current drawn from the supply, (ii) voltage across the load R_L, (iii) the current fed to R_L and (iv) the current in the tapped portion of the divider.*

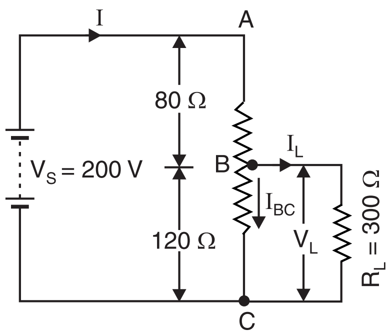


Fig. 2.149

Solution. It is a loaded voltage divider.

(*i*) $R_{BC} = 120\ \Omega \parallel 300\ \Omega = \dfrac{120 \times 300}{120 + 300} = 85.71\ \Omega$

$$V_{AB} = \frac{R_{AB}}{R_{AB} + R_{BC}} \times V_S = \frac{80}{80 + 85.71} \times 200 = 96.55 \text{ V}$$

$\therefore$ The current I drawn from the supply is

$$I = \frac{V_{AB}}{R_{AB}} = \frac{96.55}{80} = \mathbf{1.21\ A}$$

(*ii*) $$V_{BC} = \frac{R_{BC}}{R_{AB} + R_{BC}} \times V_S = \frac{85.71}{80 + 85.71} \times 200 = \mathbf{103.45\ V}$$

(*iii*) $\therefore$ Current fed to load, $I_L = \dfrac{V_{BC}}{R_L} = \dfrac{103.45}{300} = \mathbf{0.35\ A}$

(*iv*) Current in the tapped portion of the divider is

$$I_{BC} = I - I_L = 1.21 - 0.35 = \mathbf{0.86 A}$$

Tutorial Problems

1. Redraw the circuit shown in Fig. 2.150 using the common ground symbol.

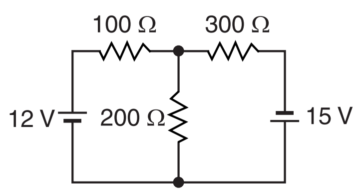
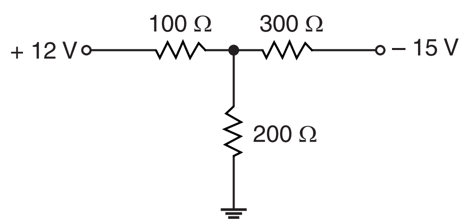


Fig. 2.150

Ans.



2. Redraw the circuit shown in Fig. 2.151 using the common ground symbol.

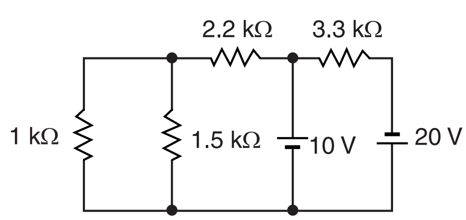
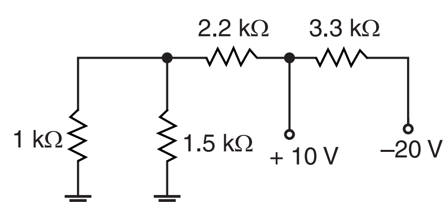


Fig. 2.151

Ans.



3. Draw the circuit shown in Fig. 2.152 by eliminating the common ground symbols.

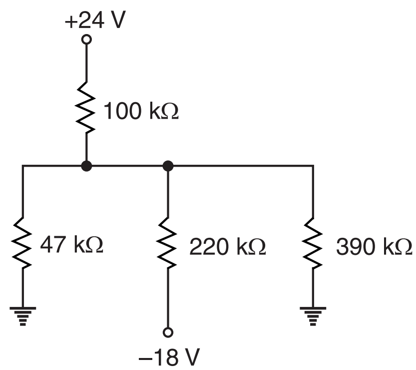
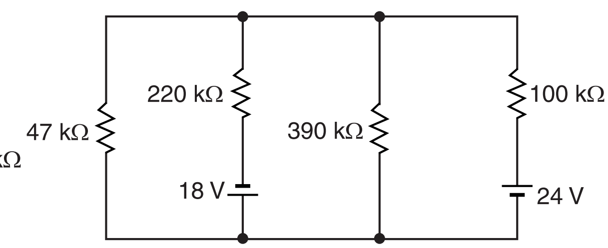


Fig. 2.152

Ans.



4. A voltage of 200 V is applied to a tapped resistor of 500 Ω. Find the resistance between the tapped points connected to a circuit reading 0.1 A at 25 V. Also calculate the total power consumed. **[79Ω ; 83.3W]**

Objective Questions

1. Two resistances are joined in parallel whose resultant resistance is 6/5 ohms. One of the resistance wire is broken and the effective resistance becomes 2 ohms. Then the resistance of the wire that got broken is

(*i*) 6/5 ohms (*ii*) 3 ohms
(*iii*) 2 ohms (*iv*) 3/5 ohms

2. The smallest resistance obtained by connecting 50 resistances of 1/4 ohm each is

(*i*) 50/4 Ω (*ii*) 4/50 Ω
(*iii*) 200 Ω (*iv*) 1/200 Ω

3. Five resistances are connected as shown in Fig. 2.153. The effective resistance between points *A* and *B* is

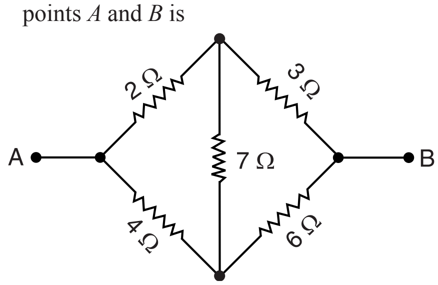


Fig. 2.153

(*i*) 10/3 Ω (*ii*) 20/3 Ω
(*iii*) 15 Ω (*iv*) 6 Ω

4. A 200 W and a 100 W bulb both meant for operation at 220 V are connected in series. When connected to a 220 V supply, the power consumed by them will be

(*i*) 33 W (*ii*) 100 W
(*iii*) 66 W (*iv*) 300 W

5. A wire has a resistance of 12 ohms. It is bent in the form of a circle. The effective resistance between two points on any diameter is

(*i*) 6 Ω (*ii*) 24 Ω
(*iii*) 16 Ω (*iv*) 3 Ω

6. A primary cell has an e.m.f. of 1.5 V. When short-circuited, it gives a current of 3 A. The internal resistance of the cell is

(*i*) 4.5 Ω (*ii*) 2 Ω
(*iii*) 0.5 Ω (*iv*) 1/4.5 Ω

7. Fig. 2.154 shows a part of a closed electrical circuit. Then $V_A - V_B$ is

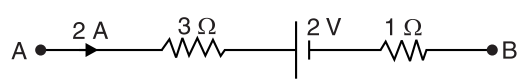


Fig. 2.154

(*i*) − 8 V (*ii*) 6 V
(*iii*) 10 V (*iv*) 3 V

8. The current *I* in the electric circuit shown in Fig. 2.155 is

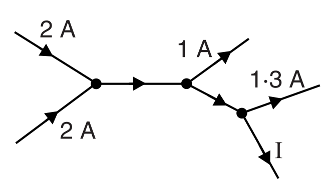


Fig. 2.155

(*i*) 1.3 A (*ii*) 3.7 A
(*iii*) 1A (*iv*) 1.7 A

9. Three 2 ohm resistors are connected to form a triangle. The resistance between any two corners is

(*i*) 6Ω (*ii*) 2Ω
(*iii*) 3/4Ω (*iv*) 4/3Ω

10. A current of 2 A flows in a system of conductors shown in Fig. 2.156. The potential difference $V_A - V_B$ will be

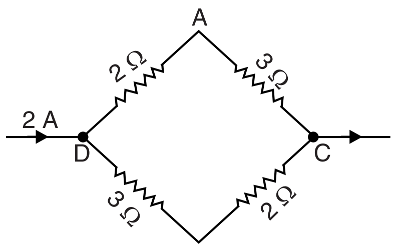


Fig. 2.156

(*i*) +2 V (*ii*) +1 V
(*iii*) –1 V (*iv*) –2 V

11. A uniform wire of resistance *R* is divided into 10 equal parts and all of them are connected in parallel. The equivalent resistance will be

(*i*) 0.01 *R* (*ii*) 0.1 *R*
(*iii*) 10 *R* (*iv*) 100 *R*

12. A cell of negligible resistance and e.m.f. 2 volts is connected to series combination of 2, 3 and 5 ohms. The potential difference in volts between the terminals of 3-ohm resistance will be

(*i*) 0.6 V (*ii*) $\frac{2}{3}$ V
(*iii*) 3 V (*iv*) 6 V

13. The equivalent resistance between points *X* and *Y* in Fig. 2.157 is

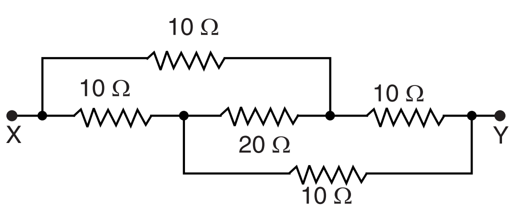


Fig. 2.157

(*i*) 10 Ω (*ii*) 22 Ω
(*iii*) 20 Ω (*iv*) 50 Ω

14. If each resistance in the network shown in Fig. 2.158 is *R*, what is the equivalent resistance between terminals *A* and *B* ?

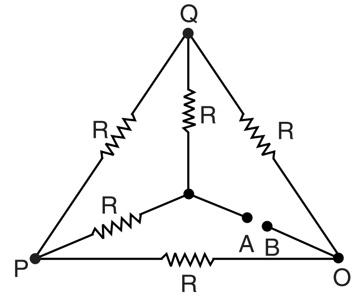


Fig. 2.158

(*i*) 5 *R* (*ii*) 3 *R*
(*iii*) 6 *R* (*iv*) *R*

15. Fig. 2.159 represents a part of a closed circuit. The potential difference between *A* and *B* (*i.e.* $V_A - V_B$) is

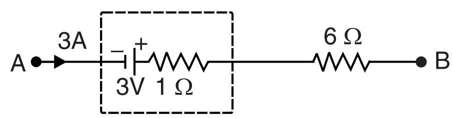


Fig. 2.159

(*i*) 24 V (*ii*) 0 V
(*iii*) 18 V (*iv*) 6 V

16. In the arrangement shown in Fig. 2.160, the potential difference between *B* and *D* will be zero if the unknown resistance *X* is

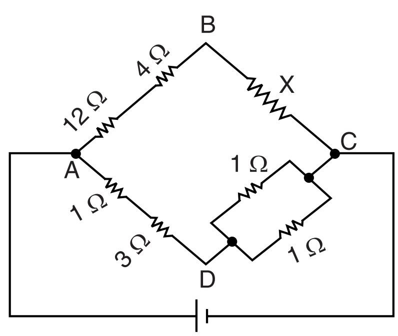


Fig. 2.160

(*i*) 4 Ω (*ii*) 2 Ω
(*iii*) 20 Ω (*iv*) 3 Ω

17. Resistances of 6 Ω each are connected in a manner shown in Fig. 2.161. With the current 0.5A as shown in the figure, the potential difference $V_P - V_Q$ is

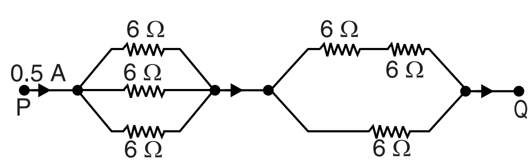


Fig. 2.161

(*i*) 3.6 V (*ii*) 6 V
(*iii*) 3 V (*iv*) 7.2 V

18. An electric fan and a heater are marked 100 W, 220 V and 1000 W, 220 V respectively. The resistance of the heater is

(*i*) zero
(*ii*) greater than that of fan
(*iii*) less than that of fan
(*iv*) equal to that of fan

19. In the circuit shown in Fig. 2.162, the final voltage drop across the capacitor C is

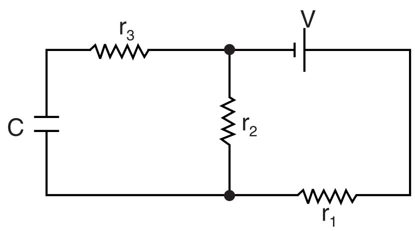


Fig. 2.162

(*i*) $\frac{V r_1}{r_1 + r_2}$ (*ii*) $\frac{V r_2}{r_1 + r_2}$

(*iii*) $\frac{V(r_1 + r_2)}{r_2}$ (*iv*) $\frac{V(r_2 + r_1)}{r_1 + r_2 + r_3}$

20. A primary cell has an e.m.f. of 1.5 V. When short circuited, it gives a current of 3 A. The internal resistance of the cell is

(*i*) 4.5 Ω (*ii*) 2 Ω
(*iii*) 0.5 Ω (*iv*) (1/4.5) Ω

Answers

1.	(*ii*)	**2.**	(*iv*)	**3.**	(*i*)	**4.**	(*iii*)	**5.**	(*iv*)
6.	(*iii*)	**7.**	(*iii*)	**8.**	(*iv*)	**9.**	(*iv*)	**10.**	(*ii*)
11.	(*i*)	**12.**	(*i*)	**13.**	(*i*)	**14.**	(*iv*)	**15**	(*iii*)
16.	(*ii*)	**17.**	(*iii*)	**18.**	(*iii*)	**19.**	(*ii*)	**20.**	(*iii*)

3

D.C. Network Theorems

Introduction

Any arrangement of electrical energy sources, resistances and other circuit elements is called an electrical network. The terms *circuit* and *network* are used synonymously in electrical literature. In the text so far, we employed two network laws *viz* Ohm's law and Kirchhoff's laws to solve network problems. Occasions arise when these laws applied to certain networks do not yield quick and easy solution. To overcome this difficulty, some network theorems have been developed which are very useful in analysing both simple and complex electrical circuits. Through the use of these theorems, it is possible either to simplify the network itself or render the analytical solution easy. In this chapter, we shall focus our attention on important d.c. network theorems and techniques with special reference to their utility in solving network problems.

3.1. Network Terminology

While discussing network theorems and techniques, one often comes across the following terms:

(*i*) **Linear circuit.** A linear circuit is one whose parameters (*e.g.* resistances) are constant *i.e.* they do not change with current or voltage.

(*ii*) **Non-linear circuit.** A non-linear circuit is one whose parameters (*e.g.* resistances) change with voltage or current.

(*iii*) **Bilateral circuit.** A bilateral circuit is one whose properties are the same in either direction. For example, transmission line is a bilateral circuit because it can be made to perform its function equally well in either direction.

(*iv*) **Active element.** An active element is one which supplies electrical energy to the circuit. Thus in Fig. 3.1, E_1 and E_2 are the active elements because they supply energy to the circuit.

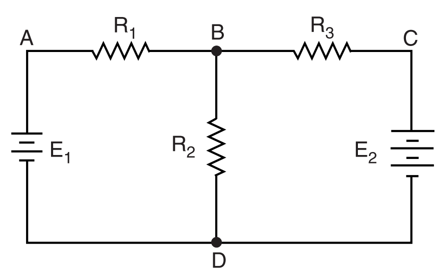


Fig. 3.1

(*v*) **Passive element.** A passive element is one which receives electrical energy and then either converts it into heat (resistance) or stores in an electric field (capacitance) or magnetic field (inductance). In Fig. 3.1, there are three passive elements, namely R_1, R_2 and R_3. These passive elements (*i.e.* resistances in this case) receive energy from the active elements (*i.e.* E_1 and E_2) and convert it into heat.

(*vi*) **Node.** A node of a network is an equipotential surface at which *two or more* circuit elements are joined. Thus in Fig. 3.1, circuit elements R_1 and E_1 are joined at A and hence A is the node. Similarly, B, C and D are nodes.

(*vii*) **Junction.** A junction is that point in a network where *three or more* circuit elements are joined. In Fig. 3.1, there are only two junction points *viz.* B and D. That B is a junction is clear from the fact that three circuit elements R_1, R_2 and R_3 are joined at it. Similarly, point D is a junction because it joins three circuit elements R_2, E_1 and E_2.

(*viii*) **Branch.** A branch is that part of a network which lies between two junction points. Thus referring to Fig. 3.1, there are a total of three branches *viz.* *BAD*, *BCD* and *BD*. The branch

BAD consists of R_1 and E_1 ; the branch *BCD* consists of R_3 and E_2 and branch *BD* merely consists of R_2.

(*ix*) **Loop.** A loop is any closed path of a network. Thus in Fig. 3.1, *ABDA*, *BCDB* and *ABCDA* are the loops.

(*x*) **Mesh.** A mesh is the most elementary form of a loop and cannot be further divided into other loops. In Fig. 3.1, both loops *ABDA* and *BCDB* qualify as meshes because they cannot be further divided into other loops. However, the loop *ABCDA* cannot be called a mesh because it encloses two loops *ABDA* and *BCDB*.

(*xi*) **Network and circuit.** Strictly speaking, the term network is used for a circuit containing passive elements only while the term circuit implies the presence of both active and passive elements. However, there is no hard and fast rule for making these distinctions and the **terms "network" and "circuit" are often used interchangeably**.

(*xii*) **Parameters.** The various elements of an electric circuit like resistance (R), inductance (L) and capacitance (C) are called parameters of the circuit. These parameters may be lumped or distributed.

(*xiii*) **Unilateral circuit.** A unilateral circuit is one whose properties change with the direction of its operation. For example, a diode rectifier circuit is a unilateral circuit. It is because a diode rectifier cannot perform rectification in both directions.

(*xiv*) **Active and passive networks.** An active network is that which contains active elements as well as passive elements. On the other hand, a passive network is that which contains passive elements only.

3.2. Network Theorems and Techniques

Having acquainted himself with network terminology, the reader is set to study the various network theorems and techniques. In this chapter, we shall discuss the following network theorems and techniques :

(*i*) Maxwell's mesh current method
(*ii*) Nodal analysis
(*iii*) Superposition theorem
(*iv*) Thevenin's theorem
(*v*) Norton's theorem
(*vi*) Maximum power transfer theorem
(*vii*) Reciprocity theorem
(*viii*) Millman's theorem
(*ix*) Compensation theorem
(*x*) Delta/star or star/delta transformation
(*xi*) Tellegen's theorem

3.3. Important Points About Network Analysis

While analysing network problems by using network theorems and techniques, the following points may be noted :

(*i*) There are two general approaches to network analysis *viz.* (*a*) **direct method** (*b*) **network reduction method**. In direct method, the network is left in its original form and different voltages and currents in the circuit are determined. This method is used for simple circuits. Examples of direct method are Kirchhoff's laws, Mesh current method, nodal analysis, superposition theorem etc. In network reduction method, the original network is reduced to a simpler equivalent circuit. This method is used for complex circuits and gives a better insight into the performance of the circuit. Examples of network reduction method are Thevenin's theorem, Norton's theorem, star/delta or delta/star transformation etc.

(*ii*) *The above theorems and techniques are applicable only to networks that have linear, bilateral circuit elements.*

(*iii*) The network theorem or technique to be used will depend upon the network arrangement. The general rule is this. *Use that theorem or technique which requires a smaller number of independent equations to obtain the solution or which can yield easy solution.*

(*iv*) *Analysis of a circuit usually means to determine all the currents and voltages in the circuit.*

3.4. Maxwell's Mesh Current Method

In this method, Kirchhoff's voltage law is applied to a network to write mesh equations in terms of **mesh currents** *instead of branch currents*. Each mesh is assigned a separate mesh current. This mesh current is assumed to flow *clockwise* around the perimeter of the mesh without splitting at a junction into branch currents. Kirchhoff's voltage law is then applied to write equations in terms of unknown mesh currents. The branch currents are then found by taking the algebraic sum of the mesh currents which are common to that branch.

Explanation. Maxwell's mesh current method consists of following steps :

(*i*) Each mesh is assigned a separate mesh current. For convenience, all mesh currents are assumed to flow in *clockwise direction. For example, in Fig. 3.2, meshes *ABDA* and *BCDB* have been assigned mesh currents I_1 and I_2 respectively. The mesh currents take on the appearance of a mesh fence and hence the name mesh currents.

(*ii*) If two mesh currents are flowing through a circuit element, the actual current in the circuit element is the algebraic sum of the two. Thus in Fig. 3.2, there are two mesh currents I_1 and I_2 flowing in R_2. If we go from *B* to *D*, current is $I_1 - I_2$ and if we go in the other direction (*i.e.* from *D* to *B*), current is $I_2 - I_1$.

(*iii*) **Kirchhoff's voltage law is applied to write equation for each mesh in terms of mesh currents. Remember, while writing mesh equations, rise in potential is assigned positive sign and fall in potential negative sign.

(*iv*) If the value of any mesh current comes out to be negative in the solution, it means that true direction of that mesh current is anticlockwise *i.e.* opposite to the assumed clockwise direction.

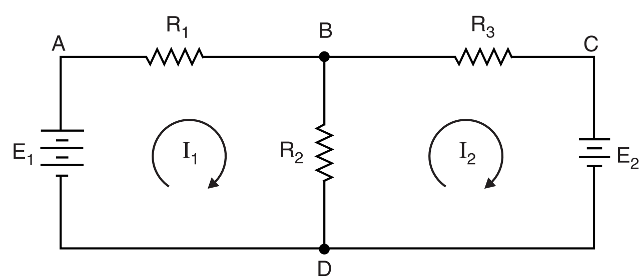


Fig. 3.2

Applying Kirchhoff's voltage law to Fig. 3.2, we have,

Mesh ABDA.

$$-I_1R_1 - (I_1 - I_2)\,R_2 + E_1 = 0$$

or $$I_1\,(R_1 + R_2) - I_2R_2 = E_1 \qquad ...(i)$$

* It is convenient to consider all mesh currents in one direction (clockwise or anticlockwise). The same result will be obtained if mesh currents are given arbitrary directions.

** Since the circuit unknowns are currents, the describing equations are obtained by applying *KVL* to the meshes.

Mesh BCDB.

$$-I_2R_3 - E_2 - (I_2 - I_1)R_2 = 0$$

or` $$-I_1R_2 + (R_2 + R_3)I_2 = -E_2 \qquad ...(ii)$$

Solving eq. (*i*) and eq. (*ii*) simultaneously, mesh currents I_1 and I_2 can be found out. Once the mesh currents are known, the branch currents can be readily obtained. *The advantage of this method is that it usually reduces the number of equations to solve a network problem*.

Note. Branch currents are the real currents because they actually flow in the branches and can be measured. However, mesh currents are fictitious quantities and cannot be measured except in those instances where they happen to be identical with branch currents. Thus in branch *DAB*, branch current is the same as mesh current and both can be measured. But in branch *BD*, mesh currents (I_1 and I_2) cannot be measured. Hence mesh current is a concept rather than a reality. However, it is a useful concept to solve network problems as it leads to the reduction of number of mesh equations.

3.5. Shortcut Procedure for Network Analysis by Mesh Currents

We have seen above that Maxwell mesh current method involves lengthy mesh equations. Here is a shortcut method to write mesh equations simply by inspection of the circuit. Consider the circuit shown in Fig. 3.3. The circuit contains resistances and independent voltage sources and has three meshes. Let the three mesh currents be I_1, I_2 and I_3 flowing in the clockwise direction.

Loop 1. Applying *KVL* to this loop, we have,

$$100 - 20 = I_1(60 + 30 + 50) - I_2 \times 50 - I_3 \times 30$$

or $$80 = 140I_1 - 50I_2 - 30I_3 \qquad ...(i)$$

We can write eq. (*i*) in a shortcut form as :

$$E_1 = I_1R_{11} - I_2R_{12} - I_3R_{13}$$

Here E_1 = Algebraic sum of e.m.f.s in Loop (1) in the direction of I_1

= 100 – 20 = 80 V

R_{11} = Sum of resistances in Loop (1)

= Self*-resistance of Loop (1)

= 60 + 30 + 50 = 140 Ω

R_{12} = Total resistance common to Loops (1) and (2)

= Common resistance between Loops (1) and (2) = 50 Ω

R_{13} = Total resistance common to Loops (1) and (3) = 30 Ω

It may be seen that the sign of the term involving self-resistances is positive while the sign of common resistances is negative. It is because the positive directions for mesh currents were all chosen clockwise. Although mesh currents are abstract currents, yet mesh current analysis offers the advantage that resistor polarities do not have to be considered when writing mesh equations.

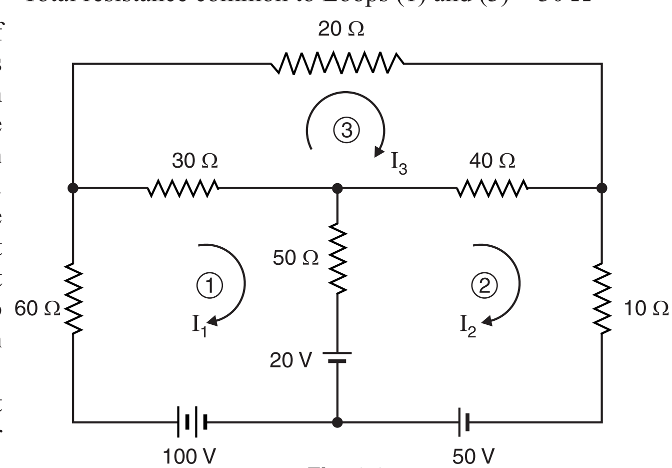


Fig. 3.3

Loop 2. We can use shortcut method to find the mesh equation for Loop (2) as under :

* The sum of all resistances in a loop is called self-resistance of that loop. Thus in Fig. 3.3, self-resistance of Loop (1) = 60 + 30 + 50 = 140 Ω.

$$E_2 = -I_1R_{21} + I_2R_{22} - I_3R_{23}$$

or $$50 + 20 = -50I_1 + 100I_2 - 40I_3 \quad ...(ii)$$

Here, E_2 = Algebraic sum of e.m.f.s in Loop (2) in the direction of I_2

= 50 + 20 = 70 V

R_{21} = Total resistance common to Loops (2) and (1) = 50 Ω

R_{22} = Sum of resistances in Loop (2) = 50 + 40 + 10 = 100 Ω

R_{23} = Total resistance common to Loops (2) and (3) = 40 Ω

Again the sign of self-resistance of Loop (2) (R_{22}) is positive while the sign of the terms of common resistances (R_{21}, R_{23}) is negative.

Loop 3. We can again use shortcut method to find the mesh equation for Loop (3) as under :

$$E_3 = -I_1R_{31} - I_2R_{32} + I_3R_{33}$$

or $$0 = -30I_1 - 40I_2 + 90I_3 \quad ...(iii)$$

Again the sign of self-resistance of Loop (3) (R_{33}) is positive while the sign of the terms of common resistances (R_{31}, R_{32}) is negative.

Mesh analysis using matrix form. The three mesh equations are rewritten below :

$$E_1 = I_1R_{11} - I_2R_{12} - I_3R_{13}$$

$$E_2 = -I_1R_{21} + I_2R_{22} - I_3R_{23}$$

$$E_3 = -I_1R_{31} - I_2R_{32} + I_3R_{33}$$

The matrix equivalent of above given equations is :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

It is reminded again that (*i*) all self-resistances are positive (*ii*) all common resistances are negative and (*iii*) by their definition, $R_{12} = R_{21}$; $R_{23} = R_{32}$ and $R_{13} = R_{31}$.

Example 3.1. *In the network shown in Fig. 3.4 (i), find the magnitude and direction of each branch current by mesh current method.*

Solution. Assign mesh currents I_1 and I_2 to meshes *ABDA* and *BCDB* respectively as shown in Fig. 3.4 (*i*).

Mesh ABDA. Applying *KVL*, we have,

$$-40I_1 - 20(I_1 - I_2) + 120 = 0$$

or $$60I_1 - 20I_2 = 120 \quad ...(i)$$

Mesh BCDB. Applying *KVL*, we have,

$$-60I_2 - 65 - 20(I_2 - I_1) = 0$$

or $$-20I_1 + 80I_2 = -65 \quad ...(ii)$$

Multiplying eq. (*ii*) by 3 and adding it to eq. (*i*), we get,

$$220I_2 = -75 \qquad \therefore \quad I_2 = -75/220 = -0{\cdot}341 \text{ A}$$

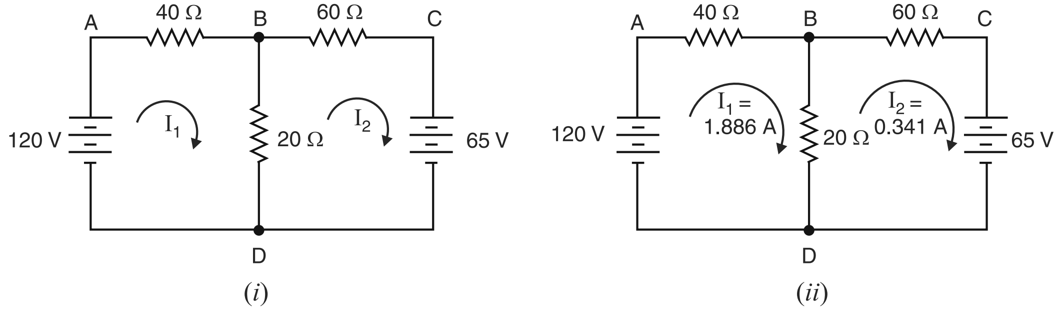


Fig. 3.4

The minus sign shows that true direction of I_2 is anticlockwise. Substituting $I_2 = -0{\cdot}341$A in eq. (*i*), we get, $I_1 = 1{\cdot}886$ A. The actual direction of flow of currents is shown in Fig. 3.4 (*ii*).

By determinant method

$$60I_1 - 20I_2 = 120$$
$$-20I_1 + 80I_2 = -65$$

$$\therefore \quad I_1 = \frac{\begin{vmatrix} 120 & -20 \\ -65 & 80 \end{vmatrix}}{\begin{vmatrix} 60 & -20 \\ -20 & 80 \end{vmatrix}} = \frac{(120\times 80)-(-65\times -20)}{(60\times 80)-(-20\times -20)} = \frac{8300}{4400} = 1{\cdot}886 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 60 & 120 \\ -20 & -65 \end{vmatrix}}{\text{Denominator}} = \frac{(60\times -65)-(-20\times 120)}{4400} = \frac{-1500}{4400} = -0{\cdot}341 \text{ A}$$

Referring to Fig. 3.4 (*ii*), we have,

Current in branch $DAB = I_1 =$ **1·886 A** ; Current in branch $DCB = I_2 =$ **0·341 A**

Current in branch $BD = I_1 + I_2 = 1{\cdot}886 + 0{\cdot}341 =$ **2·227 A**

Example 3.2. *Calculate the current in each branch of the circuit shown in Fig. 3.5.*

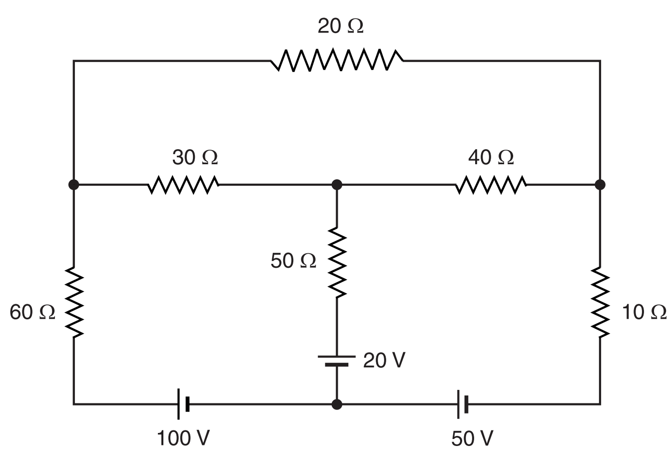


Fig. 3.5

Solution. Assign mesh currents I_1, I_2 and I_3 to meshes *ABHGA*, *HEFGH* and *BCDEHB* respectively as shown in Fig. 3.6.

Mesh ABHGA. Applying *KVL*, we have,

$$-60I_1 - 30(I_1 - I_3) - 50(I_1 - I_2) - 20 + 100 = 0$$

or $$140I_1 - 50I_2 - 30I_3 = 80$$

or $$14I_1 - 5I_2 - 3I_3 = 8 \quad ...(i)$$

Mesh GHEFG. Applying *KVL*, we have,

$$20 - 50(I_2 - I_1) - 40(I_2 - I_3) - 10I_2 + 50 = 0$$

or $$-50I_1 + 100I_2 - 40I_3 = 70$$

or $$-5I_1 + 10I_2 - 4I_3 = 7 \quad ...(ii)$$

Mesh BCDEHB. Applying *KVL*, we have,

$$-20I_3 - 40(I_3 - I_2) - 30(I_3 - I_1) = 0$$

or $$30I_1 + 40I_2 - 90I_3 = 0$$

or $$3I_1 + 4I_2 - 9I_3 = 0 \quad ...(iii)$$

Solving for equations (*i*), (*ii*) and (*iii*), we get, $I_1 = 1{\cdot}65$ A ; $I_2 = 2{\cdot}12$ A ; $I_3 = 1{\cdot}5$ A

By determinant method

$$14I_1 - 5I_2 - 3I_3 = 8$$
$$-5I_1 + 10I_2 - 4I_3 = 7$$
$$3I_1 + 4I_2 - 9I_3 = 0$$

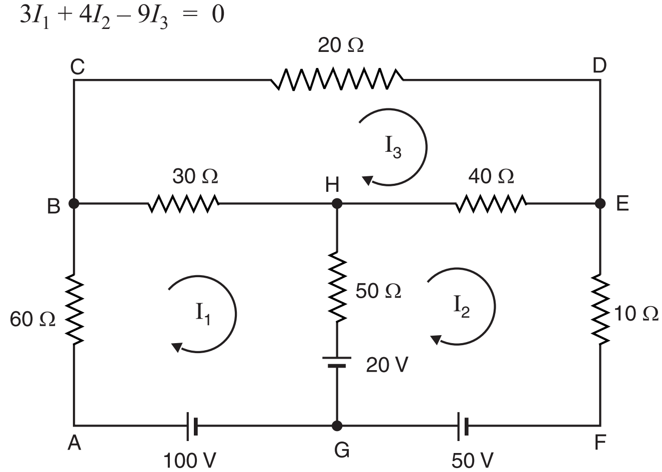


Fig. 3.6

$$\therefore \quad I_1 = \frac{\begin{vmatrix} 8 & -5 & -3 \\ 7 & 10 & -4 \\ 0 & 4 & -9 \end{vmatrix}}{\begin{vmatrix} 14 & -5 & -3 \\ -5 & 10 & -4 \\ 3 & 4 & -9 \end{vmatrix}} = \frac{8\begin{vmatrix} 10 & -4 \\ 4 & -9 \end{vmatrix} + 5\begin{vmatrix} 7 & -4 \\ 0 & -9 \end{vmatrix} - 3\begin{vmatrix} 7 & 10 \\ 0 & 4 \end{vmatrix}}{14\begin{vmatrix} 10 & -4 \\ 4 & -9 \end{vmatrix} + 5\begin{vmatrix} -5 & -4 \\ 3 & -9 \end{vmatrix} - 3\begin{vmatrix} -5 & 10 \\ 3 & 4 \end{vmatrix}}$$

$$= \frac{8[(10\times -9) - (4\times -4)] + 5[(7\times -9) - (0\times -4)] - 3[(7\times 4) - (0\times 10)]}{14[(10\times -9) - (4\times -4)] + 5[(-5\times -9) - (3\times -4)] - 3[(-5\times 4) - (3\times 10)]}$$

$$= \frac{-592 - 315 - 84}{-1036 + 285 + 150} = \frac{-991}{-601} = 1{\cdot}65 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 14 & 8 & -3 \\ -5 & 7 & -4 \\ 3 & 0 & -9 \end{vmatrix}}{\text{Denominator}} = \frac{14[(-63)-(0)]-8[(45)-(-12)]-3[(0)-(21)]}{-601}$$

$$= \frac{-882-456+63}{-601} = \frac{-1275}{-601} = 2{\cdot}12 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 14 & -5 & 8 \\ -5 & 10 & 7 \\ 3 & 4 & 0 \end{vmatrix}}{\text{Denominator}} = \frac{14[(0)-(28)]+5[(0)-(21)]+8[(-20)-(30)]}{-601}$$

$$= \frac{-392-105-400}{-601} = \frac{-897}{-601} = 1{\cdot}5 \text{ A}$$

∴ Current in 60 Ω = I_1 = **1·65 A** from A to B

Current in 30 Ω = $I_1 - I_3$ = 1·65 – 1·5 = **0·15 A** from B to H

Current in 50 Ω = $I_2 - I_1$ = 2·12 – 1·65 = **0·47 A** from G to H

Current in 40 Ω = $I_2 - I_3$ = 2·12 – 1·5 = **0·62 A** from H to E

Current in 10 Ω = I_2 = **2·12 A** from E to F

Current in 20 Ω = I_3 = **1·5 A** from C to D

Example 3.3. *By using mesh resistance matrix, determine the current supplied by each battery in the circuit shown in Fig. 3.7.*

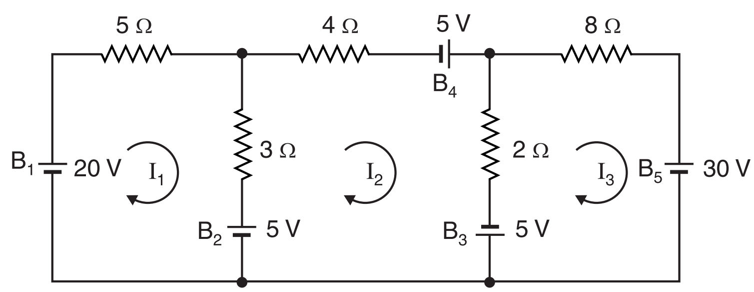


Fig. 3.7

Solution. Since there are three meshes, let the three mesh currents be I_1, I_2 and I_3, all assumed to be flowing in the clockwise direction. The different quantities of the mesh-resistance matrix are :

$$R_{11} = 5 + 3 = 8\ \Omega \quad ; \quad R_{22} = 4 + 2 + 3 = 9\ \Omega \quad ; \quad R_{33} = 8 + 2 = 10\ \Omega$$

$$R_{12} = R_{21} = -3\ \Omega \quad ; \quad R_{13} = R_{31} = 0 \quad ; \quad R_{23} = R_{32} = -2\ \Omega$$

$$E_1 = 20 - 5 = 15 \text{ V} \quad ; \quad E_2 = 5 + 5 + 5 = 15 \text{ V} \quad ; \quad E_3 = -30 - 5 = -35 \text{ V}$$

Therefore, the mesh equations in the matrix form are :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

or

$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

By determinant method, we have,

$$I_1 = \frac{\begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix}}{\begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix}} = \frac{1530}{598} = 2{\cdot}56 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix}}{\text{Denominator}} = \frac{1090}{598} = 1{\cdot}82 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix}}{\text{Denominator}} = \frac{-1875}{598} = -3{\cdot}13 \text{ A}$$

The negative sign with I_3 indicates that actual direction of I_3 is opposite to that assumed in Fig. 3.7. Note that batteries B_1, B_3, B_4 and B_5 are discharging while battery B_2 is charging.

$\therefore$ Current supplied by battery $B_1 = I_1 =$ **2·56 A**

Current supplied to battery $B_2 = I_1 - I_2 = 2{\cdot}56 - 1{\cdot}82 =$ **0·74 A**

Current supplied by battery $B_3 = I_2 + I_3 = 1{\cdot}82 + 3{\cdot}13 =$ **4·95 A**

Current supplied by battery $B_4 = I_2 =$ **1·82 A**

Current supplied by battery $B_5 = I_3 =$ **3·13 A**

Example 3.4. *By using mesh resistance matrix, calculate the current in each branch of the circuit shown in Fig. 3.8.*

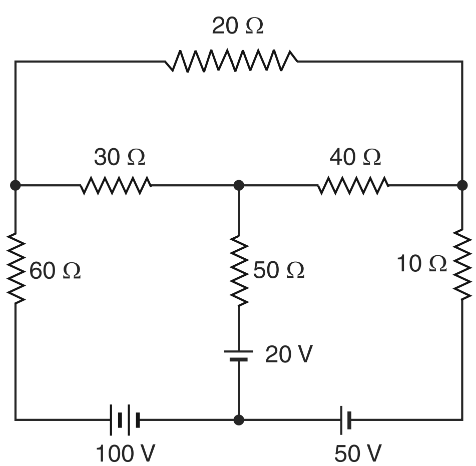


Fig. 3.8

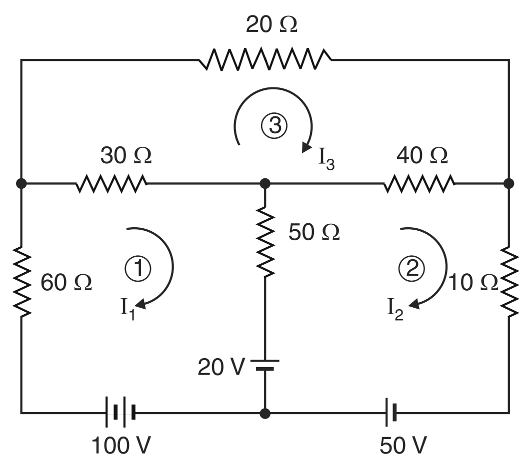


Fig. 3.9

Solution. Since there are three meshes, let the three mesh currents be I_1, I_2 and I_3, all assumed to be flowing in the clockwise direction as shown in Fig. 3.9. The different quantities of the mesh resistance-matrix are :

$$R_{11} = 60 + 30 + 50 = 140\ \Omega\ ;\ R_{22} = 50 + 40 + 10 = 100\ \Omega\ ;\ R_{33} = 30 + 20 + 40 = 90\ \Omega$$

$$R_{12} = R_{21} = -50\ \Omega\ ;\ R_{13} = R_{31} = -30\ \Omega\ ;\ R_{23} = R_{32} = -40\ \Omega$$

$$E_1 = 100 - 20 = 80\text{ V}\ ;\ E_2 = 50 + 20 = 70\text{ V}\ ;\ E_3 = 0\text{ V}$$

Therefore, the mesh equations in the matrix form are :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

or
$$\begin{bmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 70 \\ 0 \end{bmatrix}$$

By determinant method, we have,

$$I_1 = \frac{\begin{vmatrix} 80 & -50 & -30 \\ 70 & 100 & -40 \\ 0 & -40 & 90 \end{vmatrix}}{\begin{vmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{vmatrix}} = \frac{991000}{601000} = 1{\cdot}65\text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 140 & 80 & -30 \\ -50 & 70 & -40 \\ -30 & 0 & 90 \end{vmatrix}}{\text{Denominator}} = \frac{1275000}{601000} = 2{\cdot}12\text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 140 & -50 & 80 \\ -50 & 100 & 70 \\ -30 & -40 & 0 \end{vmatrix}}{\text{Denominator}} = \frac{897000}{601000} = 1{\cdot}5\text{ A}$$

∴ Current in 60 Ω = I_1 = **1·65 A in the direction of I_1**

Current in 30 Ω = $I_1 - I_3$ = **0·15 A in the direction of I_1**

Current in 50 Ω = $I_2 - I_1$ = **0·47 A in the direction of I_2**

Current in 40 Ω = $I_2 - I_3$ = **0·62 A in the direction of I_2**

Current in 10 Ω = I_2 = **2·12 A in the direction of I_2**

Current in 20 Ω = I_3 = **1·5 A in the direction of I_3**

Example 3.5. *Find mesh currents i_1 and i_2 in the electric circuit shown in Fig. 3.10.*

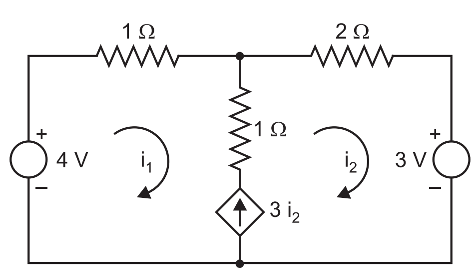


Fig. 3.10

Solution. We shall use mesh current method for the solution. Mesh analysis requires that all the sources in a circuit be voltage sources. If a circuit contains any current source, convert it into equivalent voltage source.

Outer mesh. Applying *KVL* to this mesh, we have,

$$-i_1 \times 1 - 2i_2 - 3 + 4 = 0 \quad \text{or} \quad i_1 + 2i_2 = 1 \qquad ...(i)$$

First mesh. Applying *KVL* to this mesh, we have,

$$-i_1 \times 1 - (i_1 - i_2) \times 1 - 3i_2 + 4 = 0 \quad \text{or} \quad i_1 + i_2 = 2 \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), we have $i_1 = \mathbf{3A}$; $i_2 = \mathbf{-1\ A}$

Example 3.6. *Using mesh current method, determine current I_x in the circuit shown in Fig. 3.11.*

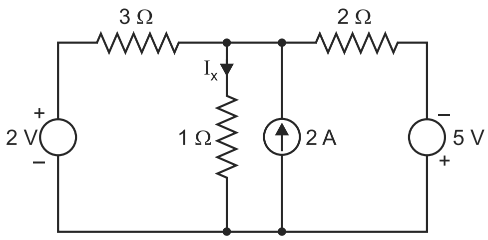


Fig. 3.11

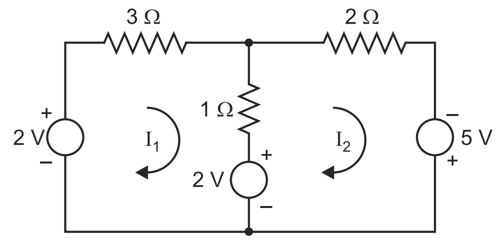


Fig. 3.12

Solution. First convert 2A current source in parallel with 1Ω resistance into equivalent voltage source of voltage 2A × 1Ω = 2V in series with 1Ω resistance. The circuit then reduces to that shown in Fig. 3.12. Assign mesh currents I_1 and I_2 to meshes 1 and 2 in Fig. 3.12.

Mesh 1. Applying *KVL* to this mesh, we have,

$$-3I_1 - 1 \times (I_1 - I_2) - 2 + 2 = 0 \quad \text{or} \quad I_2 = 4I_1$$

Mesh 2. Applying *KVL* to this mesh, we have,

$$-2I_2 + 5 + 2 - (I_2 - I_1) \times 1 = 0$$

or $\quad -2(4I_1) + 7 - (4I_1 - I_1) = 0 \quad (\because I_2 = 4I_1)$

$$\therefore \quad I_1 = \frac{7}{11}\text{A and } I_2 = 4I_1 = 4 \times \frac{7}{11} = \frac{28}{11}\text{A}$$

$$\therefore \quad \text{Current in } 3\Omega \text{ resistance, } I_1 = \frac{7}{11}\text{A} \ ; \ \text{Current in } 2\Omega \text{ resistance, } I_2 = \frac{28}{11}\text{A}$$

Referring to the original Fig. 3.11, we have,

$$I_x = I_1 + (2 - I_2) = \frac{7}{11} + \left(2 - \frac{28}{11}\right) = \mathbf{\frac{1}{11}\ A}$$

Example 3.7. *Using mesh current method, find the currents in resistances R_3, R_4, R_5 and R_6 of the circuit shown in Fig. 3.13 (i).*

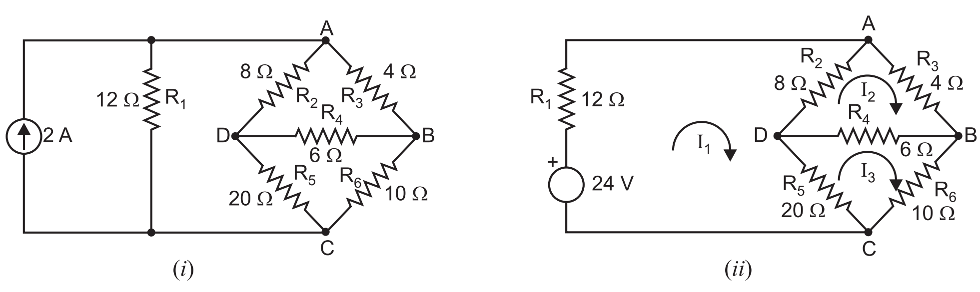


Fig. 3.13

Solution. First convert 2 A current source in parallel with 12Ω resistance into equivalent voltage source of voltage = 2A × 12Ω = 24V in series with 12Ω resistance. The circuit then reduces to the one shown in Fig. 3.13 (*ii*). Assign the mesh currents I_1, I_2 and I_3 to three meshes 1, 2 and 3 shown in Fig. 3.13 (*ii*).

Mesh 1. Applying *KVL* to this mesh, we have,

$$-12I_1 - 8 \times (I_1 - I_2) - 20 \times (I_1 - I_3) + 24 = 0$$

or $$10I_1 - 2I_2 - 5I_3 = 6 \qquad ...(i)$$

Mesh 2. Applying *KVL* to this mesh, we have,

$$-4I_2 - 6 \times (I_2 - I_3) - 8(I_2 - I_1) = 0$$

or $$-4I_1 + 9I_2 - 3I_3 = 0 \qquad ...(ii)$$

Mesh 3. Applying *KVL* to this mesh, we have,

$$-10I_3 - 20 \times (I_3 - I_1) - 6 \times (I_3 - I_2) = 0$$

or $$-10I_1 - 3I_2 + 18I_3 = 0 \qquad ...(iii)$$

From eqs. (*i*), (*ii*) and (*iii*), $I_1 = 1.125$ A ; $I_2 = 0.75$ A ; $I_3 = 0.75$ A

∴ Current in R_3 (= 4Ω) = I_2 = **0.75 A *from* A to B**

Current in R_4 (= 6Ω) = $I_2 - I_3 = 0.75 - 0.75$ = **0A**

Current in R_5 (= 20Ω) = $I_1 - I_3 = 1.125 - 0.75$ = **0.375A *from* D to C**

Current in R_6 (= 10Ω) = I_3 = **0.75A *from* B to C**

Example 3.8. *Use mesh current method to determine currents through each of the components in the circuit shown in Fig. 3.14 (i).*

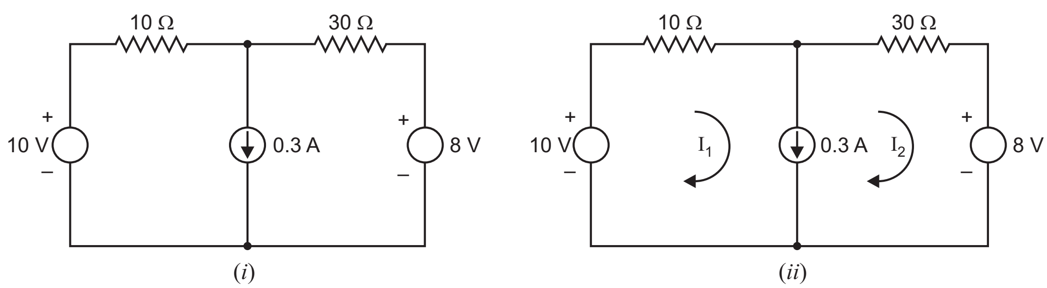


Fig. 3.14

Solution. Suppose voltage across current source is *v*. Assign mesh currents I_1 and I_2 in the meshes 1 and 2 respectively as shown in Fig. 3.14 (*ii*).

Mesh 1. Applying *KVL* to this mesh, we have,

$$10 - 10I_1 + v = 0 \qquad ...(i)$$

Mesh 2. Applying *KVL* to this mesh, we have,

$$-30I_2 - 8 - v = 0 \qquad ...(ii)$$

Adding eqs. (*i*) and (*ii*), $2 - 10I_1 - 30I_2 = 0$ ...(*iii*)

Also current in the branch containing current source is

$$I_1 - I_2 = 0.3 \qquad ...(iv)$$

From eqs. (*iii*) and (*iv*), $I_1 = 0.275$ A ; $I_2 = -0.025$A

∴ Current in 10Ω = I_1 = **0.275A**

Current in 30Ω = I_2 = **– 0.025 A**

Current in current source = $I_1 - I_2 = 0.275 - (-0.025)$ = **0.3A**

Note that negative sign means current is in the opposite direction to that assumed in the circuit.

Tutorial Problems

1. Use mesh analysis to find the current in each resistor in Fig. 3.15.

[in 100 Ω = 0·1 A from *L* to *R* ; in 20 Ω = 0·4 A from *R* to *L* ; in 10 Ω = 0·5 A downward]

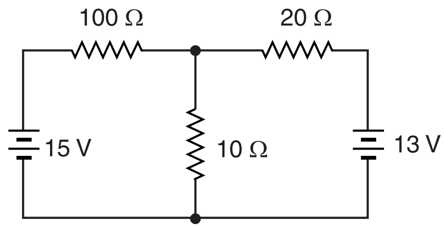


Fig. 3.15

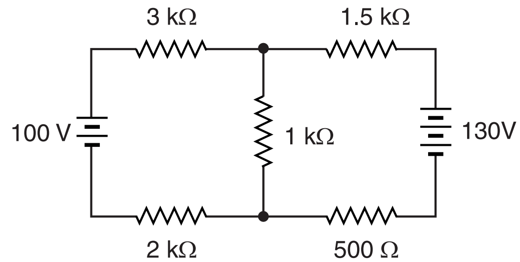


Fig. 3.16

2. Using mesh analysis, find the voltage drop across the 1 kΩ resistor in Fig. 3.16. **[50 V]**
3. Using mesh analysis, find the currents in 50 Ω, 250 Ω and 100 Ω resistors in the circuit shown in Fig. 3.17. **[*I*(50 Ω) = 0·171 A → ; *I*(250 Ω) = 0·237 A ← ; *I*(100 Ω) = 0·408 A ↓]**

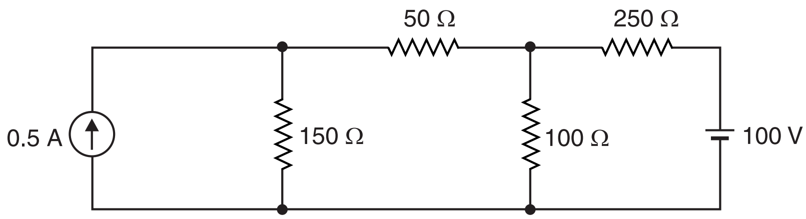


Fig. 3.17

4. For the network shown in Fig. 3.18, find the mesh currents I_1, I_2 and I_3. **[5 A, 1 A, 0·5 A]**

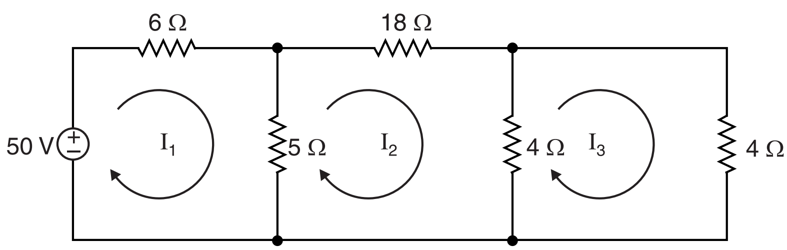


Fig. 3.18

5. In the network shown in Fig. 3.19, find the magnitude and direction of current in the various branches by mesh current method. **[*FAB* = 4 A ; *BF* = 3 A ; *BC* = 1 A ; *EC* = 2 A ; *CDE* = 3 A]**

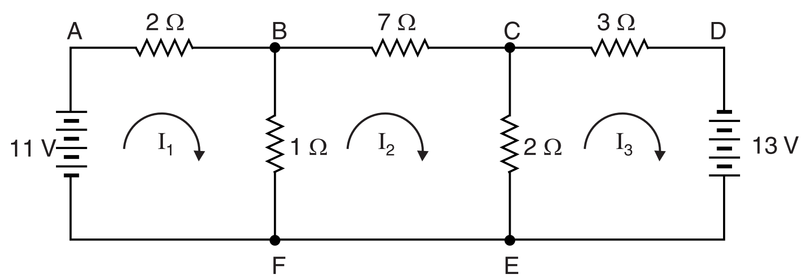


Fig. 3.19

3.6. Nodal Analysis

Consider the circuit shown in Fig. 3.20. The branch currents in the circuit can be found by Kirchhoff's laws or Maxwell's mesh current method. There is another method, called *nodal analysis* for determining branch currents in a circuit. In this method, one of the nodes (Remember a node is a point in a network where two or more circuit elements meet) is taken as the *reference node*. The potentials of all the points in the circuit are measured w.r.t. this reference node. In Fig. 3.20, A, B, C and D are four nodes and the node D has been taken as the *reference node. The fixed-voltage nodes are called *dependent nodes*. Thus in Fig. 3.20, A and C are fixed nodes because $V_A = E_1 = 120$ V and $V_C = 65$ V. The voltage from D to B is V_B and its magnitude depends upon the parameters of circuit elements and the currents through these elements. Therefore, node B is called *independent node*. Once we calculate the potential at the independent node (or nodes), each branch current can be determined because the voltage across each resistor will then be known.

Hence **nodal analysis** *essentially aims at choosing a reference node in the network and then finding the unknown voltages at the independent nodes w.r.t. reference node*. For a circuit containing N nodes, there will be N–1 node voltages, some of which may be known if voltage sources are present.

Circuit analysis. The circuit shown in Fig. 3.20 has only one independent node B. Therefore, if we find the voltage V_B at the independent node B, we can determine all branch currents in the circuit. We can express each current in terms of e.m.f.s, resistances (or conductances) and the voltage V_B at node B. Note that we have taken point D as the reference node.

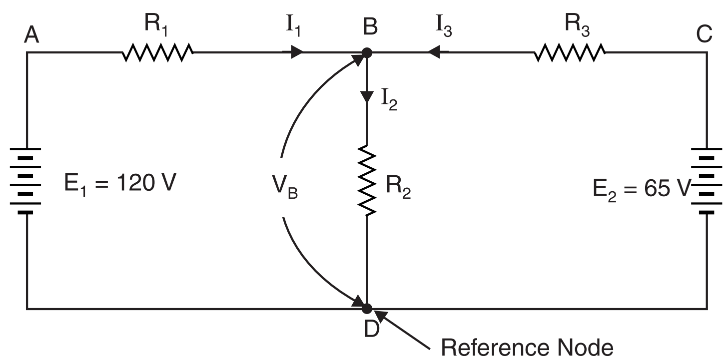


Fig. 3.20

The voltage V_B can be found by applying **Kirchhoff's current law at node B.

$$I_1 + I_3 = I_2 \quad ...(i)$$

In mesh $ABDA$, the voltage drop across R_1 is $E_1 - V_B$.

$$\therefore \quad I_1 = \frac{E_1 - V_B}{R_1}$$

In mesh $CBDC$, the voltage drop across R_3 is $E_2 - V_B$.

$$\therefore \quad I_3 = \frac{E_2 - V_B}{R_3}$$

Also
$$I_2 = \frac{V_B}{R_2}$$

Putting the values of I_1, I_2 and I_3 in eq. (i), we get,

$$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_2} \quad ...(ii)$$

All quantities except V_B are known. Hence V_B can be found out. Once V_B is known, all branch currents can be calculated. It may be seen that nodal analysis requires only one equation [eq. (ii)] for determining the branch currents in this circuit. However, Kirchhoff's or Maxwell's solution would have needed two equations.

* An obvious choice would be ground or common, if such a point exists.

** Since the circuit unknowns are voltages, the describing equations are obtained by applying *KCL* at the nodes.

Notes.

(*i*) We can mark the directions of currents at will. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed.

(*ii*) We can also express the currents in terms of conductances.

$$I_1 = \frac{E_1 - V_B}{R_1} = (E_1 - V_B)G_1 \; ; \; I_2 = \frac{V_B}{R_2} = V_B G_2 \; ; \; I_3 = \frac{E_2 - V_B}{R_3} = (E_2 - V_B)G_3$$

3.7. Nodal Analysis with Two Independent Nodes

Fig. 3.21 shows a network with two independent nodes *B* and *C*. We take node *D* (or *E*) as the reference node. We shall use Kirchhoff's current law for nodes *B* and *C* to find V_B and V_C. Once the values of V_B and V_C are known, we can find all the branch currents in the network.

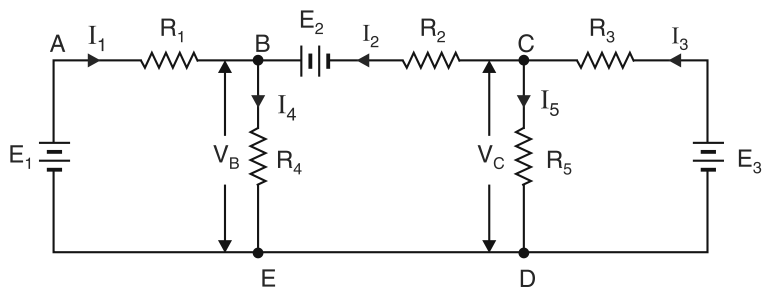


Fig. 3.21

Each current can be expressed in terms of e.m.f.s, resistances (or conductances), V_B and V_C.

$$E_1 = V_B + I_1R_1 \quad \therefore I_1 = \frac{E_1 - V_B}{R_1}$$

$$E_3 = V_C + I_3R_3 \quad \therefore I_3 = \frac{E_3 - V_C}{R_3}$$

$$E_2^* = V_B - V_C + I_2R_2 \quad \therefore I_2 = \frac{E_2 - V_B + V_C}{R_2}$$

Similarly, $$I_4 = \frac{V_B}{R_4} \; ; I_5 = \frac{V_C}{R_5}$$

At node B. $$I_1 + I_2 = I_4$$

or $$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B + V_C}{R_2} = \frac{V_B}{R_4} \qquad ...(i)$$

At node C. $$I_2 + I_5 = I_3$$

or $$\frac{E_2 - V_B + V_C}{R_2} + \frac{V_C}{R_5} = \frac{E_3 - V_C}{R_3} \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), we can find V_B and V_C since all other quantities are known. Once we know the values of V_B and V_C, we can find all the branch currents in the network.

Note. We can also express currents in terms of conductances as under :

$I_1 = (E_1 - V_B)\,G_1$; $I_2 = (E_2 - V_B + V_C)\,G_2$

$I_3 = (E_3 - V_C)\,G_3$; $I_4 = V_B\,G_4$; $I_5 = V_C\,G_5$

* As we go from *C* to *B*, we have,

$$V_C - I_2R_2 + E_2 = V_B$$

$$\therefore \quad E_2 = V_B - V_C + I_2R_2$$

Example 3.9. *Find the currents in the various branches of the circuit shown in Fig. 3.22 by nodal analysis.*

Solution. Mark the currents in the various branches as shown in Fig. 3.22. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed. Take point *E* (or *F*) as the reference node. We shall find the voltages at nodes *B* and *C*.

At node B. $I_2 + I_3 = I_1$

or $$\frac{V_B}{10} + \frac{{}^*V_B - V_C}{15} = \frac{100 - V_B}{20}$$

or $$13V_B - 4V_C = 300 \quad \ldots(i)$$

At node C. $I_4 + I_5 = I_3$

or $$\frac{V_C}{10} + \frac{V_C + 80}{10} = \frac{V_B - V_C}{15}$$

or $$V_B - 4V_C = 120 \quad \ldots(ii)$$

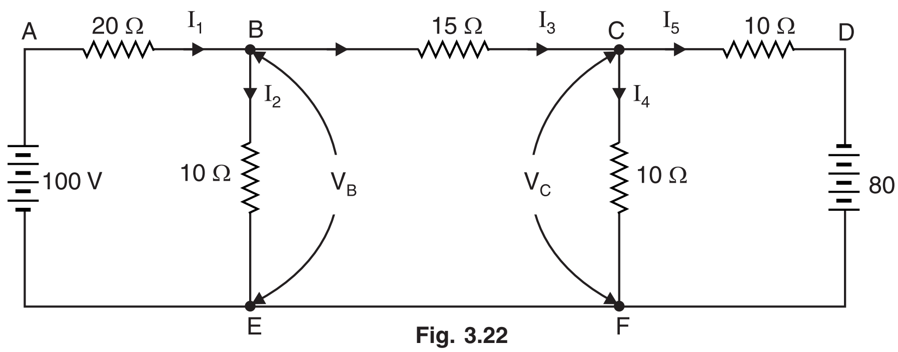


Fig. 3.22

Subtracting eq. (*ii*) from eq. (*i*), we get, $12V_B = 180 \quad \therefore \quad V_B = 180/12 = 15$ V

Putting $V_B = 15$ volts in eq. (*i*), we get, $V_C = -26.25$ volts.

By determinant method

$$13V_B - 4V_C = 300$$

$$V_B - 4V_C = 120$$

$\therefore$ $$V_B = \frac{\begin{vmatrix} 300 & -4 \\ 120 & -4 \end{vmatrix}}{\begin{vmatrix} 13 & -4 \\ 1 & -4 \end{vmatrix}} = \frac{(300 \times -4) - (120 \times -4)}{(13 \times -4) - (1 \times -4)} = \frac{-720}{-48} = 15 \text{ V}$$

and $$V_C = \frac{\begin{vmatrix} 13 & 300 \\ 1 & 120 \end{vmatrix}}{\text{Denominator}} = \frac{(13 \times 120) - (1 \times 300)}{-48} = \frac{1260}{-48} = -26{\cdot}25 \text{ V}$$

$\therefore$ $$\text{Current } I_1 = \frac{100 - V_B}{20} = \frac{100 - 15}{20} = \mathbf{4{\cdot}25\ A}$$

$$\text{Current } I_2 = V_B/10 = 15/10 = \mathbf{1{\cdot}5\ A}$$

$$\text{Current } I_3 = \frac{V_B - V_C}{15} = \frac{15 - (-26.25)}{15} = \mathbf{2{\cdot}75\ A}$$

* Note that the current I_3 is assumed to flow from *B* to *C*. Therefore, with this assumption, $V_B > V_C$.

$$\text{Current } I_4 = V_C/10 = -26{\cdot}25/10 = \mathbf{-2{\cdot}625\ A}$$

$$\text{Current } I_5 = \frac{V_C + 80}{10} = \frac{-26.25 + 80}{10} = \mathbf{5{\cdot}375\ A}$$

The negative sign for I_4 shows that actual current flow is opposite to that of assumed.

Example 3.10. *Use nodal analysis to find the currents in various resistors of the circuit shown in Fig. 3.23 (i).*

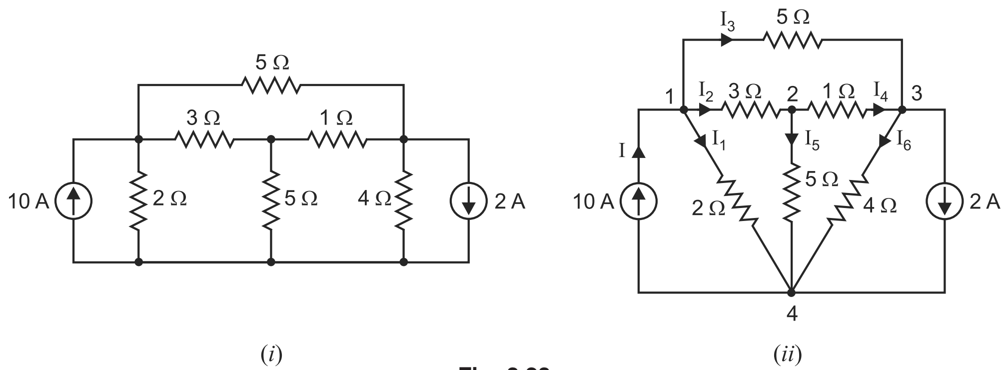


Fig. 3.23

Solution. The given circuit is redrawn in Fig. 3.23 (*ii*) with nodes marked 1, 2, 3 and 4. Let us take node 4 as the reference node. We shall apply *KCL* at nodes 1, 2 and 3 to obtain the solution.

At node 1. Applying *KCL*, we have,

$$I_1 + I_2 + I_3 = I$$

or $$\frac{V_1}{2} + \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{5} = 10$$

or $$31V_1 - 10V_2 - 6V_3 = 300 \quad \text{...}(i)$$

At node 2. Applying *KCL*, we have,

$$I_2 = I_4 + I_5$$

or $$\frac{V_1 - V_2}{3} = \frac{V_2 - V_3}{1} + \frac{V_2}{5}$$

or $$5V_1 - 23V_2 + 15V_3 = 0 \quad \text{...}(ii)$$

At node 3. Applying *KCL*, we have,

$$I_3 + I_4 = I_6 + 2$$

or $$\frac{V_1 - V_3}{5} + \frac{V_2 - V_3}{1} = \frac{V_3}{4} + 2$$

or $$4V_1 + 20V_2 - 29V_3 = 40 \quad \text{...}(iii)$$

From eqs. (*i*), (*ii*) and (*iii*), $V_1 = \frac{6572}{545}\text{V}$; $V_2 = \frac{556}{109}\text{V}$; $V_3 = \frac{2072}{545}\text{V}$

$\therefore$ $$\text{Current } I_1 = \frac{V_1}{2} = \frac{6572}{545} \times \frac{1}{2} = \mathbf{6.03\ A}$$

$$\text{Current } I_2 = \frac{V_1 - V_2}{3} = \frac{1}{3}\left[\frac{6572}{545} - \frac{556}{109}\right] = \mathbf{2.32A}$$

$$\text{Current } I_3 = \frac{V_1 - V_3}{5} = \frac{1}{5}\left[\frac{6572}{545} - \frac{2072}{545}\right] = \mathbf{1.65\ A}$$

$$\text{Current } I_4 = \frac{V_2 - V_3}{1} = \frac{556}{109} - \frac{2072}{545} = \mathbf{1.3A}$$

$$\text{Current } I_5 = \frac{V_2}{5} = \frac{556}{109} \times \frac{1}{5} = \mathbf{1.02A}$$

$$\text{Current } I_6 = \frac{V_3}{4} = \frac{2072}{545} \times \frac{1}{4} = \mathbf{0.95A}$$

Example 3.11. *Find the total power consumed in the circuit shown in Fig. 3.24.*

Solution. Mark the direction of currents in the various branches as shown in Fig. 3.24. Take D as the reference node. If voltages V_B and V_C at nodes B and C respectively are known, then all the currents can be calculated.

At node B. $I_1 + I_3 = I_2$

or $$\frac{15 - V_B}{1} + \frac{V_C - V_B}{0.5} = \frac{V_B}{1}$$

or $$15 - V_B + 2(V_C - V_B) - V_B = 0$$

or $$4V_B - 2V_C = 15 \quad ...(i)$$

At node C. $I_3 + I_4 = I_5$

or $$\frac{V_C - V_B}{0.5} + \frac{V_C}{2} = \frac{20 - V_C}{1}$$

or $$2(V_C - V_B) + 0{\cdot}5V_C - (20 - V_C) = 0$$

or $$3{\cdot}5V_C - 2V_B = 20$$

or $$4V_B - 7V_C = -40 \quad ...(ii)$$

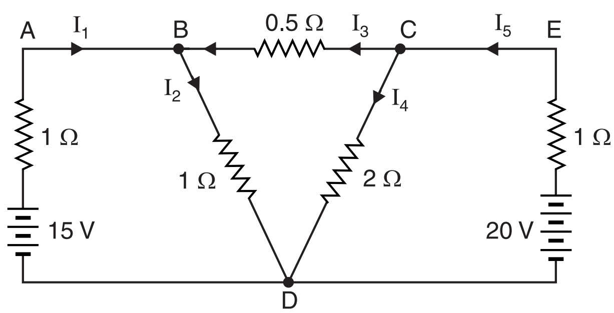


Fig. 3.24

Subtracting eq. (ii) from eq. (i), we get, $5V_C = 55$

$\therefore$ $V_C = 55/5 = 11$ volts

Putting $V_C = 11$ V in eq. (i), we get, $V_B = 9{\cdot}25$ V

$\therefore$ $$\text{Current } I_1 = \frac{15 - V_B}{1} = \frac{15 - 9.25}{1} = 5{\cdot}75 \text{ A}$$

$$\text{Current } I_2 = V_B/1 = 9{\cdot}25/1 = 9{\cdot}25 \text{ A}$$

$$\text{Current } I_3 = \frac{V_C - V_B}{0.5} = \frac{11 - 9.25}{0.5} = 3{\cdot}5 \text{ A}$$

$$\text{Current } I_4 = V_C/2 = 11/2 = 5{\cdot}5 \text{ A}$$

$$\text{Current } I_5 = \frac{20 - V_C}{1} = \frac{20 - 11}{1} = 9 \text{ A}$$

$\therefore$ Power loss in the circuit $= I_1^2 \times 1 + I_2^2 \times 1 + I_3^2 \times 0{\cdot}5 + I_4^2 \times 2 + I_5^2 \times 1$

$= (5{\cdot}75)^2 \times 1 + (9{\cdot}25)^2 \times 1 + (3{\cdot}5)^2 \times 0{\cdot}5 + (5{\cdot}5)^2 \times 2 + (9)^2 \times 1$

$=$ **266·25 W**

Example 3.12. *Using nodal analysis, find node-pair voltages V_B and V_C and branch currents in the circuit shown in Fig. 3.25. Use conductance method.*

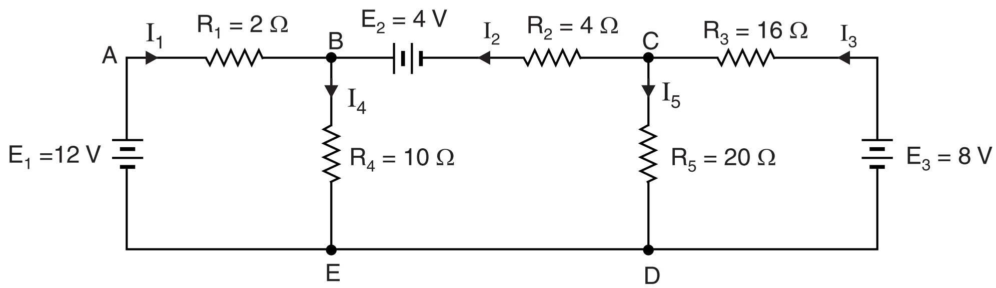


Fig. 3.25

Solution. Mark the currents in the various branches as shown in Fig. 3.25. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed. Take point *D* (or *E*) as the reference node. We shall find the voltages at nodes *B* and *C* and hence the branch currents.

$$G_1 = \frac{1}{R_1} = \frac{1}{2} = 0{\cdot}5 \text{ S} \; ; \; G_2 = \frac{1}{R_2} = \frac{1}{4} = 0{\cdot}25 \text{ S} \; ; \; G_3 = \frac{1}{R_3} = \frac{1}{16} = 0{\cdot}0625 \text{ S} \; ;$$

$$G_4 = \frac{1}{R_4} = \frac{1}{10} = 0.1 \text{ S} \; ; \quad G_5 = \frac{1}{R_5} = \frac{1}{20} = 0{\cdot}05 \text{ S}$$

At node *B*. $I_1 + I_2 = I_4$

or $(E_1 - V_B)G_1 + (E_2 - V_B + V_C)G_2 = V_B G_4$

or $E_1 G_1 + E_2 G_2 = V_B(G_1 + G_2 + G_4) - V_C G_2$

or $(12 \times 0.5) + (4 \times 0.25) = V_B(0{\cdot}5 + 0{\cdot}25 + 0{\cdot}1) - V_C \times 0{\cdot}25$

or $7 = 0{\cdot}85\, V_B - 0{\cdot}25\, V_C$ ...(*i*)

At node *C*. $I_3 = I_2 + I_5$

or $(E_3 - V_C)G_3 = (E_2 - V_B + V_C)G_2 + V_C \times G_5$

or $E_3 G_3 - E_2 G_2 = -V_B G_2 + V_C(G_2 + G_3 + G_5)$

or $(8 \times 0.0625) - (4 \times 0.25) = -V_B(0{\cdot}25) + V_C(0{\cdot}25 + 0{\cdot}0625 + 0{\cdot}05)$

or $-0{\cdot}5 = -0{\cdot}25\, V_B + 0{\cdot}362\, V_C$ ...(*ii*)

From equations (*i*) and (*ii*), we get, $V_B =$ **9.82 V** ; $V_C =$ **5·4V**

$\therefore$ $I_1 = (E_1 - V_B)G_1 = (12 - 9{\cdot}82) \times 0{\cdot}5 =$ **1·09 A**

$I_2 = (E_2 - V_B + V_C)G_2 = (4 - 9{\cdot}82 + 5{\cdot}4) \times 0{\cdot}25 =$ **–0·105A**

$I_3 = (E_3 - V_C)G_3 = (8 - 5{\cdot}4) \times 0{\cdot}0625 =$ **0·162A**

$I_4 = V_B G_4 = 9{\cdot}82 \times 0{\cdot}1 =$ **0·982A**

$I_5 = V_C G_5 = 5{\cdot}4 \times 0.05 =$ **0·27A**

The negative sign for I_2 means that the actual direction of this current is opposite to that shown in Fig. 3.25.

Example 3.13. *Using nodal analysis, find the different branch currents in the circuit shown in Fig. 3.26 (i).*

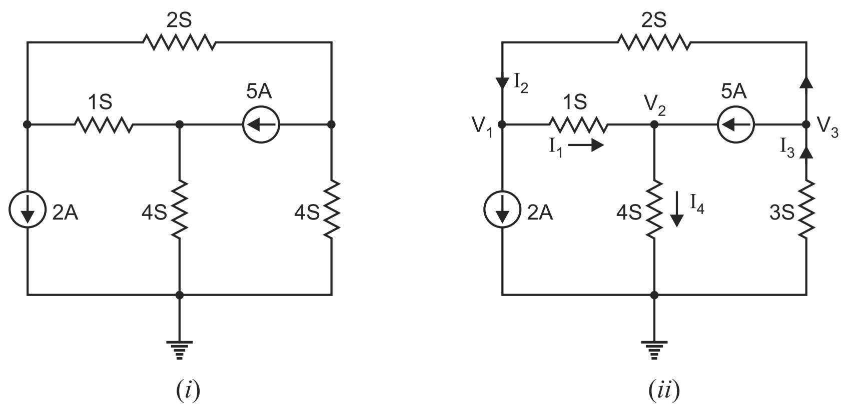


Fig. 3.26

Solution. Mark the currents in the various branches as shown in Fig. 3.26 (*ii*). Take ground as the reference node. We shall find the voltages at the other three nodes.

At first node. Applying *KCL* to the first node from left,

$$I_2 = I_1 + 2$$

or $$(V_3 - V_1)2 = (V_1 - V_2)1 + 2$$

or $$3V_1 - V_2 - 2V_3 = -2 \quad ...(i)$$

At second node. Applying *KCL* to the second node from left,

$$I_1 + 5 = I_4$$

or $$(V_1 - V_2)1 + 5 = V_2 \times 4$$

or $$V_1 - 5V_2 = -5 \quad ...(ii)$$

At third node. Applying *KCL* to the third node from left,

$$I_3 = 5 + I_2$$

or $$-V_3 \times 3 = 5 + (V_3 - V_1)2$$

or $$2V_1 - 5V_3 = 5 \quad ...(iii)$$

Solving eqs. (*i*), (*ii*) and (*iii*), we have, $V_1 = -\frac{3}{2}\text{V}$; $V_2 = \frac{7}{10}\text{V}$ and $V_3 = \frac{-8}{5}\text{V}$

$\therefore$ $$I_1 = (V_1 - V_2)1 = \left(-\frac{3}{2} - \frac{7}{10}\right)1 = \mathbf{-2.2A}$$

$$I_2 = (V_3 - V_1)2 = \left(-\frac{8}{5} + \frac{3}{2}\right)2 = \mathbf{-0.2A}$$

$$I_3 = -V_3 \times 3 = \frac{8}{5} \times 3 = \mathbf{4.8\ A}$$

$$I_4 = V_2 \times 4 = \frac{7}{10} \times 4 = \mathbf{2.8A}$$

The negative value of any current means that actual direction of current is opposite to that originally assumed.

Example 3.14. *Find the current I in Fig. 3.27 (i) by changing the two voltage sources into their equivalent current sources and then using nodal method. All resistances are in ohms.*

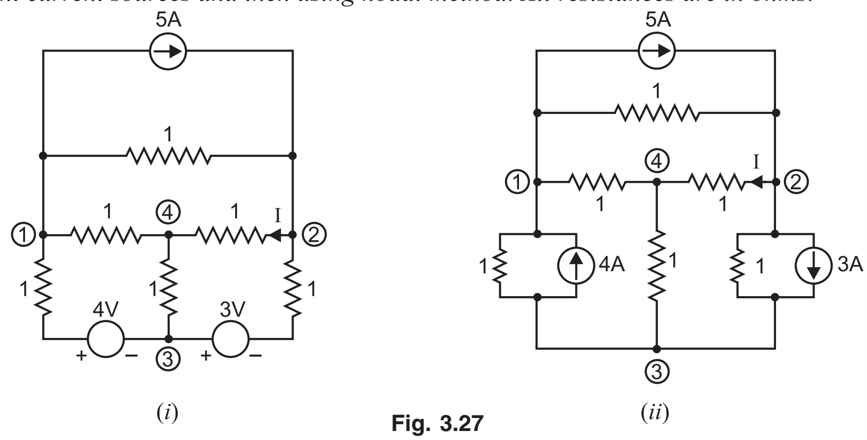


Fig. 3.27

Solution. Since we are to find *I*, it would be convenient to take node 4 as the reference node. The two voltage sources are converted into their equivalent current sources as shown in Fig. 3.27. (*ii*). We shall apply *KCL* at nodes 1, 2 and 3 in Fig. 3.27 (*ii*) to obtain the required solution.

At node 1. Applying *KCL*, we have,

$$\frac{V_3 - V_1}{1} + 4 = \frac{V_1}{1} + \frac{V_1 - V_2}{1} + 5$$

or $$3V_1 - V_2 - V_3 = -1 \qquad ...(i)$$

At node 2. Applying *KCL*, we have,

$$5 + \frac{V_1 - V_2}{1} = \frac{V_2}{1} + \frac{V_2 - V_3}{1} + 3$$

or $$V_1 - 3V_2 + V_3 = -2 \qquad ...(ii)$$

At node 3. Applying *KCL*, we have,

$$\frac{V_2 - V_3}{1} + 3 - \frac{V_3}{1} = \frac{V_3 - V_1}{1} + 4$$

or $$V_1 + V_2 - 3V_3 = 1 \qquad ...(iii)$$

From eqs. (*i*), (*ii*) and (*iii*), we get, $V_2 = 0.5$ V.

$\therefore$ $$\text{Current } I = \frac{V_2 - 0}{1} = \frac{0.5 - 0}{1} = \mathbf{0.5A}$$

Example 3.15. *Use nodal analysis to find the voltage across and current through 4 Ω resistor in Fig. 3.28 (i).*

Solution. We must first convert the 2V voltage source to an equivalent current source. The value of the equivalent current source is $I = 2V/2\Omega = 1$ A. The circuit then becomes as shown in Fig. 3.28 (*ii*).

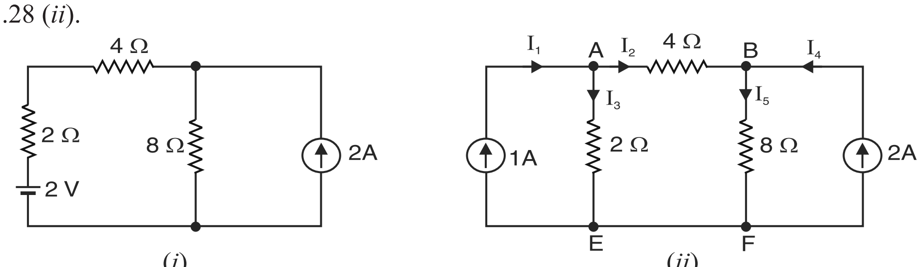


Fig. 3.28

Mark the currents in the various branches as shown in Fig. 3.28 (*ii*). Take point *E* (or *F*) as the reference node. We shall calculate the voltages at nodes *A* and *B*.

At node A. $I_1 = I_2 + I_3$

or $1 = \frac{{}^*V_A - V_B}{4} + \frac{V_A}{2}$

or $3V_A - V_B = 4$ ...(*i*)

At node B. $I_2 + I_4 = I_5$

or $\frac{V_A - V_B}{4} + 2 = \frac{V_B}{8}$

or $2V_A - 3V_B = -16$ ...(*ii*)

Solving equations (*i*) and (*ii*), we find $V_A = 4$V and $V_B = 8$V. Note that $V_B > V_A$, contrary to our initial assumption. Therefore, actual direction of current is from node *B* to node *A*.

By determinant method

$$3V_A - V_B = 4$$
$$2V_A - 3V_B = -16$$

$$\therefore \quad V_A = \frac{\begin{vmatrix} 4 & -1 \\ -16 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix}} = \frac{(-12)-(16)}{(-9)-(-2)} = \frac{-28}{-7} = 4\text{V}$$

$$V_B = \frac{\begin{vmatrix} 3 & 4 \\ 2 & -16 \end{vmatrix}}{\text{Denominator}} = \frac{(-48)-(8)}{-7} = \frac{-56}{-7} = 8\text{V}$$

Voltage across 4Ω resistor = $V_B - V_A = 8 - 4 =$ **4V**

Current through 4Ω resistor $= \frac{4\text{V}}{4\Omega} =$ **1A**

We can also find the currents in other resistors.

$$I_3 = \frac{V_A}{2} = \frac{4}{2} = 2\text{A}$$

$$I_5 = \frac{V_B}{8} = \frac{8}{8} = 1\text{A}$$

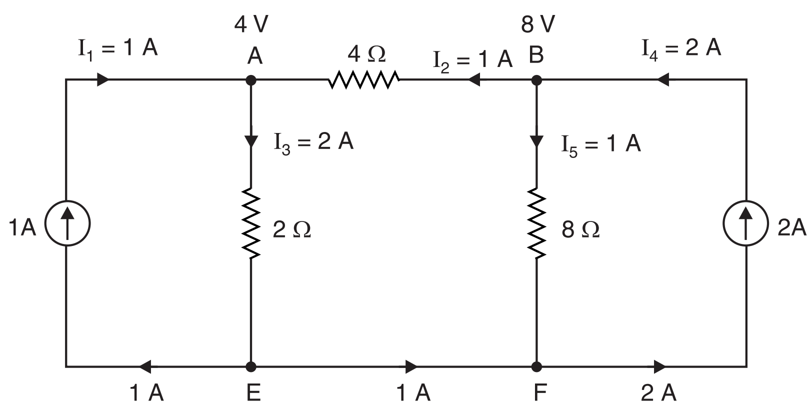


Fig. 3.29

* We assume that $V_A > V_B$. On solving the circuit, we shall see whether this assumption is correct or not.

Fig. 3.29 shows the various currents in the circuit. You can verify Kirchhoff's current law at each node.

Example 3.16. *Use nodal analysis to find current in the 4 kΩ resistor shown in Fig. 3.30.*

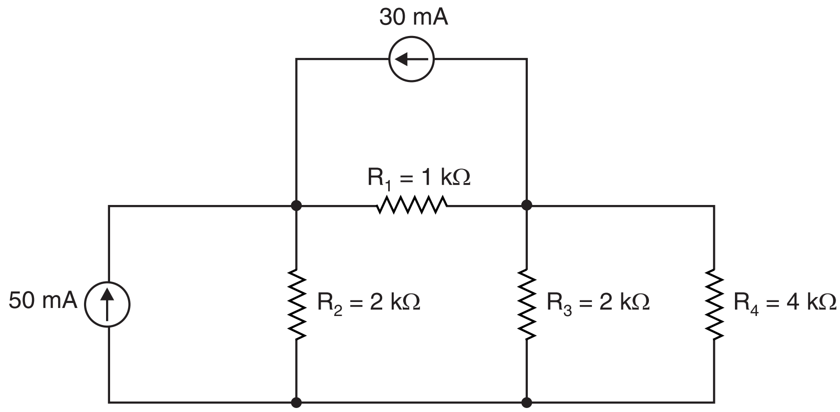


Fig. 3.30

Solution. We shall solve this example by expressing node currents in terms of conductance than expressing them in terms of resistance. The conductance of each resistor is

$$G_1 = \frac{1}{R_1} = \frac{1}{1\times10^3} = 10^{-3}\text{ S} \;;\; G_2 = \frac{1}{R_2} = \frac{1}{2\times10^3} = 0.5\times10^{-3}\text{ S}$$

$$G_3 = \frac{1}{R_3} = \frac{1}{2\times10^3} = 0.5\times10^{-3}\text{ S} \;;\; G_4 = \frac{1}{R_4} = \frac{1}{4\times10^3} = 0.25\times10^{-3}\text{ S}$$

Mark the currents in the various branches as shown in Fig. 3.31. Take point *E* (or *F*) as the reference node. We shall find voltages at nodes *A* and *B*.

At node A. $I_5 + I_6 = I_1 + I_2$

or $50\times10^{-3} + 30\times10^{-3} = G_1(V_A - V_B) + G_2 V_A$

or $80\times10^{-3} = 10^{-3}(V_A - V_B) + 0.5\times10^{-3} V_A$

or $1.5V_A - V_B = 80$ ...(*i*)

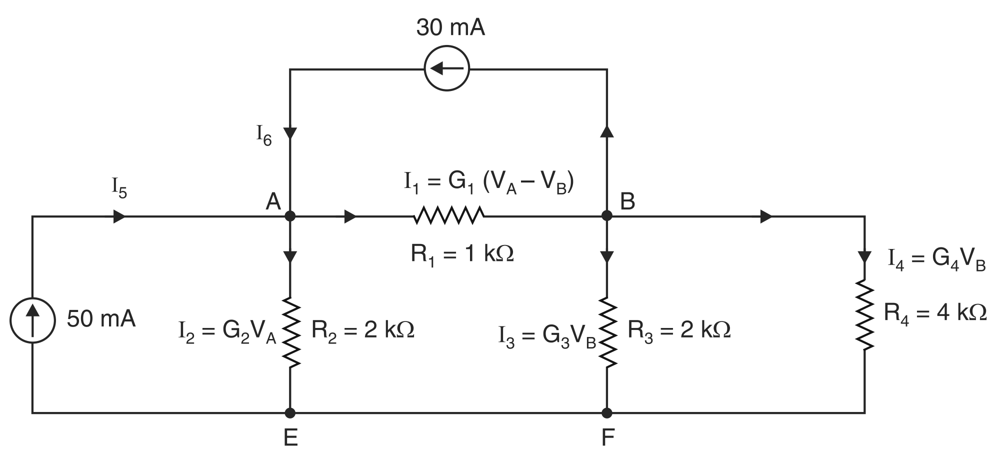


Fig. 3.31

At node B. $I_1 = I_6 + I_3 + I_4$

or $G_1(V_A - V_B) = 30\times10^{-3} + G_3V_B + G_4V_B$

or $\quad 10^{-3}(V_A - V_B) = 30 \times 10^{-3} + 0.5 \times 10^{-3}\, V_B + 0{\cdot}25 \times 10^{-3}\, V_B$

or $\quad V_A - 1{\cdot}75\, V_B = 30 \quad ...(ii)$

Solving equations (*i*) and (*ii*), we get, $V_B = 21{\cdot}54$ V.

By determinant method

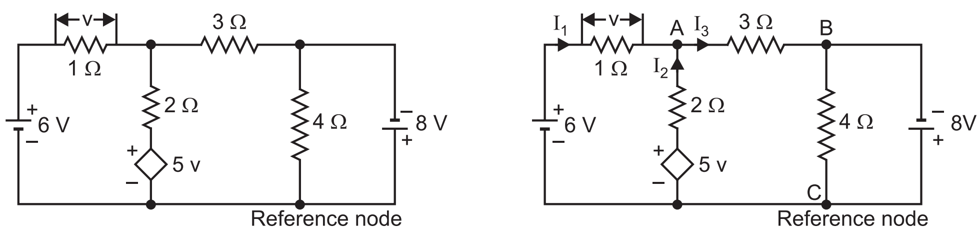
$$1{\cdot}5\, V_A - V_B = 80$$

$$V_A - 1{\cdot}75\, V_B = 30$$

$$\therefore \quad V_B = \frac{\begin{vmatrix} 1.5 & 80 \\ 1 & 30 \end{vmatrix}}{\begin{vmatrix} 1.5 & -1 \\ 1 & -1.75 \end{vmatrix}} = \frac{(45)-(80)}{(-2.625)-(-1)} = \frac{-35}{-1.625} = 21{\cdot}54 \text{ V}$$

$\therefore$ Current in 4 kΩ resistor, $I_4 = G_4 V_B = 0{\cdot}25 \times 10^{-3} \times 21{\cdot}54 = 5{\cdot}39 \times 10^{-3}$ A = **5·39 mA**

Example 3.17. *For the circuit shown in Fig. 3.32 (i), find (i) voltage v and (ii) current through 2Ω resistor using nodal method.*



(*i*) (*ii*)

Fig. 3.32

Solution. Mark the direction of currents in the various branches as shown in Fig. 3.32 (*ii*). Let us take node *C* as the reference node. It is clear from Fig. 3.32 (*ii*) that $V_B = -8$V ($\because V_C = 0$V). Also, $v = 6 - V_A$.

Applying *KCL* to node A, we have,

$$I_1 + I_2 = I_3$$

or $$\frac{6 - V_A}{1} + \frac{5v - V_A}{2} = \frac{V_A - V_B}{3}$$

or $$\frac{6 - V_A}{1} + \frac{5(6 - V_A) - V_A}{2} = \frac{V_A - (-8)}{3}$$

On solving, we get, $V_A = \dfrac{55}{13}$ V

(*i*) Voltage $v = 6 - V_A = 6 - \dfrac{55}{13} = \dfrac{\mathbf{23}}{\mathbf{13}}$ V

(*ii*) Current through 2Ω, $I_2 = \dfrac{5v - V_A}{2} = \dfrac{5(23/13) - (55/13)}{2} = \dfrac{\mathbf{30}}{\mathbf{13}}$ **A**

3.8. Shortcut Method for Nodal Analysis

There is a shortcut method for writing node equations similar to the form for mesh equations. Consider the circuit with three independent nodes *A*, *B* and *C* as shown in Fig. 3.33.

The node equations in shortcut form for nodes *A*, *B* and *C* can be written as under :

$$V_A G_{AA} + V_B G_{AB} + V_C G_{AC} = I_A$$
$$V_A G_{BA} + V_B G_{BB} + V_C G_{BC} = I_B$$
$$V_A G_{CA} + V_B G_{CB} + V_C G_{CC} = I_C$$

Let us discuss the various terms in these equations.

G_{AA} = Sum of all conductances connected to node A
= $G_1 + G_2$ in Fig. 3.33.

The term G_{AA} is called *self-conductance* at node A. Similarly, G_{BB} and G_{CC} are self-conductances at nodes B and C respectively. *Note that product of node voltage at a node and self-conductance at that node is always a* **positive** *quantity.* Thus $V_A G_{AA}$, $V_B G_{BB}$ and $V_C G_{CC}$ are all positive.

G_{AB} = Sum of all conductances *directly* connected between nodes A and B
= G_2 in Fig. 3.33

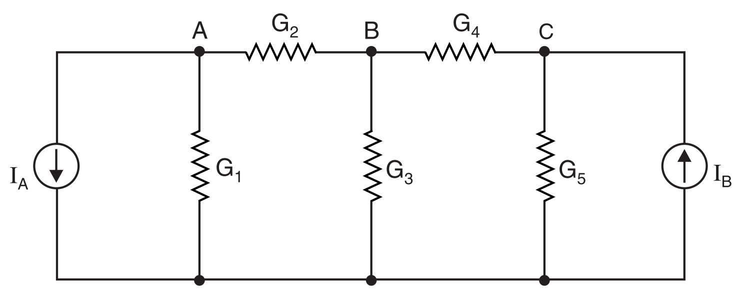


Fig. 3.33

The term G_{AB} is called *common conductance* between nodes A and B. Similarly, the term G_{BC} is common conductance between nodes B and C and G_{CA} is common conductance between nodes C and A. *The product of connecting node voltage with common conductance is always a* **negative** *quantity.* Thus $V_B G_{AB}$ is a negative quantity. Here connecting node voltage is V_B and common conductance is G_{AB}. Note that $G_{AB} = G_{BA}$, $G_{AC} = G_{CA}$ and so on.

Note the direction of current provided by current source connected to the node. *A current leaving the node is shown as negative and a current entering a node is positive. If a node has no current source connected to it, set the term equal to zero.*

Node A. Refer to Fig. 3.33. At node A, $G_{AA} = G_1 + G_2$ and is a positive quantity. The product $V_B G_{AB}$ is a negative quantity. The current I_A is leaving the node A and will be assigned a negative sign. Therefore, node equation at node A is

$$V_A G_{AA} - V_B G_{AB} = -I_A$$

or $$V_A(G_1 + G_2) - V_B(G_2) = -I_A$$

Similarly, for **nodes B and C,** the node equations are :

$$V_B(G_2 + G_3 + G_4) - V_A(G_2) - V_C(G_4) = 0$$
$$V_C(G_4 + G_5) - V_B(G_4) = I_B$$

Example 3.18. *Solve the circuit shown in Fig. 3.34 using nodal analysis.*

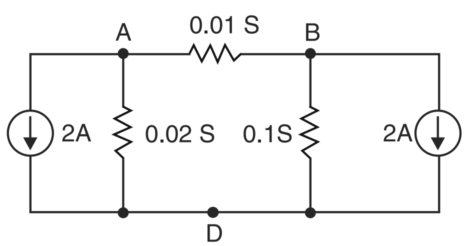


Fig. 3.34

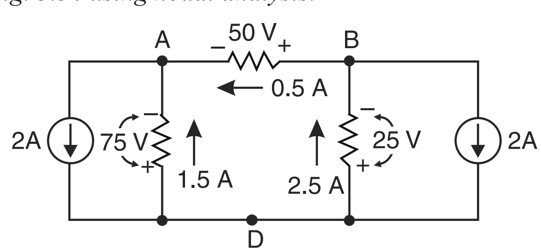


Fig. 3.35

Solution. Here point D is chosen as the reference node and A and B are the independent nodes.

Node A. $V_A(0.02 + 0.01) - V_B(0.01) = -2$

or $0.03\ V_A - 0.01\ V_B = -2$ ...(*i*)

Node B. $V_B(0.01 + 0.1) - V_A(0.01) = -2$

or $-0.01\ V_A + 0.11\ V_B = -2$ ...(*ii*)

From equations (*i*) and (*ii*), we have, $V_A = -75\text{V}$ and $V_B = -25\text{V}$

Fig. 3.35 shows the circuit redrawn with solved voltages.

Current in 0.02 S = $VG = 75 \times 0{\cdot}02 =$ **1·5A**

Current in 0.1 S = $VG = 25 \times 0{\cdot}1 =$ **2·5A**

Current in 0.01 S = $VG = 50 \times 0{\cdot}01 =$ **0·5 A**

The directions of currents will be as shown in Fig. 3.35.

Example 3.19. *Solve the circuit shown in Fig. 3.36 using nodal analysis.*

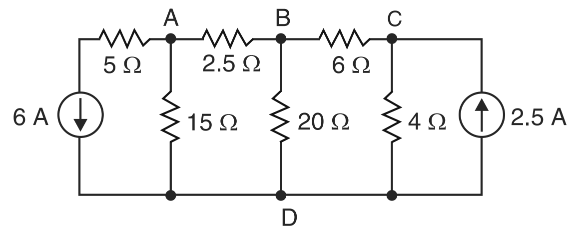


Fig. 3.36

Solution. Here A, B and C are the independent nodes and D is the reference node.

Node A. $$V_A^*\left(\frac{1}{15}+\frac{1}{2.5}\right) - V_B\left(\frac{1}{2.5}\right) = -6$$

or $$0{\cdot}467\ V_A - 0{\cdot}4\ V_B = -6 \quad ...(i)$$

Node B. $$V_B\left(\frac{1}{2.5}+\frac{1}{20}+\frac{1}{6}\right) - V_A\left(\frac{1}{2.5}\right) - V_C\left(\frac{1}{6}\right) = 0$$

or $$-0{\cdot}4\ V_A + 0{\cdot}617\ V_B - 0{\cdot}167\ V_C = 0 \quad ...(ii)$$

Node C. $$V_C\left(\frac{1}{6}+\frac{1}{4}\right) - V_B\left(\frac{1}{6}\right) = 2{\cdot}5$$

or $$-0{\cdot}167\ V_B + 0{\cdot}417\ V_C = 2{\cdot}5 \quad ...(iii)$$

From equations (*i*), (*ii*) and (*iii*), $V_A = -30$ V ; $V_B = -20$ V ; $V_C = -2$ V

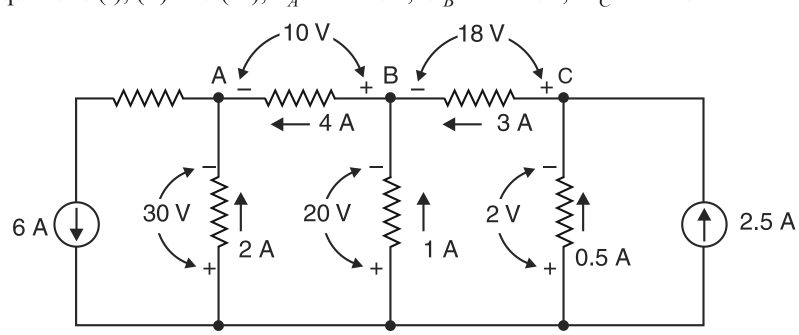


Fig. 3.37

* Note that 5Ω is omitted from the equation for node A because it is in series with the current source.

Fig. 3.37 shows the circuit redrawn with solved voltages.

$$\text{Current in } 15\ \Omega = 30/15 = \mathbf{2\ A}$$
$$\text{Current in } 20\ \Omega = 20/20 = \mathbf{1\ A}$$
$$\text{Current in } 4\ \Omega = 2/4 = \mathbf{0.5\ A}$$
$$\text{Current in } 6\ \Omega = 18/6 = \mathbf{3\ A}$$
$$\text{Current in } 2.5\ \Omega = 10/2.5 = \mathbf{4\ A}$$
$$\text{Current in } 5\ \Omega = 4 + 2 = \mathbf{6\ A}$$

The directions of currents will be as shown in Fig. 3.37.

Example 3.20. *Find the value of I_x in the circuit shown in Fig. 3.38 using nodal analysis. The various values are :*

$G_u = 10\ S\ ;\ G_v = 1S\ ;\ G_w = 2S\ ;$

$G_x = 1S\ ;\ G_y = 1S\ ;\ G_z = 1S$ *and* $I = 100\ A$.

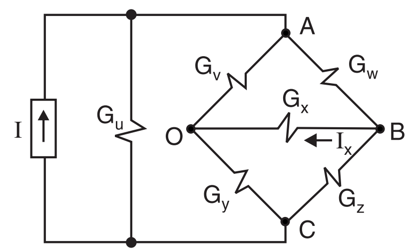


Fig. 3.38

Solution.

Node A. $(G_u + G_v + G_w)V_A - G_wV_B - G_uV_C = I$

Node B. $-G_wV_A + (G_w + G_x + G_z)V_B - G_zV_C = 0$

Node C. $-G_uV_A - G_zV_B + (G_u + G_y + G_z)V_C = -I$

Putting the various values in these equations, we have,

$$13\ V_A - 2\ V_B - 10\ V_C = I$$
$$-2\ V_A + 4\ V_B - V_C = 0$$
$$-10\ V_A - V_B + 12\ V_C = -I$$

Now V_B can be calculated as the ratio of two determinants N_B/D where

$$D = \begin{vmatrix} 13 & -2 & -10 \\ -2 & 4 & -1 \\ -10 & -1 & 12 \end{vmatrix} = 624 - 20 - 20 - (400 + 48 + 13) = 123$$

and

$$N_B = \begin{vmatrix} 13 & I & -10 \\ -2 & 0 & -1 \\ -10 & -I & 12 \end{vmatrix} = 10I - 20I - (13I - 24I) = I$$

$$\therefore \quad V_B = \frac{N_B}{D} = \frac{I}{123}$$

$$\text{Current } I_x = G_xV_B = 1 \times \frac{I}{123} = 1 \times \frac{100}{123} = \mathbf{0.813A}$$

Tutorial Problems

1. Using nodal analysis, find the voltages at nodes *A*, *B* and *C* w.r.t. the reference node shown by the ground symbol in Fig. 3.39.

   [V_A = –30V ; V_B = –20V ; V_C = –2V]

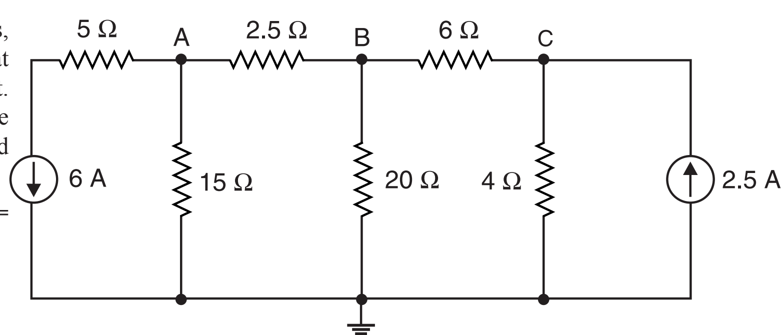


Fig. 3.39

2. Using nodal analysis, find the current through 0.05 S conductance in Fig. 3.40. **[0.264 A]**

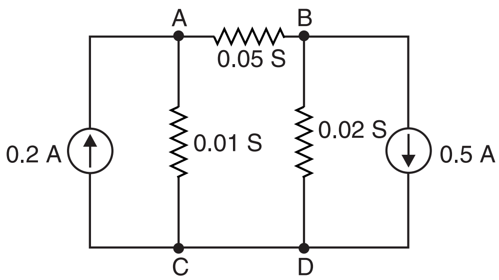


Fig. 3.40

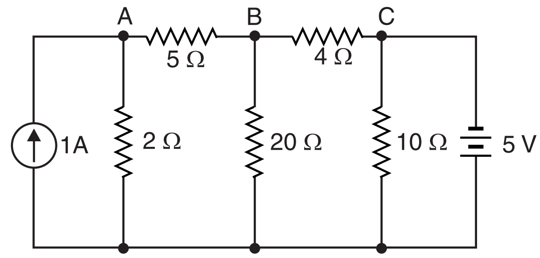


Fig. 3.41

3. Using nodal analysis, find the current flowing in the battery in Fig. 3.41. **[1.21 A]**

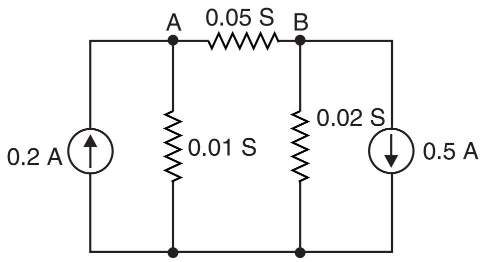


Fig. 3.42

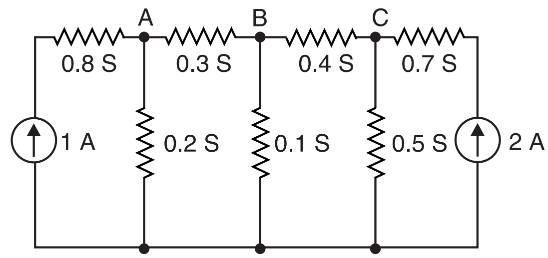


Fig. 3.43

4. In Fig. 3.42, find the node voltages. **[V_A = –6.47 V; V_B = – 11.8V]**
5. In Fig. 3.42, find current through 0.05 S conductance. Use nodal analysis. **[264 mA]**
6. In Fig. 3.43, find the node voltages. **[V_A = 4.02 V; V_B = 3.37 V ; V_C = 3.72 V]**
7. By using nodal analysis, find current in 0.3 S in Fig. 3.43. **[196 mA]**
8. Using nodal analysis, find current in 0.4 S conductance in Fig. 3.43. **[141 mA]**

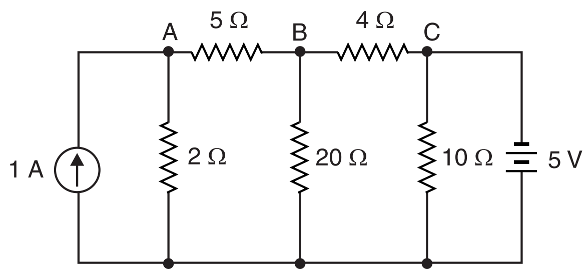


Fig. 3.44

9. Find node voltages in Fig. 3.44. **[V_A = 0.806 V; V_B = –2.18 V; V_C = –5 V]**
10. Using nodal analysis, find current through the battery in Fig. 3.44. **[1. 21A]**

3.9. Superposition Theorem

Superposition is a general principle that allows us to determine the effect of several energy sources (voltage and current sources) acting simultaneously in a circuit by considering the effect of each source acting alone, and then combining (superposing) these effects. This theorem as applied to d.c. circuits may be stated as under :

In a linear, bilateral d.c. network containing more than one energy source, the resultant potential difference across or current through any element is equal to the algebraic sum of potential differences or currents for that element produced by each source acting alone with all other independent ideal voltage sources replaced by short circuits and all other independent ideal current sources replaced by open circuits (non-ideal sources are replaced by their internal resistances).

Procedure. The procedure for using this theorem to solve d.c. networks is as under :

(*i*) Select one source in the circuit and replace all other ideal voltage sources by short circuits and ideal current sources by open circuits.

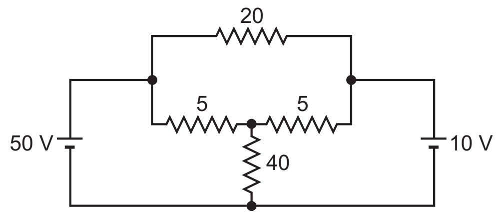
(*ii*) Determine the voltage across or current through the desired element/branch due to single source selected in step (*i*).

(*iii*) Repeat the above two steps for each of the remaining sources.

(*iv*) *Algebraically* add all the voltages across or currents through the element/branch under consideration. The sum is the actual voltage across or current through that element/branch when all the sources are acting simultaneously.

Note. This theorem is called *superposition* because we superpose or algebraically add the components (currents or voltages) due to each independent source acting alone to obtain the total current in or voltage across a circuit element.

Example 3.21. *Using superposition theorem, find the current through the 40 Ω resistor in the circuit shown in Fig. 3.45 (i). All resistances are in ohms.*

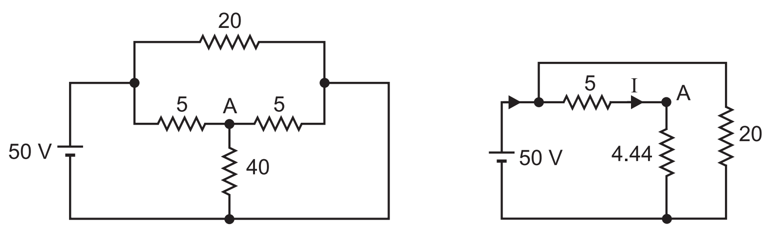


(*i*)

Fig. 3.45

Solution. In Fig. 3.45 (*ii*), 10V battery is replaced by a short so that 50V battery is acting alone. It can be seen that right-hand 5 Ω resistance is in parallel with 40 Ω resistance and their combined resistance = 5 Ω || 40 Ω = 4.44 Ω as shown in Fig. 3.45 (*iii*). The 4.44 Ω resistance is in series with left-hand 5 Ω resistance giving total resistance of (5 + 4.44) = 9.44 Ω to this path. As can be seen from Fig. 3.45 (*iii*), there are two parallel branches of resistances 20 Ω and 9.44 Ω across the 50 V battery. Therefore, current through 9.44 Ω branch is I = 50/9.44 = 5.296 A. Thus in Fig. 3.45 (*ii*), the current I (= 5.296 A) at point A divides between 5 Ω resistance and 40 Ω resistance. By current-divider rule, current I_1 in 40 Ω resistance is

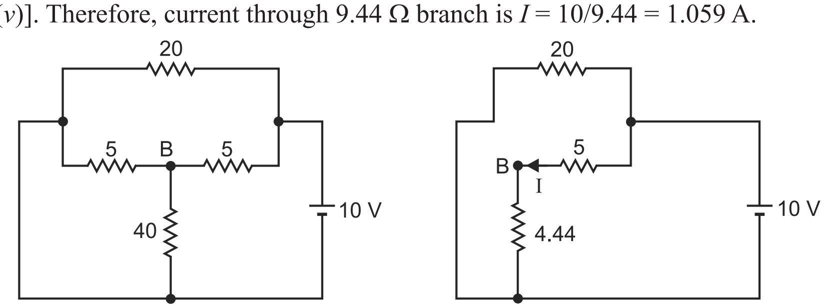
$$I_1 = I \times \frac{5}{5+40} = 5.296 \times \frac{5}{45} = 0.589 \text{ A downward}$$



(*ii*) (*iii*)

Fig. 3.45

In Fig. 3.45 (*iv*), the 50 V battery is replaced by a short so that 10 V battery is acting alone. Again, there are two parallel branches of resistances 20 Ω and 9.44 Ω across the 10V battery [See Fig. 3.45 (*v*)]. Therefore, current through 9.44 Ω branch is I = 10/9.44 = 1.059 A.



(*iv*) (*v*)

Fig. 3.45

Thus in Fig. 3.45 (*iv*), the current I (= 1.059 A) at point B divides between 5 Ω resistance and 40 Ω resistance. By current-divider rule, current in 40 Ω resistance is

$$I_2 = 1.059 \times \frac{5}{5+40} = 0.118 \text{ A downward}$$

∴ By superposition theorem, the total current in 40 Ω

$$= I_1 + I_2 = 0.589 + 0.118 = \mathbf{0.707\ A\ downward}$$

Example 3.22. *In the circuit shown in Fig. 3.46 (i), the internal resistances of the batteries are 0·12 Ω and 0·08 Ω. Calculate (i) current in load (ii) current supplied by each battery.*

Solution. In Fig. 3.46 (*ii*), the right-hand 12 V source is replaced by its internal resistance so that left-hand battery of 12 V is acting alone. The various branch currents due to left-hand battery of 12 V alone [See Fig. 3.46 (*ii*)] are :

$$\text{Total circuit resistance} = 0.12 + \frac{0.08 \times 0.5}{0.08 + 0.5} = 0{\cdot}189\ \Omega$$

$$\text{Total circuit current, } I_1' = 12/0{\cdot}189 = 63{\cdot}5\text{ A}$$

$$\text{Current in } 0{\cdot}08\ \Omega,\ I_2' = 63.5 \times \frac{0.5}{0.08 + 0.5} = 54{\cdot}74\text{ A}$$

$$\text{Current in } 0{\cdot}5\ \Omega,\ I_3' = 63.5 \times \frac{0.08}{0.08 + 0.5} = 8{\cdot}76\text{ A}$$

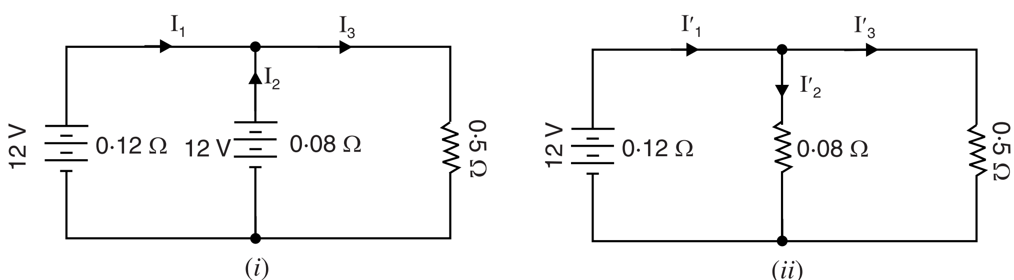


Fig. 3.46

In Fig. 3.46 (*iii*), left-hand 12 V source is replaced by its internal resistance so that now right-hand 12 V source is acting alone.

$$\text{Total circuit resistance} = 0.08 + \frac{0.12 \times 0.5}{0.12 + 0.5} = 0{\cdot}177\ \Omega$$

$$\text{Total circuit current, } I_2'' = 12/0{\cdot}177 = 67{\cdot}8\text{ A}$$

$$\text{Current in } 0{\cdot}12\ \Omega,\ I_1'' = 67.8 \times \frac{0.5}{0.12 + 0.5} = 54{\cdot}6\text{ A}$$

$$\text{Current in } 0{\cdot}5\ \Omega,\ I_3'' = 67.8 \times \frac{0.12}{0.12 + 0.5} = 13{\cdot}12\text{ A}$$

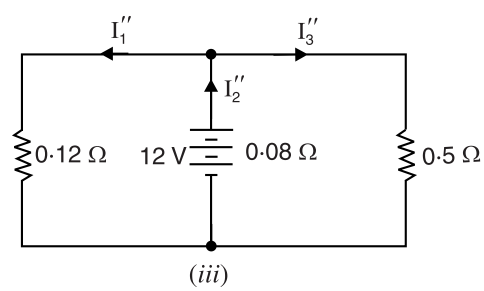


Fig. 3.46

The actual current values of I_1 (current in first battery), I_2 (current in second battery) and I_3 (load current) can be found by algebraically adding the component values.

$$I_1 = I_1' - I_1'' = 63{\cdot}5 - 54{\cdot}6 = \mathbf{8{\cdot}9\ A}$$

$$I_2 = I_2'' - I_2' = 67{\cdot}8 - 54{\cdot}74 = \mathbf{13{\cdot}06\ A}$$

$$I_3 = I_3' + I_3'' = 8{\cdot}76 + 13{\cdot}12 = \mathbf{21{\cdot}88\ A}$$

Example 3.23. *By superposition theorem, find the current in resistance R in Fig. 3.47 (i).*

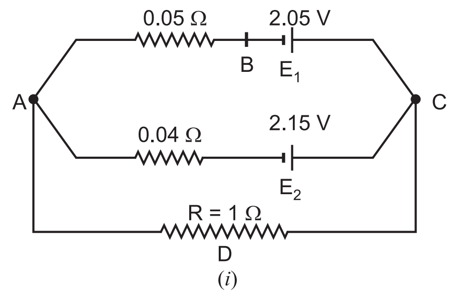


Fig. 3.47

Solution. In Fig. 3.47 (*ii*), battery E_2 is replaced by a short so that battery E_1 is acting alone. It is clear that resistances of 1Ω (= R) and 0.04Ω are in parallel across points A and C.

$$\therefore \quad R_{AC} = 1\Omega \,||\, 0.04\Omega = \frac{1 \times 0.04}{1 + 0.04} = 0.038\ \Omega$$

This resistance (*i.e.*, R_{AC}) is in series with 0.05 Ω.

Total resistance to battery $E_1 = 0.038 + 0.05 = 0.088\Omega$

∴ Current supplied by battery E_1 is

$$I = \frac{E_1}{0.088} = \frac{2.05}{0.088} = 23.2\text{A}$$

The current I(= 23.2A) is divided between the parallel resistances of 1Ω (= R) and 0.04Ω.

∴ Current in 1Ω (= R) resistance is

$$I_1 = 23.2 \times \frac{0.04}{1 + 0.04} = 0.892 \text{ A from } C \text{ to } A$$

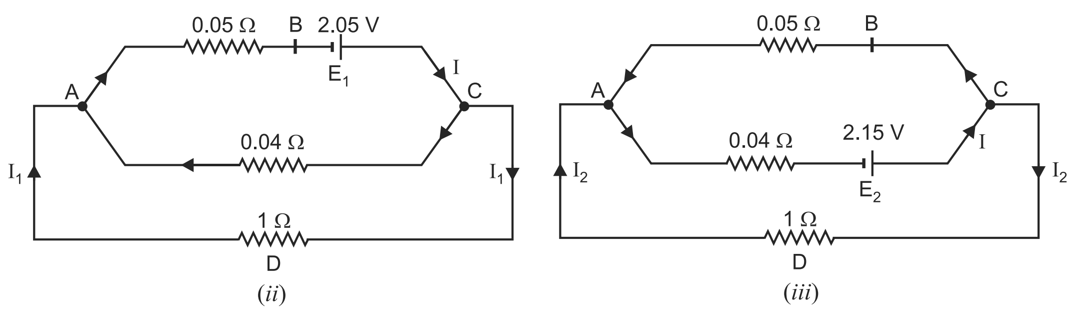


Fig. 3.47

In Fig. 3.47 (*iii*), battery E_1 is replaced by a short so that battery E_2 is acting alone.

Total resistance offered to battery E_2

$$= (1\Omega \,||\, 0.05\Omega) + 0.04\Omega$$

$$= \frac{1 \times 0.05}{1 + 0.05} + 0.04 = 0.088\Omega$$

∴ Current supplied by battery E_2 is

$$I = \frac{2.15}{0.088} = 24.4\text{A}$$

The current I(= 24.4A) is divided between two parallel resistances of 1Ω (= R) and 0.05Ω.

∴ Current in 1Ω (= R) resistance is

$$I_2 = 24.4 \times \frac{0.05}{1 + 0.05} = 1.16\text{A from } C \text{ to } A$$

∴ Current through 1Ω resistance when both batteries are present

$$= I_1 + I_2 = 0.892 + 1.16 = \mathbf{2.052A}$$

Example 3.24. *Using the superposition principle, find the voltage across 1kΩ resistor in Fig. 3·48. Assume the sources to be ideal.*

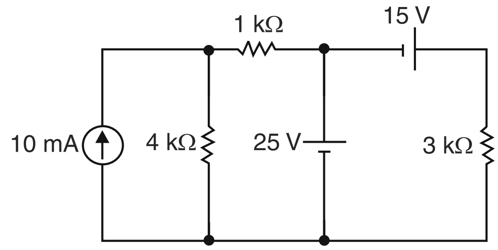


Fig. 3.48

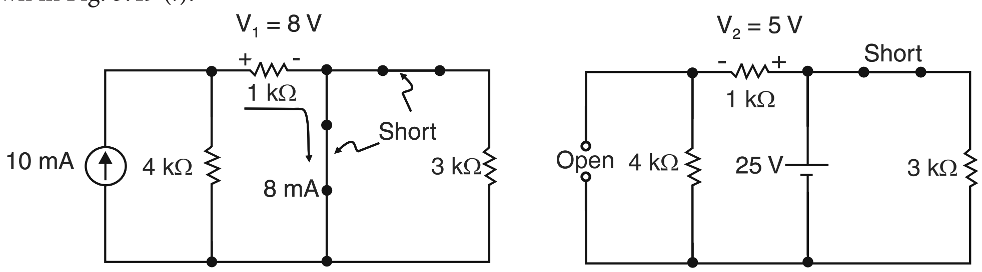
Solution. ***(i)*** The voltage across 1kΩ resistor due to **current source acting alone** is found by replacing 25-V and 15-V sources by short circuit as shown in Fig. 3.49 (*i*). Since 3 kΩ resistor is shorted out, the current in 1 kΩ resistor is, by current divider rule,

$$I_{1\,k\,\Omega} = \left(\frac{4}{1+4}\right)10 = 8 \text{ mA}$$

$\therefore$ Voltage V_1 across 1 kΩ resistor is

$$V_1 = (8 \text{ mA})\,(1 \text{ k}\Omega) = {}^{+}8\text{V}^{-}$$

The + and – symbols indicate the polarity of the voltage due to current source acting alone as shown in Fig. 3.49 (*i*).



Voltage across the 1-k Ω resistor due to the 10-mA current source acting alone.

(*i*)

Voltage across the 1-k Ω resistor due to the 25-V voltage source acting alone.

(*ii*)

Fig. 3.49

(ii) The voltage across the 1 k Ω resistor due to **25 V source acting alone** is found by replacing the 10 mA current source by an open circuit and 15 V source by a short circuit as shown in Fig. 3.49 (*ii*). Since the 25 V source is across the series combination of the 1 kΩ and 4 k Ω resistors, the voltage V_2 across 1 kΩ resistor can be found by the voltage divider rule.

$$\therefore \qquad V_2 = \left(\frac{1}{4+1}\right)25 = {}^{-}5\text{V}^{+}$$

Note that 3 kΩ resistor has no effect on this computation.

(iii) The voltage V_3 across 1 kΩ resistor due to **15 V source acting alone** is found by replacing the 25 V source by a short circuit and the 10 mA current source by an open circuit as shown in Fig. 3.49 (*iii*). The short circuit prevents any current from flowing in the 1 k Ω resistor.

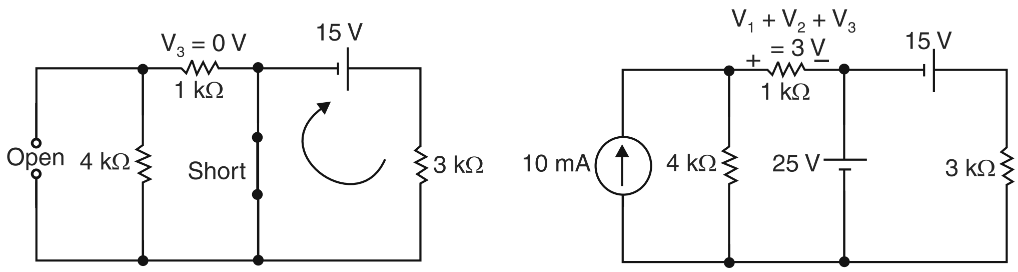
$$\therefore \qquad V_3 = 0$$

(iv) Applying superposition principle, the voltage across the 1kΩ resistor due to all the three sources acting simultaneously [See Fig. 3.49 (*iv*)] is

$$\begin{aligned} V_{1\,k\,\Omega} &= V_1 + V_2 + V_3 \\ &= {}^{+}8\text{ V}^{-} + {}^{-}5\text{ V}^{+} + 0\text{ V} \\ &= {}^{+}\mathbf{3\ V}^{-} \end{aligned}$$

Note that V_1 and V_2 have opposite polarities so that the sum (net) voltage is actually

$$= 8 - 5 = 3\text{ V}$$



Voltage across the 1-k Ω resistor due to the 15-V voltage source acting alone.

(*iii*)

Voltage across the 1-k Ω resistor due to all sources acting simultaneously.

(*iv*)

Fig. 3.49

Example 3.25. *To what voltage should adjustable source E be set in order to produce a current of 0.3 A in the 400 Ω resistor shown in Fig. 3.50?*

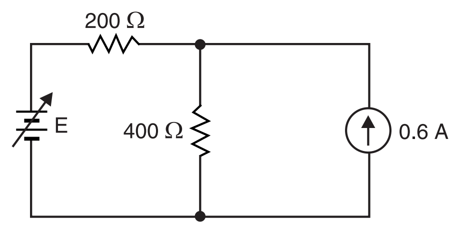
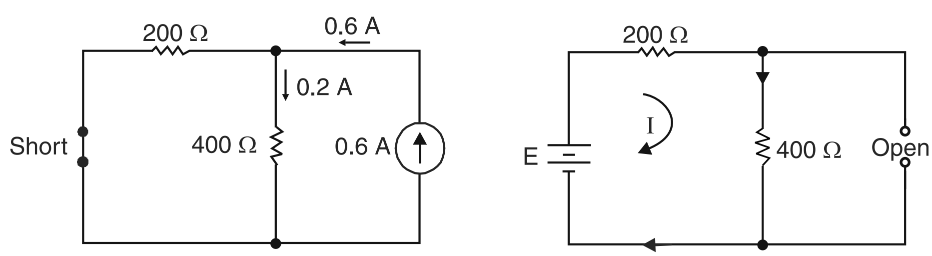


Fig. 3.50

Solution. We first find the current I_1 in 400 Ω resistor due to the 0.6 A current source alone. This current can be found by replacing E by a short circuit as shown in Fig. 3.51 (*i*). Applying current divider rule to Fig. 3.51 (*i*),

$$I_1 = \left(\frac{200}{200 + 400}\right) 0.6 = 0.2\text{ A}$$



(*i*)

(*ii*)

Fig. 3.51

In order that current in the 400 Ω resistor is equal to 0.3 A, the current produced in the resistor by the voltage source acting alone must be = 0.3 − 0.2 = 0.1 A. The current in the 400 Ω resistor due to voltage source alone can be calculated by open-circuiting the current source as shown in Fig 3.51 (*ii*). Referring to Fig. 3.51 (*ii*) and applying Ohm's law, we have,

$$I = \frac{E}{200 + 400} = \frac{E}{600}$$

or

$$0.1 = \frac{E}{600} \qquad \therefore\ E = 600 \times 0.1 = \mathbf{60\ V}$$

Example 3.26. *Use superposition theorem to find current I in the circuit shown in Fig. 3.52 (i). All resistances are in ohms.*

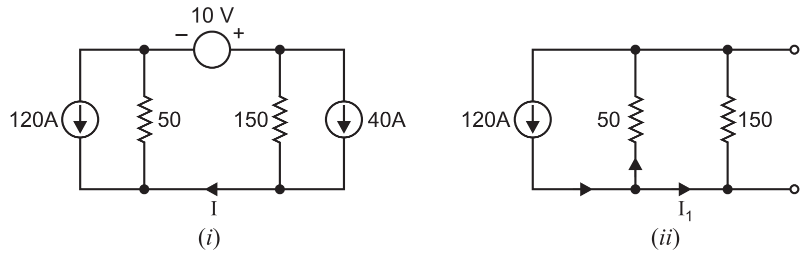


Fig. 3.52

Solution. In Fig. 3.52 (*ii*), the 10V voltage source has been replaced by a short and the 40A current source by an open so that now only 120A current source is acting alone. By current-divider rule, I_1 is given by ;

$$I_1 = 120 \times \frac{50}{50+150} = 30 \text{ A}$$

In Fig. 3.52 (*iii*), 40A current source is acting alone; 10 V voltage source being replaced by a short and 120A current source by an open. By current-divider rule, I_2 is given by ;

$$I_2 = 40 \times \frac{150}{50+150} = 30\text{A}$$

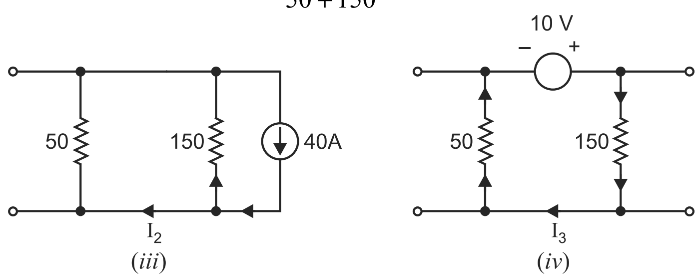


Fig. 3.52

In Fig. 3.52 (*iv*), 10V voltage source is acting alone. By Ohm's law, I_3 is given by ;

$$I_3 = \frac{10}{50+150} = 0.05\text{A}$$

Currents I_1 and I_2, being equal and opposite, cancel out so that :

$$I = I_3 = \mathbf{0.05\ A}$$

Example 3.27. *Using superposition theorem, find the current in the branch AC of the network ABCD shown in Fig. 3.53 (i).*

Solution. Let the current in section *AC* be *I* as shown in Fig. 3.53 (*i*). We shall determine the value of this current by superposition theorem.

First consider 20A load acting alone

Let I_1 and I_2 be the currents through *AB* and *AC* respectively as shown in Fig. 3.53 (*ii*). Then the current distribution will be as shown. We shall apply Kirchhoff's voltage law to loops *ADCA* and *ABCA*.

Loop *ADCA*. Applying *KVL*, we have,

$$-(20 - I_1 - I_2) \times 0.15 + 0.1\, I_2 = 0$$

or $$0.15\, I_1 + 0.25\, I_2 = 3 \quad ...(i)$$

Loop *ABCA*. Applying *KVL*, we have,

$$-0.1\ I_1 + (20 - I_1) \times 0.05 + 0.1\ I_2 = 0$$

or $$0.15\ I_1 - 0.1\ I_2 = 1 \qquad ...(ii)$$

From equations (*i*) and (*ii*), we get, $I_2 = 40/7$A.

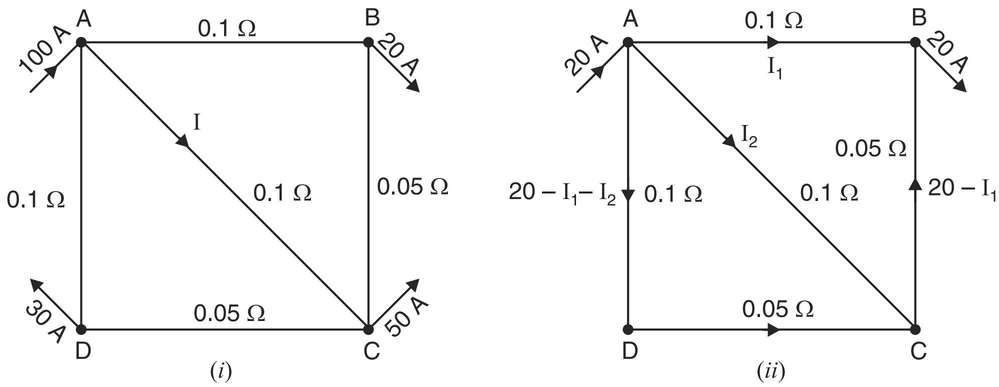


Fig. 3.53

Consider now 50 A load acting alone

Let I_1' and I_2' be the currents through *AB* and *AC* respectively. Then the current distribution will be as shown in Fig. 3.54 (*i*).

Loop *ABCA*. Applying *KVL*, we have,

$$-0.15\ I_1' + 0.1\ I_2' = 0$$

or $$0.15\ I_1' - 0.1\ I_2' = 0 \qquad ...(iii)$$

Loop *ADCA*. Applying *KVL*, we have,

$$-(50 - I_1' - I_2') \times 0.15 + 0.1\ I_2' = 0$$

or $$0.15\ I_1' + 0.25\ I_2' = 7.5 \qquad ...(iv)$$

From equations (*iii*) and (*iv*), we get, $I_2' = 150/7$A.

Consider now 30A load acting alone

Let the currents circulate as shown in Fig. 3.54 (*ii*). It is required to find I_2''.

Loop *ABCA*. Applying *KVL*, we have,

$$-0.15\ I_1'' + 0.1\ I_2'' = 0$$

or $$0.15\ I_1'' - 0.1\ I_2'' = 0 \qquad ...(v)$$

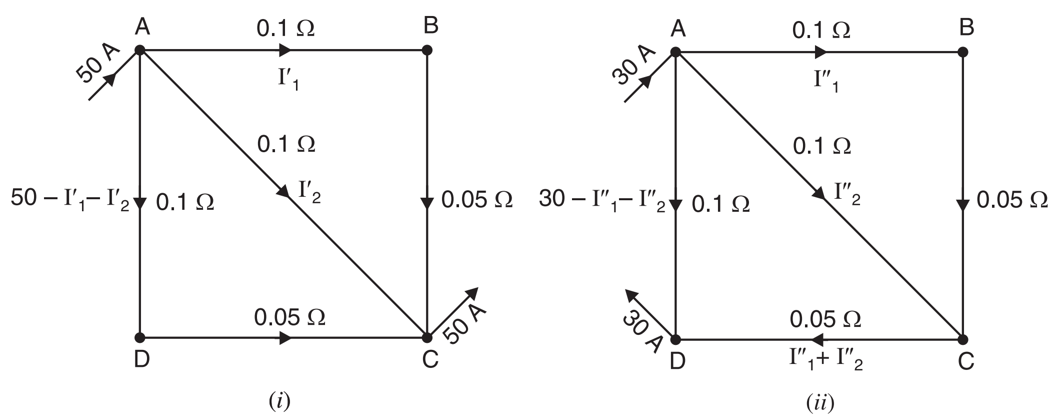


Fig. 3.54

Loop *ADCA*. Applying *KVL*, we have,

$$-(30 - I_1'' - I_2'') \times 0.1 + 0.05\,(I_1'' + I_2'') + 0.1I_2'' = 0$$

or $$0.15\, I_1'' + 0.25\, I_2'' = 3 \qquad ...(vi)$$

From equations (*v*) and (*vi*), we get, $I_2'' = 60/7$A.

According to superposition theorem, the total current in *AC* is equal to the algebraic sum of the component values.

$$I = I_2 + I_2' + I_2''$$
$$= 40/7 + 150/7 + 60/7$$
$$= 250/7 = \mathbf{35.7A}$$

Example 3.28. *Using superposition theorem, find the current in the each branch of the network shown in Fig. 3.55 (i).*

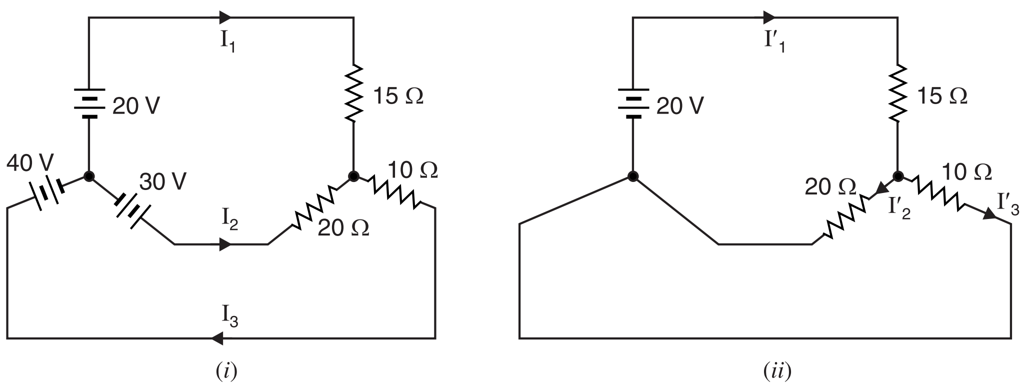


Fig. 3.55

Solution. Since there are three sources of e.m.f., three circuits [Fig. 3.55 (*ii*), Fig. 3.56 (*i*) and (*ii*)] are required for analysis by superposition theorem.

In Fig. 3.55 (*ii*), it is shown that only 20 V source is acting.

$$\text{Total resistance across source} = 15 + \frac{20 \times 10}{20 + 10} = 21.67\Omega$$

$$\therefore \quad \text{Total circuit current, } I'_1 = 20/21.67 = 0.923 \text{ A}$$

$$\text{Current in } 20\ \Omega,\ I'_2 = 0.923 \times 10/30 = 0.307\text{A}$$

$$\text{Current in } 10\ \Omega,\ I'_3 = 0.923 \times 20/30 = 0.616 \text{ A}$$

In Fig. 3.56 (*i*), only 40V source is acting in the circuit.

$$\text{Total resistance across source} = 10 + \frac{20 \times 15}{20 + 15} = 18.57\Omega$$

$$\text{Total circuit current, } I_3'' = 40/18.57 = 2.15\text{A}$$

$$\text{Current in } 20\ \Omega,\ I_2'' = 2.15 \times 15/35 = 0.92 \text{ A}$$

$$\text{Current in } 15\ \Omega,\ I_1'' = 2.15 \times 20/35 = 1.23 \text{ A}$$

In Fig. 3.56 (*ii*), only 30 *V* source is acting in the circuit.

$$\text{Total resistance across source} = 20 + 10 \times 15/(10 + 15) = 26\ \Omega$$

$$\text{Total circuit current, } I_2''' = 30/26 = 1.153 \text{ A}$$

$$\text{Current in } 15\ \Omega,\ I_1''' = 1.153 \times 10/25 = 0.461 \text{ A}$$

$$\text{Current in } 10\ \Omega,\ I_3''' = 1.153 \times 15/25 = 0.692 \text{ A}$$

The actual values of currents I_1, I_2 and I_3 shown in Fig. 3.55 (*i*) can be found by algebraically adding the component values.

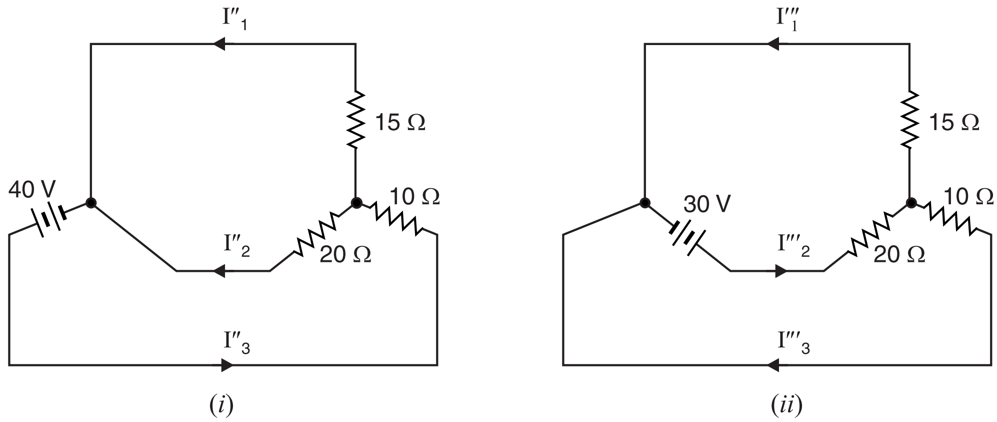


Fig. 3.56

$$I_1 = I_1' - I_1'' - I_1''' = 0.923 - 1.23 - 0.461 = \mathbf{-0.768\ A}$$

$$I_2 = -I_2' - I_2'' + I_2''' = -0.307 - 0.92 + 1.153 = \mathbf{-0.074\ A}$$

$$I_3 = I_3' - I_3'' + I_3''' = 0.616 - 2.15 + 0.692 = \mathbf{-0.842\ A}$$

The negative signs with I_1, I_2 and I_3 show that their actual directions are opposite to that assumed in Fig. 3.55 (*i*).

Example 3.29. *Use superposition theorem to find the voltage V in Fig. 3.57 (i).*

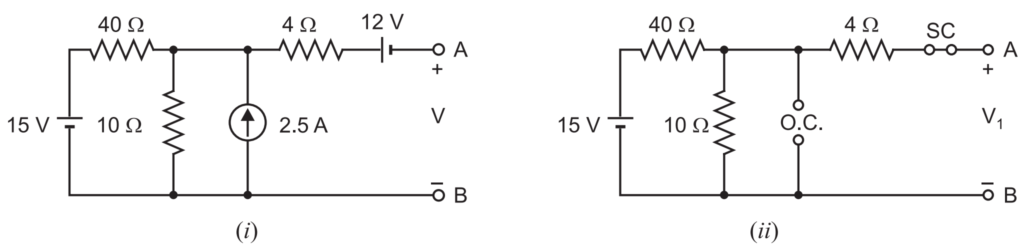


Fig. 3.57

Solution. In Fig. 3.57 (*ii*), 12 V battery is replaced by a short and 2.5A current source by an open so that 15V battery is acting alone. Therefore, voltage V_1 across open terminals *A* and *B* is

$$V_1 = \text{Voltage across } 10\Omega \text{ resistor}$$

By voltage-divider rule, V_1 is given by ;

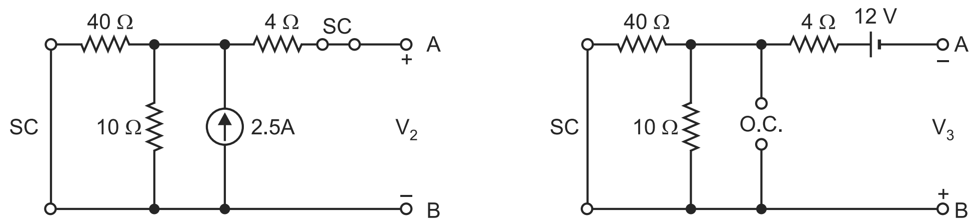
$$V_1 = 15 \times \frac{10}{40+10} = 3\text{V}$$

In Fig. 3.57 (*iii*), 15 V and 12 V batteries are replaced by shorts so that 2.5A current source is acting alone. Therefore, voltage V_2 across open terminals *A* and *B* is

$$V_2 = \text{Voltage across } 10\ \Omega \text{ resistor}$$

By current-divider rule, current in 10 Ω = $2.5 \times \frac{40}{50}$ = 2A

$\therefore \qquad V_2 = 2 \times 10 = 20\text{V}$



(*iii*) (*iv*)

Fig. 3.57

In Fig. 3.57 (*iv*), 15 V battery is replaced by a short and 2.5 A current source by an open so that 12V battery is acting alone. Therefore, voltage V_3 across open terminals *A* and *B* is

$$V_3 = -{}^*12\text{V}$$

The minus sign is given because the negative terminal of the battery is connected to point *A* and positive terminal to point *B*.

∴ Voltage across open terminals *AB* when all sources are present is

$$V = V_1 + V_2 + (-V_3) = 3 + 20 - 12 = \mathbf{11V}$$

Example 3.30. *Using superposition theorem, find the current in 23 Ω resistor in the circuit shown in Fig. 3.58.*

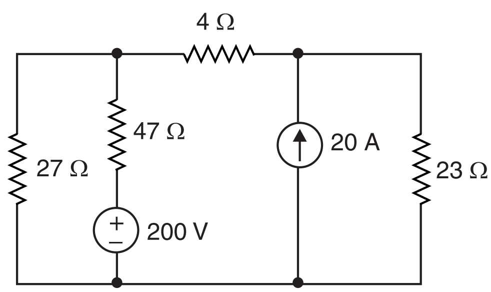


Fig. 3.58

Solution.

200 V source acting alone. We first consider the case when 200 V voltage source is acting alone as shown in Fig. 3.59. Note that current source is replaced by an open. The total resistance R_T presented to the voltage source is 47 Ω in series with the parallel combination of 27 Ω and (23 + 4) Ω. Therefore, the value of R_T is given by ;

$$R_T = 47 + [27 \parallel (23+4)] = 47 + \frac{27 \times 27}{27+27} = 47 + 13.5 = 60.5\ \Omega$$

∴ Current supplied by 200 V source is given by ;

$$I_T = \frac{V}{R_T} = \frac{200}{60.5} = 3.31\ \text{A}$$

At the node *A*, I_T (= 3.31 A) divides between the parallel resistors of 27 Ω and (23 + 4) Ω.

∴ Current through 23 Ω, $I_1 = 3.31 \times \frac{27}{27+27} = 1.65$ A downward

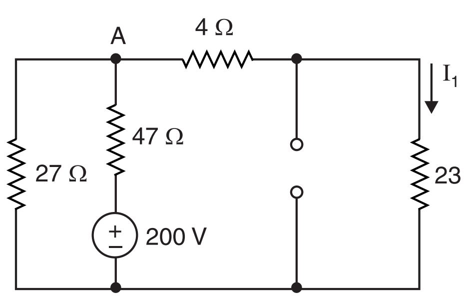


Fig. 3.59

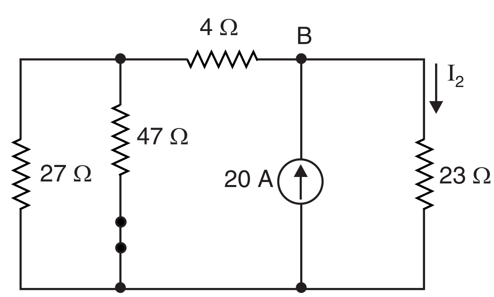


Fig. 3.60

* The total circuit resistance at terminals *AB* = 4 + (40||10) = 12Ω. The circuit behaves as a 12V battery having internal resistance of 12Ω with terminals *A* and *B* open.

20 A current source acting alone. We now consider the case when the current source is acting alone as shown in Fig. 3.60. Note that voltage source is replaced by a short because its internal resistance is assumed zero. The equivalent resistance R_{eq} to the left of the current source is

$$R_{eq} = 4 + (27 \| 47) = 4 + \frac{27 \times 47}{27 + 47} = 4 + 17.15 = 21.15\ \Omega$$

At node B, 20 A divides between two parallel resistors R_{eq} and 23 Ω. By current divider rule,

$$\text{Current in } 23\Omega \text{ resistor, } I_2 = 20 \times \frac{R_{eq}}{R_{eq} + 23} = 20 \times \frac{21.15}{21.15 + 23} = 9.58 \text{ A}$$

Note that I_2 in 23Ω resistor is downward.

∴ Total current in 23 Ω = $I_1 + I_2$ = 1.65 + 9.58 = **11.23 A**

Example 3.31. *Fig. 3.61 shows the circuit with two independent sources and one dependent source. Find the power delivered to the 3 Ω resistor.*

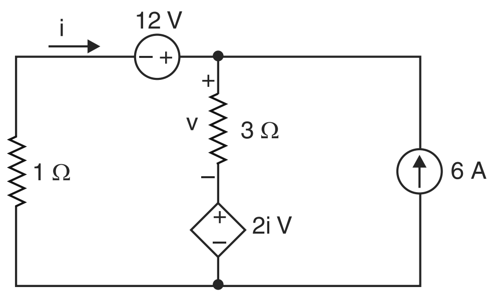


Fig. 3.61

Solution. While applying superposition theorem, two points must be noted carefully. First, we *cannot* find the power due to each independent source acting alone and add the results to obtain total power. It is because the relation for power is non-linear ($P = I^2R$ or V^2/R). Secondly, when the circuit also has dependent source, only independent sources act one at a time while dependent sources remain unchanged. Let us come back to the problem. Suppose v_1 is the voltage across 3 Ω resistor when 12 V source is acting alone and v_2 is the voltage across 3 Ω resistor when 6 A source is acting alone. Therefore, $v = v_1 + v_2$.

When 12 V source is acting alone. When 12 V source is acting alone, the circuit becomes as shown in Fig. 3.62. Note that 6A source is replaced by an open. Applying *KVL* to the loop *ABCDA* in Fig. 3.62, we have,

$$12 - v_1 - 2i_1 - i_1 \times 1 = 0$$

or $\quad 12 - 3i_1 - 2i_1 - i_1 = 0 \quad \therefore \quad i_1 = 12/6 = 2$ A

∴ $\quad v_1 = 3i_1 = 3 \times 2 = 6$V

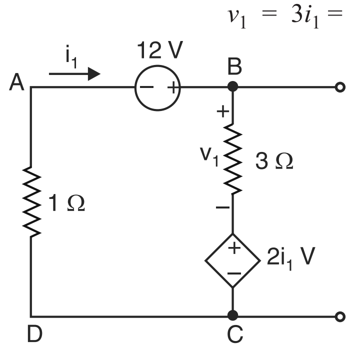


Fig. 3.62

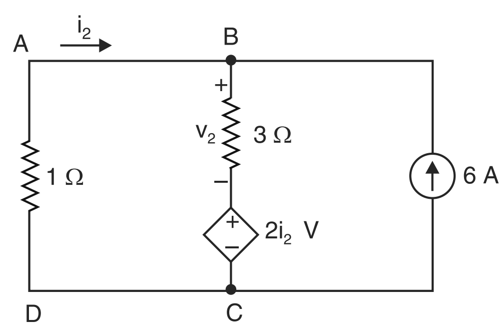


Fig. 3.63

When 6A source is acting alone. When 6A source is acting alone, the circuit becomes as shown in Fig. 3.63. Note that 12V source is replaced by a short because internal resistance of the source is assumed zero. Applying *KVL* to the loop *ABCDA* in Fig. 3.63, we have,

$$-3(i_2 + 6) - 2i_2 - i_2 \times 1 = 0$$

or $\quad -3i_2 - 18 - 2i_2 - i_2 = 0 \quad \therefore\ i_2 = -18/6 = -3$ A

∴ $\quad v_2 = 3(i_2 + 6) = 3 \times 3 = 9$ V

$\therefore \quad v = v_1 + v_2 = 6 + 9 = 15$ V

$\therefore$ Power delivered to 3Ω, $P = \dfrac{v^2}{3} = \dfrac{(15)^2}{3} = $ **75 W**

Example 3.32. *Using superposition principle, find the current through G_C conductance in the circuit shown in Fig. 3.64. Given that $G_A = 0.3$ S ; $G_B = 0.4$ S and $G_C = 0.1$ S.*

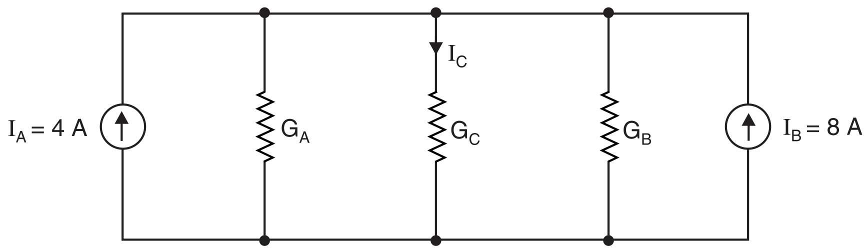


Fig. 3.64

Solution.

Current source I_A acting alone. We first consider the case when current source I_A is acting alone as shown in Fig. 3.65. Note that current source I_B is replaced by an open.

Total conductance, $G_T = G_A + G_C + G_B = 0.3 + 0{\cdot}1 + 0{\cdot}4 = 0{\cdot}8$S

Voltage across G_C, $V' = \dfrac{I_A}{G_T} = \dfrac{4}{0.8} = 5$ V

$\therefore$ Current through G_C, $I'_C = V'G_C = 5 \times 0{\cdot}1 = 0{\cdot}5$ A

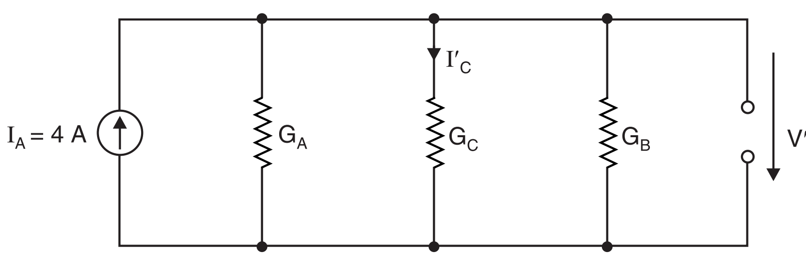


Fig. 3.65

Current source I_B acting alone. We now consider the case when current source I_B acts alone as shown in Fig. 3.66. Note that current source I_A is replaced by an open.

Voltage across G_C, $V'' = \dfrac{I_B}{G_T} = \dfrac{8}{0.8} = 10$ V

Current through G_C, $I''_C = V''G_C = 10 \times 0{\cdot}1 = 1$ A

$\therefore$ Total current through G_C, $I_C = I'_C + I''_C = 0{\cdot}5 + 1 =$ **1·5 A**

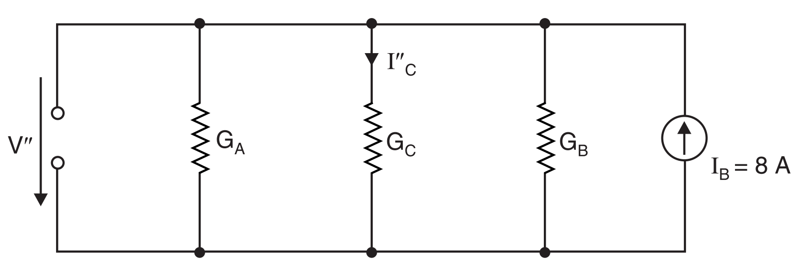


Fig. 3.66

Note. It is important to note that superposition theorem applies to currents and voltages; it does not mean that powers from two sources can be superimposed. It is because power varies as the *square* of the voltage or the current and this relationship is nonlinear.

Example 3.33. *Using superposition theorem, find the value of output voltage V_0 in the circuit shown in Fig. 3.67.*

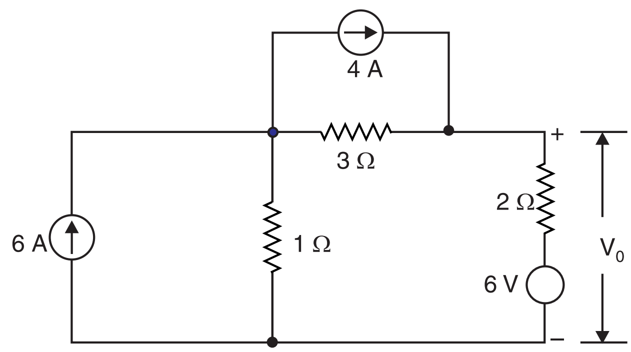


Fig. 3.67

Solution. The problem will be divided into three parts using one source at a time.

6A source acting alone. We first consider the case when 6 A source is acting alone as shown in Fig. 3.68. Note that voltage source is replaced by a short and the current source of 4 A is replaced by an open. According to current-divider rule, current i_1 through 2 Ω resistor is

$$i_1 = 6 \times \frac{1}{1+2+3} = 1\text{A} \quad \therefore \quad V_{01} = 1 \times 2 = 2\text{V}$$

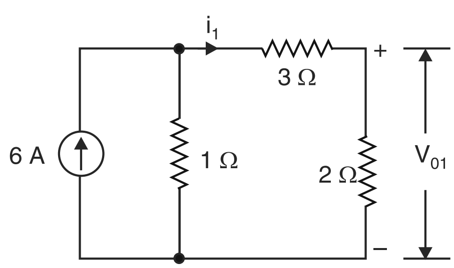


Fig. 3.68

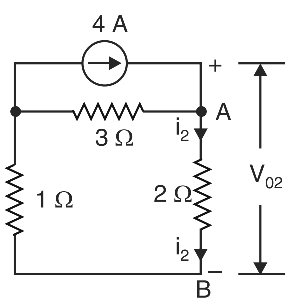


Fig. 3.69

4A source acting alone. We now consider the case when 4A source is acting alone as shown in Fig. 3.69. Note that voltage source is replaced by a short and current source of 6A is replaced by an open. At point *A*, the current 4A finds two parallel paths; one of resistance 3 Ω and the other of resistance = 2 + 1 = 3 Ω. Therefore, current i_2 through 2 Ω resistor is

$$i_2 = 4/2 = 2\text{A} \quad \therefore \quad V_{02} = 2 \times 2 = 4\text{V}$$

6 V source acting alone. Finally, we consider the case when 6 V source is acting alone as shown in Fig. 3.70. Note that each current source is replaced by an open. The circuit current is 1A and voltage drop across 2 Ω resistor = 2 × 1 = 2V.

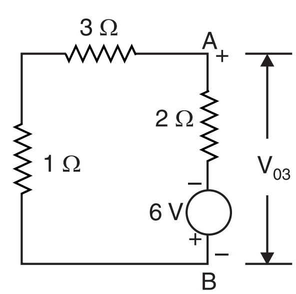


Fig. 3.70

It is clear from Fig. 3.70 that :

$$V_A - 2\text{V} + 6\text{V} = V_B \quad \therefore \quad V_A - V_B = V_{03} = -4\text{V}$$

According to superposition theorem, we have,

$$V_0 = V_{01} + V_{02} + V_{03} = 2 + 4 - 4 = \mathbf{2V}$$

Example 3.34. *Using superposition theorem, find voltage across 4Ω resistance in Fig. 3.71 (i).*

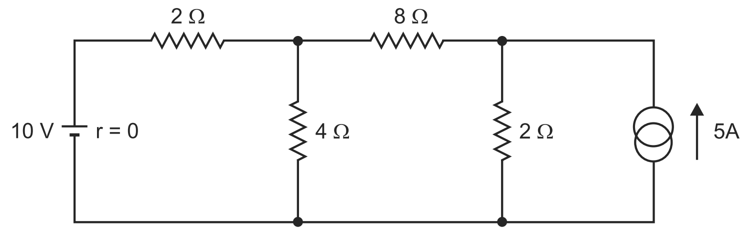


Fig. 3.71 (*i*)

Solution. In Fig. 3.71 (*ii*), the 5A current source is replaced by an open so that 10V source is acting alone. Referring to Fig. 3.71 (*ii*), the total circuit resistance R_T offered to 10V source is

$$R_T = 2\Omega + [4\Omega \parallel (2+8)\Omega] = 2 + \frac{4 \times 10}{4+10} = 4.857\Omega$$

∴ Current I supplied by 10 V source is given by ;

$$I = \frac{10\text{V}}{R_T} = \frac{10\text{V}}{4.857\Omega} = 2.059 \text{ A}$$

At point A in Fig. 3.71 (*ii*), the current 2.059 A divides into two parallel paths consisting of 4Ω resistance and (8 + 2) = 10Ω resistance.

∴ By current-divider rule, current I_1 in 4Ω due to 10 V alone is

$$I_1 = 2.059 \times \frac{10}{4+10} = 1.471 \text{ A in downward direction}$$

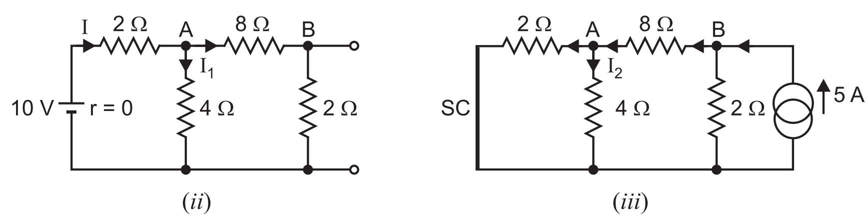


Fig. 3.71

In Fig. 3.71 (*iii*), the 10V battery is replaced by a short so that 5A current source is acting alone. At point B in Fig. 3.71 (*iii*), current 5A divides into two parallel paths consisting of 2Ω resistance and 8Ω + (2Ω||4Ω) = 8 + (2 × 4)/(2 + 4) = 9.333Ω.

∴ By current-divider rule, current in 8Ω resistance is

$$I_{8\Omega} = 5 \times \frac{2}{2+9.333} = 0.8824 \text{ A}$$

At point A in Fig. 3.71 (*iii*), current 0.8824A divides into two parallel paths consisting of 2Ω resistance and 4Ω resistance.

∴ By current-divider rule, current I_2 in 4Ω due to 5A alone is

$$I_2 = 0.8824 \times \frac{2}{2+4} = 0.294 \text{ A in downward direction}$$

By superposition theorem, total current in 4 Ω

$= I_1 + I_2 = 1.471 + 0.294 = 1.765\text{A}$ in downward direction

$\therefore$ Voltage across 4Ω $= 1.765 \times 4 =$ **7.06V**

Note. We can also find I_2 in another way. Current in left-hand side 2Ω resistance will be $2I_2$ because $2\Omega \parallel 4\Omega$. By *KCL*, current in 8Ω resistance is

$$I_{8\Omega} = I_2 + 2I_2 = 3I_2$$

$$\text{Resistance to } I_{8\Omega} \text{ flow} = 8\Omega + (4\Omega \parallel 2\Omega) = 8 + \frac{2 \times 4}{2+4} = 9.333\ \Omega$$

Now 5A divides between two parallel paths of resistances 9.333 Ω and 2 Ω.

$$\therefore \quad I_{8\Omega} = 5 \times \frac{2}{2 + 9.333} = 0.8824\ \text{A}$$

$$\text{or} \quad 3I_2 = 0.8824 \quad \therefore I_2 = \frac{0.8824}{3} = 0.294\ \text{A}$$

Tutorial Problems

1. Use the superposition theorem to find the current in R_1 (= 60 Ω) in the circuit shown in Fig. 3.72. **[0.125 A from left to right]**
2. Use the superposition theorem to find the current through R_1 (= 1k Ω) in the circuit shown in Fig 3.73. **[2 mA from right to left]**

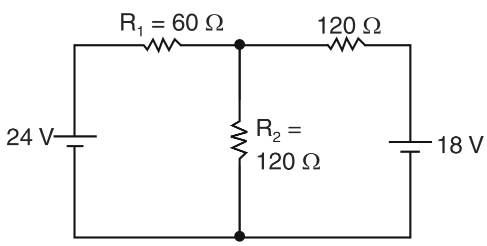


Fig. 3.72

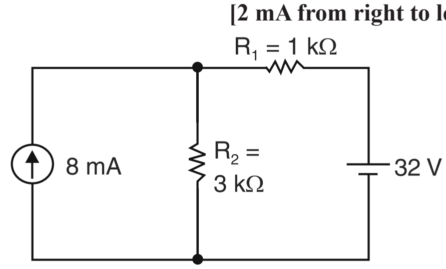


Fig. 3.73

3. Use the superposition theorem to find the current through R_1 (= 10 Ω) in the circuit shown in Fig. 3.74. **[4.6 A from left to right]**

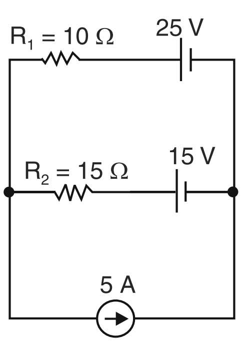


Fig. 3.74

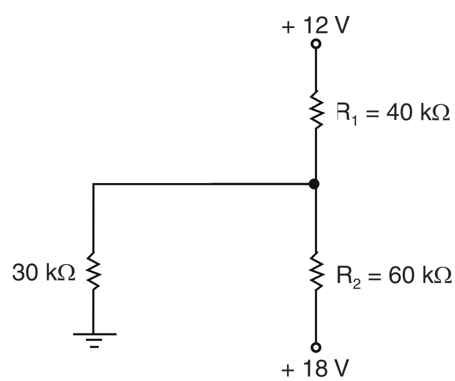


Fig. 3.75

4. Use superposition principle to find the current through resistance R_1 (= 40 kΩ) in the circuit shown in Fig. 3.75. **[1 mA downward]**
5. Use superposition principle to find the voltage across R_1 (= 1 k Ω) in the circuit shown in Fig. 3.76. Be sure to indicate the polarity of the voltage. **[− (11 V) +]**

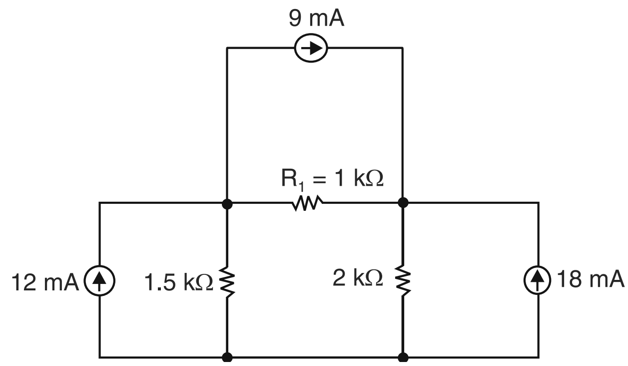


Fig. 3.76

6. Using superposition principle, find the current through 10 Ω resistor in Fig. 3.77. **[0.5 A↓]**

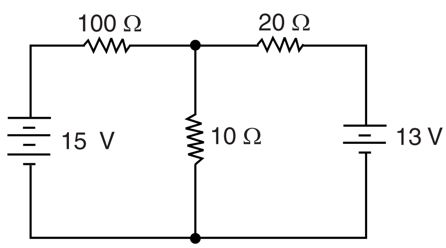


Fig. 3.77

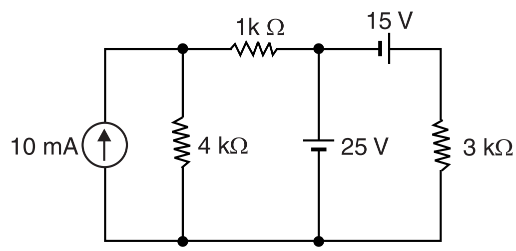


Fig. 3.78

7. Using superposition principle, find the voltage across 4 kΩ resistor in Fig. 3.78. **[$28\ V^{+}_{-}$]**

8. Referring to Fig. 3.79, the internal resistance R_S of the current source is 100 Ω. The internal resistance R_S of the voltage source is 10 Ω. Use superposition principle to find the power dissipated in 50 Ω resistor. **[8.26 W]**

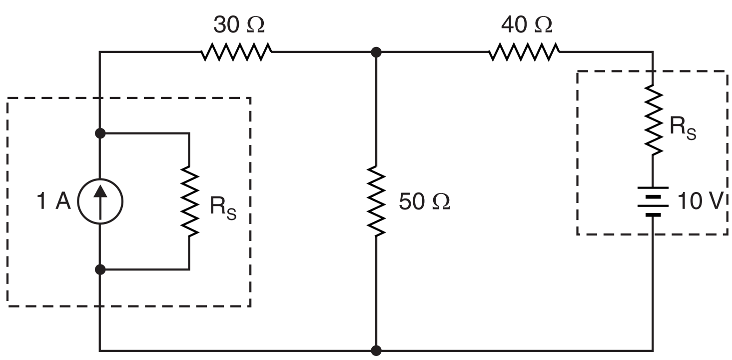


Fig. 3.79

9. Find v using superposition principle if $R = 2\Omega$ in Fig. 3.80. **[8 V]**

10. State whether true or false.

 (*i*) Superposition theorem is applicable to multiple source circuits.

 (*ii*) Superposition theorem is restricted to linear circuits. [(*i*) **True** (*ii*) **True**]

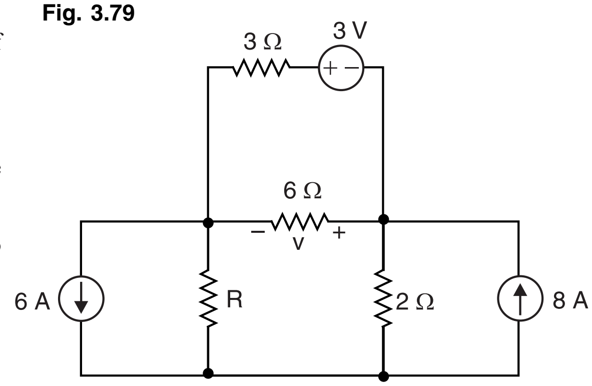


Fig. 3.80

11. Find i using superposition theorem in Fig. 3.81. **[–6 A]**

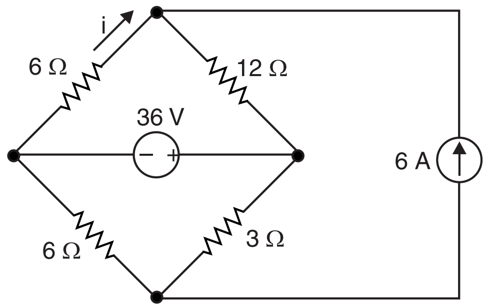


Fig. 3.81

3.10. Thevenin's Theorem

Fig. 3.82 (*i*) shows a network enclosed in a box with two terminals *A* and *B* brought out. The network in the box may consist of any number of resistors and e.m.f. sources connected in any manner. But according to Thevenin, the entire circuit behind terminals *A* and *B* can be replaced by a single source of e.m.f. V_{Th} (called Thevenin voltage) in series with a single resistance R_{Th} (called Thevenin resistance) as shown in Fig. 3.82 (*ii*). The values of V_{Th} and R_{Th} are determined as mentioned in Thevenin's theorem. Once *Thevenin's equivalent circuit* is obtained [See Fig. 3.82 (*ii*)], then current *I* through any load resistance R_L connected across *AB* is given by ;

$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

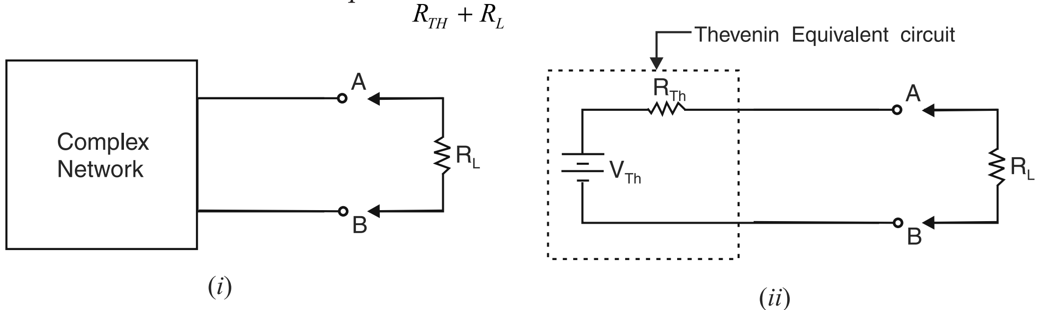


Fig. 3.82

Thevenin's theorem as applied to d.c. circuits is stated below :

Any linear, bilateral network having terminals A and B can be replaced by a single source of e.m.f. V_{Th} in series with a single resistance R_{Th}.

(*i*) *The e.m.f. V_{Th} is the voltage obtained across terminals A and B with load, if any removed i.e. it is open-circuited voltage between terminals A and B.*

(*ii*) *The resistance R_{Th} is the resistance of the network measured between terminals A and B with load removed and sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.*

Note how truly remarkable the implications of this theorem are. No matter how complex the circuit and no matter how many voltage and / or current sources it contains, it is equivalent to a single voltage source in series with a single resistance (*i.e.* **equivalent to a single real voltage source).** *Although Thevenin equivalent circuit is not the same as its original circuit, it acts the same in terms of output voltage and current.*

Explanation. Consider the circuit shown in Fig. 3.83 (*i*). As far as the circuit behind terminals *AB* is concerned, it can be replaced by a single source of e.m.f. V_{Th} in series with a single resistance R_{Th} as shown in Fig. 3.84 (*ii*).

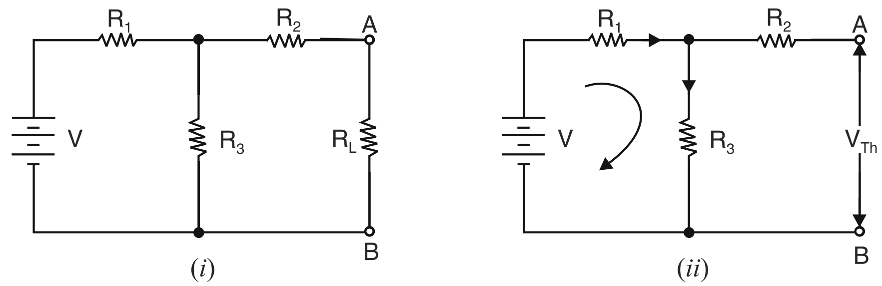


Fig. 3.83

(*i*) Finding V_{Th}. The e.m.f. V_{Th} is the voltage across terminals *AB* with load (*i.e.* R_L) removed as shown in Fig. 3.83 (*ii*). With R_L disconnected, there is no current in R_2 and V_{Th} is the voltage appearing across R_3.

$$\therefore \quad V_{Th} = \text{Voltage across } R_3 = \frac{V}{R_1 + R_3} \times R_3$$

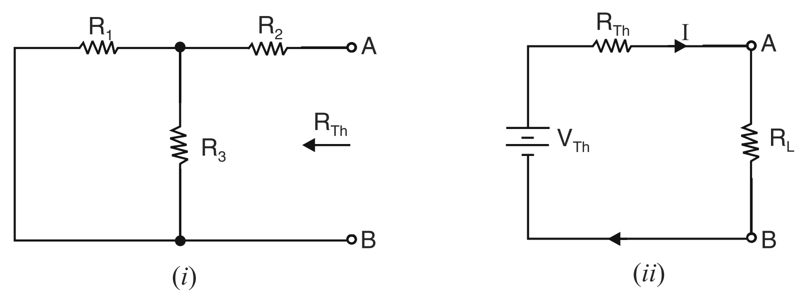


Fig. 3.84

(*ii*) Finding R_{Th}. To find R_{Th}, remove the load R_L and replace the battery by a short-circuit because its internal resistance is assumed zero. Then resistance between terminals *A* and *B* is equal to R_{Th} as shown in Fig. 3.84 (*i*). Obviously, at the terminals *AB* in Fig. 3.84 (*i*), R_1 and R_3 are in parallel and this parallel combination is in series with R_2.

$$\therefore \quad R_{Th} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

When load R_L is connected between terminals *A* and *B* [See Fig. 3.84 (*ii*)], then current in R_L is given by ;

$$I = \frac{V_{Th}}{R_{Th} + R_L}$$

3.11. Procedure for Finding Thevenin Equivalent Circuit

(*i*) Open the two terminals (*i.e.*, remove any load) between which you want to find Thevenin equivalent circuit.

(*ii*) Find the open-circuit voltage between the two open terminals. It is called Thevenin voltage V_{Th}.

(*iii*) Determine the resistance between the two open terminals with all ideal voltage sources shorted and all ideal current sources opened (a non-ideal source is replaced by its internal resistance). It is called Thevenin resistance R_{Th}.

(*iv*) Connect V_{Th} and R_{Th} in series to produce Thevenin equivalent circuit between the two terminals under consideration.

(*v*) Place the load resistor removed in step (*i*) across the terminals of the Thevenin equivalent circuit. The load current can now be calculated using only Ohm's law and it has the same value as the load current in the original circuit.

Note. Thevenin's theorem is sometimes called *Helmholtz's theorem*.

Example 3.35. *Using Thevenin's theorem, find the current in 6 Ω resistor in Fig. 3·85 (i).*

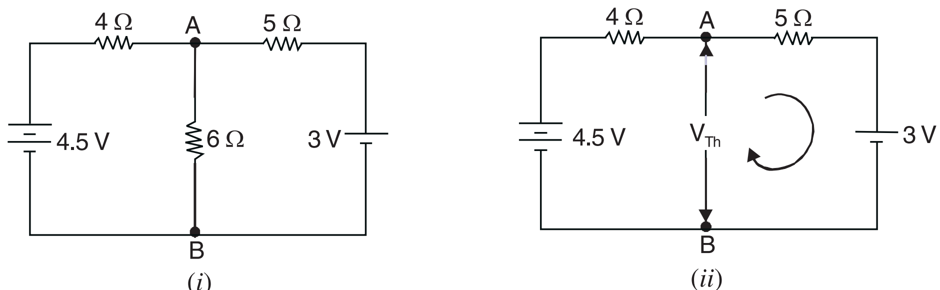


Fig. 3.85

Solution. Since internal resistances of batteries are not given, it will be assumed that they are zero. We shall find Thevenin's equivalent circuit at terminals *AB* in Fig. 3.85 (*i*).

V_{Th} = Voltage across terminals *AB* with load (*i.e.* 6 Ω resistor) removed as shown in Fig. 3·85 (*ii*).

$= {}^*4{\cdot}5 - 0.167 \times 4 = 3{\cdot}83$ V

R_{Th} = Resistance at terminals *AB* with load (*i.e.* 6 Ω resistor) removed and battery replaced by a short as shown in Fig. 3·86 (*i*).

$$= \frac{4 \times 5}{4 + 5} = 2.22\ \Omega$$

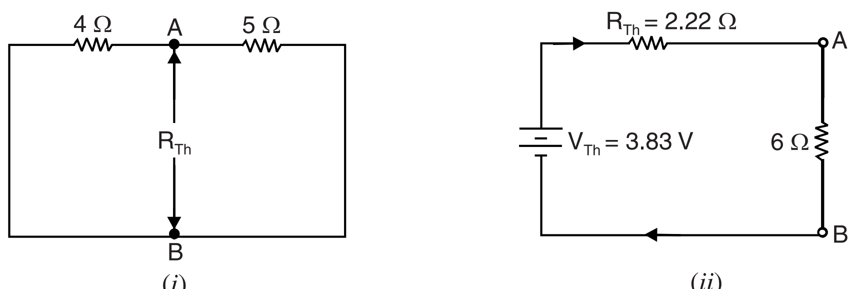


Fig. 3.86

Thevenin's equivalent circuit at terminals *AB* is V_{Th} (= 3·83 *V*) in series with R_{Th} (= 2·22 Ω). When load (*i.e.* 6 Ω resistor) is connected between terminals *A* and *B*, the circuit becomes as shown in Fig. 3·86 (*ii*).

$$\therefore \quad \text{Current in 6 Ω resistor} = \frac{V_{Th}}{R_{Th} + 6} = \frac{3.83}{2.22 + 6} = \mathbf{0.466A}$$

Example 3.36. *Using Thevenin's theorem, find p.d. across terminals AB in Fig. 3·87 (i).*

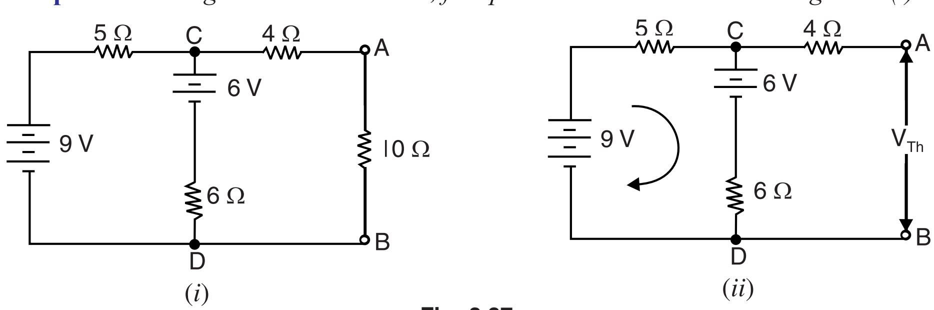


Fig. 3.87

* Net e.m.f. in the circuit shown in Fig. 3·85 (*ii*) is 4·5 − 3 = 1·5 V and total circuit resistance is 9 Ω.

∴ Circuit current = 1·5/9 = 0·167 A

The voltage across *AB* is equal to 4·5 V *less* drop in 4 Ω resistor.

$\therefore \quad V_{Th} = 4{\cdot}5 - 0{\cdot}167 \times 4 = 3{\cdot}83$ V

Solution. We shall find Thevenin's equivalent circuit at terminals *AB* in Fig. 3.87 (*i*).

V_{Th} = Voltage across terminals *AB* with load (*i.e.* 10 Ω resistor) removed as shown in Fig. 3.87 (*ii*).

= Voltage across terminals *CD*

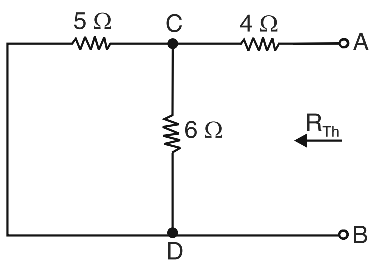
= 9 – drop in 5 Ω resistor

= 9* – 5 × 0·27 = 7·65 V

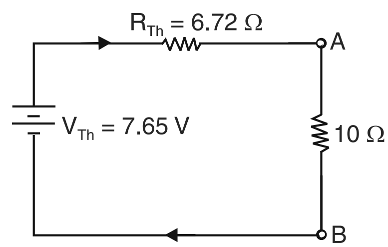
R_{Th} = Resistance at terminals *AB* with load (*i.e.* 10 Ω resistor) removed and batteries replaced by a short as shown in Fig. 3.88 (*i*).

$$= 4 + \frac{5\times 6}{5+6} = 6.72\Omega$$

Thevenin's equivalent circuit to the left of terminals *AB* is V_{Th} (= 7·65 *V*) in series with R_{Th} (= 6·72 Ω). When load (*i.e.* 10 Ω resistor) is connected between terminals *A* and *B*, the circuit becomes as shown in Fig. 3.88 (*ii*).



(*i*)



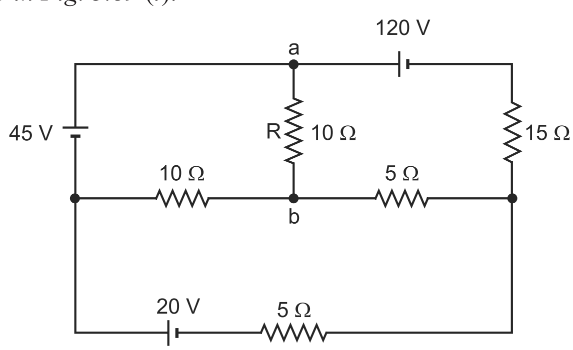
(*ii*)

Fig. 3.88

$$\therefore \quad \text{Current in 10 }\Omega\text{ resistor} = \frac{V_{Th}}{R_{Th}+10} = \frac{7.65}{6.72+10} = 0.457\text{ A}$$

P.D. across 10 Ω resistor = 0·457 × 10 = **4·57 V**

Example 3.37. *Using Thevenin's theorem, find the current through resistance R connected between points a and b in Fig. 3.89* (*i*).



(*i*)

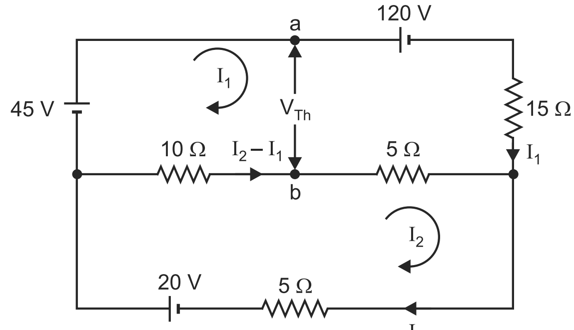
Fig. 3.89

Solution. **(*i*) Finding V_{Th}.** Thevenin voltage V_{Th} is the voltage across terminals *ab* with

* The net e.m.f. in the loop of circuit shown in Fig. 3·87 (*ii*) is 9 – 6 = 3V and total resistance is 5 + 6 = 11 Ω.

∴ Circuit current = 3/11 = 0·27 A

resistance R (= 10Ω) removed as shown in Fig. 3.89 (*ii*). It can be found by Maxwell's mesh current method.



(*ii*)

Fig. 3.89

Mesh 1. $45 - 120 - 15I_1 - 5(I_1 - I_2) - 10(I_1 - I_2) = 0$

or $\quad 30I_1 - 15I_2 = -75 \quad$ ...(*i*)

Mesh 2. $-10(I_2 - I_1) - 5(I_2 - I_1) - 5I_2 + 20 = 0$

or $\quad -15I_1 + 20I_2 = 20 \quad$ ...(*ii*)

From eqs. (*i*) and (*ii*), $\quad I_1 = -3.2\text{A}$; $I_2 = -1.4$ A

Now, $\quad V_a - 45 - 10(I_2 - I_1) = V_b$

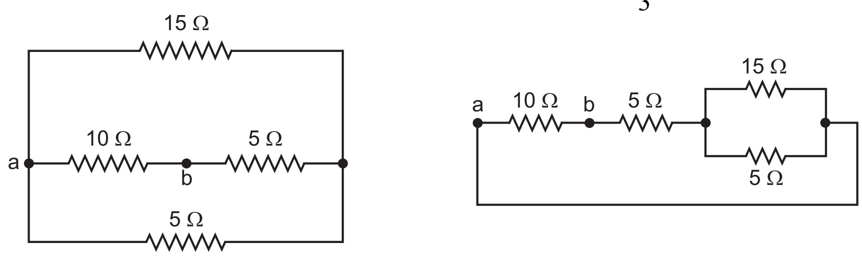
or $\quad V_a - V_b = 45 + 10\,(I_2 - I_1) = 45 + 10\,[-1.4 - (-3.2)] = 63\text{V}$

∴ $\quad V_{Th} = V_{ab} = V_a - V_b = 63\text{V}$

***(ii)* Finding R_{Th}.** Thevenin resistance R_{Th} is the resistance at terminals *ab* with resistance R (= 10Ω) removed and batteries replaced by a short as shown in Fig. 3.89 (*iii*). Using laws of series and parallel resistances, the circuit is reduced to the one shown in Fig. 3.89 (*iv*).

∴ $\quad R_{Th}$ = Resistance at terminals *ab* in Fig 3.89 (*iv*).

$$= 10\Omega \parallel [5\Omega + (15\Omega \parallel 5\Omega)] = 10\Omega \parallel (5\Omega + 3.75\Omega) = \frac{14}{3}\Omega$$



(*iii*) (*iv*)

Fig. 3.89

$$\therefore \text{ Current in } R\ (= 10\Omega) = \frac{V_{Th}}{R_{Th} + R} = \frac{63}{(14/3) + 10} = \mathbf{4.295A}$$

Example 3.38. *A Wheatstone bridge ABCD has the following details : AB = 10 Ω, BC = 30 Ω, CD = 15 Ω and DA = 20 Ω. A battery of e.m.f. 2 V and negligible resistance is connected between A and C with A positive. A galvanometer of 40 Ω resistance is connected between B and D. Using Thevenin's theorem, determine the magnitude and direction of current in the galvanometer.*

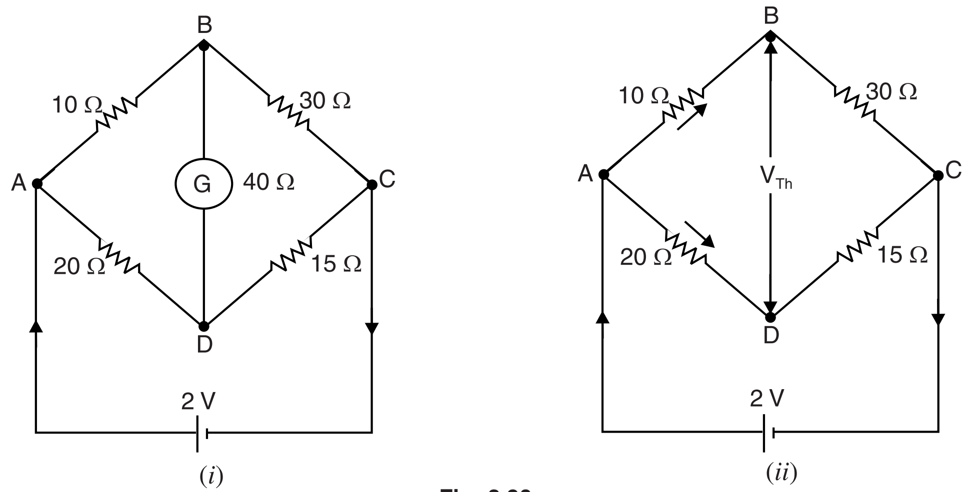


Fig. 3.90

Solution. We shall find Thevenin's equivalent circuit at terminals *BD* in Fig. 3.90 (*i*).

***(i)* Finding V_{Th}.** To find V_{Th} at terminals *BD*, remove the load (*i.e.* 40 Ω galvanometer) as shown in Fig. 3.90 (*ii*). The voltage between terminals *B* and *D* is equal to V_{Th}.

$$\text{Current in branch } ABC = \frac{2}{10+30} = 0.05 \text{ A}$$

$$\text{P.D. between } A \text{ and } B, V_{AB} = 10 \times 0.05 = 0.5 \text{ V}$$

$$\text{Current in branch } ADC = \frac{2}{20+15} = 0.0571\text{A}$$

$$\text{P.D. between } A \text{ and } D, V_{AD} = 0.0571 \times 20 = 1.142 \text{ V}$$

$$\therefore \quad \text{P.D. between } B \text{ and } D, V_{Th} = V_{AD} - V_{AB} = 1.142 - 0.5 = 0.642 \text{ V}$$

Obviously, point *B** is positive w.r.t. point *D i.e.* current in the galvanometer, when connected between *B* and *D*, will flow from *B* to *D*.

***(ii)* Finding R_{Th}.** In order to find R_{Th}, remove the load (*i.e.* 40 Ω galvanometer) and replace the battery by a short (as its internal resistance is assumed zero) as shown in Fig. 3.91 (*i*). Then resistance measured between terminals *B* and *D* is equal to R_{Th}.

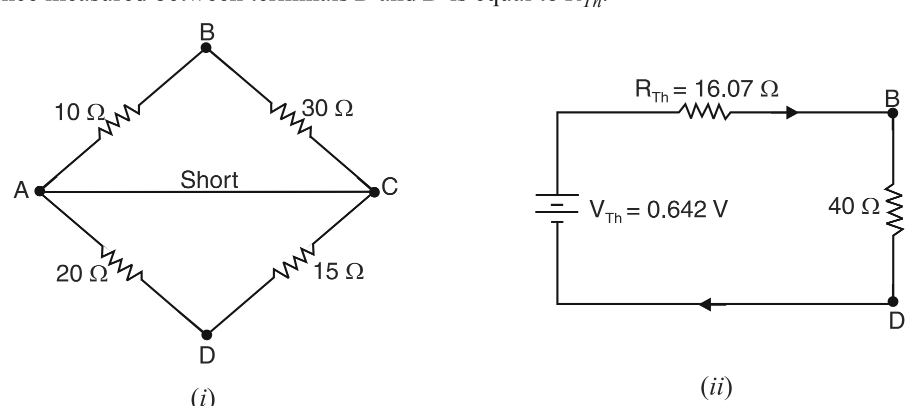


Fig. 3.91

* The potential at point *D* is 1·142 V lower than at *A*. Also potential of point *B* is 0·5 V lower than *A*. Hence point *B* is at higher potential than point *D*.

$$R_{Th} = \text{Resistance at terminals } BD \text{ in Fig. 3.91 } (i).$$

$$= \frac{10\times 30}{10+30} + \frac{20\times 15}{20+15} = 7.5 + 8.57 = 16.07\Omega$$

Thevenin's equivalent circuit at terminals *BD* is V_{Th} (= 0.642 V) in series with R_{Th} (= 16·07 Ω). When galvanometer is connected between *B* and *D*, the circuit becomes as shown in Fig. 3.91 (*ii*).

$$\therefore \quad \text{Galvanometer current} = \frac{V_{Th}}{R_{Th}+40} = \frac{0.642}{16.07+40}$$

$$= 11.5 \times 10^{-3}\text{ A} = \mathbf{11.5\ mA}\ \textit{from B to D}$$

Example 3·39. *Find the Thevenin equivalent circuit lying to the right of terminals x − y in Fig. 3.92.*

Solution. In this example, there is no external circuitry connected to *x* − *y* terminals.

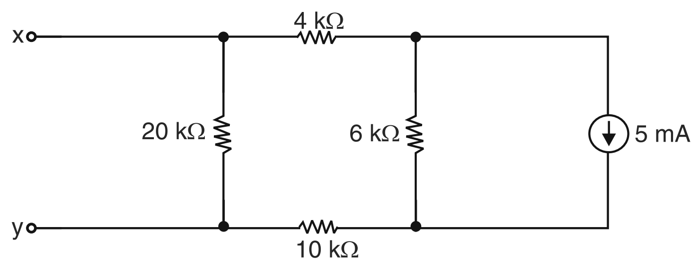


Fig. 3.92

(*i*) **Finding R_{Th}.** To find Thevenin equivalent resistance R_{Th}, we open-circuit the current source as shown in Fig. 3.93 (*i*). Note that 4 kΩ, 6 kΩ and 10 kΩ resistors are then in series and have a total resistance of 20 kΩ. Thus R_{Th} is the parallel combination of that 20 kΩ resistance and the other 20 kΩ resistor as shown in Fig. 3·93 (*ii*).

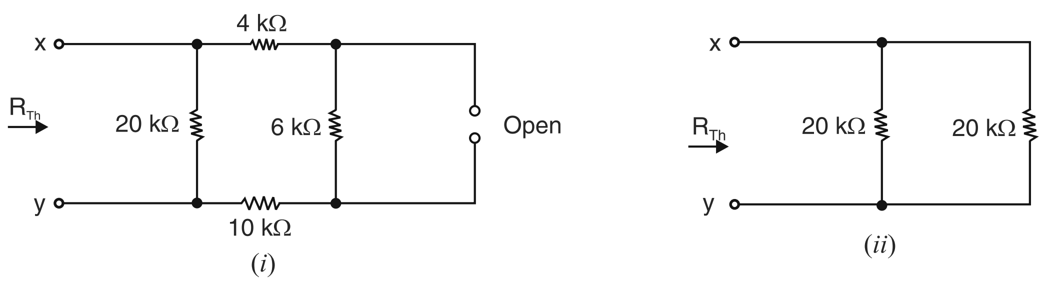


Fig. 3.93

$$\therefore \quad R_{Th} = 20\text{ k}\Omega \parallel 20\text{ k}\Omega = \frac{20\times 20}{20+20} = 10\text{ k}\Omega$$

(*ii*) **Finding V_{Th}.** Fig. 3.94 (*i*) shows the computation of Thevenin equivalent voltage V_{Th}. Note that V_{Th} is the voltage drop across the 20 kΩ resistor. The current from the 5 mA source divides between 6 kΩ resistor and the series string of 10 kΩ + 20 kΩ + 4 kΩ = 34 kΩ. Thus, by the current divider-rule, the current in 20 kΩ resistor is

$$I_{20}\text{ k}\Omega = \left(\frac{6}{34+6}\right)\times 5 = 0.75\text{ mA}$$

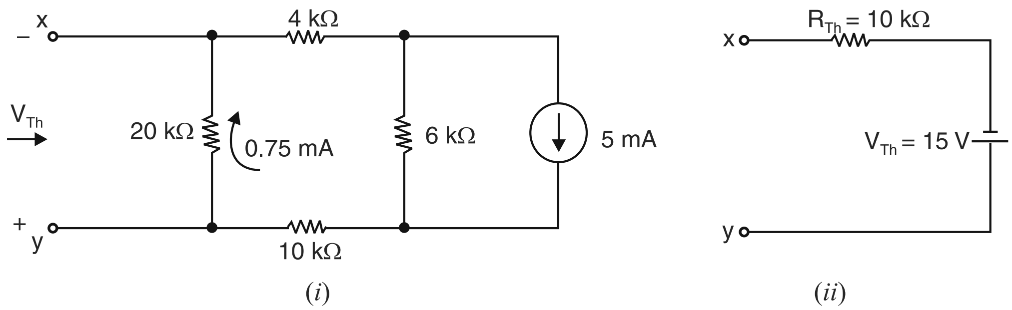


Fig. 3.94

Voltage across 20 kΩ resistor is given by ;

$$V_{Th} = (0.75 \text{ mA})\ (20 \text{ k}\Omega) = 15 \text{ V}$$

Notice that terminal *y* is positive with respect to terminal *x*. **Fig. 3.94 (*ii*) shows the Thevenin equivalent circuit.** The polarity of V_{Th} is such that terminal *y* is positive with respect to terminal *x*, as required.

Example 3.40. *Calculate the power which would be dissipated in a 50 Ω resistor connected across xy in the network shown in Fig. 3.95.*

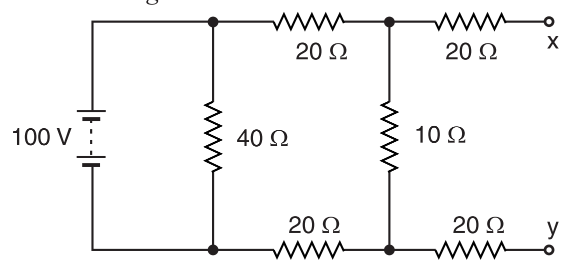


Fig. 3.95

Solution. We shall find Thevenin equivalent circuit to the left of terminals *xy*. With *xy* terminals open, the current in 10 Ω resistor is given by ;

$$*I = \frac{100}{20+10+20} = 2\text{A}$$

∴ Open circuit voltage across *xy* is given by ;

$$V_{Th} = I \times 10 = 2 \times 10 = 20\text{V}$$

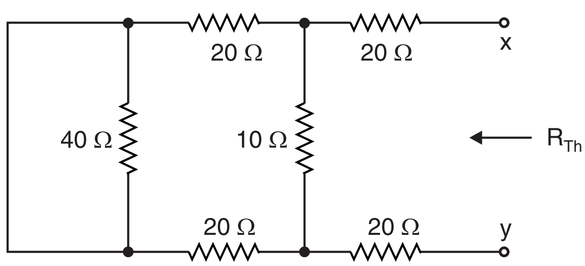


Fig. 3.96

In order to find R_{Th} replace the battery by a short since its internal resistance is assumed to be zero [See Fig. 3.96].

$$R_{Th} = \text{Resistance looking into the terminals } xy \text{ in Fig. 3.96.}$$

$$= 20 + [(20 + 20) \parallel 10] + 20$$

* It is clear that (20 + 10 + 20) Ω is in parallel with 40 Ω resistor across 100 V source.

$$= 20 + \frac{*40 \times 10}{40 + 10} + 20 = 20 + 8 + 20 = 48\ \Omega$$

Therefore, Thevenin's equivalent circuit behind terminals *xy* is V_{Th} (= 20V) in series with R_{Th} (= 48 Ω). When load R_L (= 50 Ω) is connected across *xy*, the circuit becomes as shown in Fig. 3.97.

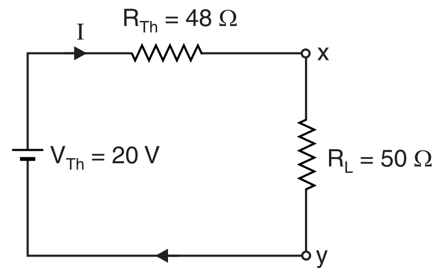


Fig. 3.97

∴ Current I in 50 Ω resistor is

$$I = \frac{V_{Th}}{R_{Th} + R_L} = \frac{20}{48 + 50} = \frac{20}{98}\,\text{A}$$

∴ Power dissipated in 50 Ω resistor is

$$P = I^2 R_L = \left(\frac{20}{98}\right)^2 \times 50 = \mathbf{2{\cdot}08\ W}$$

Example 3.41. *Calculate the current in the 50 Ω resistor in the network shown in Fig. 3.98.*

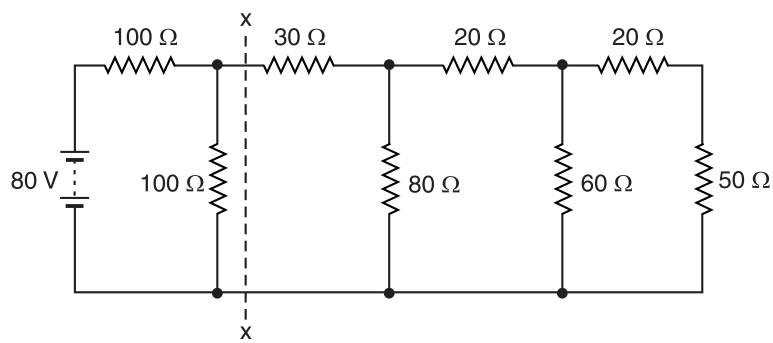
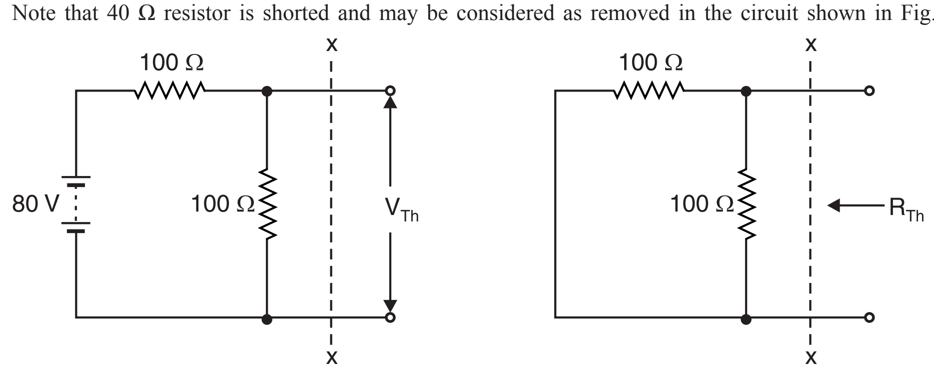


Fig. 3.98

Solution. We shall simplify the circuit shown in Fig. 3.98 by the repeated use of Thevenin's theorem. We first find Thevenin's equivalent circuit to the left of ***xx*.

* Note that 40 Ω resistor is shorted and may be considered as removed in the circuit shown in Fig. 3.96.

**



(*a*) (*b*)

$$V_{Th} = \text{Current in } 100\ \Omega \times 100\ \Omega = \frac{80}{100 + 100} \times 100 = 40\text{V}$$

$$R_{Th} = \text{Resistance looking into the open terminals in Fig. } (b)$$

$$= 100 \parallel 100 = \frac{100 \times 100}{100 + 100} = 50\Omega$$

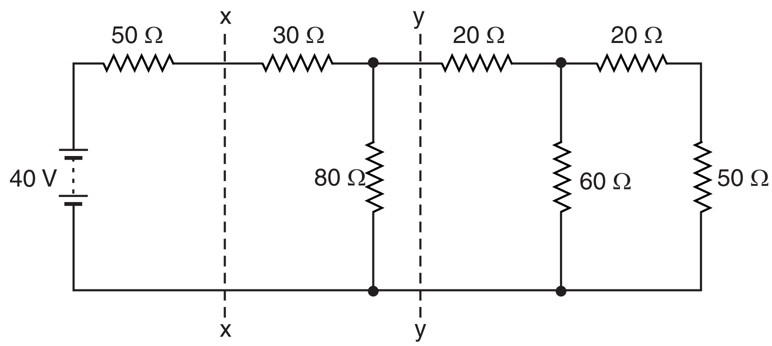


Fig. 3.99

$$V_{Th} = \frac{80}{100+100} \times 100 = 40\text{V}$$

$$R_{Th} = 100 \parallel 100 = \frac{100 \times 100}{100+100} = 50\Omega$$

Therefore, we can replace the circuit to the left of *xx* in Fig. 3.98 by its Thevenin's equivalent circuit *viz*. V_{Th} (= 40V) in series with R_{Th} (= 50 Ω). The original circuit of Fig. 3.98 then reduces to the one shown in Fig. 3.99.

We shall now find Thevenin's equivalent circuit to the left of *yy* in Fig. 3.99.

$$V'_{Th} = \frac{40}{50+30+80} \times 80 = 20 \text{ V}$$

$$R'_{Th} = (50 + 30) \parallel 80 = \frac{80 \times 80}{80+80} = 40 \ \Omega$$

We can again replace the circuit to the left of *yy* in Fig. 3.99 by its Thevenin's equivalent circuit. Therefore, the original circuit reduces to that shown in Fig. 3.100.

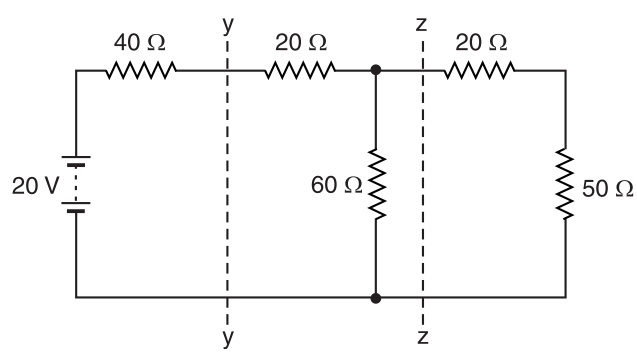


Fig. 3.100

Using the same procedure to the left of *zz*, we have,

$$V''_{Th} = \frac{20}{40+20+60} \times 60 = 10 \text{ V}$$

$$R''_{Th} = (40 + 20) \parallel 60 = \frac{60 \times 60}{60+60} = 30\Omega$$

The original circuit then reduces to that shown in Fig. 3.101.

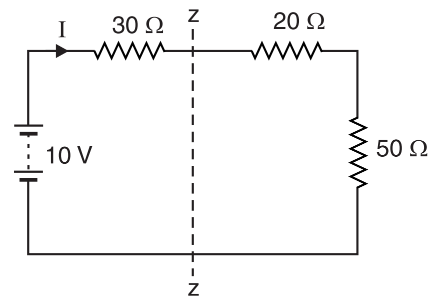


Fig. 3.101

By Ohm's law, current I in 50 Ω resistor is

$$I = \frac{10}{30+20+50} = \mathbf{0.1\ A}$$

Example 3.42. *Calculate the current in the 10 Ω resistor in the network shown in Fig. 3·102.*

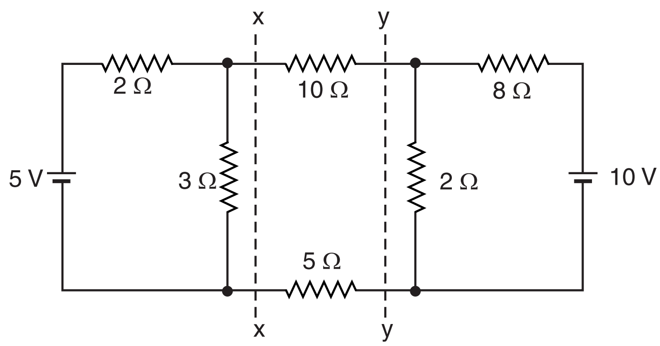


Fig. 3.102

Solution. We can replace circuits to the left of xx and right of yy by the Thevenin's equivalent circuits. It is easy to see that to the left of xx, the Thevenin's equivalent circuit is a voltage source of 3V (= V_{Th}) in series with a resistor of *1·2 Ω (= R_{Th}). Similarly, to the right of yy, the Thevenin's equivalent circuit is a voltage source of 2V (= V_{Th}) in series with a resistor of **1·6 Ω (= R_{Th}). The original circuit then reduces to that shown in Fig. 3.103.

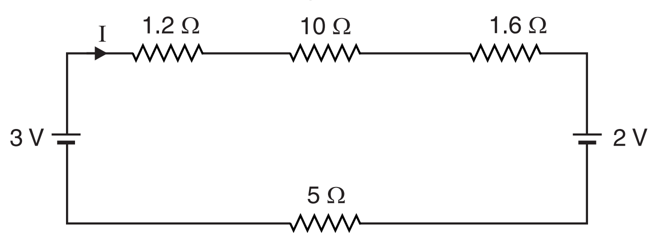


Fig. 3.103

∴ Current through 10 Ω resistor is given by ;

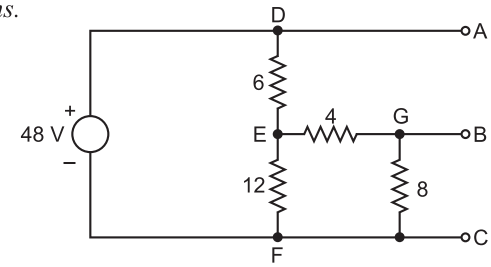
$$I = \frac{\text{Net voltage}}{\text{Total resistance}} = \frac{3-2}{1.2+10+1.6+5} = 56{\cdot}2 \times 10^{-3}\text{ A} = \mathbf{56{\cdot}2\ mA}$$

---

* $R_{Th} = 2 \parallel 3 = \frac{2\times3}{2+3} = 1.2\Omega$

** $R_{Th} = 2 \parallel 8 = \frac{2\times8}{2+8} = 1.6\Omega$

Example 3.43. *Calculate the values of V_{Th} and R_{Th} between terminals A and B in Fig. 3.104 (i). All resistances are in ohms.*



(*i*)

Fig. 3.104

Solution. (*i*) Finding V_{Th}. Between points *E* and *F* [See Fig. 3.104 (*i*)], 12Ω || (4 + 8)Ω.

$\therefore \quad R_{EF} = 12\Omega \parallel (4+8)\Omega = 12\Omega \parallel 12\Omega = 6\Omega$

By voltage-divider rule, we have,

$$V_{DE} = 48 \times \frac{6}{6+6} = 24\text{V} \; ; \; V_{EF} = 48 \times \frac{6}{6+6} = 24\text{V}$$

Now V_{EF} (= 24V) is divided between 4Ω and 8Ω resistances in series.

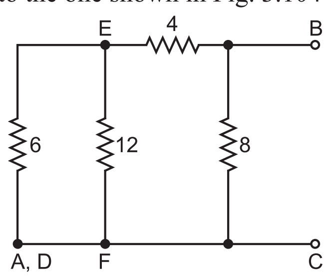
$$\therefore \quad V_{EG} = 24 \times \frac{4}{4+8} = 8\text{V}$$

In going from *A* to *B* via *D, E* and *G,* there is fall in potential from *D* to *E*, fall in potential from *E* to *G* and rise in potential from *B* to *A*. Therefore, by *KVL*,

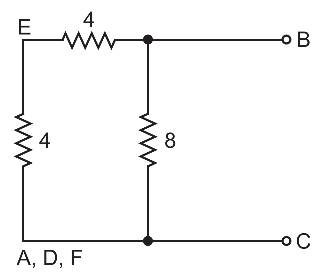
$$V_{BA} - V_{DE} - V_{EG} = 0 \quad \text{or} \quad V_{BA} = V_{DE} + V_{EG} = 24 + 8 = 32\text{V}$$

$$\therefore \quad V_{Th} = V_{BA} = \mathbf{32V} \; ; A \text{ positive } w.r.t\ B.$$

(*ii*) Finding R_{Th}. R_{Th} is the resistance between open terminals *AB* with voltage source replaced by a short as shown in Fig. 3.104 (*ii*). Shorting voltage source brings points *A*, *D* and *F* together. Now combined resistance of parallel combination of 6Ω and 12Ω = 6Ω || 12Ω = 4Ω and the circuit reduces to the one shown in Fig. 3.104 (*iii*).



(*ii*)



(*iii*)

Fig. 3.104

$$\therefore \quad R_{Th} = R_{AB} \text{ in Fig. 3.104 } (iii) = 8\Omega \parallel (4+4)\Omega = \mathbf{4\Omega}$$

Example 3.44. *The circuit shown in Fig. 3.105 consists of a current source I = 10 A paralleled by G = 0·1S and a voltage source E = 200 V with a 10 Ω series resistance. Find Thevenin equivalent circuit to the left of terminals AB.*

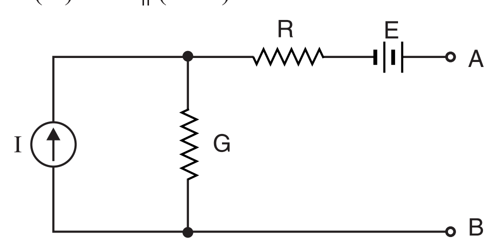


Fig. 3.105

Solution. With terminals *A* and *B* open-circuited, the current source will send a current through conductance *G* as shown in Fig. 3.106.

$\therefore$ Voltage across G, $V_G = \dfrac{I}{G} = \dfrac{10}{0.1} = 100$ V

Thevenin voltage, V_{Th} = Open-circuited voltage at terminals AB in Fig. 3.106.

$= E + V_G = 200 + 100 = 300$ V

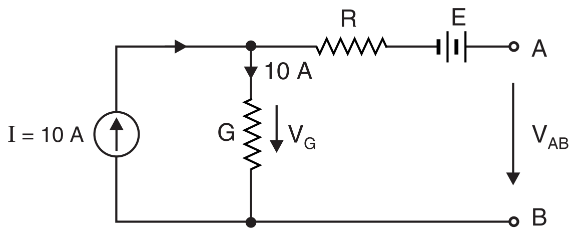


Fig. 3.106

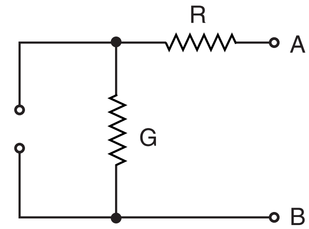


Fig. 3.107

In order to find Thevenin resistance R_{Th}, replace the voltage source by a short and current source by an open. The circuit then becomes as shown in Fig. 3.107.

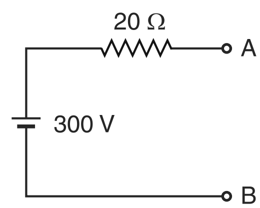


Fig. 3.108

R_{Th} = Resistance looking into terminals AB in Fig. 3.107.

$$= R + \frac{1}{G} = 10 + \frac{1}{0.1} = 10 + 10 = 20\Omega$$

Therefore, Thevenin equivalent circuit consists of **300V voltage source in series with a resistance of 20 Ω** as shown in Fig. 3.108.

Example 3.45. *Using Thevenin's theorem, find the voltage across 3Ω resistor in Fig. 3.109 (i).*

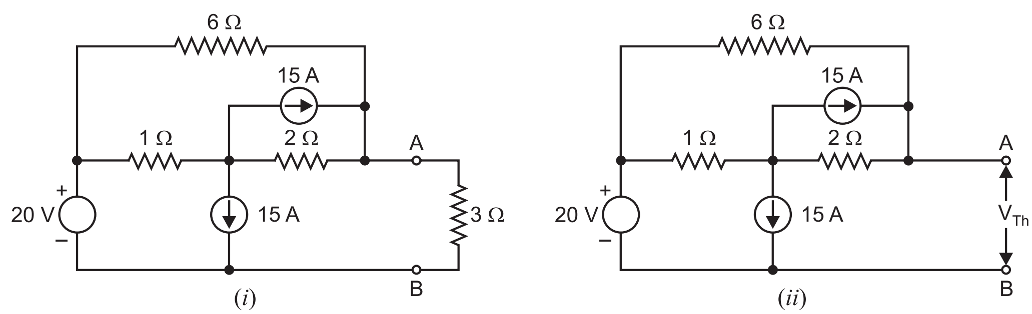


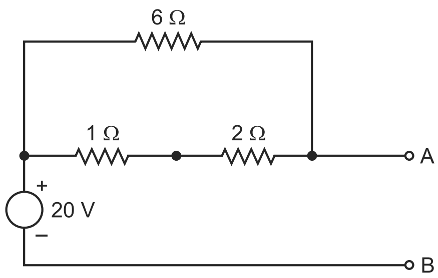
Fig. 3.109

Solution. (*i*) Finding V_{Th}. Thevenin voltage V_{Th} is the voltage at the open-circuited load terminals AB (*i.e.,* when 3Ω is removed) as shown in Fig. 3.109 (*ii*). It can be found by superposition theorem. First, open circuit both 15A current sources so that 20V voltage source is acting alone as shown in Fig. 3.109 (*iii*). It is clear that :

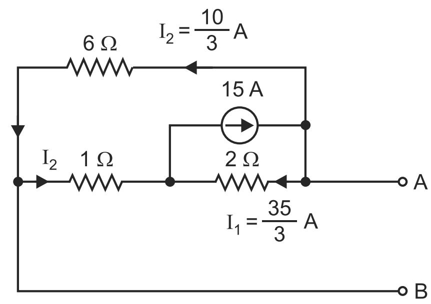
V_{AB1} = *20V

Next, open one 15A current source and replace 20V source by a short so that the second 15A source is acting alone as shown in Fig. 3.109 (*iv*). By current-divider rule, the currents in the various branches will be as shown in Fig. 3.109 (*iv*).

* The circuit behaves as a 20V source having internal resistance of (1 + 2)Ω || 6Ω with terminals AB open.



(*iii*)



(*iv*)

Fig. 3.109

Referring to Fig. 3.109 (*iv*), we have,

$$V_A - I_1 \times 2 + I_2 \times 1 = V_B$$

$$\therefore \quad V_A - V_B = I_1 \times 2 - I_2 \times 1 = \frac{35}{3}\times 2 - \frac{10}{3}\times 1 = 20 \text{ V}$$

$$\therefore \quad V_{AB2} = V_A - V_B = 20\text{V}$$

Finally, open the second 15A source and replace the 20V source by a short as shown in Fig. 3.109 (*v*). By current-divider rule, the currents in the various branches will be as shown in Fig. 3.109 (*v*).

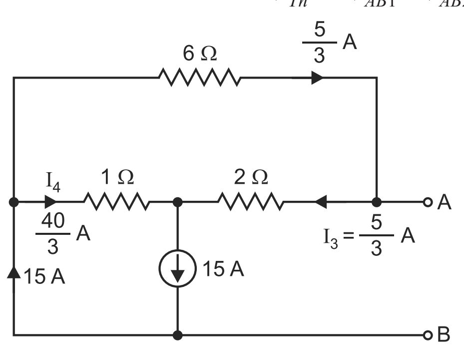
Now, $$V_A - I_3 \times 2 + I_4 \times 1 = V_B$$

$$\therefore \quad V_A - V_B = I_3 \times 2 - I_4 \times 1 = \frac{5}{3}\times 2 - \frac{40}{3}\times 1 = -10\text{V}$$

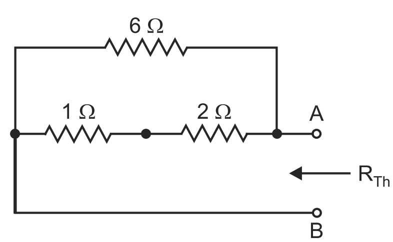
$$\therefore \quad V_{AB3} = V_A - V_B = -10\text{V}$$

By superposition theorem, the open-circuited voltage at terminals *AB* (*i.e.*, V_{Th}) with all sources present is

$$V_{Th} = V_{AB1} + V_{AB2} + V_{AB3} = 20 + 20 - 10 = 30\text{V}$$



(*v*)



(*vi*)

Fig. 3.109

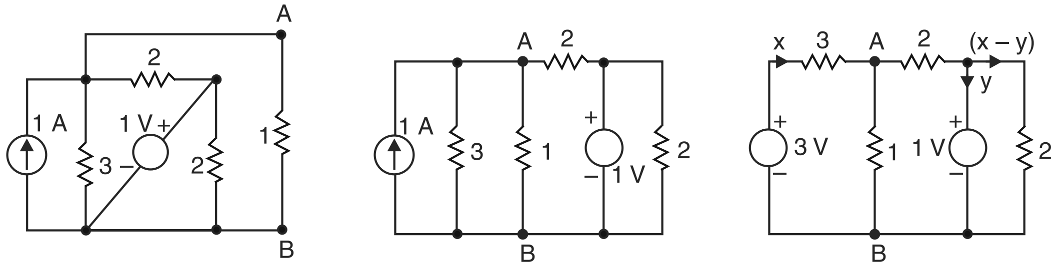
(*ii*) **Finding R_{Th}.** Thevenin resistance R_{TH} is the resistance at terminals *AB* when 3Ω is removed and current sources replaced by open and voltage source replaced by short as shown in Fig. 3.109 (*vi*).

$$\therefore \quad R_{Th} = (1\Omega + 2\Omega) \parallel 6\Omega = 2\Omega$$

$$\therefore \quad \text{Current in } 3\Omega, I = \frac{V_{Th}}{R_{Th} + 3} = \frac{30}{2+3} = 6\text{A}$$

$$\therefore \quad \text{Voltage across } 3\Omega = I \times 3 = 6 \times 3 = \mathbf{18V}$$

Example 3.46. *Using Thevenin's theorem, determine the current in 1 Ω resistor across AB of the network shown in Fig. 3.110 (i). All resistances are in ohms.*



(*i*) (*ii*) (*iii*)

Fig. 3.110

Solution. The circuit shown in Fig. 3.110 (*i*) can be redrawn as shown in Fig. 3.110 (*ii*). If we convert the current source into equivalent voltage source, the circuit becomes as shown in Fig. 3.110 (*iii*). In order to find V_{Th}, remove 1 Ω resistor from the terminals *AB*. Then voltage at terminals *AB* is equal to V_{Th} (See Fig. 3.111 (*i*)). Applying *KVL* to the first loop in Fig. 3.111 (*i*), we have,

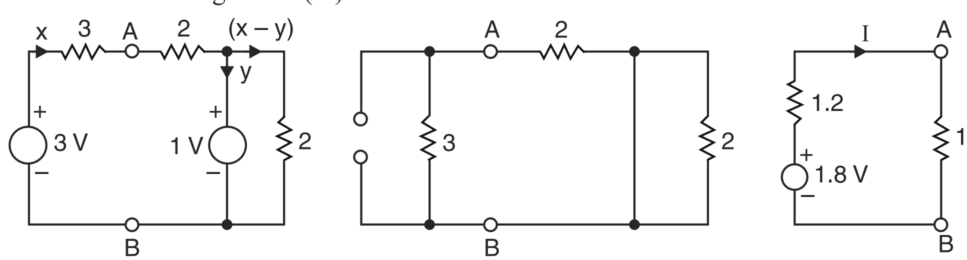
$$3 - (3+2)\,x - 1 = 0 \qquad \therefore \qquad x = 0{\cdot}4 \text{ A}$$

$$\therefore \qquad V_{Th} = V_{AB} = 3 - 3x = 3 - 3 \times 0{\cdot}4 = 1{\cdot}8 \text{ V}$$

In order to find R_{Th}, replace the voltage sources by short circuits and current sources by open circuits in Fig. 3.110 (*ii*). The circuit then becomes as shown in Fig. 3.111 (*ii*). Then resistance at terminals *AB* is equal to R_{Th}.

Clearly, $$R_{Th} = 2 \,\|\, 3 = \frac{2 \times 3}{2+3} = 1.2\ \Omega$$

Thevenin's equivalent circuit is 1·8 V voltage source in series with 1·2 Ω resistor. When 1 Ω resistor is connected across the terminals *AB* of the Thevenin's equivalent circuit, the circuit becomes as shown in Fig. 3.111 (*iii*).



(*i*) (*ii*) (*iii*)

Fig. 3.111

$$\therefore \qquad \text{Current in } 1\ \Omega = \frac{V_{Th}}{R_{Th}+1} = \frac{1.8}{1.2+1} = \mathbf{0{\cdot}82\ A}$$

Example 3.47. *At no-load, the terminal voltage of a d.c. generator is 120 V. When delivering its rated current of 40 A, its terminal voltage drops to 112 V. Represent the generator by its Thevenin equivalent.*

Solution. If *R* is the internal resistance of the generator, then,

$$E = V + IR \qquad \text{or} \qquad R = \frac{E-V}{I} = \frac{120-112}{40} = 0.2\Omega$$

Therefore, V_{Th} = No-load voltage = 120 V and $R_{Th} = R = 0{\cdot}2\ \Omega$.

Hence Thevenin equivalent circuit of the generator is **120 V source in series with 0·2 Ω resistor.**

Example 3.48. *Calculate V_{Th} and R_{Th} between the open terminals A and B of the circuit shown in Fig. 3.112 (i). All resistance values are in ohms.*

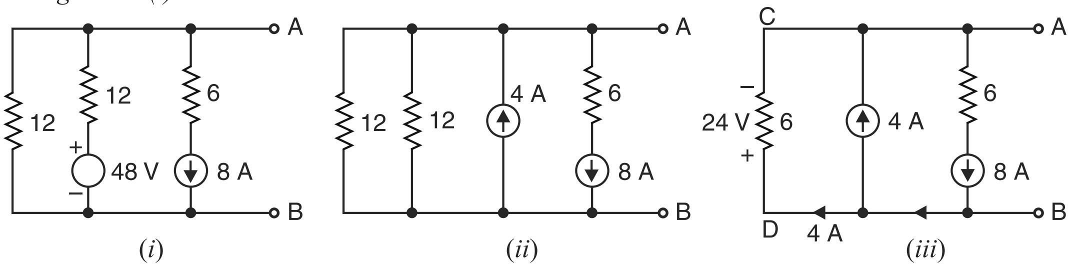


Fig. 3.112

Solution. If we replace the 48 V voltage source into equivalent current source, the circuit becomes as shown in Fig. 3.112 (*ii*). The two 12 Ω resistors are in parallel and can be replaced by 6 Ω resistor. The circuit then reduces to the one shown in Fig. 3.112 (*iii*). It is clear that 4 A current flows through 6 Ω resistor.

∴ V_{Th} = Voltage across terminals *AB* in Fig. 3.112 (*iii*)

= Voltage across 6 Ω resistor = 4 × 6 = 24 V

Note that terminal *A* is negative w.r.t. *B*. Therefore, V_{Th} = **– 24 V.**

R_{Th} = Resistance between terminals *AB* in Fig. 3.112 (*i*) with 48V source replaced by a short and 8 A source replaced by an open

= 12 || 12 = **6 Ω**

Example 3.49. *Find the voltage across R_L in Fig. 3.113 when (i) R_L = 1 kΩ (ii) R_L = 2 kΩ (iii) R_L = 9 kΩ. Use Thevenin's theorem to solve the problem.*

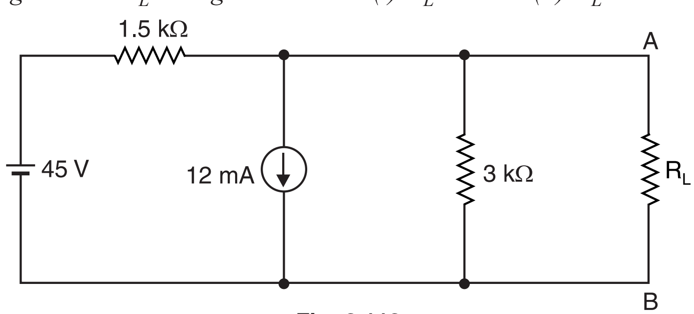


Fig. 3.113

Solution. It is required to find the voltage across R_L when R_L has three different values. We shall find Thevenin's equivalent circuit to the left of the terminals *AB*. The solution involves two steps.

The first step is to find the open-circuited voltage V_{Th} at terminals *AB*. For this purpose, we shall use the superposition principle. With the current source removed (opened), we find voltage V_1 due to the 45 V source acting alone as shown in Fig. 3.114 (*i*). Since V_1 is the voltage across the 3 kΩ resistor, we have by voltage-divider rule :

$$V_1 = 45 \times \frac{3\,k\Omega}{1.5\,k\Omega + 3\,k\Omega} = 30V^{+}_{-}$$

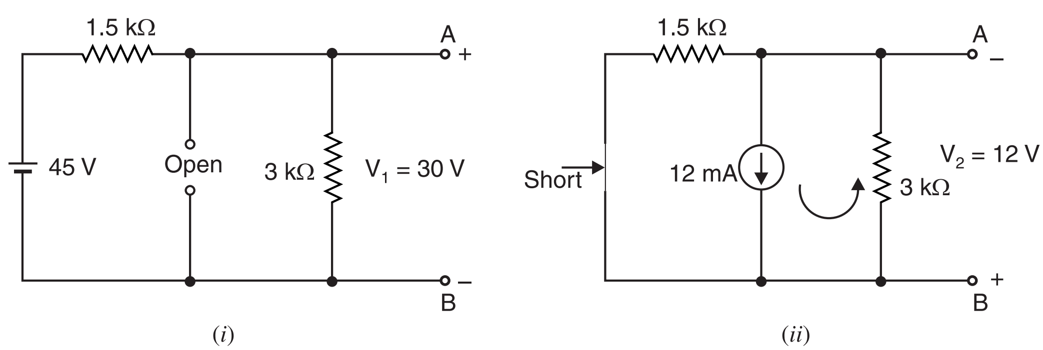


Fig. 3.114

The voltage V_2 due to the current source acting alone is found by shorting 45 V voltage source as shown in Fig. 3.114 (*ii*). By current-divider rule,

$$\text{Current in 3 k}\Omega\text{ resistor} = 12 \times \frac{1.5\text{ k}\Omega}{1.5\text{ k}\Omega + 3\text{ k}\Omega} = 4\text{ mA}$$

$$\therefore \quad V_2 = 4\text{ mA} \times 3\text{ k}\Omega = 12\text{ V}_{+}^{-}$$

Note that V_1 and V_2 have opposite polarities.

$$\therefore \quad \text{Thevenin's voltage, } V_{Th} = V_1 - V_2 = 30 - 12 = 18\text{ V}_{-}^{+}$$

The second step is to find Thevenin's resistance R_{Th}. For this purpose, we replace the 45 V voltage source by a short circuit and the 12 mA current source by an open circuit as shown in Fig. 3.115. As can be seen in the figure, R_{Th} is equal to parallel equivalent resistance of 1·5 kΩ and 3 kΩ resistors.

$$\therefore \quad R_{Th} = 1{\cdot}5\text{ k}\Omega \parallel 3\text{ k}\Omega = 1\text{ k}\Omega$$

Fig. 3.116 shows Thevenin's equivalent circuit.

$$\text{Voltage across } R_L, \quad V_L = 18 \times \frac{R_L}{1\text{ k}\Omega + R_L}$$

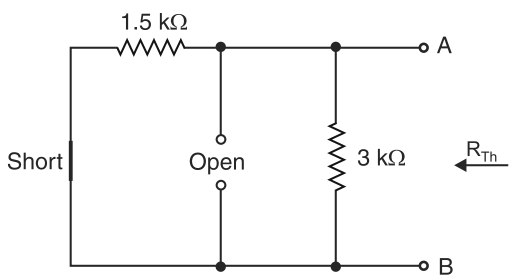


Fig. 3.115

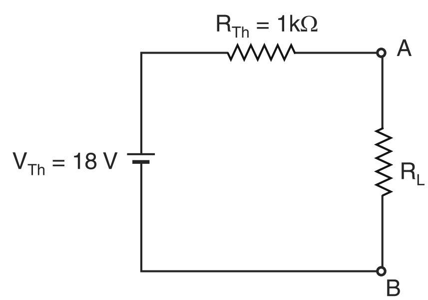


Fig. 3.116

(*i*) When $R_L = 1\text{ k}\Omega$; $V_L = 18 \times \dfrac{1\text{k}\Omega}{1\text{k}\Omega + 1\text{k}\Omega} =$ **9 V**

(*ii*) When $R_L = 2\text{ k}\Omega$; $V_L = 18 \times \dfrac{2\text{k}\Omega}{1\text{k}\Omega + 2\text{k}\Omega} =$ **12 V**

(*iii*) When $R_L = 9\text{ k}\Omega$; $V_L = 18 \times \dfrac{9\text{k}\Omega}{1\text{k}\Omega + 9\text{k}\Omega} =$ **16·2 V**

Example 3.50. *Find Thevenin's equivalent circuit to the left of terminals AB in Fig. 3.117.*

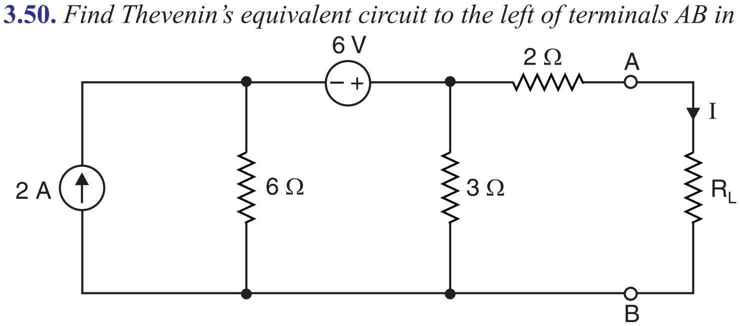


Fig. 3.117

Solution. To find V_{Th}, remove R_L from terminals AB. The circuit then becomes as shown in Fig. 3.118 (*i*).

$\therefore \quad V_{Th}$ = Voltage across terminals AB in Fig. 3.118 (*i*)

= Voltage across 3 Ω resistor in Fig. 3.118 (*i*)

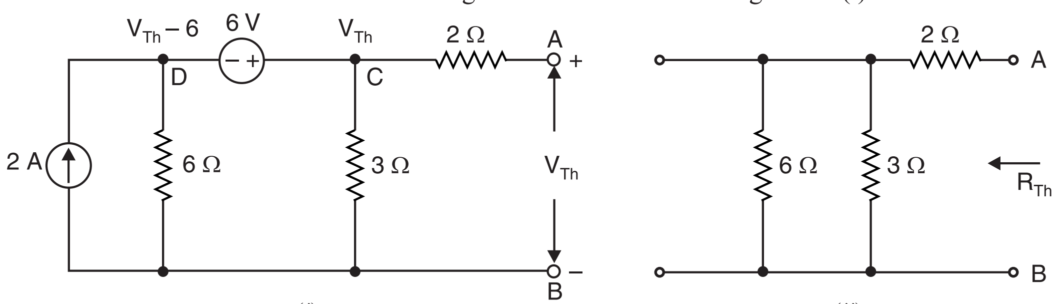


Fig. 3.118

Note that voltage at point C is V_{Th} and voltage at point D is $V_{Th} - 6$. Therefore, nodal equation becomes :

$$\frac{V_{Th}-6}{6}+\frac{V_{Th}}{3} = 2 \quad \text{or} \quad V_{Th} = 6 \text{ V}$$

In order to find R_{Th}, remove R_L and replace voltage source by a short and current source by an open in Fig. 3.117. The circuit then becomes as shown in Fig. 3.118 (*ii*).

$\therefore \quad R_{Th}$ = Resistance looking into terminals AB in Fig. 3.118 (*ii*).

$$= 2+(3\,||\,6) = 2+\frac{3\times 6}{3+6} = 4\,\Omega$$

Therefore, Thevenin equivalent circuit to the left of terminals AB is a **voltage source of 6 V (= V_{Th}) in series with a resistor of 4 Ω (= R_{Th}).** When load R_L is connected across the output terminals of Thevenin equivalent circuit, the circuit becomes as shown in Fig. 3.119. We can use Ohm's law to find current in the load R_L.

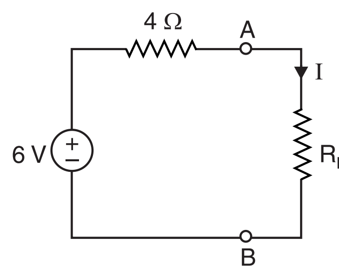


Fig. 3.119

$$\therefore \quad \text{Current in } R_L,\ I = \frac{V_{Th}}{R_{Th}+R_L} = \frac{6}{4+R_L}$$

Example 3.51. *Find Thevenin's equivalent circuit in Fig. 3.120 when we view from (i) between points A and C (ii) between points B and C.*

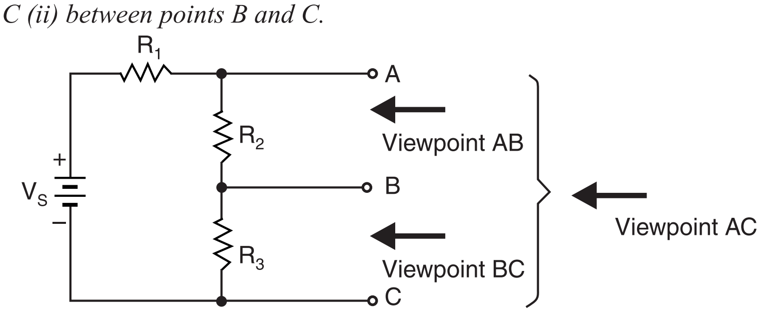


Fig. 3.120

Solution. The Thevenin equivalent for any circuit depends on the location of the two points from between which circuit is "viewed". Any given circuit can have more than one Thevenin equivalent, depending on how the viewpoints are designated. For example, if we view the circuit in Fig. 3.120

from between points A and C, we obtain a completely different result than if we view it from between points A and B or from between points B and C.

(*i*) Viewpoint AC. When the circuit is viewed from between points A and C,

V_{Th} = Voltage between open-circuited points A and C in Fig. 3.121 (*i*).

= Voltage across $(R_2 + R_3)$ in Fig. 3.121 (*i*)

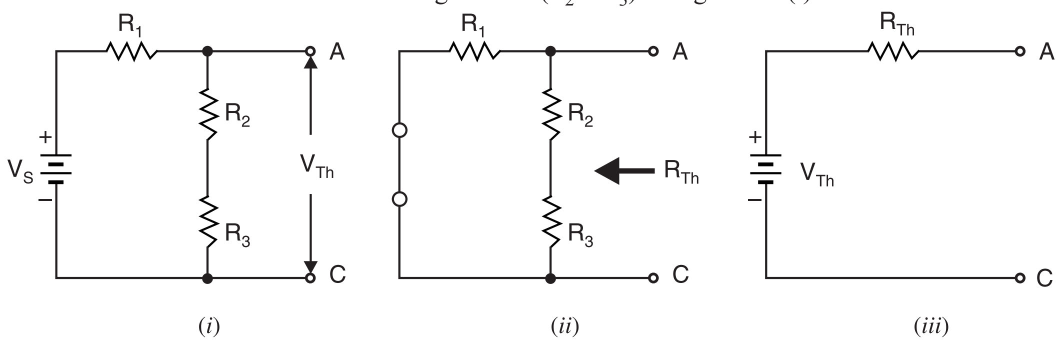


Fig. 3.121

$$= \frac{V_S}{R_1 + R_2 + R_3} \times (R_2 + R_3) = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right) V_S$$

In order to find R_{Th}, replace the voltage source by a short. Then resistance looking into the open-circuited terminals A and C [See Fig. 3.121 (*ii*)] is equal to R_{Th}.

$$\therefore \qquad R_{Th} = R_1 \parallel (R_2 + R_3) = \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3}$$

The resulting Thevenin equivalent circuit is shown in Fig. 3.121 (*iii*).

(*ii*) Viewpoint BC. When the circuit is viewed from between points B and C,

V_{Th} = Voltage between open-circuited points B and C in Fig.3.122 (*i*).

= Voltage across R_3

$$= \frac{V_S}{R_1 + R_2 + R_3} \times R_3 = \left(\frac{R_3}{R_1 + R_2 + R_3}\right) V_S$$

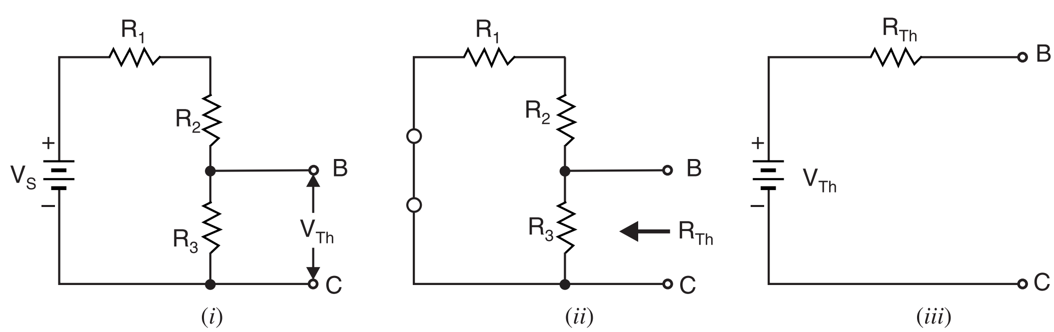


Fig. 3.122

In order to find R_{Th}, replace the voltage source by a short. Then resistance looking into the open-circuited terminals B and C [See Fig. 3.122 (*ii*)] is equal to R_{Th}.

$$\therefore \qquad R_{Th} = (R_1 + R_2) \parallel R_3 = \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3}$$

The resulting Thevenin equivalent circuit is shown in Fig. 3.122 (*iii*).

Example 3.52. *Calculate (i) V_{Th} and (ii) R_{Th} between the open terminals A and B in the circuit shown in Fig. 3.123 (i). All resistance values are in ohms.*

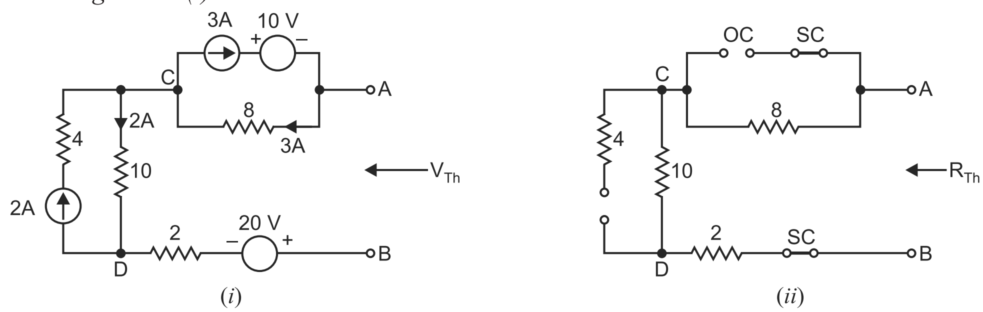


Fig. 3.123

Solution. Since terminals *A* and *B* are open, it is clear from the circuit that 10V and 20V voltage sources are ineffective in producing current in the circuit. However, current sources will circulate currents in their respective loops. Therefore, 2A current circulating in its loop will produce a voltage drop across 10 Ω resistance = 2A × 10 Ω = 20 V. Similarly, 3A current will produce a voltage drop across 8 Ω resistance = 3A × 8Ω = 24V. Tracing the circuit from *A* to *B via* points *C* and *D* [See Fig. 3.123 (*i*)], we have,

$$V_A - 24 - 20 + 20 = V_B$$

or $$V_A - V_B = 24 + 20 - 20 = 24\text{V}$$

∴ $$V_{Th} = V_{AB} = V_A - V_B = \mathbf{24V}$$

In order to find R_{Th}, open circuit the current sources and replace the voltage sources by a short as shown in Fig. 3.123 (*ii*). The resistance at the open-circuited terminals *AB* is R_{Th}.

∴ $$R_{Th} = \text{Resistance at terminals } AB \text{ in Fig. 3.123 } (ii)$$

$$= 8\Omega + 10\Omega + 2\Omega = \mathbf{20\Omega}$$

Example 3.53. *Find the current in the 25 Ω resistor in Fig. 3.124 (i) when E = 3 V.*

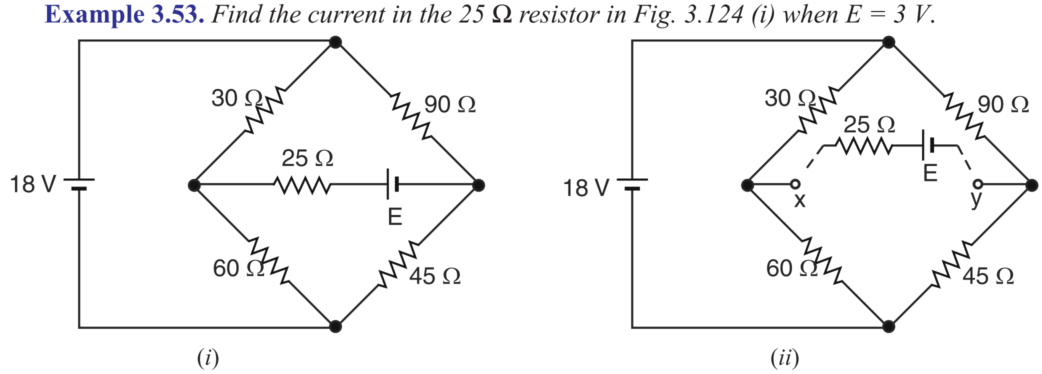


Fig. 3.124

Solution. Finding V_{Th}. Remove the voltage source *E* and the 25 Ω resistor, leaving the terminals *x – y* open-circuited as shown in Fig. 3.124 (*ii*). The circuit shown in Fig. 3.124 (*ii*) can be redrawn as shown in Fig. 3.125. The voltage between terminals *xy* in Fig. 3.125 is equal to V_{Th}. We can use voltage-divider rule to find voltage drops across 60 Ω and 45 Ω resistors.

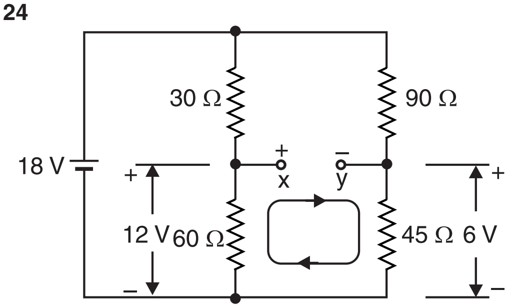


Fig. 3.125

$$\text{Voltage across } 60\ \Omega = 18 \times \frac{60}{60+30} = 12\text{V}$$

$$\text{Voltage across } 45\ \Omega = 18 \times \frac{45}{90+45} = 6\text{V}$$

Applying *KVL* around the loop shown in Fig. 3.125, we have,

$$12 - V_{xy} - 6 = 0 \qquad \therefore \qquad V_{xy} = 6\text{ V}$$

$$\text{But} \quad V_{xy} = V_{Th}. \text{ Therefore,} \quad V_{Th} = 6\text{ V.}$$

Finding R_{Th}. In order to find R_{Th}, replace the voltage source by a short. Then resistance at open-circuited terminals *xy* (See Fig. 3.126) is equal to R_{Th}. Note that in Fig. 3.126, 30 Ω and 60 Ω resistors are in parallel and so are 90 Ω and 45 Ω resistors.

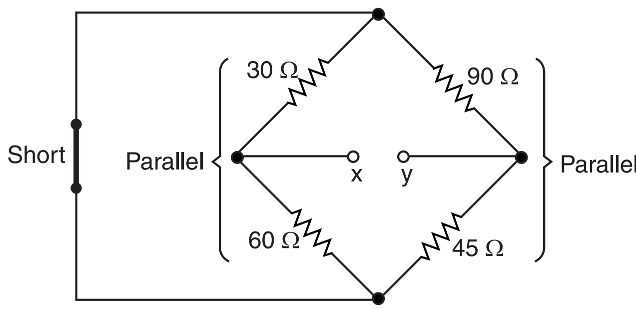


Fig. 3.126

The circuit shown in Fig. 3.126 can be redrawn as shown in Fig. 3.127 (*i*). This further reduces to the circuit shown in Fig. 3.127 (*ii*).

$$\therefore \qquad R_{Th} = 20 + 30 = 50\ \Omega$$

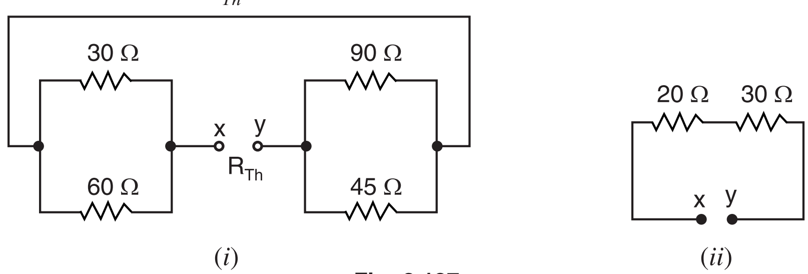


Fig. 3.127

Therefore, the Thevenin equivalent circuit is a voltage source of 6 V in series with 50 Ω resistor. When we reconnect *E* and 25 Ω resistor, the circuit becomes as shown in Fig. 3.128. Note that V_{Th} and *E* are in series opposition.

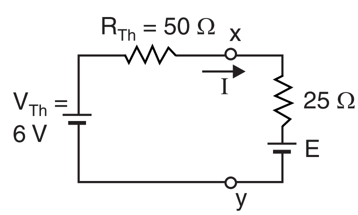


Fig. 3.128

$$\therefore \qquad \text{Current in } 25\ \Omega, I = \frac{V_{Th} - E}{R_{Th} + 25} = \frac{6-3}{50+25}$$

$$= 40 \times 10^{-3}\text{ A} = \mathbf{40\ mA}$$

Example 3.54. *Find the current in the feeder AC of the distribution circuit shown in Fig. 3.129 (i) by using Thevenin's theorem. Also determine the currents in other branches.*

Solution. To determine current in the feeder *AC*, we shall find Thevenin voltage V_{Th} and Thevenin resistance R_{Th} at terminals *AC*.

(*i*) With *AC* removed, the voltage between *A* and *C* will be equal to V_{Th} as shown in Fig. 3.129 (*ii*). Assuming that current *I* flows in *AB*, then current distribution in the network will be as shown in Fig. 3.129 (*ii*).

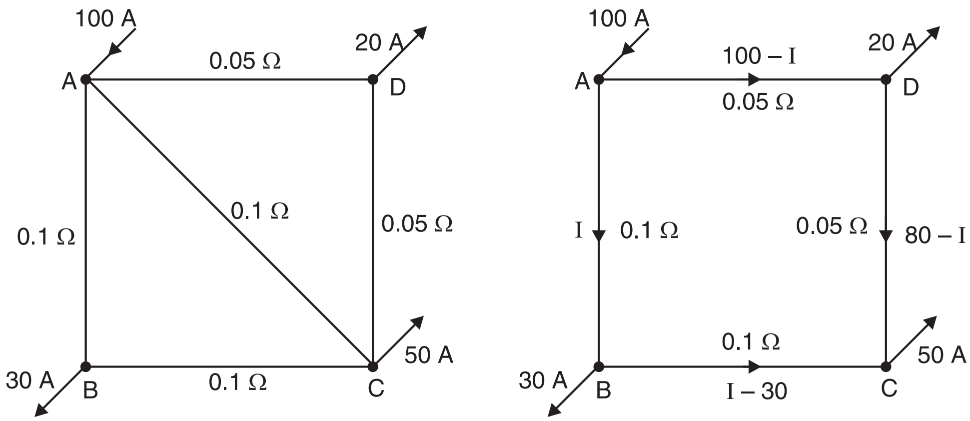
Voltage drop along ADC = Voltage drop across ABC

or $0{\cdot}05\ (100 - I) + 0{\cdot}05\ (80 - I) = 0.1\ I + 0{\cdot}1\ (I - 30)$

or $0{\cdot}3\ I = 12 \quad \therefore \quad I = 12/0{\cdot}3 = 40$ A

$\therefore$ P.D. between A and C, V_{Th} = Voltage drop from A to C

$= 0{\cdot}05\ (100 - 40) + 0{\cdot}05\ (80 - 40) = 5$ V

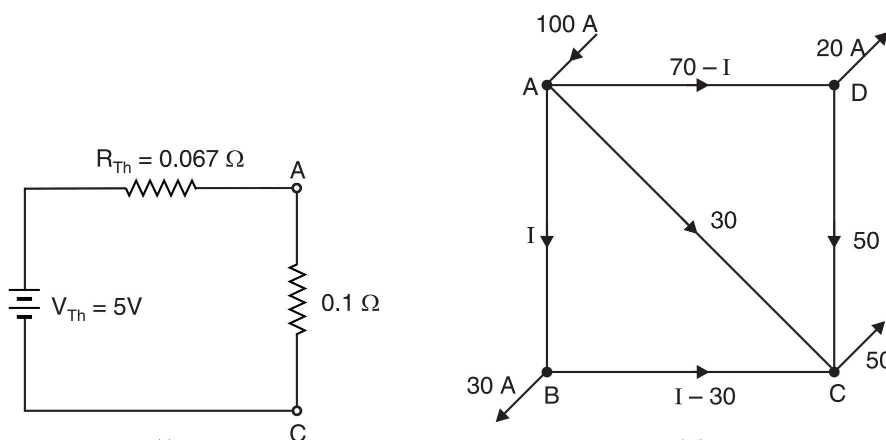


(*i*) (*ii*)

Fig. 3.129

(ii) With AC removed, the resistance between terminals A and C is equal to R_{Th}. Referring to Fig. 3.129 (*ii*), there are two parallel paths *viz* ADC $(= 0{\cdot}05 + 0{\cdot}05 = 0{\cdot}1\ \Omega)$ and ABC $(= 0{\cdot}1 + 0{\cdot}1 = 0{\cdot}2\ \Omega)$ between terminals A and C.

$$\therefore \quad R_{Th} = \frac{0.2 \times 0.1}{0.2 + 0.1} = 0{\cdot}067\ \Omega$$



(*i*) (*ii*)

Fig. 3.130

The Thevenin equivalent circuit at terminals AC will be V_{Th} (= 5 V) in series with R_{Th} (= 0·067 Ω). When feeder AC (= 0·1 Ω) is connected between A and C, the circuit becomes as shown in Fig. 3.130 (*i*).

$$\therefore \quad \text{Current in } AC = \frac{V_{Th}}{R_{Th} + 0.1} = \frac{5}{0.067 + 0.1} = \mathbf{30\ A}$$

To find currents in other branches, refer to Fig. 3.130 (*ii*). With current in *AC* calculated (*i.e.* 30A) and current in *AB* assumed to be *I*, the current distribution will be as shown in Fig. 3.130 (*ii*). It is clear that voltage drop along the path *ADC* is equal to the voltage drop along the path *ABC i.e.*

$$0{\cdot}05\ (70 - I) + 0{\cdot}05\ (50 - I) = 0{\cdot}1\ I + 0{\cdot}1\ (I - 30)$$

or $\quad 0{\cdot}3\ I = 9$

$\therefore \quad I = 9/0{\cdot}3 =$ **30 A**

The current distribution in the various branches will be as shown in Fig. 3.131. Note that branch *BC* of the circuit carries no current.

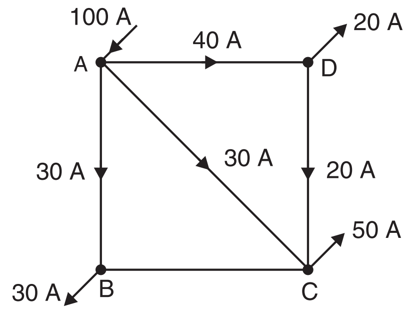


Fig. 3.131

Example 3.55. *Using Thevenin's theorem, calculate current in 1000Ω resistor connected between terminals A and B in Fig. 3.132 (i).*

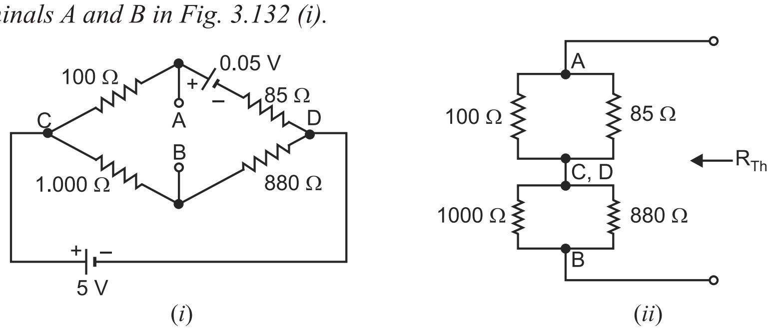


Fig. 3.132

Solution. (*i*) Finding V_{Th}. Thevenin voltage V_{Th} is the voltage across open circuited terminals *AB* in Fig. 3.132 (*i*). Refer to Fig. 3.132 (*i*).

By voltage-divider rule, we have,

$$V_{BD} = 5 \times \frac{880}{1000 + 880} = 2.340426\text{V}$$

Current in branch *CAD* is $I = \dfrac{5 - 0.05}{100 + 85} = 0.026757\text{A}$

Now, $V_A - 0.05 - 0.026757 \times 85 = V_D$

$$\therefore \quad V_{AD} = V_A - V_D = 0.05 + 0.026757 \times 85 = 2.324324 \text{ V}$$

Clearly, point *B* is at higher potential than point *A*.

$$\therefore \quad V_{Th} = V_{BA} = 2.340426 - 2.324324 = 0.0161\text{V}$$

(*ii*) Finding R_{Th}. Thevenin resistance R_{Th} is the resistance at open circuited terminals *AB* with 5V battery replaced by a short as shown in Fig. 3.132 (*ii*).

$$\therefore \quad R_{Th} = (100\Omega \parallel 85\Omega) + (1000\Omega \parallel 880\Omega)$$

$$= \frac{100 \times 85}{100 + 85} + \frac{1000 \times 880}{1000 + 880} = 514\Omega$$

$\therefore$ Current in 1000Ω connected between terminals *A* and *B*

$$= \frac{V_{Th}}{R_{Th} + 1000} = \frac{0.0161}{514 + 1000} = 10.634 \times 10^{-6} \text{ A}$$

$$= \mathbf{10.634\ \mu A} \textit{ from B to A}$$

Example 3.56. *Calculate the values of V_{Th} and R_{Th} between the open terminals A and B of the circuit shown in Fig. 3.133 (i). All resistance values are in ohms.*

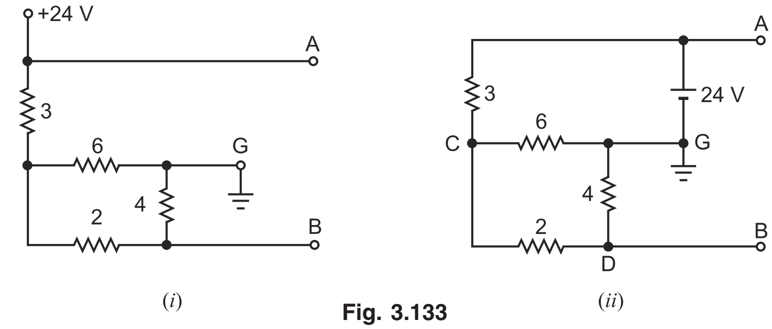


Fig. 3.133

Solution. If we eliminate the ground symbols in the circuit shown in Fig. 3.133 (*i*), we get the circuit shown in Fig. 3.133 (*ii*). Referring to Fig. 3.133 (*ii*),

Total resistance offered to 24V battery

$$= 3\Omega + (6\Omega \parallel 6\Omega) = 3\Omega + 3\Omega = 6\Omega$$

Current delivered by 24V battery = 24/6 = 4A

The distribution of currents in the various branches of the circuit is shown in Fig. 3.133 (*iii*).

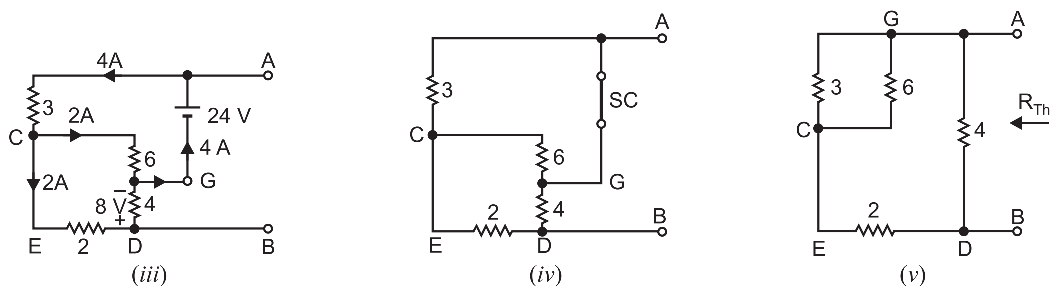


Fig. 3.133

Referring to Fig. 3.133 (*iii*) and tracing the circuit from point *A* to point *B via* points *C* and *D*, we have,

$$V_A - 3 \times 4 - 2 \times 6 + 4 \times 2 = V_B \quad \therefore \quad V_A - V_B = 3 \times 4 + 2 \times 6 - 4 \times 2 = 16\text{V}$$

$$\therefore \quad V_{Th} = V_{AB} = V_A - V_B = \mathbf{16V}$$

In order to find R_{Th}, we replace the 24V source by a short and the circuit becomes as shown in Fig. 3.133 (*iv*). This circuit further reduces to the one shown in Fig. 3.133 (*v*).

$$\therefore \quad R_{Th} = R_{AB} = [(3\Omega \parallel 6\Omega) + 2\Omega] \parallel 4\Omega = [2\Omega + 2\Omega] \parallel 4\Omega = \mathbf{2\Omega}$$

Example 3.57. *Using Thevenin theorem, find current in 1 Ω resistor in the circuit shown in Fig. 3.134 (i).*

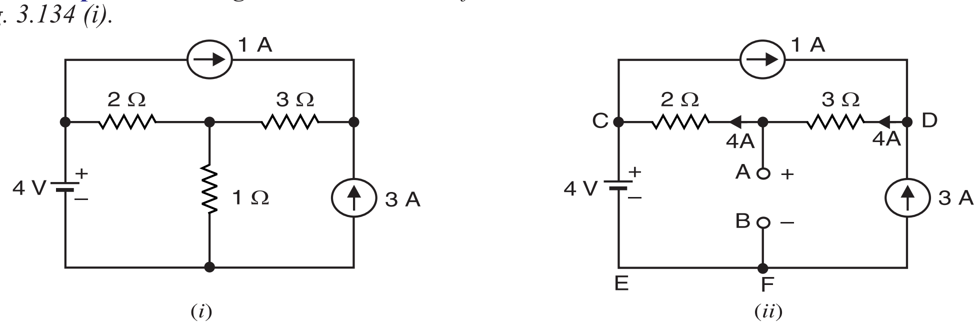


Fig. 3.134

Solution. In order to find V_{Th}, remove the load as shown in Fig. 3.134 (*ii*). Then voltage between the open-circuited terminals *A* and *B* is equal to V_{Th}. It is clear from Fig. 3.134 (*ii*) that 4 A (= 3 + 1) flows from *D* to *C*. Applying *KVL* to the loop *ECABFE*, we have,

$$4 + 2 \times 4 - V_{AB} = 0 \qquad \therefore \qquad V_{AB} = V_{Th} = 12 \text{ V}$$

$$R_{Th} = \text{Resistance looking into terminals } AB \text{ in Fig. 3.134 } (iii) = 2\ \Omega$$

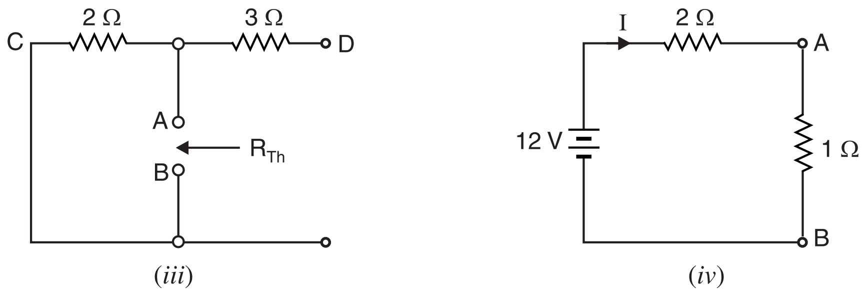


Fig. 3.134

When load (*i.e.* 1 Ω resistor) is reconnected, circuit becomes as shown in Fig. 3.134 (*iv*).

$$\therefore \qquad \text{Current in } 1\ \Omega = \frac{12}{2+1} = \mathbf{4\ A}$$

3.12. Thevenin Equivalent Circuit

(Circuits containing both independent and dependent sources)

Sometimes we come across circuits which contain both independent and dependent sources. One such example is shown in Fig. 3.135. The procedure for finding Thevenin equivalent circuit (*i.e.* finding v_{Th} and R_{Th}) in such cases is as under :

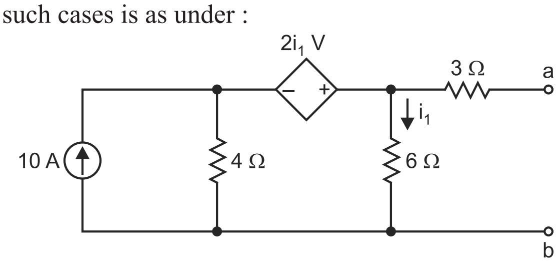


Fig. 3.135

(*i*) The open-circuit voltage v_{oc} (= v_{Th}) at terminals *ab* is determined as usual with sources present.

(*ii*) We cannot find R_{Th} at terminals *ab* simply by calculating equivalent resistance because of the presence of the dependent source. Instead, we place a short circuit across the terminals *ab* and find the value of short-circuit current i_{sc} at terminals *ab*.

(*iii*) Therefore, Thevenin resistance *$R_{Th} = v_{oc}/i_{sc} (= v_{Th}/i_{sc})$. It is the same procedure as adopted for Norton's theorem.

Note. In case the circuit contains dependent sources *only*, the procedure of finding $v_{oc} (= v_{Th})$ and R_{Th} is as under :

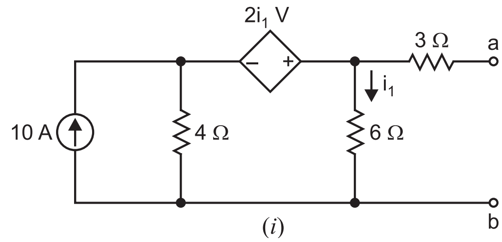
(*a*) In this case, $v_{oc} = 0$ and $i_{sc} = 0$ because no independent source is present.

(*b*) We cannot use the relation $R_{Th} = v_{oc}/i_{sc}$ as we do in case the circuit contains both independent and dependent sources.

* Alternatively, we can find R_{Th} in another way. We excite the circuit at terminals *ab* from external 1A current source and measure v_{ab}. Then $R_{Th} = v_{ab}/1\Omega$.

(*c*) In order to find R_{Th}, we excite the circuit at terminals *ab* by connecting 1A source to the terminals *a* and *b* and calculate the value v_{ab}. Then $R_{Th} = v_{ab}/1\Omega$.

Example 3.58. *Find the values of v_{Th} and R_{Th} at terminals ab for the circuit shown in Fig. 3.136 (i).*



(*i*)

Fig. 3.136

Solution. We first put a short circuit across terminals *a* and *b* and find short-circuit current i_{sc} at terminals *ab* as shown in Fig. 3.136 (*ii*). Applying *KCL* at node *C*,

$$10 = i_1 + i_2 + i_{sc}$$

or $$i_2 = 10 - i_1 - i_{sc}$$

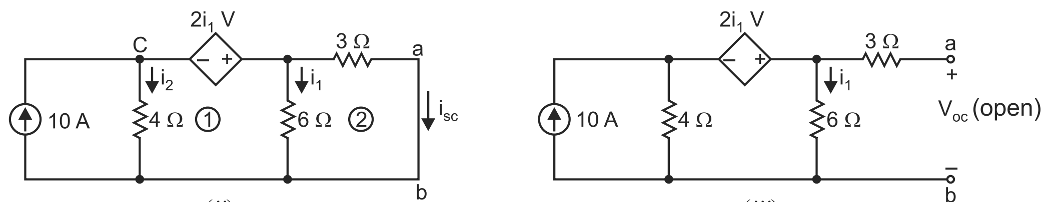
Applying *KVL* to loops 1 and 2, we have,

$$-4i_2 + 6i_1 - 2i_1 = 0 \qquad \text{... Loop 1}$$

or $$-4(10 - i_1 - i_{sc}) + 4i_1 = 0 \qquad ...(i)$$

Also $$-6i_1 + 3i_{sc} = 0 \qquad ...(ii) \text{ ... Loop 2}$$

From eqs. (*i*) and (*ii*), $i_{sc} = 5\text{A}$.



(*ii*) (*iii*)

Fig. 3.136

In order to find v_{oc} (= v_{Th}), we refer to Fig. 3.136 (*iii*) where we have,

$$v_{oc} = 6i_1 \qquad ...(iii)$$

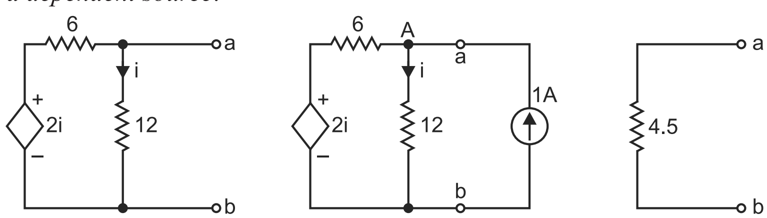
Applying *KVL* to the central loop in Fig. 3.136 (*iii*),

$$-4(10 - i_1) + 6i_1 - 2i_1 = 0 \qquad ...(iv)$$

From eqs. (*iii*) and (*iv*), we have, $v_{oc} = v_{Th} = $ **30V**

Also $$R_{Th} = \frac{v_{oc}}{i_{sc}} = \frac{30}{5} = \mathbf{6\Omega}$$

Example 3.59. *Find Thevenin equivalent circuit for the network shown in Fig. 3.137 (i) which contains only a dependent source.*



(*i*) (*ii*) (*iii*)

Fig. 3.137

Solution. In order to find R_{Th}, we connect 1A current source to terminals *a* and *b* as shown in Fig. 3.137 (*ii*). Then by finding the value of v_{ab}, we can determine the value of $R_{Th} = v_{ab}/1\Omega$. It may be seen that potential at point *A* is the same as that at *a*.

$\therefore$ $$v_{ab} = \text{Voltage across } 12\Omega \text{ resistor}$$

Applying *KCL* to point *A*, we have,

$$\frac{2i - v_{ab}}{6} + 1 = \frac{v_{ab}}{12}$$

or $\quad 4i - 3v_{ab} = -12 \quad$ or $\quad 4\left(\frac{v_{ab}}{12}\right) - 3v_{ab} = -2 \quad \therefore v_{ab} = 4.5\text{V}$

$$\therefore \quad R_{Th} = 4.5/1 = \mathbf{4.5\Omega}$$

Fig. 3.137 (*iii*) shows the Thevenin equivalent circuit.

Example 3.60. *Find Thevenin equivalent circuit at terminals ab for the circuit shown in Fig. 3.138.*

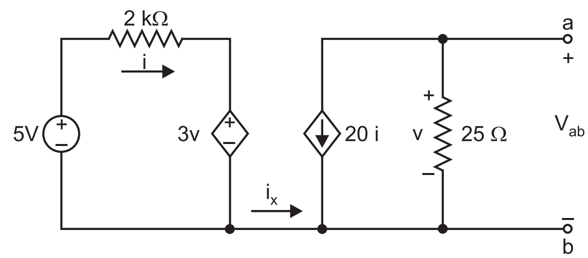


Fig. 3.138

Solution. The current i_x is zero because there is no return path for i_x. The Thevenin voltage v_{Th} will be the voltage across 25Ω resistor.

With $i_x = 0$, $v_{Th} = v = v_{ab} = (-20i)(25) = -500i$

The current i is, $i = \dfrac{5 - 3v}{2 \times 1000} = \dfrac{5 - 3v_{Th}}{2000} \qquad (\because v = v_{Th})$

$$\therefore \quad v_{Th} = -500\left(\frac{5 - 3v_{Th}}{2000}\right) \quad \text{or} \quad v_{Th} = -5\text{V}$$

In order to find Thevenin resistance R_{Th}, we find the short-circuit current i_{sc} at terminals *ab*. Then,

$$R_{Th} = \frac{v_{Th}}{i_{sc}}$$

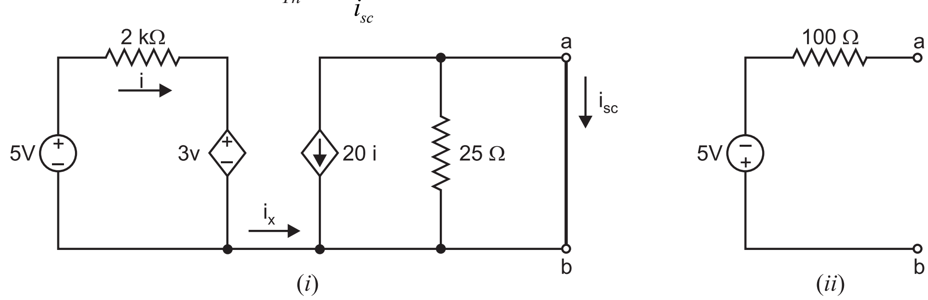


Fig. 3.139

To find i_{sc}, we short circuit the terminals *ab* as shown in Fig. 3.139 (*i*). It is clear that all the current from the dependent current source will pass through the short circuit (∵ 25Ω resistor is shunted by the short circuit).

$$\therefore \quad i_{sc} = -20i$$

Now, $i = \dfrac{5}{2000} = 2.5$ mA so that $i_{sc} = -20 \times 2.5 = -50$ mA

$$\therefore \quad R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{-5}{-50 \times 10^{-3}} = 100\ \Omega$$

Fig. 3.139 (*ii*) shows the **Thevenin equivalent circuit at terminals *ab*.**

Tutorial Problems

1. Using Thevenin's theorem, find the current in 10 Ω resistor in the circuit shown in Fig. 3.140. **[0.481 A]**

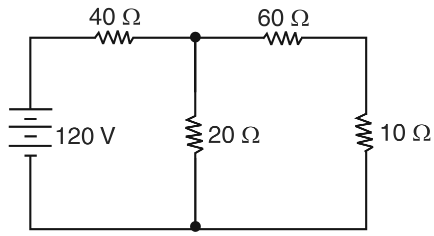


Fig. 3.140

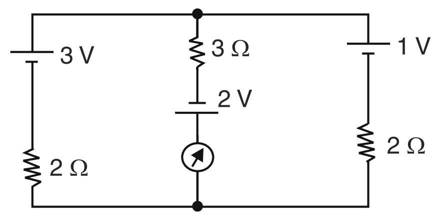


Fig. 3.141

2. Using Thevenin's theorem, find current in the ammeter shown in Fig. 3.141. **[1 A]**
3. Using Thevenin's theorem, find p.d. across branch *AB* of the network shown in Fig. 3.142. **[4.16 V]**

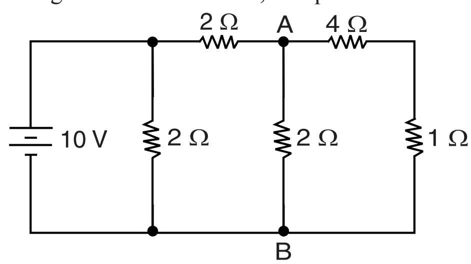


Fig. 3.142

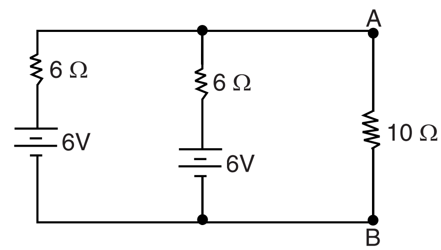


Fig. 3.143

4. Determine Thevenin's equivalent circuit to the left of *AB* in Fig. 3.143.

[A 6 V source in series with 3 Ω]

5. A Wheatstone bridge *ABCD* is arranged as follows : *AB* = 100 Ω, *BC* = 99 Ω, *CD* = 1000 Ω and *DA* = 1000 Ω. A battery of e.m.f. 10 V and negligible resistance is connected between *A* and *C* with *A* positive. A galvanometer of resistance 100 Ω is connected between *B* and *D*. Using Thevenin's theorem, determine the galvanometer current. **[38.6 μA]**
6. Find the Thevenin equivalent circuit of the circuitry, excluding R_1, connected to the terminals $x - y$ in Fig. 3.144. **[10 V in series with 9Ω ; *x* positive w.r.t. *y*]**

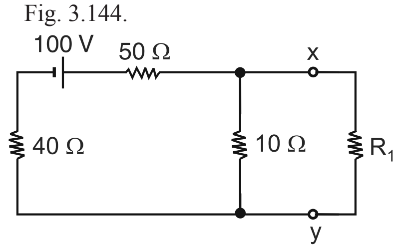


Fig. 3.144

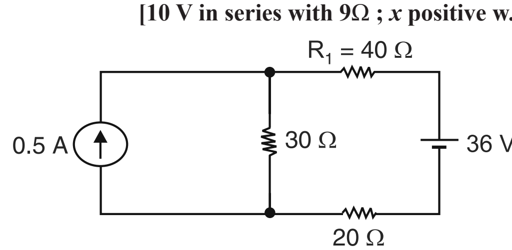


Fig. 3.145

7. Find the voltage across R_1 in Fig. 3.145 by constructing Thevenin equivalent circuit at the R_1 terminals. Be sure to indicate the polarity of the voltage. **[– (9.33V) +]**
8. By using Thevenin Theorem, find current *I* in the circuit shown in Fig. 3.146. **[2·5 A]**

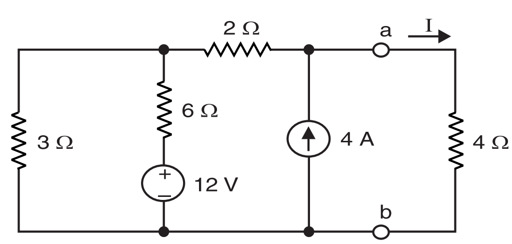


Fig. 3.146

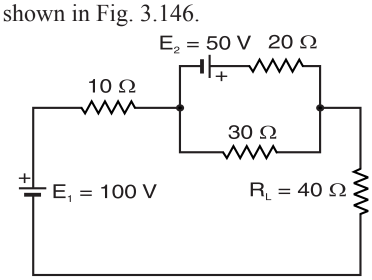


Fig. 3.147

9. Find Thevenin equivalent circuit in Fig. 3.147. **[V_{Th} = 130 V; R_{Th} = 22 Ω]**

10. Find the Thevenin equivalent circuit of the circuitry, excluding R_1, connected to terminals $x - y$ in Fig. 3.148. **[V_{Th} = 23·1 V; R_{Th} = 69 kΩ]**

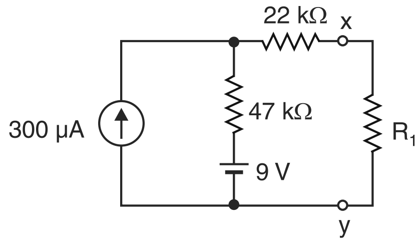


Fig. 3.148

11. Using Thevenin's theorem, find the magnitude and direction of current in 2Ω resistor in the circuit shown in Fig. 3.149. **[0.25A from *D* to *B*]**

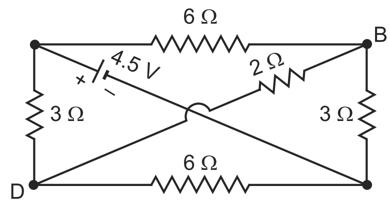


Fig. 3.149

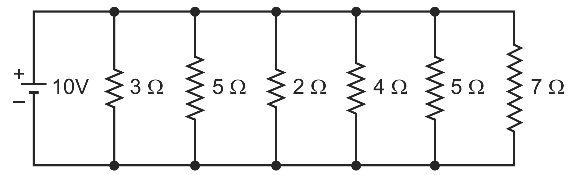


Fig. 3.150

12. Using Thevenin's theorem, find the current flowing and power dissipated in the 7Ω resistance branch in the circuit shown in Fig. 3.150. **[1.43A; 14.3W]**

13. Find Thevenin's equivalent circuit at terminals *BC* of Fig. 3.151. Hence determine current through the resistor *R* = 1Ω. **[V_{Th} = 76/7 V; R_{Th} = 32/7Ω ; 76/39A]**

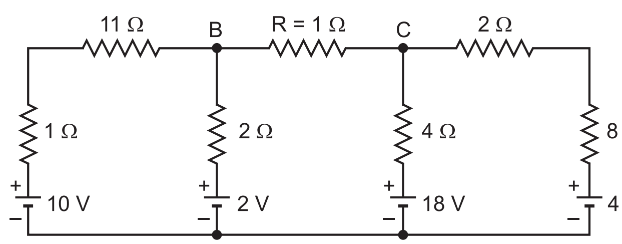
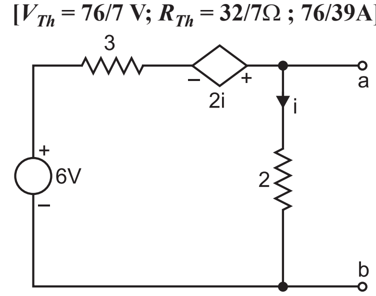


Fig. 3.151



3.152

14. Find the Thevenin equivalent circuit of the network shown in Fig. 3.152. All resistances are in ohms. **[v_{Th} = 4V; R_{Th} = 8Ω]**

15. Replace the circuit (See Fig. 3.153) to the left of terminals *a* – *b* by its Thevenin equivalent and use the result to find *v*. **[v_{Th} = 12V ; R_{Th} = 8Ω ; *v* = 4V]**

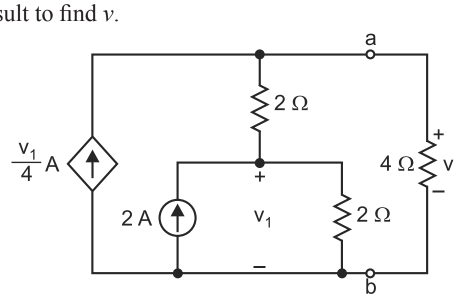


Fig. 3.153

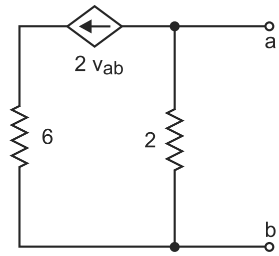


Fig. 3.154

16. Find the Thevenin equivalent circuit for the network shown in Fig. 3.154. All resistances are in ohms. **[v_{Th} = 0V; R_{Th} = 2/5Ω]**

3.13. Advantages of Thevenin's Theorem

The Thevenin equivalent circuit is *always* an equivalent voltage source (V_{Th}) in series with an equivalent resistance (R_{Th}) regardless of the original circuit that it replaces. Although the Thevenin equivalent is not the same as its original circuit, it acts the same in terms of output voltage and current. It is worthwhile to give the advantages of Thevenin's theorem.

(*i*) It reduces a complex circuit to a simple circuit *viz.* a single source of e.m.f. V_{Th} in series with a single resistance R_{Th}.

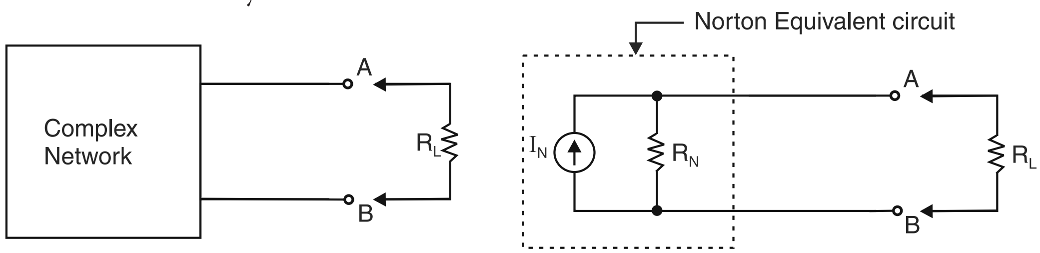
(*ii*) It greatly simplifies the portion of the circuit of lesser interest and enables us to view the action of the output part directly.

(*iii*) This theorem is particularly useful to find current in a particular branch of a network as the resistance of that branch is varied while all other resistances and sources remain constant.

(*iv*) Thevenin's theorem can be applied in successive steps. Any two points in a circuit can be chosen and all the components to one side of these points can be reduced to Thevenin's equivalent circuit.

3.14. Norton's Theorem

Fig. 3.155 (*i*) shows a network enclosed in a box with two terminals *A* and *B* brought out. The network in the box may contain any number of resistors and e.m.f. sources connected in any manner. But according to Norton, the entire circuit behind *AB* can be replaced by a current source I_N in parallel with a resistance R_N as shown in Fig. 3.155 (*ii*). The resistance R_N is the same as Thevenin resistance R_{Th}. The value of I_N is determined as mentioned in Norton's theorem. Once *Norton's equivalent circuit* is determined [See Fig. 3.155 (*ii*)], then current in any load R_L connected across *AB* can be readily obtained.



(*i*) (*ii*)

Fig. 3.155

Hence Norton's theorem as applied to d.c. circuits may be stated as under :

Any linear, bilateral network having two terminals A and B can be replaced by a current source of current output I_N in parallel with a resistance R_N.

(*i*) *The output I_N of the current source is equal to the current that would flow through AB when A and B are short-circuited.*

(*ii*) *The resistance R_N is the resistance of the network measured between A and B with load removed and the sources of e.m.f. replaced by their internal resistances. Ideal voltage sources are replaced with short circuits and ideal current sources are replaced with open circuits.*

Norton's Theorem is *converse* of Thevenin's theorem in that Norton equivalent circuit uses a current generator instead of voltage generator and the resistance R_N (which is the same as R_{Th}) in parallel with the generator instead of being in series with it. *Thus the use of either of these theorems enables us to replace the entire circuit seen at a pair of terminals by an equivalent circuit made up of a single source and a single resistor.*

Illustration. Fig. 3.156 illustrates the application of Norton's theorem. As far as the circuit behind terminals *AB* is concerned [See Fig. 3.156 (*i*)], it can be replaced by a current source I_N in parallel with a resistance R_N as shown in Fig. 3.156 (*iv*). The output I_N of the current generator is equal to the current that would flow through *AB* when terminals *A* and *B* are short-circuited as shown in Fig. 3.156 (*ii*). The load on the source when terminals *AB* are short-circuited is given by ;

$$R' = R_1 + \frac{R_2 R_3}{R_2 + R_3} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3}$$

$$\text{Source current, } I' = \frac{V}{R'} = \frac{V(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Short-circuit current, I_N = Current in R_2 in Fig. 3.156 (*ii*)

$$= I' \times \frac{R_3}{R_2 + R_3} = \frac{V R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

To find R_N, remove the load R_L and replace battery by a short because its internal resistance is assumed zero [See Fig. 3.156 (*iii*)].

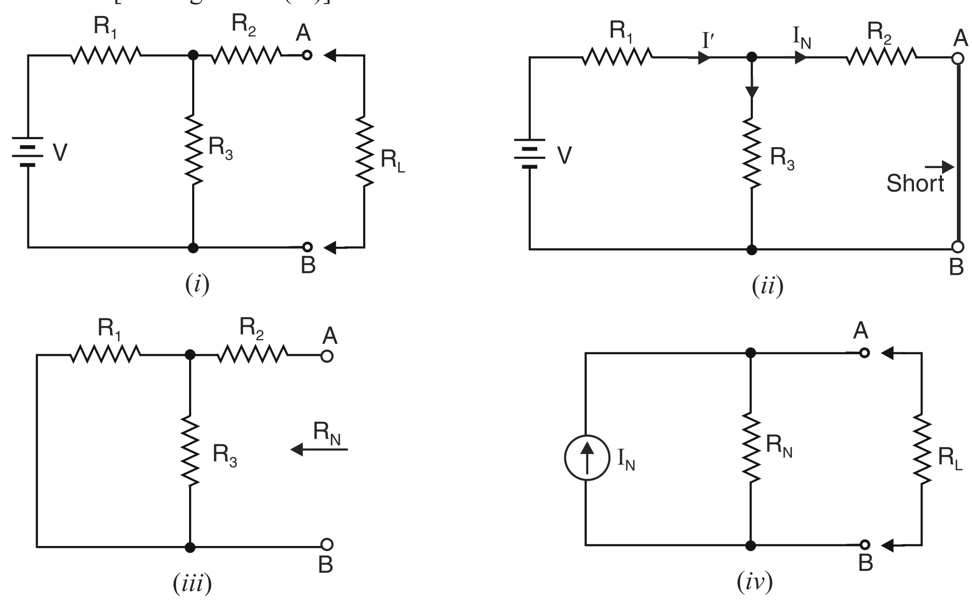


Fig. 3.156

∴ R_N = Resistance at terminals *AB* in Fig. 3.156 (*iii*).

$$= R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

Thus the values of I_N and R_N are known. The Norton equivalent circuit will be as shown in Fig. 3.156 (*iv*).

3.15. Procedure for Finding Norton Equivalent Circuit

(*i*) Open the two terminals (*i.e.* remove any load) between which we want to find Norton equivalent circuit.

(*ii*) Put a short-circuit across the terminals under consideration. Find the short-circuit current flowing in the short circuit. It is called Norton current I_N.

(*iii*) Determine the resistance between the two open terminals with all ideal voltage sources shorted and all ideal current sources opened (a non-ideal source is replaced by its internal resistance). It is called Norton's resistance R_N. It is easy to see that $R_N = R_{Th}$.

(*iv*) Connect I_N and R_N in parallel to produce Norton equivalent circuit between the two terminals under consideration.

(*v*) Place the load resistor removed in step (*i*) across the terminals of the Norton equivalent circuit. The load current can now be calculated by using current-divider rule. This load current will be the same as the load current in the original circuit.

Example 3.61. *Show that when Thevenin's equivalent circuit of a network is converted into Norton's equivalent circuit,* $I_N = V_{Th}/R_{Th}$ *and* $R_N = R_{Th}$. *Here* V_{Th} *and* R_{Th} *are Thevenin voltage and Thevenin resistance respectively.*

Solution. Fig. 3.157 (*i*) shows a network enclosed in a box with two terminals *A* and *B* brought out. Thevenin's equivalent circuit of this network will be as shown in Fig. 3.157 (*ii*). To find Norton's equivalent circuit, we are to find I_N and R_N. Referring to Fig. 3.157 (*ii*),

I_N = Current flowing through short-circuited *AB* in Fig. 3.157 (*ii*)

$= V_{Th}/R_{Th}$

R_N = Resistance at terminals *AB* in Fig. 3.157 (*ii*)

$= R_{Th}$

Fig. 3.157 (*iii*) shows Norton's equivalent circuit. Hence we arrive at the following two important conclusions :

(*i*) To convert Thevenin's equivalent circuit into Norton's equivalent circuit,

$$I_N = V_{Th}/R_{Th} \; ; \; R_N = R_{Th}$$

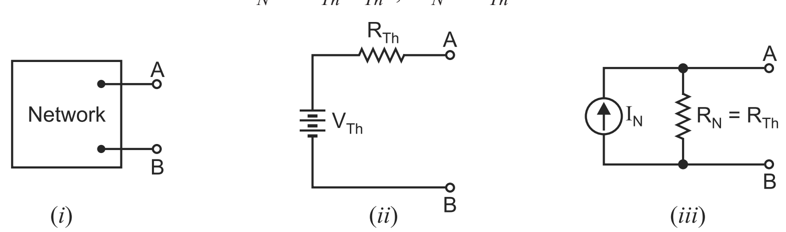


Fig. 3.157

(*ii*) To convert Norton's equivalent circuit into Thevenin's equivalent circuit,

$$V_{Th} = I_N R_N \; ; \; R_{Th} = R_N$$

Example 3.62. *Find the Norton equivalent circuit at terminals x − y in Fig. 3.158.*

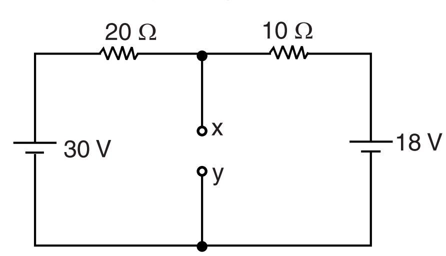


Fig. 3.158

Solution. We shall first find the Thevenin equivalent circuit and then convert it to an equivalent current source. This will then be Norton equivalent circuit.

Finding Thevenin equivalent circuit. To find V_{Th}, refer to Fig. 3.159 (*i*). Since 30 V and 18 V sources are in opposition, the circuit current *I* is given by ;

$$I = \frac{30-18}{20+10} = \frac{12}{30} = 0.4 \text{ A}$$

Applying Kirchhoff's voltage law to loop *ABCDA*, we have,

$$30 - 20 \times 0.4 - V_{Th} = 0 \quad \therefore \quad V_{Th} = 30 - 8 = 22 \text{ V}$$

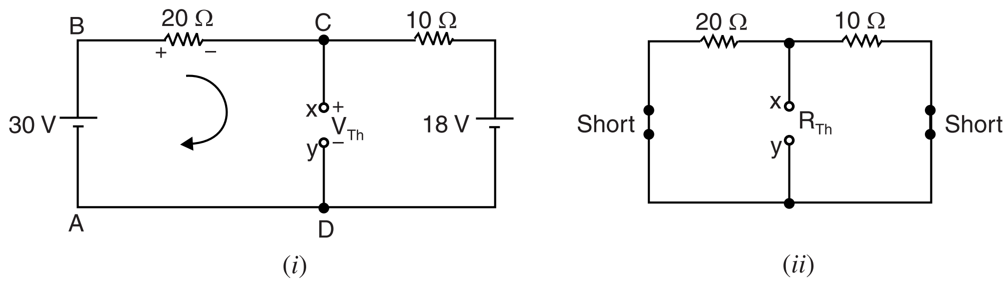


Fig. 3.159

To find R_{Th}, we short both voltage sources as shown in Fig. 3.159 (*ii*). Notice that 10 Ω and 20 Ω resistors are then in parallel.

$$\therefore \qquad R_{Th} = 10\ \Omega \parallel 20\ \Omega = \frac{10 \times 20}{10 + 20} = 6.67\ \Omega$$

Therefore, Thevenin equivalent circuit will be as shown in Fig. 3.160 (*i*). Now it is quite easy to convert it into equivalent current source.

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{22}{6.67} = 3.3\text{A} \qquad \text{[See Fig. 3.160 } (ii)\text{]}$$

$$R_N = R_{Th} = 6.67\ \Omega$$

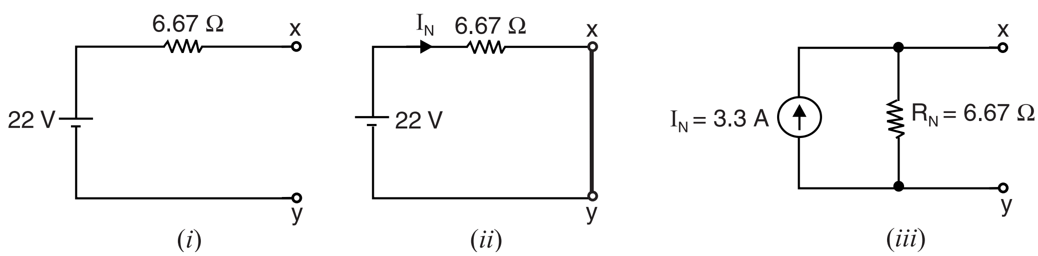


Fig. 3.160

Fig. 3.160 (*iii*) shows **Norton equivalent circuit.** Observe that the Norton equivalent resistance has the same value as the Thevenin equivalent resistance. Therefore, R_N is found exactly the same way as R_{Th}.

Example 3.63. *Using Norton's theorem, calculate the current in the 5 Ω resistor in the circuit shown in Fig. 3.161.*

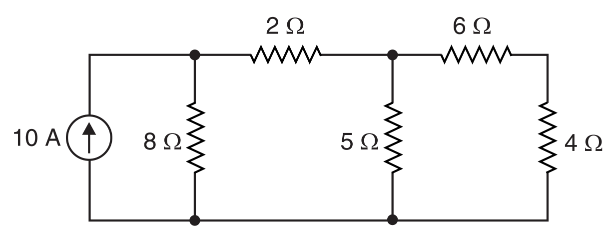


Fig. 3.161

Solution. Short the branch that contains 5 Ω resistor in Fig. 3.161. The circuit then becomes as shown in Fig. 3.162 (*i*). Referring to Fig. 3.162 (*i*), the 6 Ω and 4 Ω resistors are in series and this series combination is in parallel with the short. Therefore, these resistors have no effect on Norton current and may be considered as removed from the circuit. As a result, 10 A divides between parallel resistors of 8 Ω and 2 Ω.

$$\therefore \qquad \text{Norton current, } I_N = \text{Current in 2 Ω resistor}$$

$$= 10 \times \frac{8}{8+2} = 8\text{ A} \qquad \text{... Current-divider rule}$$

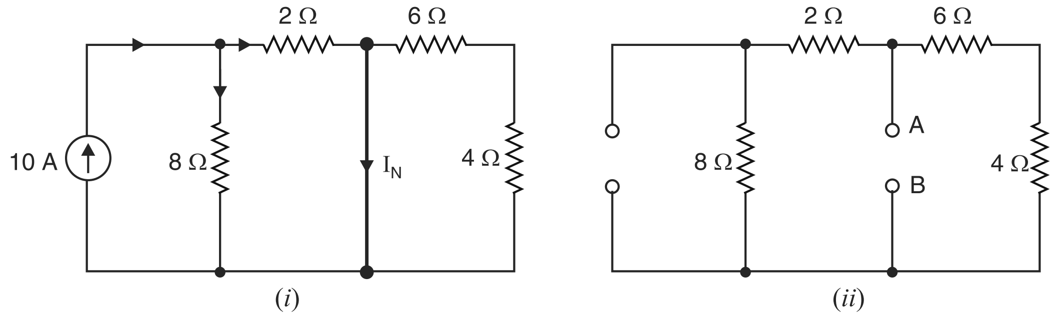


Fig. 3.162

In order to find Norton resistance $R_N (= R_{Th})$, open circuit the branch containing the 5 Ω resistor and replace the current source by an open in Fig. 3.161. The circuit then becomes as shown in Fig. 3.162 (*ii*).

Norton resistance, R_N = Resistance at terminals *AB* in Fig. 3.162 (*ii*).

$$= (2 + 8) \parallel (4 + 6) = 10 \parallel 10 = \frac{10 \times 10}{10 + 10} = 5\ \Omega$$

Therefore, Norton equivalent circuit consists of a current source of 8 A (= I_N) in parallel with a resistance of 5 Ω (= R_N) as shown in Fig. 3.163 (*i*). When the branch containing 5 Ω resistor is connected across the output terminals of Norton's equivalent circuit, the circuit becomes as shown in Fig. 3.163 (*ii*).

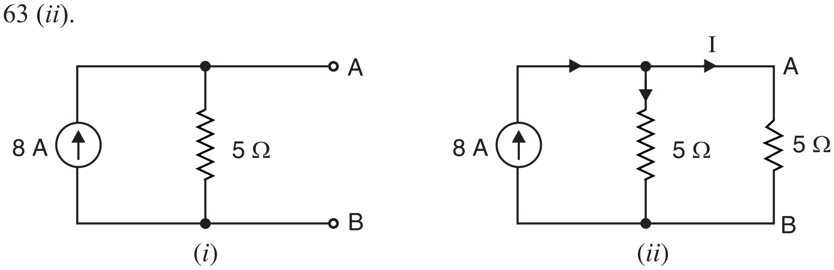


Fig. 3.163

By current-divider rule, the current *I* in 5 Ω resistor is

$$I = 8 \times \frac{5}{5 + 5} = \mathbf{4\ A}$$

Example 3.64. *Find Norton equivalent circuit for Fig. 3.164 (i). Also solve for load current and load voltage.*

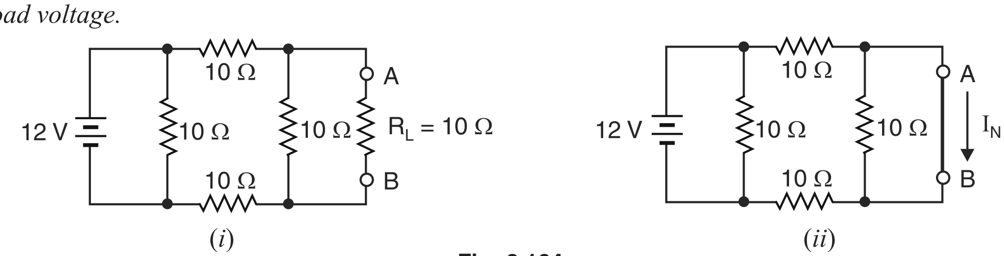


Fig. 3.164

Solution. Short the branch that contains R_L (= 10 Ω) in Fig. 3.164 (*i*). The circuit then becomes as shown in Fig. 3.164 (*ii*). The resistor that is in parallel with the battery has no effect on the Norton current (I_N). The resistor in parallel with the short also has no effect. Therefore, these resistors may be considered as removed from the circuit shown in Fig. 3.164 (*ii*). The circuit then contains two 10 Ω resistors in series.

$\therefore$ Norton current, $I_N = \frac{12}{10+10} = 0 \cdot 6\ A$

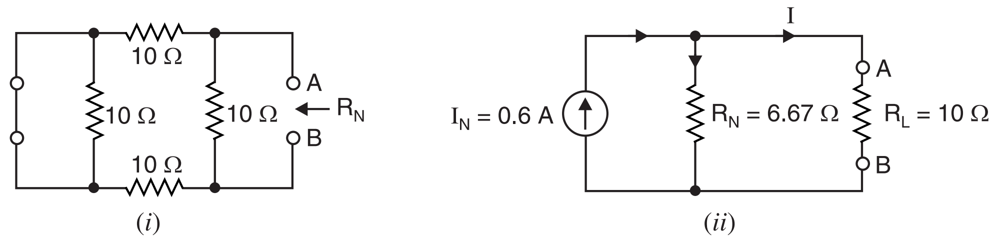


Fig. 3.165

In order to find Norton resistance R_N (= R_{Th}), open circuit the branch containing R_L and replace the voltage source by a short ($\because$ internal resistance of the voltage source is assumed zero) in Fig. 3.164 (*i*). The circuit then becomes as shown in Fig. 3.165 (*i*).

Norton resistance, $*R_N$ = Resistance at terminals *AB* in Fig. 3.165 (*i*)

$$= (10 + 10) \parallel 10 = \frac{20 \times 10}{20+10} = 6 \cdot 67\ \Omega$$

Therefore, Norton equivalent circuit consists of a **current source of 0·6 A (= I_N) in parallel with a resistance of 6·67 Ω (=R_N).** When the branch containing R_L (= 10 Ω) is connected across the output terminals of Norton equivalent circuit, the circuit becomes as shown in Fig. 3.165 (*ii*).

By current-divider rule, the current *I* in R_L is

$$I = 0.6 \times \frac{6.67}{6.67+10} = \mathbf{0 \cdot 24\ A}$$

Voltage across R_L = $I R_L = 0 \cdot 24 \times 10 = \mathbf{2 \cdot 4\ V}$

Example 3.65. *Find the Norton current for the unbalanced Wheatstone bridge shown in Fig. 3.166.*

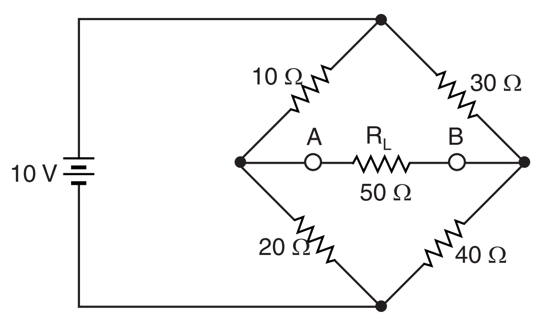


Fig. 3.166

Solution. The Norton current is found by shorting the load terminals as shown in Fig. 3.167 (*i*). This situation is more complicated than finding the Thevenin voltage. Here is an easy way to find I_N in the circuit of Fig. 3.167 (*i*). First determine the total current and then use Ohm's law to find current in the four resistors. Once the currents in the four resistors are known, Kirchhoff's current law can be used to determine Norton current I_N.

* The resistor 10 Ω that is in parallel with short is ineffective and may be considered as removed from the circuit of Fig. 3.165 (*i*). Therefore, two 10 Ω resistors are in series and this series combination is in parallel with 10 Ω resistor.

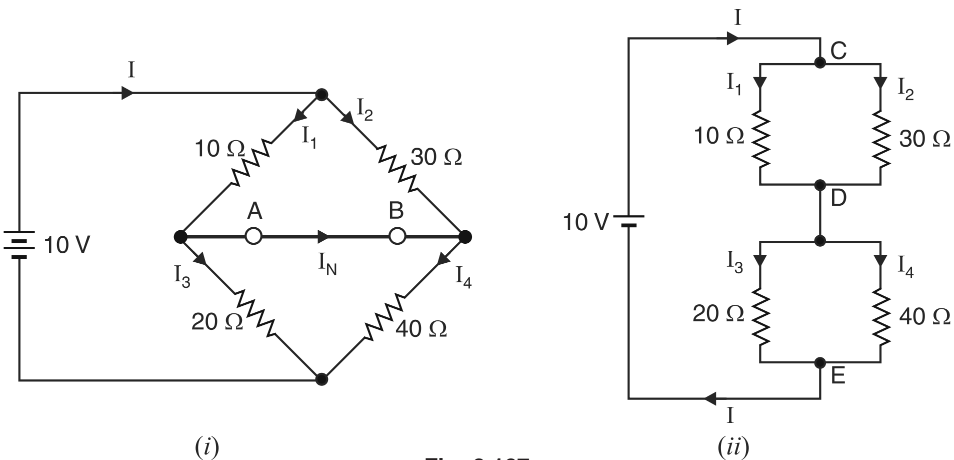


Fig. 3.167

Fig. 3.167 (*ii*) shows the equivalent circuit of Fig. 3.167 (*i*). The total circuit resistance R_T to 10 V source is

$$R_T = \frac{10 \times 30}{10 + 30} + \frac{20 \times 40}{20 + 40} = 7{\cdot}5 + 13{\cdot}33 = 20{\cdot}83\ \Omega$$

$$\text{Total circuit current, } I = \frac{10}{20.83} = 0{\cdot}48 \text{ A}$$

Referring to Fig. 3.167 (*ii*), we have,

$$V_{CD} = I \times R_{CD} = 0{\cdot}48 \times 7{\cdot}5 = 3{\cdot}6 \text{ V}$$

$$V_{DE} = I \times R_{DE} = 0{\cdot}48 \times 13{\cdot}33 = 6{\cdot}4 \text{ V}$$

$$\therefore \quad I_1 = \frac{V_{CD}}{10} = \frac{3.6}{10} = 0{\cdot}36 \text{ A} \;; \quad I_2 = \frac{V_{CD}}{30} = \frac{3.6}{30} = 0{\cdot}12 \text{ A}$$

$$I_3 = \frac{V_{DE}}{20} = \frac{6.4}{20} = 0{\cdot}32 \text{ A} \;; \quad I_4 = \frac{V_{DE}}{40} = \frac{6.4}{40} = 0{\cdot}16 \text{ A}$$

Referring to Fig. 3.167 (*i*), it is now clear that I_1(= 0·36 A) is greater than I_3(= 0·32 A). Therefore, current I_N will flow from *A* to *B* and its value is

$$I_N = I_1 - I_3 = 0{\cdot}36 - 0{\cdot}32 = \mathbf{0{\cdot}04\ A}$$

Example 3.66. *Two batteries, each of e.m.f. 12 V, are connected in parallel to supply a resistive load of 0·5 Ω. The internal resistances of the batteries are 0·12 Ω and 0·08 Ω. Calculate the current in the load and the current supplied by each battery.*

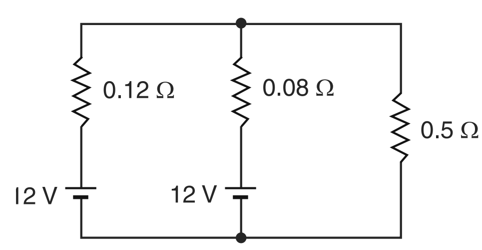
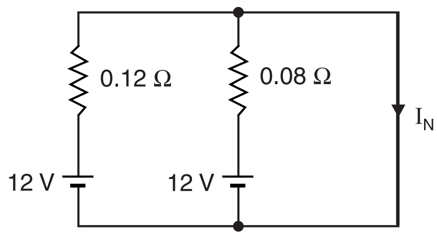


Fig. 3.168

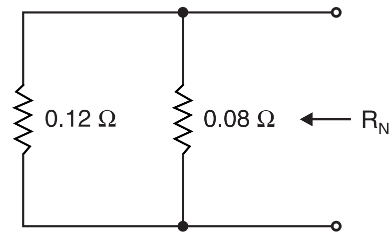
Solution. Fig. 3.168 shows the conditions of the problem. If a short circuit is placed across the load, the circuit becomes as shown in Fig. 3.169 (*i*). The total short circuit current is given by ;

$$I_N = \frac{12}{0.12} + \frac{12}{0.08}$$

$$= 100 + 150 = 250 \text{ A}$$



(*i*)



(*ii*)

Fig. 3.169

In order to find Norton resistance $R_N (= R_{Th})$, open circuit the load and replace the batteries by their internal resistances. The circuit then becomes as shown in Fig. 3.169 (*ii*). Then resistance looking into the open-circuited terminals is the Norton resistance.

∴ Norton resistance, R_N = Resistance looking into the open-circuited load terminals in Fig. 3.169 (*ii*)

$$= 0{\cdot}12 \parallel 0{\cdot}08 = \frac{0.12 \times 0.08}{0.12 + 0.08} = 0{\cdot}048\ \Omega$$

Therefore, Norton equivalent circuit consists of a current source of 250 A (= I_N) in parallel with a resistance of 0·048 Ω (= R_N). When load (= 0·5 Ω) is connected across the output terminals of Norton equivalent circuit, the circuit becomes as shown in Fig. 3.170. By current-divider rule, the current *I* in load (= 0·5 Ω) is given by ;

$$I = 250 \times \frac{0.048}{0.048 + 0.5} = \mathbf{21{\cdot}9\ A}$$

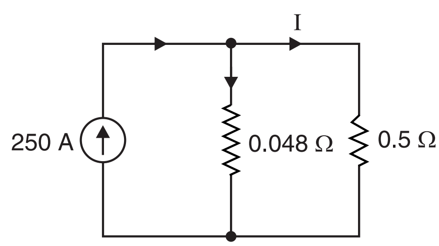


Fig. 3.170

$$\text{Battery terminal voltage} = I R_L = 21{\cdot}9 \times 0{\cdot}5$$

$$= 10{\cdot}95\ \text{V}$$

$$\text{Current in first battery} = \frac{12 - 10.95}{0.12} = \mathbf{8{\cdot}8\ A}$$

$$\text{Current in second battery} = \frac{12 - 10.95}{0.08} = \mathbf{13{\cdot}1\ A}$$

Example 3.67. *Represent the network shown in Fig. 3.171 between the terminals A and B by one source of current I_N and internal resistance R_N. Hence calculate the current that would flow in a 6 Ω resistor connected across AB.*

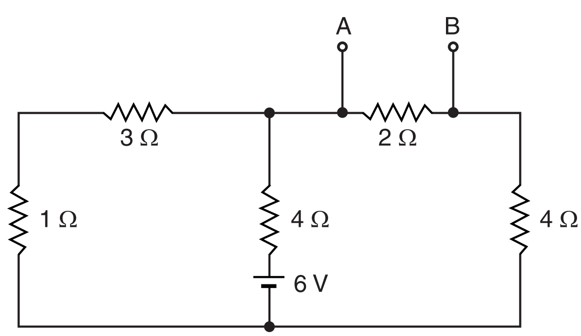


Fig. 3.171

Solution. Place short circuit across *AB* in Fig. 3.171. Then the circuit becomes as shown in Fig. 3.172 (*i*). Note that 2 Ω resistor is shorted and may be considered as removed from the circuit. The total resistance R_T presented to the 6 V source is a parallel combination of (3 + 1) Ω and 4 Ω in series with 4 Ω. Therefore, the value of R_T is given by ;

$$R_T = [(3+1) \parallel 4] + 4 = \frac{4 \times 4}{4 + 4} + 4 = 2 + 4 = 6\ \Omega$$

∴ Current supplied by 6 V source, $I = 6/6 = 1$ A

At node D, 1 A current divides between two parallel resistors of (3 + 1) Ω and 4 Ω.

∴ Norton current, $I_N = 1 \times \frac{4}{4+4} = 0{\cdot}5$ A

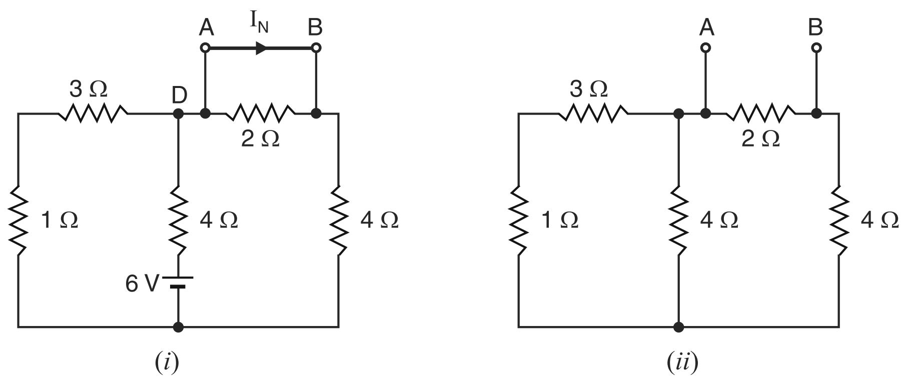


Fig. 3.172

Now Norton resistance $R_N (= R_{Th})$ is the resistance between open-circuited terminals AB with voltage source replaced by a short as shown in Fig. 3.172 (*ii*). Referring to Fig. 3.172 (*ii*), (3 + 1) Ω resistance is in parallel with 4 Ω, giving equivalent resistance of 2 Ω. Now (2 + 4) Ω resistance is in parallel with 2 Ω.

∴ $R_N = (2 + 4) \parallel 2 = 6 \parallel 2$

$= \frac{6 \times 2}{6+2} = \frac{12}{8} = 1{\cdot}5\ \Omega$

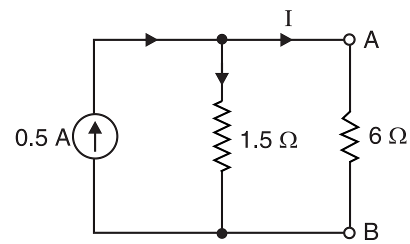


Fig. 3.173

Therefore, Norton equivalent circuit is a **current source of 0·5 A in parallel with resistance of 1·5 Ω.** When a 6 Ω resistor is connected across AB, the circuit becomes as shown in Fig. 3.173. By current-divider rule, current in 6 Ω,

$$I = 0.5 \times \frac{1.5}{1.5+6} = 0.5 \times \frac{1.5}{7.5} = \mathbf{0{\cdot}1\ A}$$

Example 3.68. *For the circuit shown in Fig. 3.174, calculate the potential difference between the points O and N and what current would flow in a 50 Ω resistor connected between these points?*

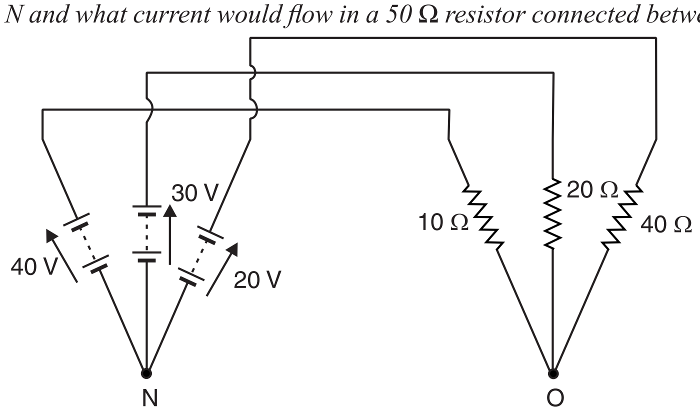


Fig. 3.174

Solution. Place a short circuit across *ON* in Fig. 3.174. Then total short circuit current in *ON* is

$$I_N = \frac{40}{10} + \frac{30}{20} + \frac{20}{40} = 4 + 1{\cdot}5 + 0{\cdot}5 = 6 \text{ A}$$

In order to find $R_N (= R_{Th})$, replace the voltage sources by short. Then R_N is equal to the resistance looking into open circuited terminals *ON*. It is easy to see that the resistors 10 Ω, 20 Ω and 40 Ω are in parallel across *ON*.

$$\therefore \quad \frac{1}{R_N} = \frac{1}{10} + \frac{1}{20} + \frac{1}{40} = \frac{7}{40}$$

or $$R_N = \frac{40}{7} = 5{\cdot}71\ \Omega$$

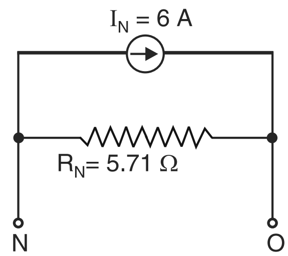


Fig. 3.175

Therefore, the original circuit reduces to that shown in Fig. 3.175.

∴ Open-circuited voltage across $ON = I_N R_N = 6 \times 5{\cdot}71 =$ **34·26 V**

When 50 Ω resistor is connected between points *O* and *N*,

Current in 50 Ω connected between O*N* = $6 \times \frac{5.71}{5.71 + 50}$ = **0·62 A**

Example 3.69. *Find Norton equivalent circuit to the left of terminals AB in the circuit shown in Fig. 3.176. The current sources are I_1 = 10 A and I_2 = 15 A. The conductances are G_1 = 0·2 S, G_2 = 0·3 S and G_3 is variable.*

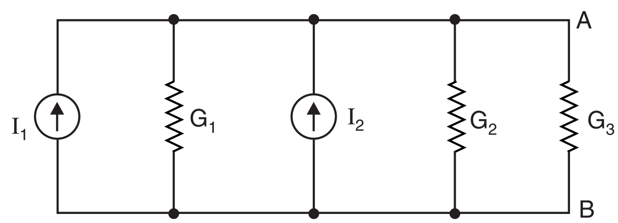


Fig. 3.176

Solution. First, disconnect branch G_3 and short circuit the terminals *AB* as shown in Fig. 3.177 (*i*). Since the short circuit has infinite conductance, the total current of 25 A (= $I_1 + I_2$) supplied by the two sources would pass through the short-circuited terminals *i.e.*

$$\text{Norton current, } I_N = I_1 + I_2 = 10 + 15 = 25 \text{ A}$$

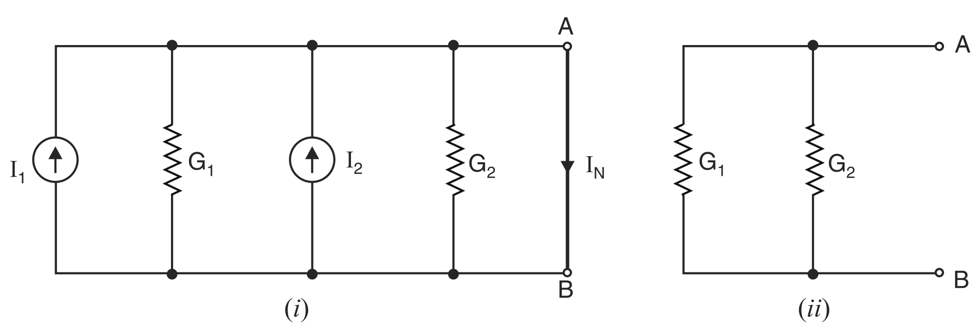


Fig. 3.177

Next, remove the short-circuit and replace the current sources by open. The circuit then becomes as shown in Fig. 3.177 (*ii*).

Norton conductance, G_N = Conductance at terminals *AB* in Fig. 3.177 (*ii*).

$= G_1 + G_2 = 0{\cdot}2 + 0{\cdot}3 = 0{\cdot}5$ S

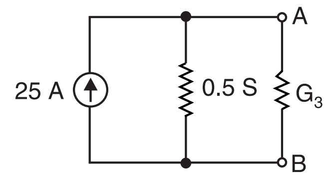


Fig. 3.178

Therefore, Norton equivalent circuit consists of a **25 A current source in parallel with a conductance of 0·5 S.** When conductance G_3 is connected across terminals *AB*, the circuit becomes as shown in Fig. 3·178. Although Norton equivalent circuit is not the same as its original circuit, it acts the same in terms of output voltage and current.

Example 3.70. *The circuit shown in Fig. 3.179 consists of a current source I = 10 A paralleled by G = 0·1 S and a voltage source E = 200 V with 10 Ω series resistance. Find Norton equivalent circuit to the left of terminals AB.*

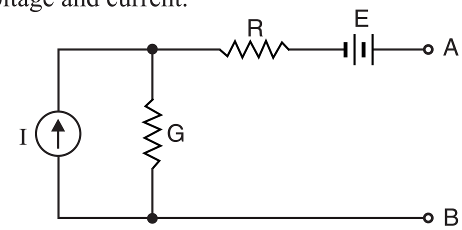


Fig. 3.179

Solution. We are to find Norton current and Norton resistance. In order to find Norton current I_N, short-circuit the terminals *AB* as shown in Fig. 3.180 (*i*). Then current that flows in *AB* is I_N. It is easy to see that current which flows in conductance *G* is *I_G = 5 A (upward).

∴ Norton current, $I_N = I + I_G = 10 + 5 = 15$ A

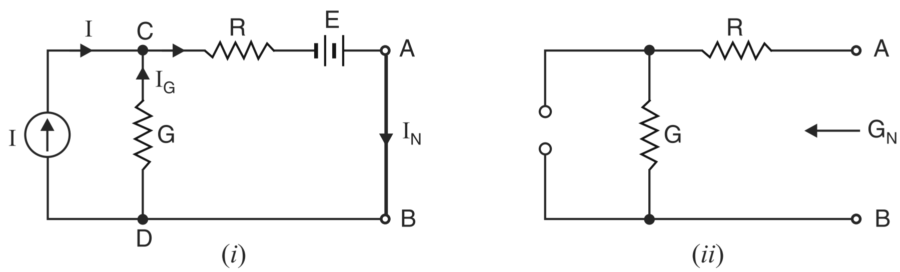


Fig. 3.180

In order to find Norton resistance, remove the short circuit and replace the voltage source by a short and current source by an open. The circuit then becomes as shown in Fig. 3.180 (*ii*).

R_N = Resistance looking into terminals *AB* in Fig. 3.180 (*ii*).

$$= R + \frac{1}{G} = 10 + \frac{1}{0.1} = 10 + 10 = 20\ \Omega$$

∴ Norton conductance, $G_N = \dfrac{1}{R_N} = \dfrac{1}{20} = 0{\cdot}05$ S

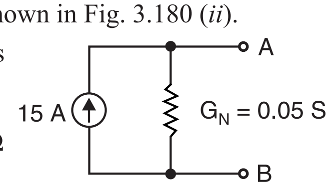


Fig. 3.181

Therefore, Norton equivalent circuit consists of a **15 A current source paralleled with 0·05 S conductance G_N** as shown in Fig. 3·181.

---

* Applying *KVL* to loop *CABDC*, we have,

$$-(I + I_G)\,R + 200 - \frac{I_G}{G} = 0$$

or $-(10 + I_G)\,10 + 200 - \dfrac{I_G}{0.1} = 0$

or $-100 - 10I_G + 200 - 10\,I_G = 0$

∴ $I_G = 100/20 = 5$ A

Example 3.71. *Draw Norton's equivalent circuit at terminals AB and determine the current flowing through 12Ω resistor for the network shown in Fig. 3.182 (i).*

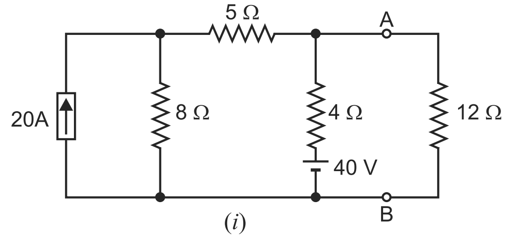


Fig. 3.182

Solution. In order to find Norton current I_N, short circuit terminals *A* and *B* after removing the load (= 12Ω). The circuit then becomes as shown in Fig. 3.182 (*ii*). The current flowing in the short circuit is the Norton current I_N. It can be found by using superposition theorem.

(*i*) **When current source is acting alone.** In this case, we short circuit the voltage source so that only current source acts in the circuit. The circuit then becomes as shown in Fig. 3.183 (*i*). It is clear that :

$$\text{Norton current, } I_{N1} = \text{*Current in } 5\Omega \text{ resistor}$$

$$= 20 \times \frac{8}{8+5} = \frac{160}{13}\text{A}$$

(*ii*) **When voltage source is acting alone.** In this case, we open circuit the current source so that only voltage source acts in the circuit. The circuit then becomes as shown in Fig. 3.183 (*ii*). It is clear that :

$$\text{Norton current, } I_{N2} = \frac{40}{4} = 10\text{A}$$

Therefore, when both voltage and current sources are present in the circuit, we have,

$$\text{Norton current, } I_N = I_{N1} + I_{N2} = \frac{160}{13} + 10 = \frac{290}{13}\text{A}$$

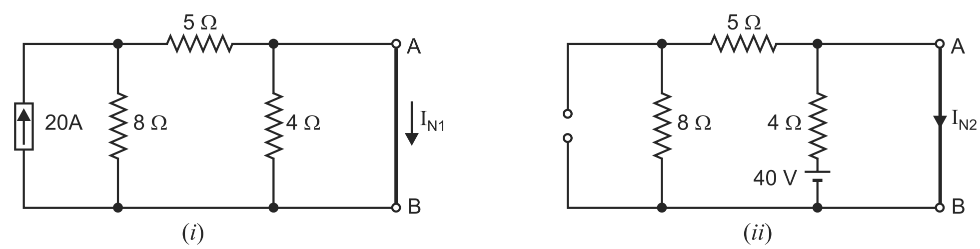


Fig. 3.183

In order to find R_N, open circuit 12Ω resistor and replace current source by open circuit and voltage source by short circuit. Then circuit becomes as shown in Fig. 3.184 (*i*).

$$\therefore \quad R_N = \text{Resistance at terminals } AB \text{ in Fig. 3.184 } (i)$$

$$= 4 \,||\, (5+8) = 4 \,||\, 13 = \frac{4 \times 13}{4+13} = \frac{52}{17}\Omega$$

* No current flows in 4Ω resistor because it is short circuited at terminals *A* and *B*. Therefore, 20A divides between 8Ω and 5Ω connected in parallel.

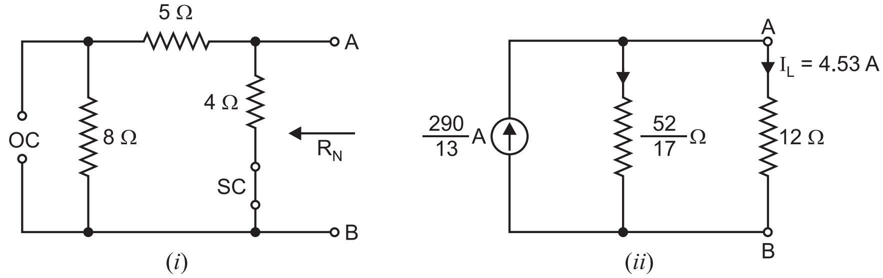


Fig. 3.184

Thus Norton equivalent circuit at terminals *AB* is a current source of current 290/13 A in parallel with 52/17Ω resistance. When load resistor of 12Ω is connected across Norton's equivalent circuit, the circuit becomes as shown in Fig. 3.184 (*ii*).

∴ Load current, $I_L = I_N \times \dfrac{R_N}{R_N + R_L} = \dfrac{290}{13} \times \dfrac{52/17}{52/17 + 12}$ = **4.53 A**

Example 3.72. *Determine the values of I and R in the circuit shown in Fig. 3.185.*

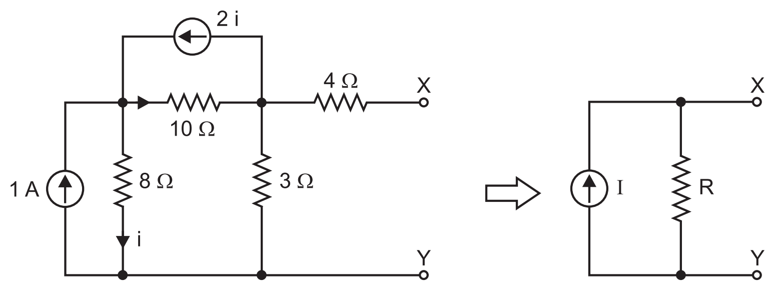


Fig. 3.185

Solution. Short the terminals *XY* in Fig. 3.185 and we get the circuit shown in Fig. 3.186 (*i*). The currents in the various branches will be as shown. In order to find the short-circuit current I_{sc} (= $I = I_N$), we apply *KVL* to loops 1 and 2 in Fig. 3.186 (*i*).

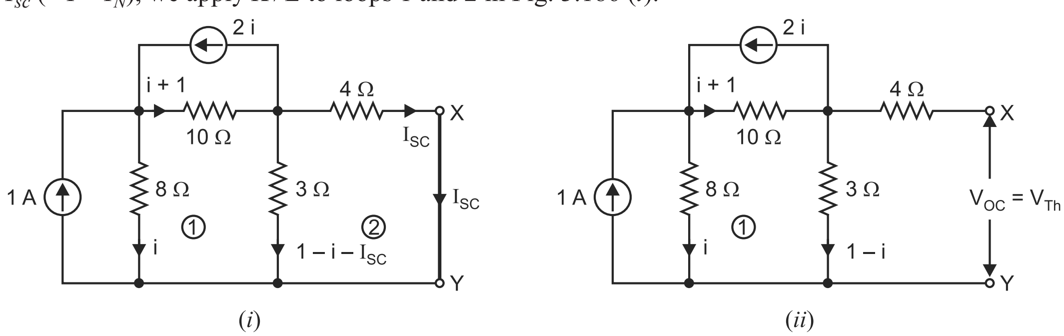


Fig. 3.186

Loop 1. $-10\,(i + 1) - 3(1 - i - I_{sc}) + 8i = 0$

or $i + 3I_{sc} = 13$ ...(*i*)

Loop 2. $-4I_{sc} + 3(1 - i - I_{sc}) = 0$

or $3i + 7I_{sc} = 3$ ...(*ii*)

From eqs. (*i*) and (*ii*), we have, I_{sc} = 18A.

In order to find the open-circuited voltage V_{oc} (= V_{Th}) at terminals *X* and *Y*, refer to Fig 3.186 (*ii*). The various branch currents are shown. Applying *KVL* to loop 1 in Fig. 3.186 (*ii*), we have,

$$-10(i + 1) - 3(1 - i) + 8i = 0 \quad \text{or} \quad i = 13\text{A}$$

$$\therefore \quad V_{oc} = \text{Voltage across } 3\Omega \text{ resistor}$$

$$= 3(1 - i) = 3(1 - 13) = -36 \text{ V}$$

$$\text{Thevenin resistance, } R(= R_N) = \frac{V_{oc}}{I_{sc}} = \frac{36}{18} = \mathbf{2\Omega}$$

$$\text{Current } I = I_N = \mathbf{-18A}$$

Note the polarity of current source I (= I_N).

Example 3.73. *With the help of Norton's theorem, find V_o in the circuit shown in Fig. 3.187 (i). All resistances are in ohms.*

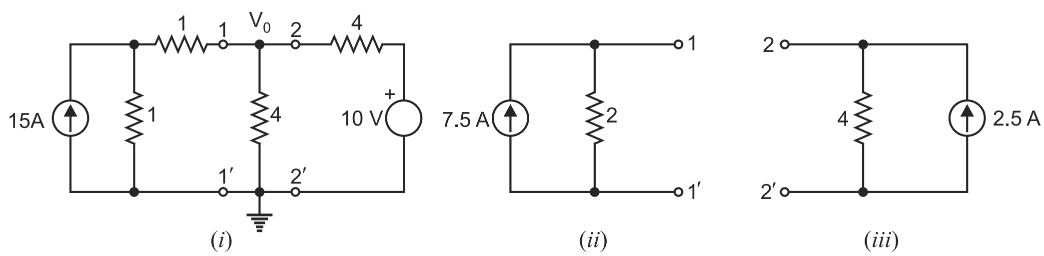


Fig. 3.187

Solution. In order to find V_o, it is profitable to find Norton equivalent circuit to the left of terminals 1 – 1′ and to the right of terminals 2 – 2′ in Fig. 3.187 (*i*). To the left of terminals 1 – 1′, $V_{oc} = 15 \times 1 = 15$ V and $R_N = 1 + 1 = 2\Omega$ so that $I_N = 15/2 = 7.5$A as shown in Fig. 3.187 (*ii*). To the right of terminals 2 – 2′, $V_{oc} = 10$ V and $R_{Th} = R_N = 4\Omega$ so that $I_N = 10/4 = 2.5$A as shown in Fig. 3.187 (*iii*). The two Norton equivalent circuits are put back at terminals 1 – 1′, and 2 – 2′ as shown in Fig. 3.187 (*iv*).

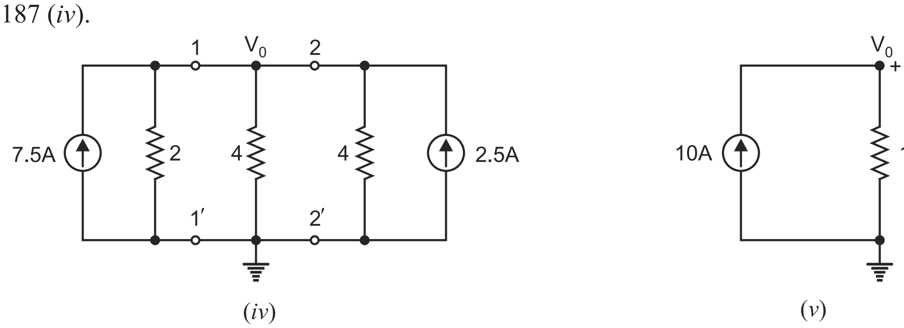


Fig. 3.187

In Fig. 3.187 (*iv*), the two current sources, being parallel and carrying currents in the same direction, can be combined into a single current source of 7.5 + 2.5 = 10A. The three resistances are in parallel and can be combined to give a single resistance = 2Ω || 4Ω || 4Ω = 1Ω. Therefore, the circuit of Fig. 3.187 (*iv*) reduces to the circuit shown in Fig. 3.187 (*v*).

$$\therefore \quad V_o = 10\text{A} \times 1\Omega = \mathbf{10V}$$

Example 3.74. *Find current in the 4 ohm resistor by any three methods for the circuit shown in Fig. 3.188(i).*

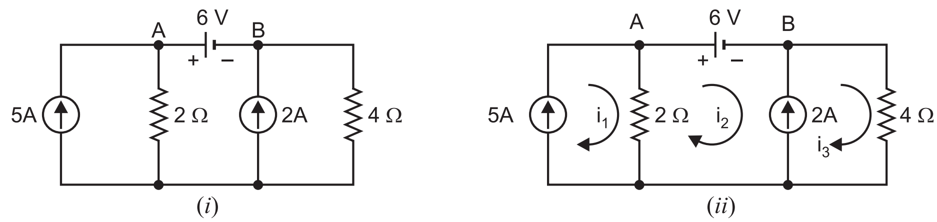


Fig. 3.188

Solution. Method 1. We shall find current in 4Ω resistor by **mesh current method.** Mark three mesh currents i_1, i_2 and i_3 in the three loops as shown in Fig. 3.188 (*ii*). The describing circuit equations are :

$$i_1 = 5\text{A due to the current source of 5A}$$

$$V_A - V_B = 6\text{V due to voltage source of 6V}$$

$$i_3 - i_2 = 2\text{A due to current source of 2A}$$

$$V_A = (i_1 - i_2)2 \;;\; V_B = i_3 \times 4$$

Now, $-6 - 4i_3 - 2(i_2 - i_1) = 0$ ... Applying *KVL*

or $-6 - 4(2 + i_2) - 2(i_2 - 5) = 0$

or $-6i_2 = 4$

$\therefore$ $i_2 = -\frac{4}{6} = -\frac{2}{3}\text{A}$ and $i_3 = i_2 + 2 = -\frac{2}{3} + 2 = \frac{4}{3}\text{A}$

$\therefore$ Current in 4Ω resistor $= i_3 = \mathbf{\frac{4}{3}A}$

Method 2. We now find current in 4Ω resistor by **Thevenin's theorem.** Remove 4Ω resistor (*i.e.* load) and the circuit becomes as shown in Fig. 3.188 (*iii*).

Current in 2Ω resistor = 5 + 2 = 7A

It is because 6V source is ineffective in producing any current.

In going from point *X* to point *Y via B* and *A*, we have,

$$V_X + 6 - 7 \times 2 = V_Y$$

or $V_X - V_Y = 7 \times 2 - 6 = 8\text{V}$

$\therefore$ $V_{Th} = V_{XY} = V_X - V_Y = 8\text{V}$

In order to find R_{Th}, short circuit the voltage source and open-circuit the current sources in Fig. 3.188 (*iii*). Then circuit becomes as shown in Fig. 3.188 (*iv*). The resistance at the open-circuited terminals *XY* in Fig. 3.188 (*iv*) is R_{Th}.

$\therefore$ $R_{Th} = 2\Omega$

$\therefore$ Current in 4Ω resistor $= \frac{V_{Th}}{R_{Th} + 4} = \frac{8}{2 + 4} = \mathbf{\frac{4}{3}A}$

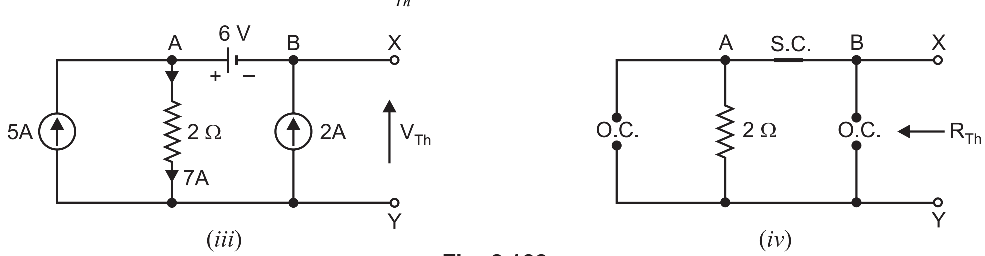


Fig. 3.188

Method 3. Finally, we find current in 4Ω resistor by **Norton's theorem.** To find I_N, short-circuit 4Ω resistor in Fig. 3.188 (*i*). The circuit then becomes as shown in Fig. 3.188 (*v*). The current distribution in the various branches will be as shown.

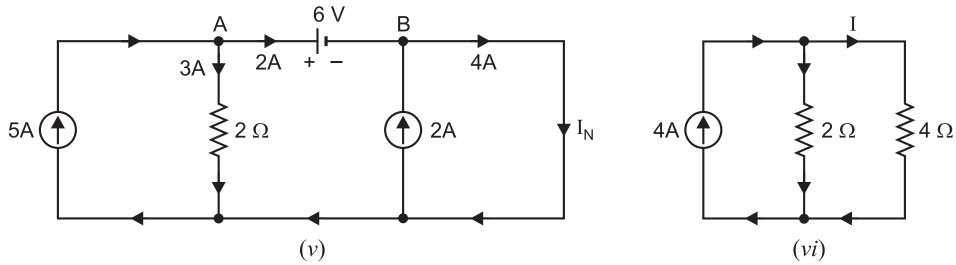


Fig. 3.188

It is clear from Fig. 3.188 (*v*) that :

$$I_N = 2 + 2 = 4\text{A}$$

$$R_N = R_{Th} = 2\Omega \qquad \text{...as calculated above}$$

When 4Ω resistor is connected to Norton equivalent circuit, it becomes as shown in Fig. 3.188 (*vi*).

∴ Current in 4Ω resistor is given by (current-divider rule) ;

$$I = 4 \times \frac{2}{2+4} = \frac{8}{6} = \mathbf{\frac{4}{3}\,A}$$

Example 3.75. *Using Norton's theorem, find current through 1Ω resistor in Fig. 3.189 (i). All resistances are in ohms,*

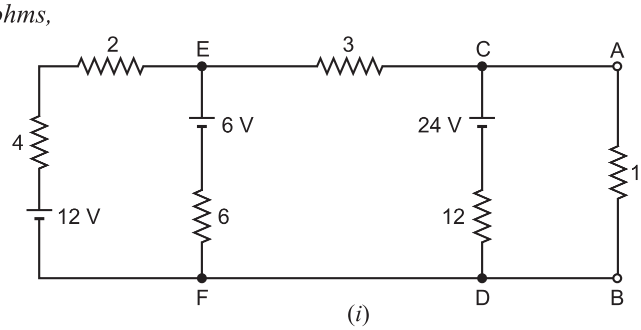


Fig. 3.189

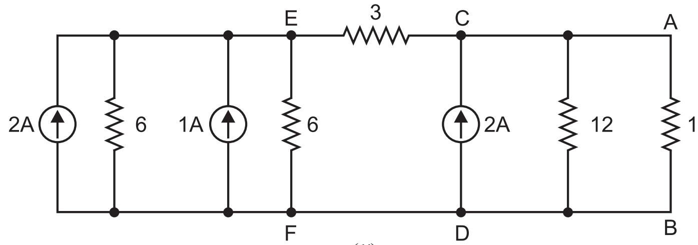
Solution. To find the answers, we convert the three voltage sources into their equivalent current sources.

(***a***) 12 V source in series with (4 + 2) = 6Ω resistance is converted into equivalent current source of 12V/6Ω = 2A in parallel with 6Ω resistance.

(***b***) 6V source in series with 6Ω resistance is converted into equivalent current source of 6V/6Ω = 1A in parallel with 6Ω resistance.

(***c***) 24V source in series with 12Ω resistance is converted into equivalent current source of 24V/12Ω = 2A in parallel with 12Ω resistance.

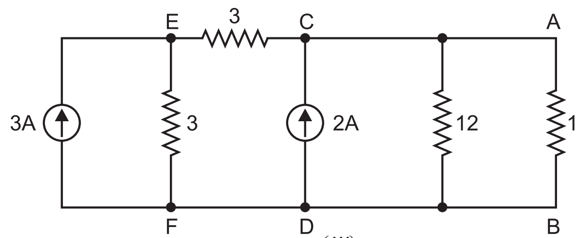
After the above source conversions, the circuit of Fig. 3.189 (*i*) becomes the circuit shown in Fig. 3.189 (*ii*).



(*ii*)

Fig. 3.189

Referring to Fig. 3.189 (*ii*), we can combine the two current sources to the left of *EF* but cannot combine 2A source across *CD* with them because 3Ω resistance is between *E* and *C*. Therefore, combining the two current sources to the left of *EF*, we have a single current source of 2 + 1 = 3A and a single resistance of 6Ω || 6Ω = 3Ω in parallel with it. As a result, Fig. 3.189 (*ii*) reduces to the circuit shown in Fig. 3.189 (*iii*).



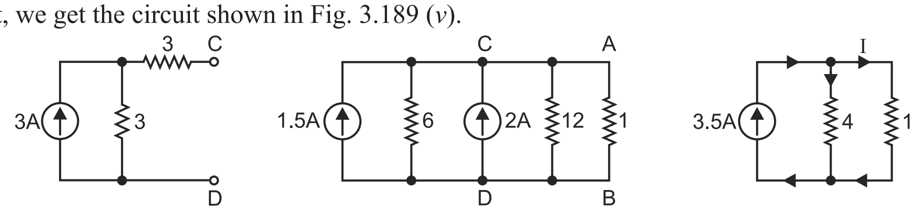
(*iii*)

Fig. 3.189

We now convert the circuit to the left of *CD* in Fig. 3.189 (*iii*) into Norton equivalent circuit. Fig. 3.189 (*iv*) shows this circuit to the left of *CD*. Its Norton equivalent circuit values are :

$$I_N = 3\times\frac{3}{3+3} = 1.5\text{A} \;;\; R_N = 3\Omega + 3\Omega = 6\Omega$$

Therefore, replacing the circuit to the left of *CD* in Fig. 3.189 (*iii*) by its Norton equivalent circuit, we get the circuit shown in Fig. 3.189 (*v*).



(*iv*) (*v*) (*vi*)

Fig. 3.189

Referring to Fig. 3.189 (*v*), we can combine the two current sources into a single current source of 1.5 + 2 = 3.5 A and a single resistance of 6Ω || 12Ω = 4Ω in parallel with it. The circuit then reduces to the one shown in Fig. 3.189 (*vi*). By current-divider rule [See Fig. 3.189 (*vi*)],

$$\text{Current in } 1\Omega \text{ resistor, } I = 3.5\times\frac{4}{4+1} = \mathbf{2.8\ A}$$

3.16. Norton Equivalent Circuit

(Circuits containing both independent and dependent sources)

Sometimes we come across circuits which contain both independent and dependent sources.

One such example is shown in Fig. 3.190. The procedure for finding Norton equivalent circuit (*i.e.* finding i_N and R_N) in such cases is as under :

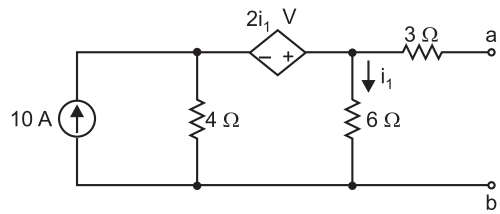


Fig. 3.190

(*i*) The open-circuited voltage $v_{oc}(= v_{Th})$ at terminals *ab* is determined as usual with sources present.

(*ii*) We cannot find R_N $(= R_{Th})$ at terminals *ab* simply by calculating equivalent resistance because of the presence of the dependent source. Instead, we place a short circuit across the terminals *ab* and find the value of short-circuit current i_{sc} $(= i_N)$ at terminals *ab*.

(*iii*) Norton resistance, $R_N = v_{oc}/i_{sc}$ $(= v_{Th}/i_{sc})$.

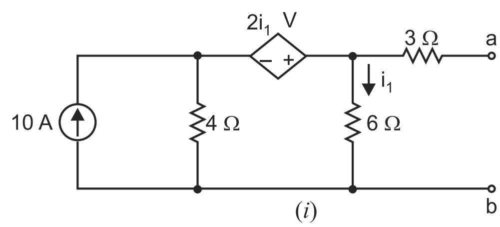
Note. In case the circuit contains dependent sources *only*, the procedure for finding v_{oc} $(= v_{Th})$ and $R_N(= R_{Th})$ is as under :

(*a*) In this case, $v_{oc} = 0$ and $i_{sc} = 0$ because no independent source is present.

(*b*) We cannot use the relation $R_N = v_{oc}/i_{sc}$ as we do in case the circuit contains both independent and dependent sources.

(*c*) In order to find R_N, we excite the circuit at terminals *ab* by connecting 1A source to the terminals *a* and *b* and calculate the value of v_{ab}. Then R_N $(= R_{Th}) = v_{ab}/1\Omega$.

Example 3.76. *Find the values of* i_N *and* R_N *at terminals ab for the circuit shown in Fig. 3.191 (i).*



(*i*)

Fig. 3.191

Solution. We first put a short circuit across terminals *a* and *b* to find short-circuit current i_{sc} $(= i_N)$ at terminals *ab* as shown in Fig. 3.191 (*ii*). Applying *KCL* at node *c*, we have,

$$10 = i_1 + i_2 + i_{sc}$$

or $$i_2 = 10 - i_1 - i_{sc}$$

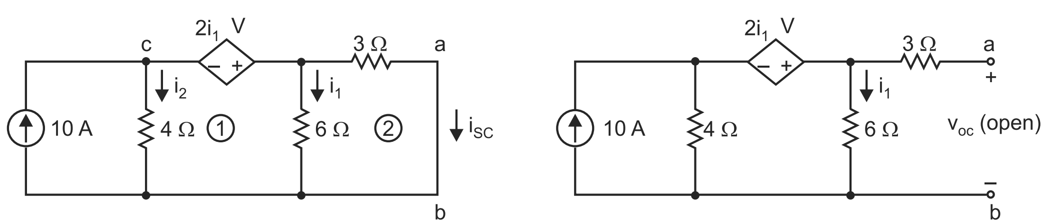
Applying *KVL* to loops 1 and 2, we have,

$$-4i_2 + 6i_1 - 2i_1 = 0 \quad \text{... Loop 1}$$

or $$-4(10 - i_1 - i_{sc}) + 4i_1 = 0 \quad ...(i)$$

Also $$-6i_1 + 3i_{sc} = 0 \quad ...(ii) \text{ ... Loop 2}$$

From eqs. (*i*) and (*ii*), $i_{sc} = i_N =$ **5A.**



(*ii*) (*iii*)

Fig. 3.191

In order to find v_{oc} $(= v_{Th})$, we refer to Fig. 3.191 (*iii*) where we have,

$$v_{oc} = 6i_1 \quad ...(iii)$$

Applying *KVL* to the central loop in Fig. 3.191 (*iii*),

$$-4\,(10 - i_1) + 6i_1 - 2i_1 \;=\; 0 \qquad ...(iv)$$

From eqs. (*iii*) and (*iv*), we have, $v_{oc} = v_{Th} = 30\text{V}$.

Also $$R_N\,(= R_{Th}) \;=\; \frac{v_{oc}}{i_{sc}} = \frac{30}{5} = 6\Omega$$

Tutorial Problems

1. Using Norton's theorem, find the current in 8 Ω resistor of the network shown in Fig. 3.192. **[1.55 A]**

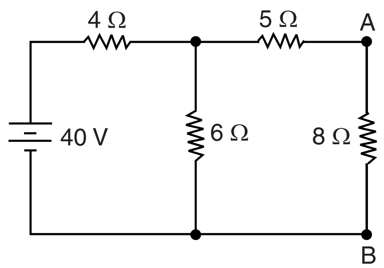


Fig. 3.192

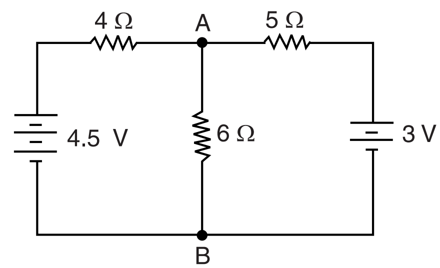


Fig. 3.193

2. Using Norton's theorem, find the current in the branch *AB* containing 6 Ω resistor of the network shown in Fig. 3.193. **[0.466 A]**
3. Show that when Thevenin's equivalent circuit of a network is converted into Norton equivalent circuit, $I_N = V_{Th}/R_{Th}$ and $R_N = R_{Th}$.
4. Find the voltage between points *A* and *B* in the network shown in Fig. 3.194 using Norton's theorem. **[2·56 V]**

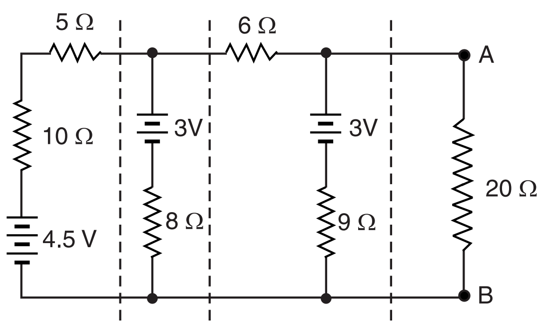


Fig. 3.194

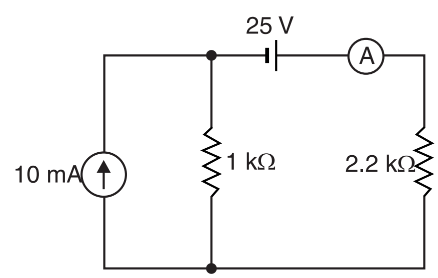


Fig. 3.195

5. The ammeter labelled A in Fig. 3.195 reads 35 mA. Is the 2·2 kΩ resistor shorted? Assume that ammeter has zero resistance. **[Shorted]**
6. Find Norton equivalent circuit to the left of terminals *a* – *b* in Fig. 3.196. **[I_N = 1·5 A; R_N = 4 Ω]**

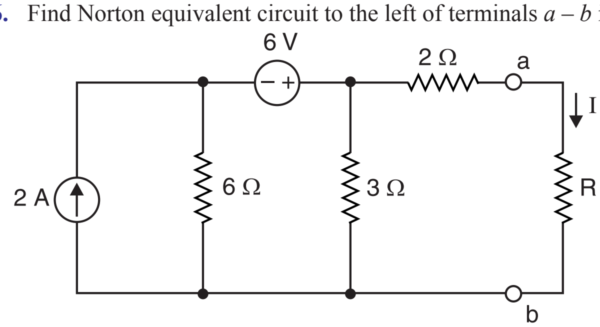


Fig. 3.196

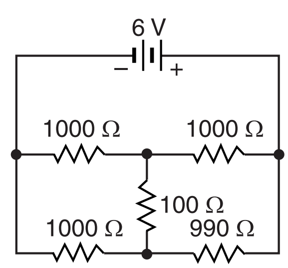


Fig. 3.197

7. What is the current in the 100 Ω resistor in Fig. 3.197 if the 990 Ω resistor is changed to 1010 Ω ? Use Norton theorem to obtain the result. **[13·45 μA]**

8. Determine the Norton equivalent circuit and the load current in R_L in Fig. 3.198. The various circuit values are :

$$E' = 64 \text{ V} ; \quad R_1 = 230\ \Omega ; \quad R_2 = 450\ \Omega ;$$
$$R_3 = 260\ \Omega ; \quad R_4 = 550\ \Omega ; \quad R_5 = 440\ \Omega ; \quad R_L = 360\ \Omega$$

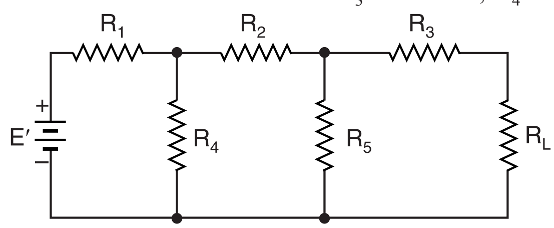


Fig. 3.198

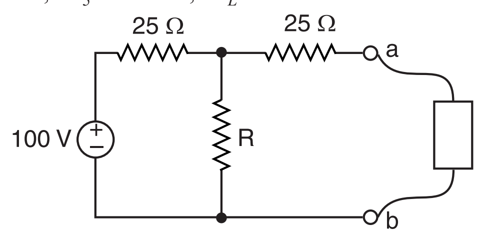


Fig. 3.199

9. In Fig. 3.199, replace the network to the left of terminals *ab* with its Norton equivalent.

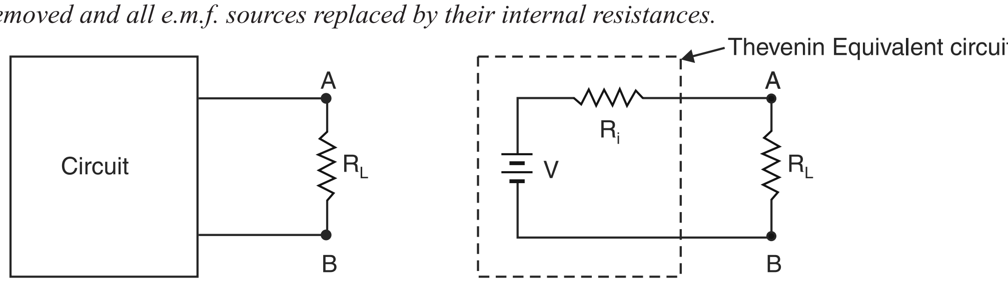
$$\left[I_N = \frac{2R}{R+12.5}\ \text{A} \ ; R_N = \frac{50\,R + 625}{R+25}\ \Omega \right]$$

10. When any source (voltage or current) is delivering maximum power to a load, prove that overall circuit efficiency is 50%.

3.17. Maximum Power Transfer Theorem

This theorem deals with transfer of maximum power from a source to load and may be stated as under :

In d.c. circuits, maximum power is transferred from a source to load when the load resistance is made equal to the internal resistance of the source as viewed from the load terminals with load removed and all e.m.f. sources replaced by their internal resistances.



(*i*) (*ii*)

Fig. 3.200

Fig. 3.200 (*i*) shows a circuit supplying power to a load R_L. The circuit enclosed in the box can be replaced by Thevenin's equivalent circuit consisting of Thevenin voltage $V = V_{Th}$ in series with Thevenin resistance $R_i (= R_{Th})$ as shown in Fig. 3.200 (*ii*). Clearly, resistance R_i is the resistance measured between terminals *AB* with R_L removed and e.m.f. sources replaced by their internal resistances. According to maximum power transfer theorem, maximum power will be transferred from the circuit to the load when R_L is made equal to R_i, the Thevenin resistance at terminals *AB*.

3.18. Proof of Maximum Power Transfer Theorem

Consider a voltage source V of internal resistance R_i delivering power to a load R_L. We shall prove that when $R_L = R_i$, the power delivered to R_L is maximum. Referring to Fig. 3.201 (*i*), we have,

$$\text{Circuit current, } I = \frac{V}{R_L + R_i}$$

$$\text{Power delivered to load, } P = I^2 R_L$$

$$= \left(\frac{V}{R_L + R_i}\right)^2 R_L \qquad ...(i)$$

For a given source, generated voltage V and internal resistance R_i are constant. Therefore, power delivered to the load depends upon R_L. In order to find the value of R_L for which the value of P is maximum, differentiate eq. (*i*) w.r.t. R_L and set the result equal to zero.

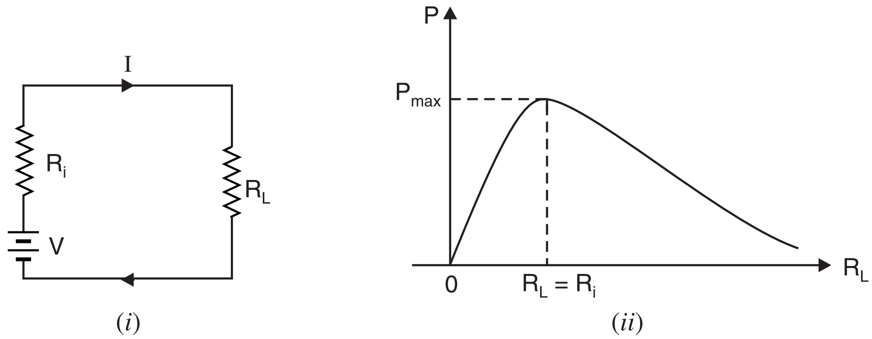


Fig. 3.201

Thus, $$\frac{dP}{dR_L} = V^2\left[\frac{(R_L + R_i)^2 - 2R_L(R_L + R_i)}{(R_L + R_i)^4}\right] = 0$$

or $$(R_L + R_i)^2 - 2\,R_L\,(R_L + R_i) = 0$$

or $$(R_L + R_i)\,(R_L + R_i - 2R_L) = 0$$

or $$(R_L + R_i)\,(R_i - R_L) = 0$$

Since $R_L + R_i$ cannot be zero,

$\therefore$ $$R_i - R_L = 0$$

or $$R_L = R_i$$

or **Load resistance = Internal resistance of the source**

Thus, for maximum power transfer, load resistance R_L must be equal to the internal resistance R_i of the source. Fig. 3.201 (*ii*) shows the graph between power delivered (P) and R_L. We may extend the maximum power transfer theorem to a linear circuit rather than a single source by means of Thevenin's theorem as under :

The maximum power is obtained from a linear circuit at a given pair of terminals when terminals are loaded by Thevenin's resistance (R_{Th}) of the circuit.

The above statement is obviously true because by Thevenin's theorem, the circuit is equivalent to a voltage source in series with internal resistance (R_{Th}) of the circuit.

Important Points. The following points are worth noting about maximum power transfer theorem :

(*i*) *The circuit efficiency at maximum power transfer is only 50%* as one-half of the total power generated is dissipated in the internal resistance R_i of the source.

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} = \frac{I^2 R_L}{I^2 (R_L + R_i)}$$

$$= \frac{R_L}{2R_L} = \frac{1}{2} = 50\% \qquad (\because R_L = R_i)$$

(*ii*) *Under the conditions of maximum power transfer, the load voltage is one-half of the open-circuited voltage at the load terminals.*

$$\text{Load voltage} = I R_L = \left(\frac{V}{R_L + R_i}\right) R_L = \frac{V R_L}{2 R_L} = \frac{V}{2}$$

(*iii*) $$\text{Max. power transferred} = \left(\frac{V}{R_L + R_i}\right)^2 R_L = \left(\frac{V}{2 R_L}\right)^2 R_L = \frac{V^2}{4 R_L}$$

Note. In case of a practical current source, the maximum power delivered is given by ;

$$P_{max} = \frac{I_N^2 R_N}{4}$$

where I_N = Norton current

R_N = Norton resistance ($= R_{Th} = R_i$)

3.19. Applications of Maximum Power Transfer Theorem

This theorem is very useful in situations where transfer of maximum power is desirable. Two important applications are listed below :

(*i*) In communication circuits, maximum power transfer is usually desirable. For instance, in a public address system, the circuit is adjusted for maximum power transfer by making load (*i.e.* speaker) resistance equal to source (*i.e.* amplifier) resistance. When source and load have the same resistance, they are said to be *matched*.

In most practical situations, the internal resistance of the source is fixed. Also, the device that acts as a load has fixed resistance. In order to make $R_L = R_i$, we use a transformer. We can use the reflected-resistance characteristic of the transformer to make the load resistance appear to have the same value as the source resistance, thereby "fooling" the source into "thinking" that there is a match (*i.e.* $R_L = R_i$). This technique is called **impedance matching.**

(*ii*) Another example of maximum power transfer is found in starting of a car engine. The power delivered to the starter motor of the car will depend upon the effective resistance of the motor and internal resistance of the battery. If the two resistances are equal (as is the case when battery is fully charged), maximum power will be transferred to the motor to turn on the engine. This is particularly desirable in winter when every watt that can be extracted from the battery is needed by the starter motor to turn on the cold engine. If the battery is weak, its internal resistance is high and the car does not start.

Note. Electric power systems are never operated for maximum power transfer because the efficiency under this condition is only 50%. This means that 50% of the generated power will be lost in the power lines. This situation cannot be tolerated because power lines must operate at much higher than 50% efficiency.

Example 3.77. *Two identical cells connected in series deliver a maximum power of 1W to a resistance of 4 Ω. What is the internal resistance and e.m.f. of each cell ?*

Solution. Let E and r be the e.m.f. and internal resistance of each cell. The total internal resistance of the battery is $2r$. For maximum power transfer,

$$2\,r = R_L = 4 \quad \therefore \quad r = R_L/2 = 4/2 = \mathbf{2\ \Omega}$$

$$\text{Maximum power} = \frac{{}^*(2E)^2}{4 R_L}$$

or $$1 = \frac{4E^2}{4R_L} \quad \therefore \quad E = \sqrt{R_L} = \sqrt{4} = \mathbf{2\ V}$$

* Here total voltage = $2E$.

Example 3.78. *Find the value of resistance R to have maximum power transfer in the circuit shown in Fig. 3.202 (i). Also obtain the amount of maximum power. All resistances are in ohms.*

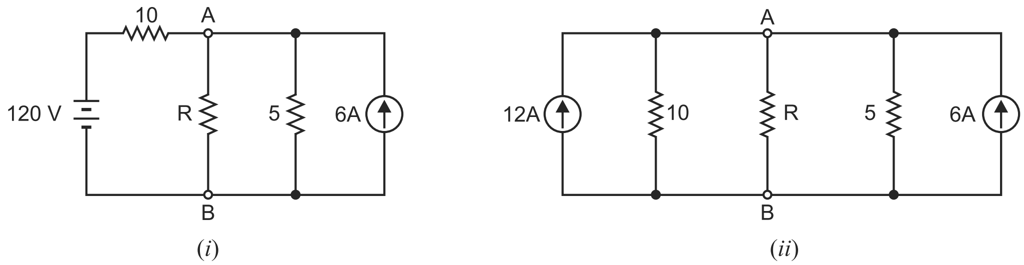


Fig. 3.202

Solution. To find the desired answers, we should find V_{Th} and R_{Th} at the load (*i.e. R*) terminals. For this purpose, we first convert 120V voltage source in series with 10Ω resistance into equivalent current source of 120/10 = 12A in parallel with 10Ω resistance. The circuit then becomes as shown in Fig. 3.202. (*ii*).

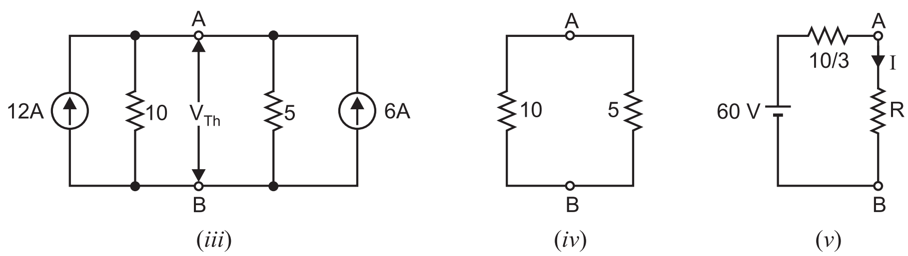


Fig. 3.202

To find V_{Th}, remove R (*i.e.* load) from the circuit in Fig. 3.202 (*ii*), and the circuit becomes as shown in Fig. 3.202 (*iii*). Then voltage across the open-circuited terminals AB is V_{Th}. Referring to Fig. 3.202 (*iii*) and applying KCL, we have,

$$\frac{V_{Th}}{10} + \frac{V_{Th}}{5} = 12 + 6 \quad \text{or} \quad V_{Th} = 60\text{V}$$

In order to find R_{Th}, remove R and replace the current sources by open in Fig. 3.202 (*ii*). Then circuit becomes as shown in Fig. 3.202 (*iv*). Then resistance at the open-circuited terminals AB is R_{Th}.

$$\therefore \qquad R_{Th} = 10\Omega \parallel 5\Omega = \frac{10 \times 5}{10 + 5} = \frac{10}{3}\Omega$$

When R is connected to the terminals of Thevenin equivalent circuit, the circuit becomes as shown in Fig. 3.202 (*v*).

For maximum power transfer, the condition is

$$R = R_{Th} = \mathbf{\frac{10}{3}\Omega}$$

Max. power transferred, $P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{V_{Th}^2}{4R} = \frac{(60)^2}{4 \times (10/3)} = \mathbf{270\ W}$

Example 3.79. *Calculate the value of R which will absorb maximum power from the circuit of Fig. 3.203 (i). Also find the value of maximum power.*

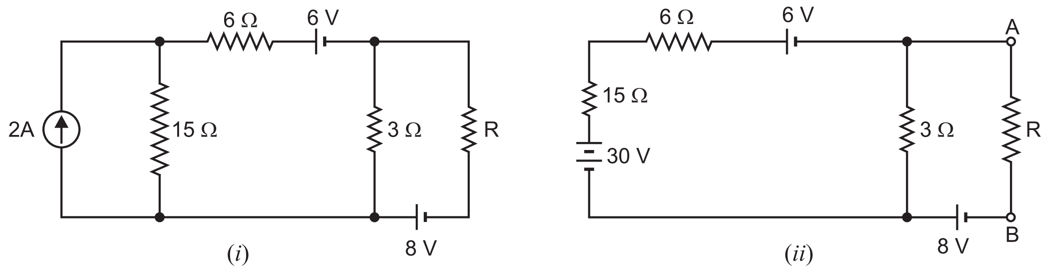


Fig. 3.203

Solution. To find the desired answers, we should find V_{Th} and R_{Th} at the load (*i.e. R*) terminals. For this purpose, we first convert 2A current source in parallel with 15Ω resistance into equivalent voltage source of 2A × 15Ω = 30 V in series with 15Ω resistance. The circuit then becomes as shown in Fig. 3.203 (*ii*).

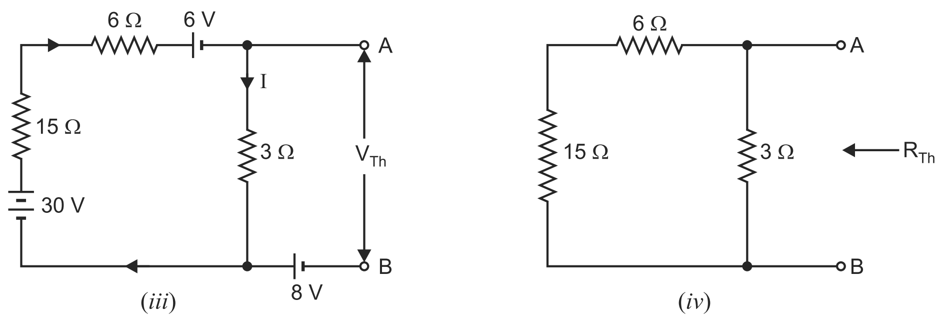


Fig. 3.203

To find V_{Th}, remove *R* (*i.e.* load) from the circuit in Fig. 3.203 (*ii*) and the circuit becomes as shown in Fig. 3.203 (*iii*). Then voltage across the open-circuited terminals *AB* is V_{Th}. Referring to Fig. 3.203 (*iii*),

Current in 3Ω resistor, $I = \dfrac{30-6}{15+6+3} = 1\text{A}$

In Fig. 3.203 (*iii*), as we go from point *A* to point *B via* 3Ω resistor, we have,

$$V_A - I \times 3 - 8 = V_B$$

or $$V_A - V_B = I \times 3 + 8 = 1 \times 3 + 8 = 11\text{V}$$

∴ $$V_{Th} = V_{AB} = V_A - V_B = 11\text{V}$$

In order to find R_{Th}, remove *R* and replace the voltage sources by short in Fig. 3.203 (*ii*). Then circuit becomes as shown in Fig. 3.203 (*iv*). Then resistance at open-circuited terminals *AB* is R_{Th}.

∴ $$R_{Th} = (15+6)\Omega \parallel 3\Omega = \frac{21 \times 3}{21+3} = \frac{21}{8}\Omega$$

For maximum power transfer, the condition is

$$R = R_{Th} = \mathbf{\frac{21}{8}\Omega}$$

Max. power transferred, $P_{max} = \dfrac{V_{Th}^2}{4R_L} = \dfrac{V_{Th}^2}{4R} = \dfrac{(11)^2}{4 \times (21/8)} = \mathbf{11.524\ W}$

Example 3.80. *Determine the value of R_L in Fig. 3.204 (i) for maximum power transfer and evaluate this power.*

Solution. The three current sources in Fig. 3.204 (*i*) are in parallel and supply current in the same direction. Therefore, they can be replaced by a single current source supplying 0·8 + 1 + 0·9 = 2·7 A as shown in Fig. 3.204 (*ii*). The circuit to the left of R_L in Fig. 3.204 (*ii*) can be replaced by Thevenin's equivalent circuit as under :

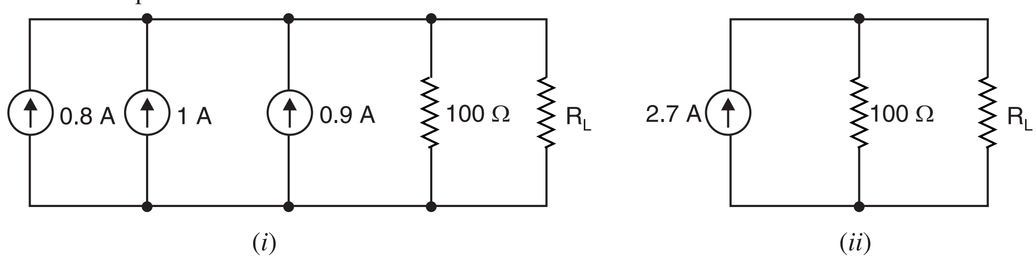


Fig. 3.204

$$V_{Th} = I_N R_N = 2{\cdot}7 \times 100 = 270 \text{ V}$$

$$R_i = R_N = 100 \ \Omega$$

The Thevenin's equivalent circuit to the left of R_L is V_{Th}(= 270 V) in series with R_i (= 100 Ω). When load R_L is connected, the circuit becomes as shown in Fig. 3.205. It is clear that maximum power will be transferred when

$$R_L = R_i = \mathbf{100 \ \Omega}$$

$$\text{Max. power} = \frac{V_{Th}^2}{4R_L} = \frac{(270)^2}{4 \times 100}$$

$$= \mathbf{182{\cdot}25 \text{ watts}}$$

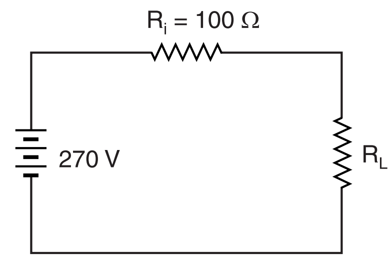


Fig. 3.205

Example 3.81. *Determine the maximum power that can be delivered by the circuit shown in Fig. 3.206 (i).*

Solution. Fig. 3.206 (*ii*) shows the Norton's equivalent circuit. Maximum power transfer occurs when $R_L = R_N = 300 \ \Omega$.

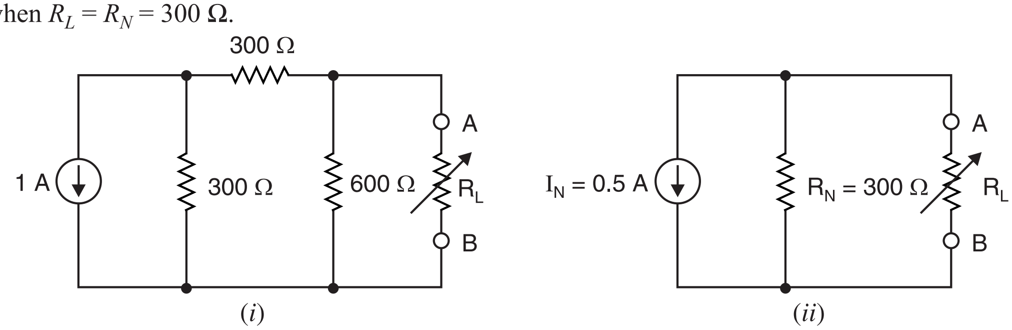


Fig. 3.206

Referring to Fig. 3.206 (*ii*), current in R_L (= 300 Ω) = $I_N/2$ = 0·5/2 = 0·25 A

∴ Max. power transferred = $(0{\cdot}25)^2 \times R_L = (0{\cdot}25)^2 \times 300$ = **18·8 W**

Example 3.82. *What percentage of maximum possible power is delivered to R_L in Fig. 3.207 (i) when $R_L = 2\,R_{Th}$?*

Solution. Fig. 3.207 (*ii*) shows the circuit when $R_L = 2\,R_{Th}$.

$$\text{Circuit current} = \frac{V_{Th}}{R_{Th} + 2R_{Th}} = \frac{V_{Th}}{3R_{Th}}$$

$$\text{Voltage across load, } V_L = \frac{V_{Th}}{3R_{Th}} \times 2R_{Th} = \frac{2}{3}V_{Th}$$

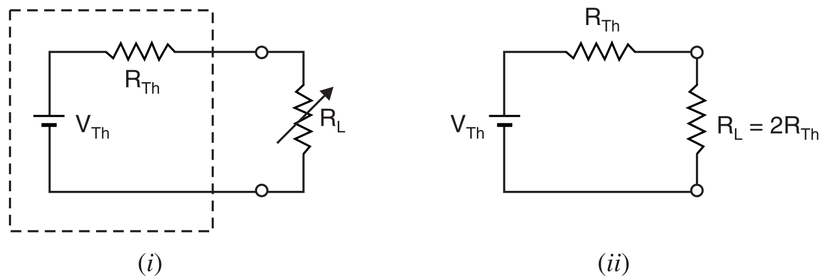


Fig. 3.207

$$\text{Power delivered to load, } P_L = \frac{V_L^2}{R_L} = \frac{\left(\frac{2}{3}V_{Th}\right)^2}{2R_{Th}} = \frac{4V_{Th}^2}{18R_{Th}}$$

Since $P_{max} = V_{Th}^2/4\,R_{Th}$, the ratio of P_L/P_{max} is

$$\frac{P_L}{P_{max}} = \frac{\frac{4V_{Th}^2}{18R_{Th}}}{\frac{V_{Th}^2}{4R_{Th}}} = \frac{16}{18}$$

$$\therefore \quad P_L = \frac{16}{18}P_{max} \times 100 = \textbf{88·89\% of } \boldsymbol{P_{max}}$$

Example 3.83. *Find the maximum power in R_L which is variable in the circuit shown in Fig. 3.208 (i).*

Solution. We shall use Thevenin theorem to obtain the result. For this purpose, remove the load R_L as shown in Fig. 3.208 (*ii*). The open-circuited voltage at terminals *AB* in Fig. 3.208 (*ii*) is equal to V_{Th}. It is clear from Fig. 3.208 (*ii*) that current in the branch containing 40 Ω and 60 Ω resistors is 1 A. Similarly, current in the branch containing two 50 Ω resistors is 1 A. It is clear that point *A* is at higher potential than point *B*. Applying *KVL* to the loop *EABCDE*, we have,

$$-40 \times 1 - V_{AB} - 2 + 50 \times 1 = 0 \qquad \therefore \qquad V_{AB} = 8 \text{ V}$$

Now V_{AB} in Fig. 3.208 (*ii*) is equal to V_{Th}. Therefore, V_{Th} = 8 V.

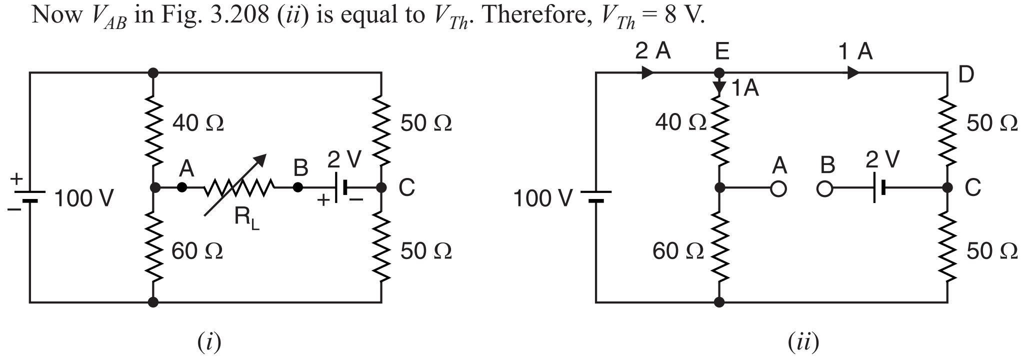


Fig. 3.208

In order to find Thevenin's resistance R_{Th}, replace 100V and 2V batteries by a short in Fig. 3.208 (*ii*). Then resistance at terminals *AB* is the R_{Th}. It is clear that 40 Ω and 60 Ω resistors are in parallel and so the two 50 Ω resistors.

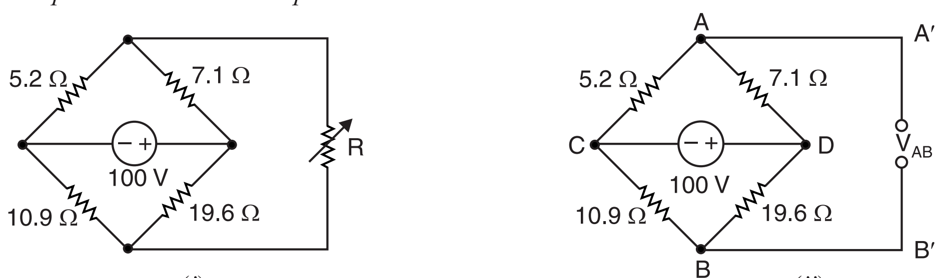
$$\therefore \quad R_{Th} = (40 \,||\, 60) + (50 \,||\, 50) = \frac{40 \times 60}{40 + 60} + \frac{50 \times 50}{50 + 50} = 24 + 25 = 49\ \Omega$$

Therefore, for maximum power, R_L should be 49 Ω. The Thevenin equivalent circuit is a voltage source of 8 V in series with a resistance of 49 Ω. When load R_L is connected across the terminals of Thevenin equivalent circuit, the total circuit resistance = 49 + 49 = 98 Ω.

$$\therefore \qquad \text{Circuit current, } I = \frac{V_{Th}}{R_{Th} + R_L} = \frac{8}{49 + 49} = \frac{8}{98} = 0{\cdot}08163 \text{ A}$$

$$\therefore \qquad P_{max} = I^2 R_L = (0{\cdot}08163)^2 \times 49 = \mathbf{0{\cdot}3265\ W}$$

Example 3.84. *For the circuit shown in Fig. 3.209 (i), find the value of R that will receieve maximum power. Determine this power.*



(*i*) (*ii*)

Fig. 3.209

Solution. We will use Thevenin's theorem to obtain the results. In order to find V_{Th}, remove the variable load *R* as shown in Fig. 3.209 (*ii*). Then open-circuited voltage across terminals *AB* is equal to V_{Th}.

$$\text{Current in branch } DAC = \frac{100}{7.1 + 5.2} = 8{\cdot}13 \text{ A}$$

$$\text{Current in branch } DBC = \frac{100}{19.6 + 10.9} = 3{\cdot}28 \text{ A}$$

It is clear from Fig. 3.209 (*ii*) that point *A* is at higher* potential than point *B*. Applying *KVL* to the loop *A'ACBB'A'*, we have,

$$-5{\cdot}2 \times 8{\cdot}13 + 10{\cdot}9 \times 3{\cdot}28 + V_{AB} = 0$$

$$\therefore \qquad V_{AB} = 6{\cdot}52 \text{ V}$$

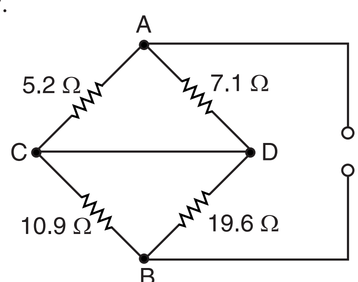
Now V_{AB} in Fig. 3.209 (*ii*) is equal to V_{Th} so that $V_{Th} = 6{\cdot}52$ V.

In order to find R_{Th}, replace the 100 V source in Fig. 3.209 (*ii*) by a short. The circuit becomes as shown in Fig. 3.209 (*iii*). The resistance across terminals *AB* is the Thevenin resistance. Referring to Fig. 3.209 (*iii*),

$$R_{AB} = R_{Th} = (5{\cdot}2 \,||\, 7{\cdot}1) + (10{\cdot}9 \,||\, 19{\cdot}6)$$
$$= 3 + 7 = 10\ \Omega$$

Therefore, for maximum power transfer, $R = R_{Th} = \mathbf{10\ \Omega}$.

$$P_{max} = \frac{(V_{Th})^2}{4R} = \frac{(6.52)^2}{4 \times 10} = \mathbf{1{\cdot}06\ W}$$



(*iii*)

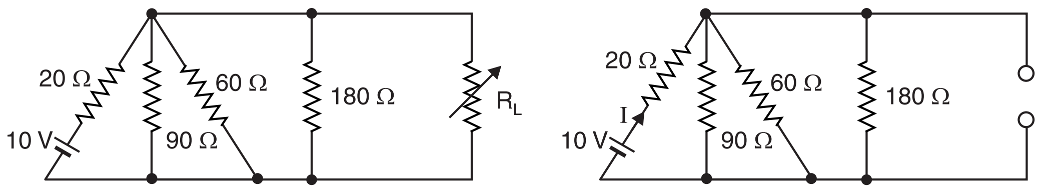
Fig. 3.209

* The fall in potential along *DA* is less than the fall in potential along *DB*. Since point *D* is common, point *A* will be at higher potential than point *B*.

Example 3.85. *For the circuit shown in Fig. 3.210 (i), what will be the value of R_L to get maximum power? Also find this power.*

Solution. We shall use Thevenin's theorem to obtain the results. In order to find V_{Th}, remove the load R_L as shown in Fig. 3.210 (*ii*). Then voltage at the open-circuited terminals *AB* is equal to V_{Th} *i.e.* $V_{AB} = V_{Th}$. The total load on 10 V source is

$$R_T = (90 \parallel 60 \parallel 180) + 20 = 30 + 20 = 50\ \Omega$$



(*i*) (*ii*)

Fig. 3.210

Current supplied by source, $I = 10/50 = 0{\cdot}2$ A

$\therefore \quad V_{AB} = V_{Th} = 10 - 20 \times 0{\cdot}2 = 6\text{V}$

In order to find R_{Th}, replace the 10 V source by a short in Fig. 3.210 (*ii*). Then,

$$R_{Th} = 20 \parallel 90 \parallel 60 \parallel 180 = 12\ \Omega$$

Therefore, the variable load R_L will receive maximum power when $R_L = R_{Th} =$ **12 Ω**.

$$\therefore \quad P_{max} = \frac{(V_{Th})^2}{4R_L} = \frac{(6)^2}{4 \times 12} = \mathbf{0{\cdot}75\ W}$$

Tutorial Problems

1. Find the value of R_L in Fig. 3.211 necessary to obtain maximum power in R_L. Also find the maximum power in R_L. **[150Ω ; 1.042 W]**

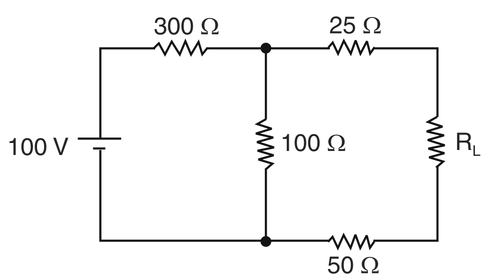


Fig. 3.211

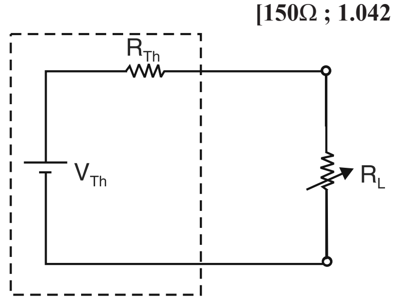


Fig. 3.212

2. If R_L in Fig. 3.211 is fixed at 100 Ω, what alternation (s) can be made in the rest of the circuit to obtain maximum power in R_L ? **[Short out 50 Ω resistor]**

3. What percentage of the maximum possible power is delivered to R_L in Fig. 3.212, when $R_L = R_{Th}/2$? **[88.9%]**

4. Determine the value of R_L for maximum power transfer in Fig. 3.213 and evaluate this power. **[100 Ω; 182·25 W]**

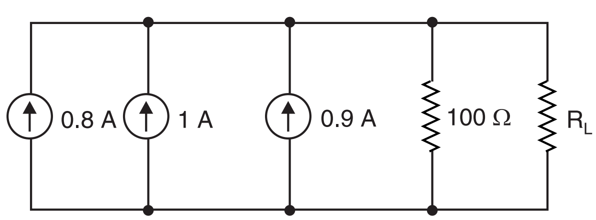


Fig. 3.213

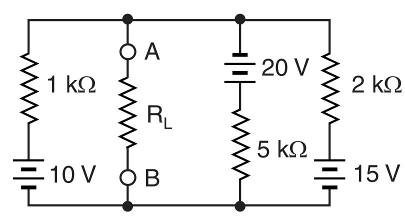


Fig. 3.214

5. What value should R_L be in Fig. 3.214 to achieve maximum power transfer to the load? **[588 Ω]**
6. For the circuit shown in Fig. 3.215, find the value of R_L for which power transferred is maximum. Also calculate this power. **[50 Ω; 0·72 W]**

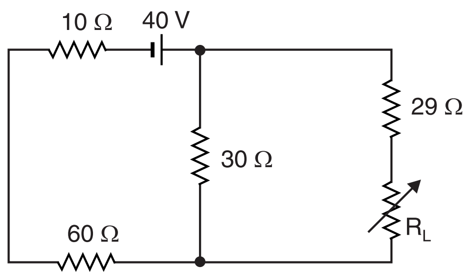


Fig. 3.215

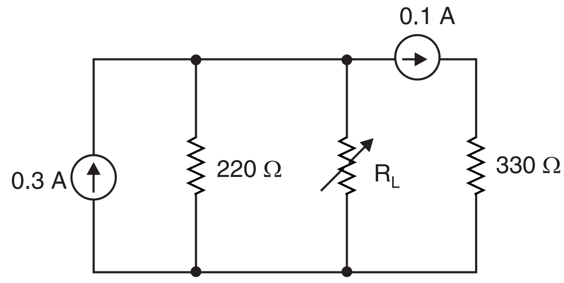


Fig. 3.216

7. Calculate the value of R_L for transference of maximum power in Fig. 3.216. Evaluate this power. **[220 Ω; 2·2 W]**

3.20. Reciprocity Theorem

This theorem permits us to transfer source from one position in the circuit to another and may be stated as under :

In any linear, bilateral network, if an e.m.f. E acting in a branch X causes a current I in branch Y, then the same e.m.f. E located in branch Y will cause a current I in branch X. However, currents in other parts of the network will not remain the same.

Explanation. Consider the circuit shown in Fig. 3.217 (*i*). The e.m.f. *E* (=100 V) acting in the branch *FAC* produces a current *I* amperes in branch *CDF* and is indicated by the ammeter. According to reciprocity theorem, if the e.m.f. *E* and ammeter are interchanged* as shown in Fig. 3.217 (*ii*), then the ammeter reading does not change *i.e.* the ammeter now connected in branch *FAC* will read *I* amperes. In fact, the essence of this theorem is that *E* and *I* are interchangeable. The ratio *E/I* is constant and is called *transfer resistance* (or impedance in case of a.c. system).

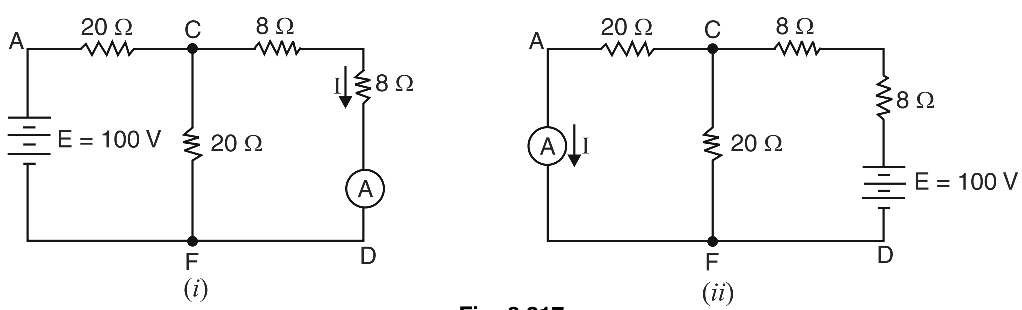


Fig. 3.217

Note. Suppose an ideal current source is connected across points *ab* of a network and this causes a voltage *v* to appear across points *cd* of the network. The reciprocity theorem states that if the current source is now connected across *cd*, the same amount of voltage *v* will appear across *ab*. This is sometimes stated as follows: An ideal current source and an ideal voltmeter can be interchanged without changing the reading of the voltmeter. However, voltages in other parts of the network will not remain the same.

Example 3.86. *Verify the reciprocity theorem for the network shown in Fig. 3.217 (i). Also find the transfer resistance.*

Solution. In Fig. 3.217 (*i*), e.m.f. *E* (= 100V) is in branch *FAC* and ammeter is in branch *CDF*. Referring to Fig. 3.217 (*i*),

* If the source of e.m.f in the original circuit has an internal resistance, this resistance must remain in the original branch and cannot be transferred to the new location of the e.m.f.

Resistance between C and F = 20 Ω || (8 + 8) Ω = 20 × 16/36 = 8·89 Ω

Total circuit resistance = 20 + 8·89 = 28·89 Ω

∴ Current supplied by battery = 100/28·89 = 3·46 A

The battery current is divided into two parallel paths *viz*. path *CF* of 20 Ω and path *CDF* of 8 + 8 = 16Ω.

Current in branch *CDF*, I = 3·46 × 20/36 = 1·923 A

Now in Fig. 3.217 (*ii*), E and ammeter are interchanged.

Referring to Fig. 3.217 (*ii*),

Resistance between C and F = 20 × 20/40 = 10 Ω

Total circuit resistance = 10 + 8 + 8 = 26 Ω

Current supplied by battery = 100/26 = 3·846 A

The battery current is divided into two parallel paths of 20 Ω each.

∴ Current in branch *CAF* = 3·846/2 = 1·923A

Hence, ammeter reading in both cases is the same. This verifies the reciprocity theorem.

Transfer resistance = E/I = 100/1·923 = **52 Ω**

Example 3.87. *Find the currents in the various branches of the circuit shown in Fig. 3.218 (i). If a battery of 9V is added in branch BCD, find current in 4 Ω resistor using reciprocity theorem and superposition theorem.*

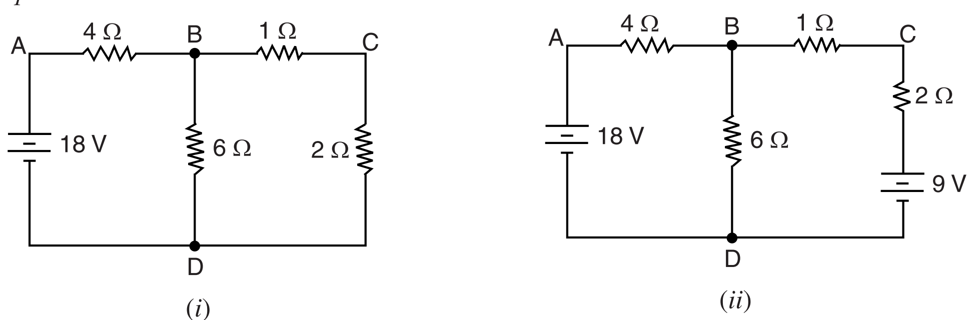


Fig. 3.218

Solution. Referring to Fig. 3.218 (*i*), we have,

Total resistance to source = 4 Ω + [6 Ω || (1 + 2) Ω] = 4 + 6 × 3/9 = 6 Ω

Current supplied by source (*i.e.* current in 4 Ω resistor or branch *DAB*)

= 18/6 = **3 A**

Current in branch *BD* = 3 × 3/9 = **1 A**

Current in branch *BCD* = 3 × 6/9 = **2 A**

In Fig. 3.218 (*i*), the current in branch *BCD* due to 18 V source acting alone is 2 A. If the 18V source is placed in branch *BCD*, then according to reciprocity theorem, the current in 4 Ω will be 2 A flowing from *B* to *A*. If a battery of 9 V is placed in branch *BCD*, then current in 4 Ω resistor due to it alone would be 2 × 9/18 = 1 A (By proportion).

Now referring to Fig. 3.218 (*ii*), the current in 4 Ω due to 18 V battery alone is 3 A flowing from *A* to *B*. The current in 4 Ω resistor due to 9 V acting alone in branch *BCD* is 1 A flowing from *B* to *A*. By superposition theorem, the current in 4 Ω is the algebraic sum of the two currents *i.e.*

Current in 4 Ω = 3 − 1 = **2 A *from A to B***

Example 3.88. *Prove the reciprocity theorem.*

Solution. We now prove the reciprocity theorem for the circuit shown in Fig. 3.219. In Fig. 3.219 (*i*), the e.m.f. E is acting in the branch FAC and the current in the branch CDF is I_2. If the same e.m.f. E now acts in branch CDF [See Fig. 3.219 (*ii*)], then current I_b in the branch FAC will be equal to I_2. We now show that $I_b = I_2$. Referring to Fig. 3.219 (*i*), we have,

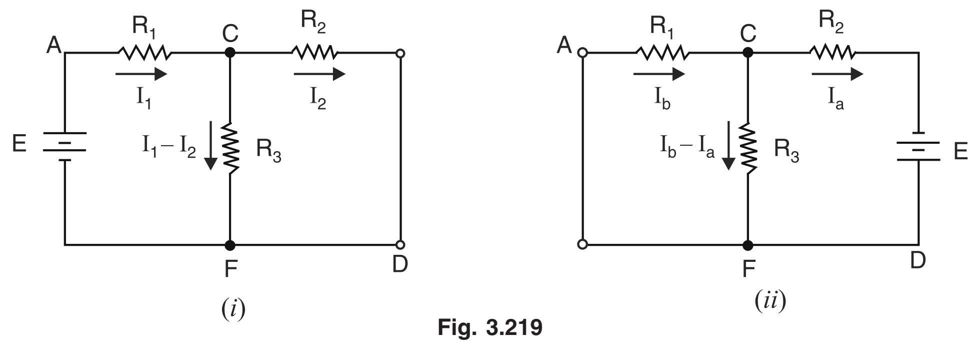


Fig. 3.219

$$E = I_1 R_T$$

where $$R_T = R_1 + (R_2 \parallel R_3) = \left[R_1 + \frac{R_2 R_3}{R_2 + R_3}\right]$$

$\therefore$ $$E = I_1\left[R_1 + \frac{R_2 R_3}{R_2 + R_3}\right]$$

$$= I_1\left[\frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2 + R_3}\right] \quad ...(i)$$

Also in Fig. 3.219 (*i*), $$0 = -(I_1 - I_2) R_3 + I_2 R_2$$

or $$I_2 = I_1\left[\frac{R_3}{R_2 + R_3}\right] \quad ...(ii)$$

Dividing eq. (*i*) by eq. (*ii*), we have,

$$\frac{E}{I_2} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \quad ...(iii)$$

Similarly, it can be shown that in Fig. 3.219 (*ii*), we have,

$$\frac{E}{I_b} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \quad ...(iv)$$

From eqs. (*iii*) and (*iv*), $I_b = I_2$

Therefore, reciprocity theorem stands proved.

3.21. Millman's Theorem

Millman's theorem is a combination of Thevenin's and Norton's theorems. It is used to reduce any number of parallel voltage/current sources to an equivalent circuit containing only one source. It has the advantage of being easier to apply to some networks than mesh analysis, nodal analysis or superposition. This theorem can be stated in terms of voltage sources or current sources or both.

1. Parallel voltage sources. Millman's theorem provides a method of calculating the common voltage across different parallel-connected voltage sources and may be stated as under :

The voltage sources that are directly connected in parallel can be replaced by a single equivalent voltage source.

Obviously, the above statement is true by virtue of Thevenin's theorem. Fig 3.220 (*i*) shows three parallel-connected voltage sources E_1, E_2 and E_3. Then common terminal voltage V_{AB} of these parallel voltage sources is given by ;

$$V_{AB} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3} = \frac{I_1 + I_2 + I_3}{G_1 + G_2 + G_3} = \frac{\Sigma I}{\Sigma G} \qquad \text{...}(i)$$

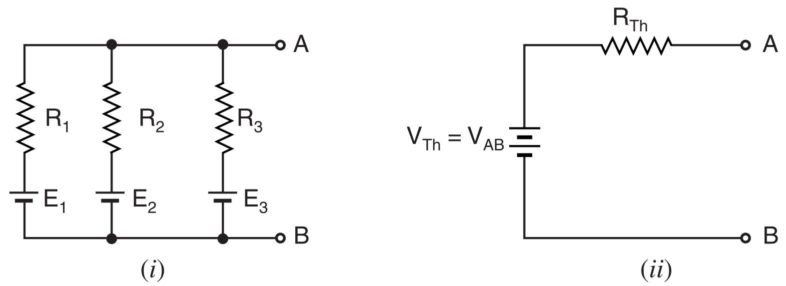


Fig. 3.220

This voltage represents the Thevenin's voltage V_{Th}. The denominator represents Thevenin's resistance R_{Th} *i.e.*

$$R_{Th} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$$

Therefore, parallel-connected voltage sources in Fig 3.220 (*i*) can be replaced by a single voltage source as shown in Fig 3.220 (*ii*). If load R_L is connected across terminals *AB*, then load current I_L is given by ;

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Note. If a branch does not contain any voltage source, the same procedure is used except that current in that branch will be zero. This is illustrated in example 3.89.

2. Parallel current sources. The Millman's theorem states as under :

The current sources that are directly connected in parallel can be replaced by a single equivalent current source. The current of this single current source is the *algebraic* sum of the individual source currents. The internal resistance of the single current source is equal to the combined resistance of the parallel combination of the source resistances.

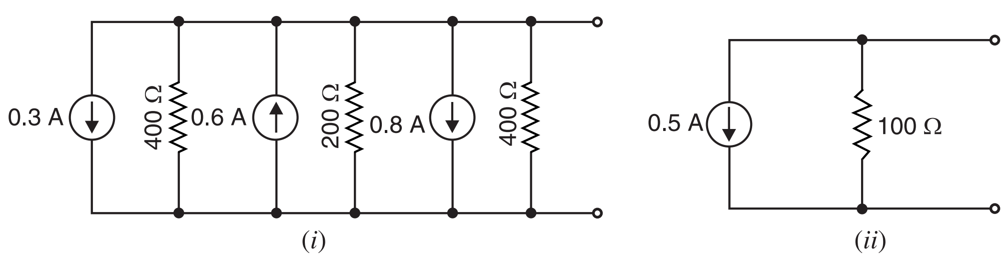


Fig. 3.221

Fig. 3.221 (*i*) shows three parallel connected current sources. The resultant current of the three sources is

$$0{\cdot}3 \downarrow + 0{\cdot}6 \uparrow + 0{\cdot}8 \downarrow = 0{\cdot}5\text{ A} \downarrow$$

The internal resistance of the single current source is equal to the equivalent resistance of three parallel resistors.

400 || 200 || 400 = 100 Ω

Thus the single equivalent current source has value 0·5 A and internal resistance 100 Ω in parallel as shown in Fig. 3.221 (*ii*).

3. Voltage sources and current sources in parallel. The Millman's theorem is also applicable if the circuit has a mixture of parallel voltage and current sources. Each parallel-connected voltage source is converted to an equivalent current source. The result is a set of parallel-connected current sources and we can replace them by a single equivalent current source. Alternatively, each parallel-connected current source can be converted to an equivalent voltage source and the set of parallel-connected voltage sources can be replaced by an equivalent voltage source.

Example 3.89. *Using Millman's theorem, determine the common voltage* V_{xy} *and the load current in the circuit shown in Fig. 3.222 (i).*

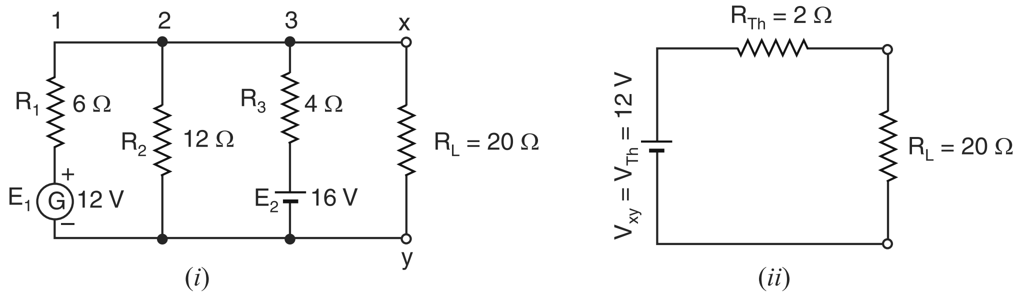


Fig. 3.222

Solution.
$$V_{xy} = V_{Th} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3}$$

$$= \frac{12/6 + 0/2 + 16/4}{1/6 + 1/12 + 1/4} = \frac{2+0+4}{0.167+0.083+0.25} = \frac{6}{0.5} = \mathbf{12V}$$

$$R_{Th} = \frac{1}{1/6 + 1/12 + 1/4} = 2\Omega$$

Therefore, the circuit shown in Fig. 3.222 (*i*) can be replaced by the one shown in Fig. 3.222 (*ii*).

$$\text{Load current} = \frac{V_{Th}}{R_{Th} + R_L} = \frac{12}{2+20} = \mathbf{0.545\ A}$$

Example 3.90. *Find the current in the 1 k Ω resistor in Fig. 3.223 by finding Millman equivalent voltage source with respect to terminals x – y.*

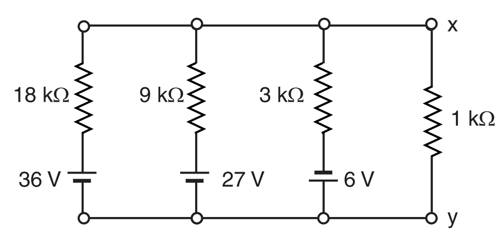


Fig. 3.223

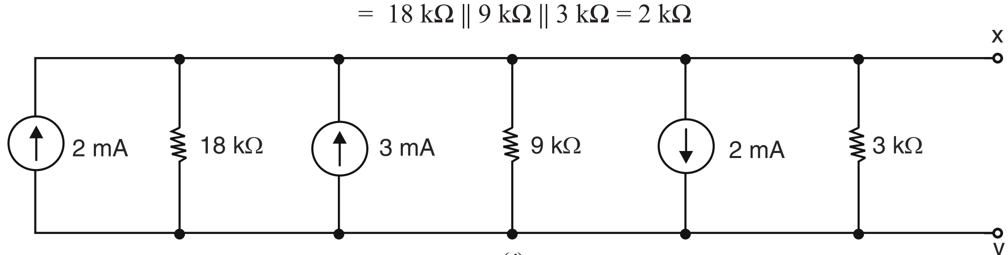
Solution. As shown Fig. 3.224 (*i*), each of the three voltage sources is converted to an equivalent current source. For example, the 36 V source in series with 18 kΩ resistor becomes a 36 V/18 kΩ = 2 mA current source in parallel with 18 kΩ. Note that the polarity of each current source is such that it produces current in the same direction as the voltage source it replaces.

The resultant current of the three current sources

$$= 2\text{ mA}\uparrow + 3\text{ mA}\uparrow + 2\text{ mA}\downarrow = 3\text{ mA}\uparrow$$

The parallel equivalent resistance of three resistors

$$= 18 \text{ k}\Omega \parallel 9 \text{ k}\Omega \parallel 3 \text{ k}\Omega = 2 \text{ k}\Omega$$

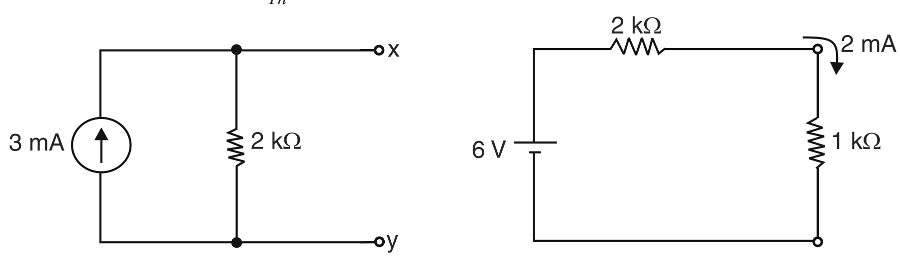


(*i*)

Fig. 3.224

Fig. 3.224 (*ii*) shows the single equivalent current source. Fig. 3.224 (*iii*) shows the voltage source that is equivalent to current source in Fig. 3.224 (*ii*).

$$V_{Th} = 3 \text{ mA} \times 2 \text{ k}\Omega = 6 \text{ V}$$



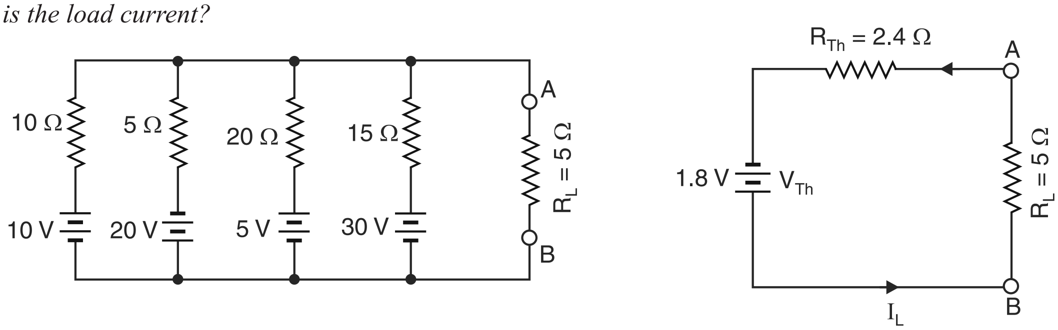
(*ii*) (*iii*)

Fig. 3.224

When the 1 kΩ resistor is connected across the $x - y$ terminals, the current is

$$I = \frac{6\text{V}}{3\text{k}\Omega} = \mathbf{2 \text{ mA}}$$

Example 3.91. *Find an equivalent voltage source for the circuit shown in Fig. 3.225 (i). What is the load current?*



(*i*) (*ii*)

Fig. 3.225

Solution. $$V_{AB} = V_{Th} = \frac{E_1/R_1 + E_2/R_2 + E_3/R_3 + E_4/R_4}{1/R_1 + 1/R_2 + 1/R_3 + 1/R_4}$$

$$= \frac{10/10 - {}^*20/5 + 5/20 + 30/15}{1/10 + 1/5 + 1/20 + 1/15} = \frac{-0.75}{0.417} = -1{\cdot}8 \text{ V}$$

Negative sign shows that terminal *A* is negative *w.r.t.* terminal *B*.

* Note that polarity is opposite as compared to other sources.

$$R_{Th} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3 + 1/R_4}$$

$$= \frac{1}{1/10 + 1/5 + 1/20 + 1/15} = 2.4\Omega$$

Therefore, equivalent voltage source consists of **1·8 V source in series with 2·4 Ω resistor** as shown in Fig. 3.225 (*ii*).

$$\therefore \quad \text{Load current, } I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{1.8}{2.4 + 5} = \mathbf{0.24A}$$

Example 3.92. *For the circuit shown in Fig. 3.225 (i) above, find the equivalent current source. Also find load current.*

Solution. Convert the voltage sources to current sources as shown in Fig. 3.226 (*i*). The arrow for each current source corresponds to the polarity of each voltage source in the original circuit.

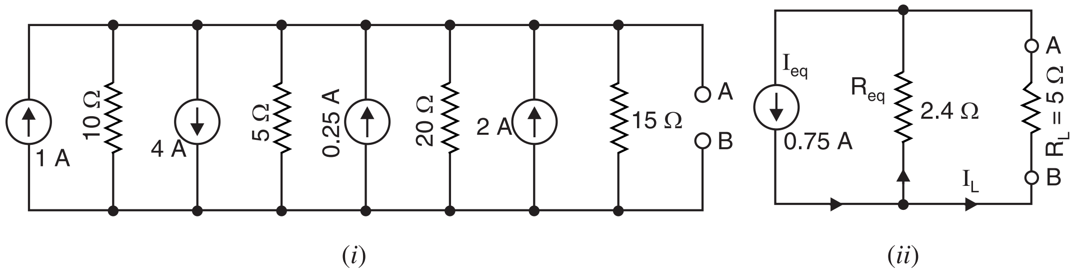


Fig. 3.226

The equivalent current source is found by algebraically adding the currents of individual sources.

$$I_{eq} = 1\text{ A}\uparrow + 4\text{ A}\downarrow + 0{\cdot}25\text{ A}\uparrow + 2\text{ A}\uparrow = 0{\cdot}75\text{ A}\downarrow$$

The downward arrow for I_{eq} shows that terminal A is negative *w.r.t.* terminal B.

$$R_{eq} = 10\ \Omega \parallel 5\ \Omega \parallel 20\ \Omega \parallel 15\ \Omega = 2{\cdot}4\ \Omega$$

Therefore, the equivalent current source consists of **0·75 A current source in parallel with 2·4 Ω resistor** as shown in Fig. 3.226 (*ii*). By current-divider rule, the load current I_L is

$$I_L = 0.75 \times \frac{2.4}{2.4 + 5} = \mathbf{0.243A}$$

Example 3.93. *Find the load current for Fig. 3.227 (i) using the dual of Millman's theorem.*

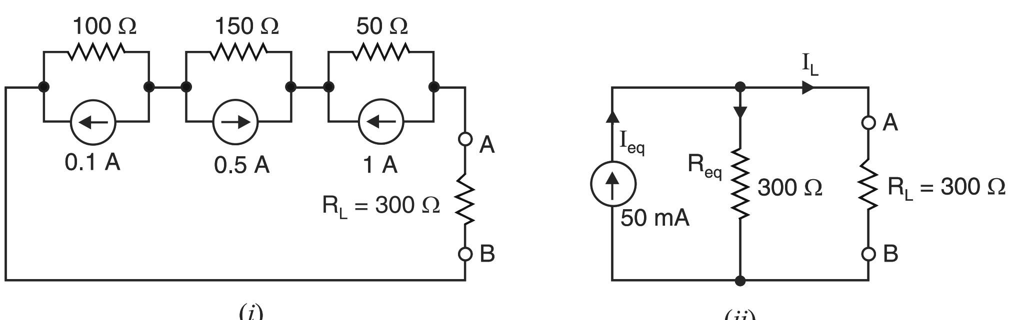


Fig. 3.227

Solution. There is a dual for Millman's theorem and it is useful for solving circuits with series current sources [See Fig. 3.227 (*i*)]. In such a case, the following equations are used to find the current and resistance of the equivalent circuit.

$$I_{eq} = \frac{I_1R_1 + I_2R_2 + I_3R_3}{R_1 + R_2 + R_3}$$

$$R_{eq} = R_1 + R_2 + R_3$$

Thus referring to Fig. 3.227 (*i*), we have,

$$I_{eq} = \frac{-0.1\times 100 + 0.5\times 150 - 1\times 50}{100+150+50} = \frac{15}{300}\text{A} = 50 \text{ mA}$$

$$R_{eq} = 100 + 150 + 50 = 300\ \Omega$$

The equivalent circuit is shown in Fig. 3.227 (*ii*). By current-divider rule, the load current I_L is

$$I_L = 50\times\frac{300}{300+300} = \mathbf{25\ mA}$$

Example 3.94. *By constructing a Millman equivalent voltage source with respect to terminals x – y, find the voltage across 40 Ω resistor in Fig. 3.228 (i).*

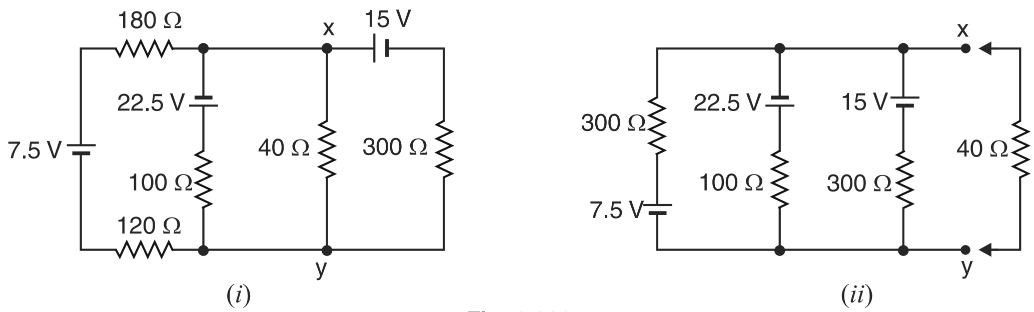


Fig. 3.228

Solution. Note that 120 Ω and 180 Ω resistors are in a series path and can therefore be combined into an equivalent resistance of 300 Ω. The circuit is *redrawn as shown in Fig. 3.228 (*ii*). It is clear that redrawn circuit has three parallel-connected voltage sources. Referring to Fig. 3.228 (*ii*), we have,

$$V_{xy} = V_{Th} = \frac{E_1/R_1 - E_2/R_2 + E_3/R_3}{1/R_1 + 1/R_2 + 1/R_3}$$

$$= \frac{7.5/300 - 22.5/100 + 15/300}{1/300 + 1/100 + 1/300} = \frac{-0.15}{0.0167} = -9\text{V}$$

Negative sign shows that terminal x is negative w.r.t. terminal y.

$$R_{Th} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3} = \frac{1}{1/300 + 1/100 + 1/300} = 60\ \Omega$$

Therefore, the equivalent voltage source consists of 9 V in series with 60 Ω resistor. When load is connected across the terminals of the equivalent voltage source, the circuit becomes as shown in Fig. 3.229.

$$\text{Load current, } I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{9}{60+40} = 0.09 \text{ A}$$

$$\text{Voltage across } 40\ \Omega = I_L R_L = 0{\cdot}09 \times 40 = \mathbf{3{\cdot}6\ V}$$

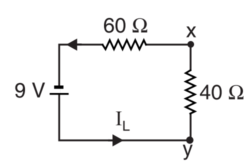


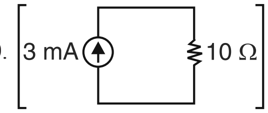
Fig. 3.229

Note that Millman's theorem is a powerful tool in the hands of engineers to solve many problems which cannot be solved easily by the usual methods of circuit analysis.

* It makes no difference on which side of each voltage source its series resistance is drawn.

Tutorial Problems

1. Find the single equivalent current source for the circuit shown in Fig. 3.230.



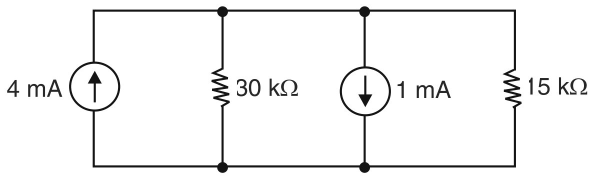


Fig. 3.230

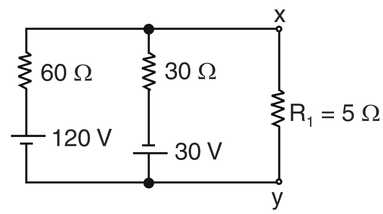
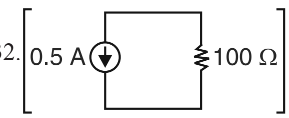


Fig. 3.231

2. By constructing a Millman equivalent voltage source at terminals $x - y$, find the voltage across R_1 (= 5 Ω) in the circuit shown in Fig. 3.231. **[4 V ±]**

3. Find the single equivalent current source for the circuit shown in Fig. 3.232.



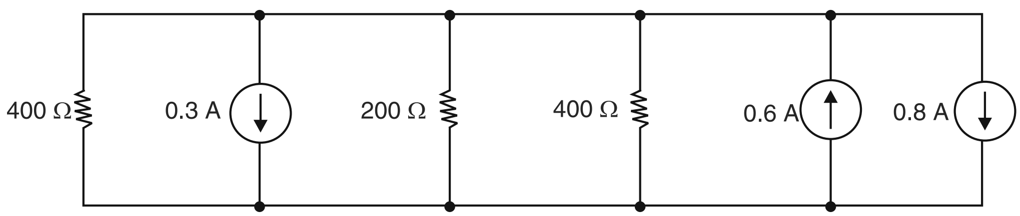


Fig. 3.232

4. What is the current flowing in the load resistor in Fig. 3.233 ? **[2·25 mA]**

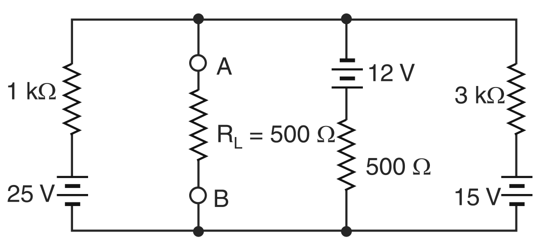


Fig. 3.233

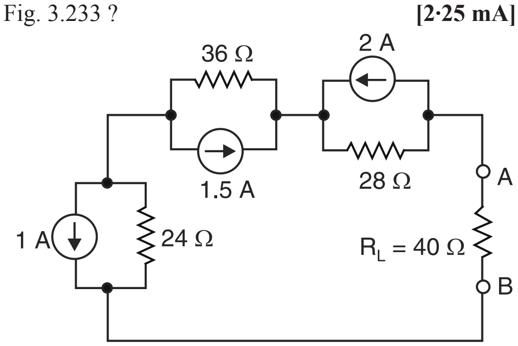


Fig. 3.234

5. What is the drop and polarity of the load in Fig. 3.234 ? **[8·13V and terminal *A* is negative]**

3.22. Compensation Theorem

It is sometimes necessary to know, when making a change in one branch of a network, what effect this change will have on the various currents and voltages throughout the network. The compensation theorem deals with this situation and may be stated for d.c. circuits as under :

The compensation theorem states that any resistance R in a branch of a network in which current I is flowing can be replaced, for the purpose of calculations, by a voltage equal to – IR. It follows from Kirchhoff's voltage law that the current *I* is unaltered if an e.m.f. – *IR* is substituted for the voltage drop *IR*.

Or

If the resistance of any branch of a network is changed from R to (R + ΔR) where the current was originally I, then the change of current at any point in the network may be calculated by assuming than an e.m.f. – IΔR has been introduced into the modified branch while all other sources have their e.m.f.s. suppressed and are represented by their internal resistances only.

Illustration. Let us illustrate the compensation theorem with a numerical example. Consider the circuit shown in Fig. 3.235 (*i*). The various branch currents in this circuit are :

$$I_1 = \frac{50}{20+5} = 2\text{ A} \;;\quad I_2 = I_3 = 1\text{ A}$$

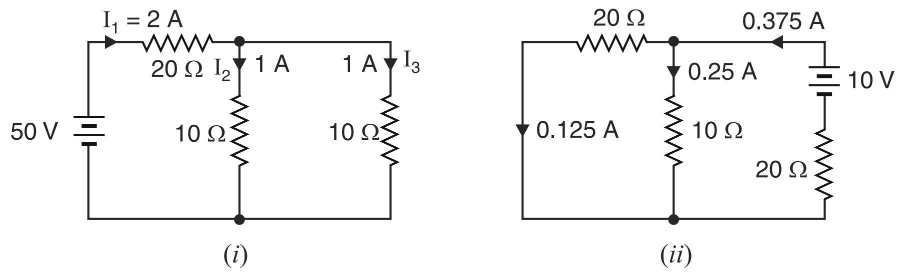


Fig. 3.235

Now suppose that the resistance of the right branch is increased to 20 Ω *i.e.* $\Delta R = 20 - 10 = 10\ \Omega$ and a voltage $V = -I_3\,\Delta R = -1 \times 10 = -10$ V is introduced in this branch and voltage source replaced by a short (∵ internal resistance is assumed zero). The circuit becomes as shown in Fig. 3.235 (*ii*). The compensating currents produced by this voltage are also indicated. When these compensating currents are algebraically added to the original currents in their respective branches, the new branch currents will be as shown in Fig. 3.236. The compensation theorem is useful in bridge and potentiometer circuits, where a slight change in one resistance results in a shift from a null condition.

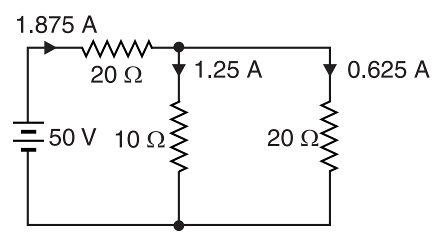


Fig. 3.236

3.23. Delta/Star and Star/Delta Transformation

There are some networks in which the resistances are neither in series nor in parallel. A familiar case is a three terminal network *e.g.* delta network or star network. In such situations, it is not possible to simplify the network by series and parallel circuit rules. However, converting delta network into star and *vice-versa* often simplifies the network and makes it possible to apply series-parallel circuit techniques.

3.24. Delta/Star Transformation

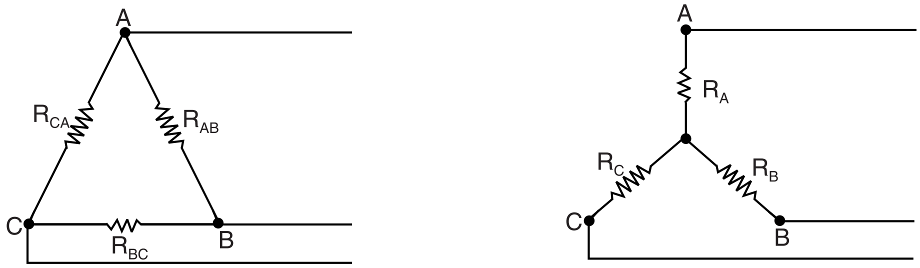
Consider three resistors R_{AB}, R_{BC} and R_{CA} connected in delta to three terminals *A, B* and *C* as shown in Fig. 3.237 (*i*). Let the equivalent star-connected network have resistances R_A, R_B and R_C. Since the two arrangements are electrically equivalent, the resistance between any two terminals of one network is equal to the resistance between the corresponding terminals of the other network.

Let us consider the terminals *A* and *B* of the two networks.

Resistance between *A* and *B* for star = Resistance between *A* and *B* for delta

or $$R_A + R_B = R_{AB} \,\|\, (R_{BC} + R_{CA})$$

or $$R_A + R_B = \frac{R_{AB}(R_{BC} + R_{CA})}{(R_{AB} + R_{BC} + R_{CA})} \qquad ...(i)$$



(*i*) (*ii*)

Fig. 3.237

Similarly, $$R_B + R_C = \frac{R_{BC}(R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} \quad ...(ii)$$

and $$R_C + R_A = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \quad ...(iii)$$

Subtracting eq. (*ii*) from eq. (*i*) and adding the result to eq. (*iii*), we have,

$$R_A = \frac{R_{AB}\,R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \quad ...(iv)$$

Similarly, $$R_B = \frac{R_{BC}\,R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \quad ...(v)$$

and $$R_C = \frac{R_{CA}\,R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \quad ...(vi)$$

How to remember ? There is an easy way to remember these relations. Referring to Fig. 3.238, star-connected resistances R_A, R_B and R_C are electrically equivalent to delta-connected resistances R_{AB}, R_{BC} and R_{CA}. We have seen above that :

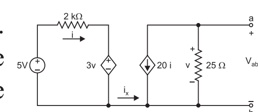


Fig. 3.238

$$R_A = \frac{R_{AB}\,R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

i.e. Any arm of star-connection $= \dfrac{\text{Product of two adjacent arms of } \Delta}{\text{Sum of arms of } \Delta}$

Thus to find the star resistance that connects to terminal *A*, *divide the product of the two delta resistors connected to A by the sum of the delta resistors*. Same is true for terminals *B* and *C*.

3.25. Star/Delta Transformation

Now let us consider how to replace the star-connected network of Fig. 3.237 (*ii*) by the equivalent delta-connected network of Fig. 3.237 (*i*).

Dividing eq. (*iv*) by (*v*), we have,

$$R_A/R_B = R_{CA}/R_{BC}$$

$$\therefore \quad R_{CA} = \frac{R_A\,R_{BC}}{R_B}$$

Dividing eq. (*iv*) by (*vi*), we have,

$$R_A / R_C = R_{AB} / R_{BC}$$

$$\therefore \qquad R_{AB} = \frac{R_A R_{BC}}{R_C}$$

Substituting the values of R_{CA} and R_{AB} in eq. (*iv*), we have,

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

Similarly,
$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B}$$

and
$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

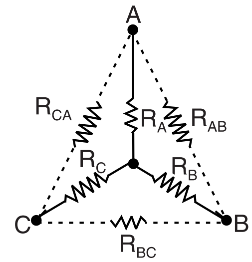


Fig. 3.239

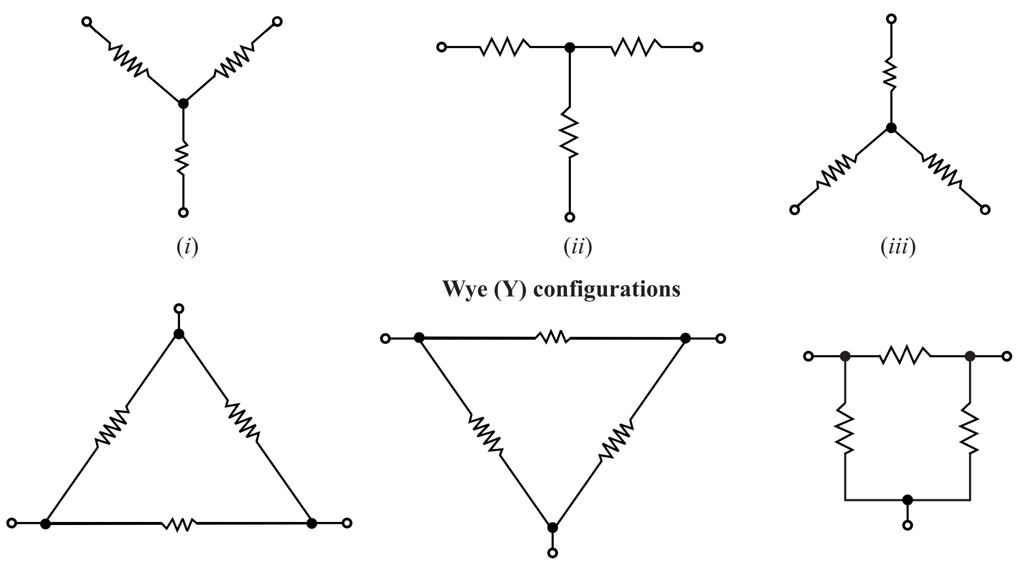
How to remember ? There is an easy way to remember these relations.

Referring to Fig. 3.239, star-connected resistances R_A, R_B and R_C are electrically equivalent to delta-connected resistances R_{AB}, R_{BC} and R_{CA}. We have seen above that :

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

i.e. Resistance between two terminals of delta = Sum of star resistances connected to those terminals *plus* product of same two resistances divided by the third star resistance

Note. Figs. 3.240 (*i*) to (*iii*) show three ways that a wye (Y) arrangement might appear in a circuit. Because the wye-connected components may appear in the equivalent form shown in Fig. 3.240 (*ii*), the arrangement is also called a *tee* (*T*) arrangement. Figs. 3.240 (*iv*) to (*vi*) show equivalent delta forms. Because the delta (Δ) arrangement may appear in the equivalent form shown in Fig. 3.240 (*vi*), it is also called a *pi* (π) arrangement. The figures show only a few of the ways the wye (*Y*) and delta (Δ) networks might be drawn in a schematic diagram. Many equivalent forms can be drawn by rotating these basic arrangements through various angles. Note that each network has three terminals.



(*i*) (*ii*) (*iii*)

Wye (Y) configurations

(*iv*) (*v*) (*vi*)

Delta (Δ) configurations

Fig. 3.240

Example 3.95. *Using delta/star transformation, find the galvanometer current in the Wheatstone bridge shown in Fig. 3.241 (i).*

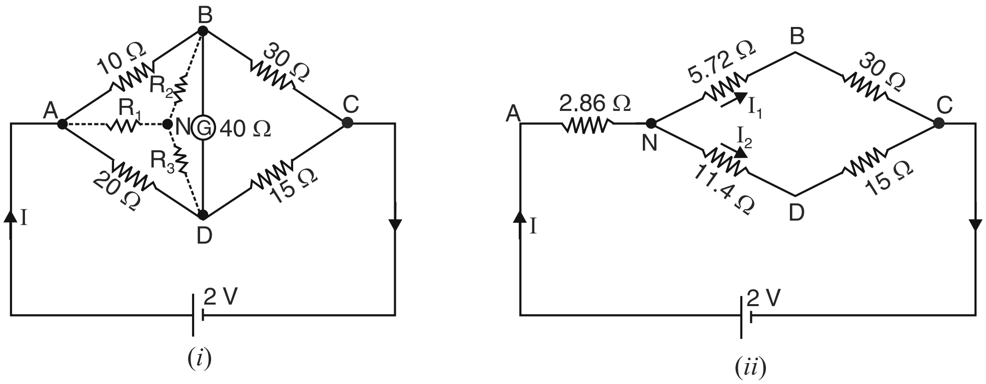


Fig. 3.241

Solution. The network *ABDA* in Fig. 3.241 (*i*) forms a delta. These delta-connected resistances can be replaced by equivalent star-connected resistances R_1, R_2 and R_3 as shown in Fig. 3.241 (*i*).

$$R_1 = \frac{R_{AB}\,R_{DA}}{R_{AB}+R_{BD}+R_{DA}} = \frac{10\times 20}{10+40+20} = 2.86\ \Omega$$

$$R_2 = \frac{R_{AB}\,R_{BD}}{R_{AB}+R_{BD}+R_{DA}} = \frac{10\times 40}{10+40+20} = 5.72\ \Omega$$

$$R_3 = \frac{R_{DA}\,R_{BD}}{R_{AB}+R_{BD}+R_{DA}} = \frac{20\times 40}{10+40+20} = 11.4\ \Omega$$

Thus the network shown in Fig. 3.241 (*i*) reduces to the network shown in Fig. 3.241 (*ii*).

$$R_{AC} = 2.86 + \frac{(30+5.72)\,(15+11.4)}{(30+5.72)+(15+11.4)} = 18.04\ \Omega$$

Battery current, $I = 2/18{\cdot}04 = 0{\cdot}11$ A

The battery current divides at N into two parallel paths.

$$\therefore \text{ Current in branch } NBC,\ I_1 = 0.11 \times \frac{26.4}{26.4+35.72} = 0{\cdot}047 \text{ A}$$

$$\text{Current in branch } NDC,\ I_2 = 0.11 \times \frac{35.72}{26.4+35.72} = 0{\cdot}063 \text{ A}$$

Potential of B *w.r.t.* C = $30 \times 0{\cdot}047 = 1{\cdot}41$ V

Potential of D *w.r.t.* C = $15 \times 0{\cdot}063 = 0{\cdot}945$ V

Clearly, point B is at higher potential than point D by

$$1{\cdot}41 - 0{\cdot}945 = 0{\cdot}465 \text{ V}$$

$$\therefore \quad \text{Galvanometer current} = \frac{\text{P.D. between } B \text{ and } D}{\text{Galvanometer resistance}}$$

$$= 0{\cdot}465 / 40 = 11{\cdot}6 \times 10^{-3} \text{ A} = \mathbf{11{\cdot}6\ mA} \textit{ from B to D}$$

Example 3.96. *With the help of star/delta transformation, obtain the value of current supplied by the battery in the circuit shown in Fig. 3.242 (i).*

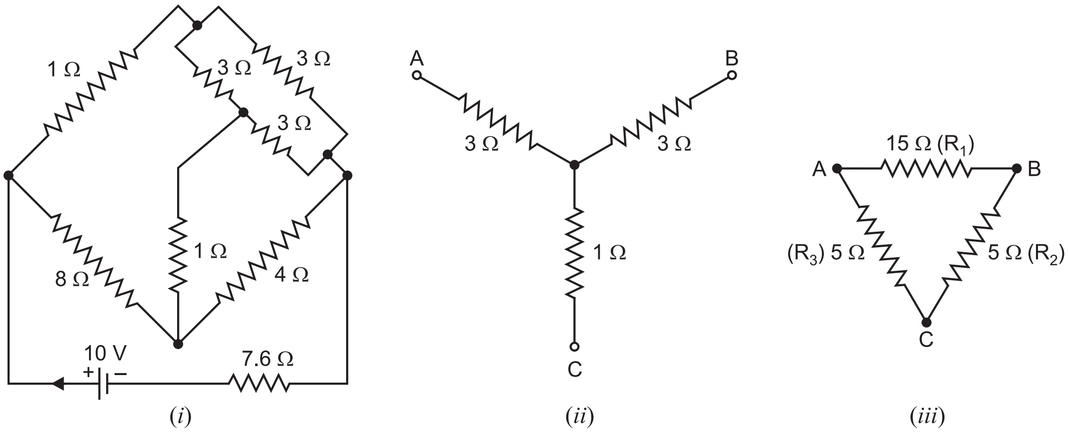


Fig. 3.242

Solution. The star-connected resistances 3 Ω, 3 Ω and 1 Ω in Fig. 3.242 (*i*), are shown separately in Fig. 3.242 (*ii*). These star-connected resistances are converted into equivalent delta-connected resistances R_1, R_2 and R_3 as shown in Fig. 3.242 (*iii*).

$$R_1 = 3+3+\frac{3\times 3}{1} = 15\ \Omega$$

$$R_2 = 3+1+\frac{3\times 1}{3} = 5\ \Omega$$

$$R_3 = 1+3+\frac{1\times 3}{3} = 5\ \Omega$$

After above star-delta conversion, the circuit reduces to the one shown in Fig. 3.242 (*iv*). This circuit can be further simplified by combining parallel resistances and the circuit becomes as shown in Fig. 3.242 (*v*).

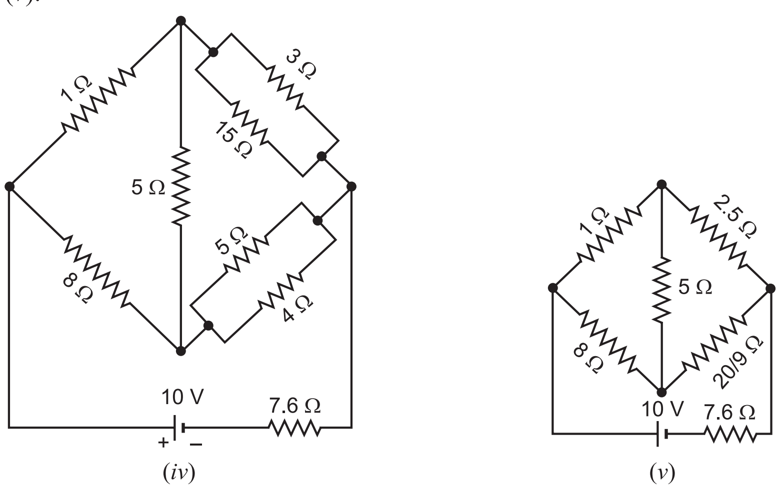


Fig. 3.242

The three delta-connected resistances 1 Ω, 5 Ω and 8 Ω in Fig. 3.242 (*v*) are shown separately in Fig. 3.242 (*vi*). These delta-connected resistances can be converted into equivalent star-connected resistances R'_1, R'_2 and R'_3 as shown in Fig. 3.242 (*vii*).

$$R'_1 = \frac{1\times8}{1+5+8} = \frac{4}{7}\Omega$$

$$R'_2 = \frac{5\times1}{1+5+8} = \frac{5}{14}\Omega$$

$$R'_3 = \frac{8\times5}{1+5+8} = \frac{20}{7}\Omega$$

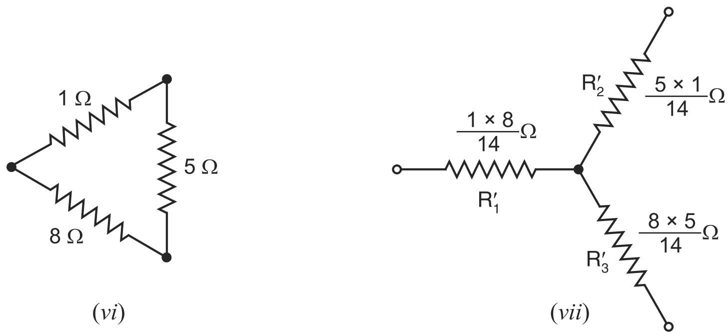


Fig. 3.242

After above delta-star conversion, the circuit reduces to the one shown in Fig. 3.242 (*viii*).

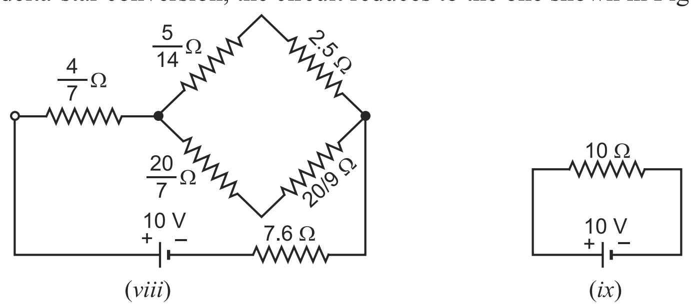


Fig. 3.242

Total resistance offered by the circuit to the battery is

$$R_T = \frac{4}{7} + \left[\left(\frac{5}{14} + 2.5\right) \| \left(\frac{20}{7} + \frac{20}{9}\right)\right] + 7.6$$

$$= \frac{4}{7} + \left(\frac{20}{7} \| \frac{320}{63}\right) + 7.6 = 10\,\Omega$$

∴ Current supplied by the battery [See Fig. 3.242 (*ix*)] is

$$I = \frac{V}{R_T} = \frac{10}{10} = \mathbf{1\ A}$$

Example 3.97. *A network of resistors is shown in Fig. 3.243 (i). Find the resistance (i) between terminals A and B (ii) B and C and (iii) C and A.*

Solution. The star-connected resistances 6 Ω, 3 Ω and 4 Ω in Fig. 3.243 (*i*) are shown separately in Fig. 3.243 (*ii*). These star-connected resistances can be converted into equivalent delta-connected resistances R_1, R_2 and R_3 as shown in Fig. 3.243 (*ii*).

$$R_1 = 4 + 6 + (4 \times 6/3) = 18\,\Omega$$

$$R_2 = 6 + 3 + (6 \times 3/4) = 13{\cdot}5\,\Omega$$

$$R_3 = 4 + 3 + (4 \times 3/6) = 9\,\Omega$$

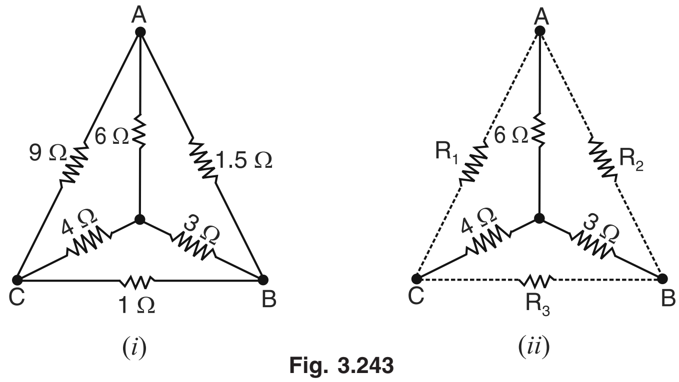


Fig. 3.243

These delta-connected resistances R_1, R_2 and R_3 come in parallel with the original delta-connected resistances. The circuit shown in Fig. 3.243 (*i*) reduces to the circuit shown in Fig. 3.244(*i*).

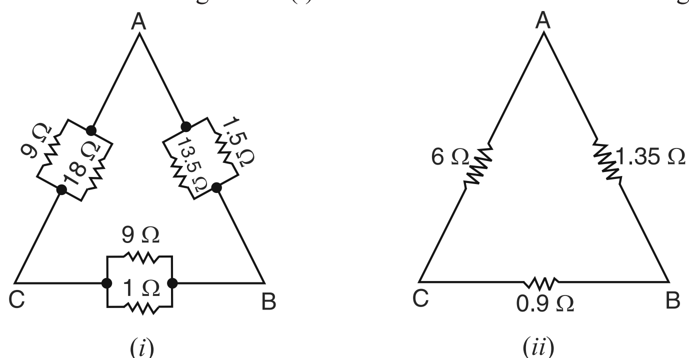


Fig. 3.244

The parallel resistances in each leg of delta in Fig. 3.244 (*i*) can be replaced by a single resistor as shown in Fig. 3.244 (*ii*) where

$$R_{AC} = 9 \times 18/27 = 6\ \Omega$$
$$R_{BC} = 9 \times 1/10 = 0{\cdot}9\ \Omega$$
$$R_{AB} = 1{\cdot}5 \times 13{\cdot}5/15 = 1{\cdot}35\ \Omega$$

(*i*) Resistance between *A* and *B* $= 1{\cdot}35\ \Omega \parallel (6 + 0{\cdot}9)\ \Omega = 1{\cdot}35 \times 6{\cdot}9/8{\cdot}25 =$ **1·13 Ω**

(*ii*) Resistance between *B* and *C* $= 0{\cdot}9\ \Omega \parallel (6 + 1{\cdot}35)\ \Omega = 0{\cdot}9 \times 7{\cdot}35/8{\cdot}25 =$ **0·8 Ω**

(*iii*) Resistance between *A* and *C* $= 6\ \Omega \parallel (1{\cdot}35 + 0{\cdot}9)\ \Omega = 6 \times 2{\cdot}25/8{\cdot}25 =$ **1.636 Ω**

Example 3.98. *Determine the load current in branch EF in the circuit shown in Fig. 3.245 (i).*

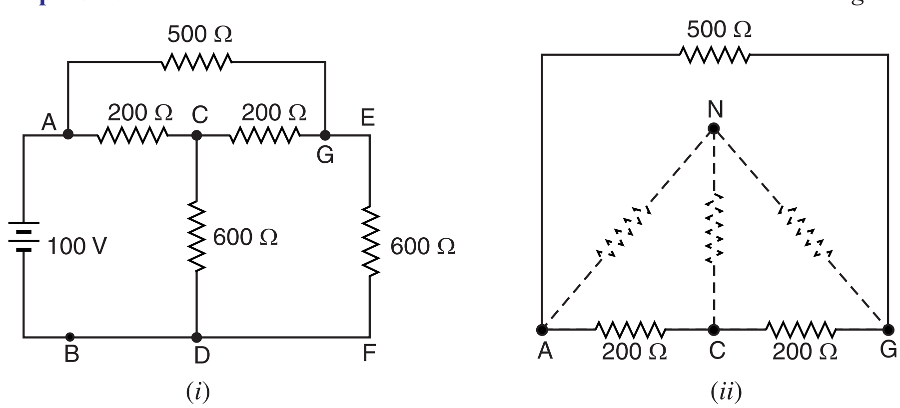


Fig. 3.245

Solution. The circuit *ACGA* forms delta and is shown separately in Fig. 3.245 (*ii*) for clarity. Changing this delta connection into equivalent star connection [See Fig. 3.245 (*ii*)], we have,

$$R_{AN} = \frac{500 \times 200}{500 + 200 + 200} = 111.11\ \Omega \quad ; \quad R_{CN} = \frac{200 \times 200}{500 + 200 + 200} = 44.44\ \Omega \ ;$$

$$R_{GN} = \frac{500 \times 200}{500 + 200 + 200} = 111.11\ \Omega$$

Thus the circuit shown in Fig. 3.245 (*i*) reduces to the circuit shown in Fig. 3.246 (*i*). The branch *NEF* (= 111·11 + 600 = 711·11 Ω) is in parallel with branch *NCD* (= 44·44 + 600 = 644·44 Ω) and the equivalent resistance of this parallel combination is

$$= \frac{711.11 \times 644.44}{711.11 + 644.44} = 338\ \Omega$$

The circuit shown in Fig. 3.246 (*i*) reduces to the circuit shown in Fig. 3.246 (*ii*).

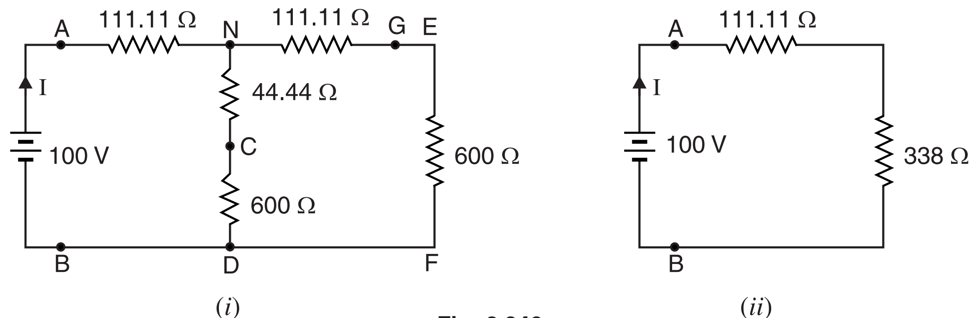


Fig. 3.246

$$\therefore \qquad \text{Battery current, } I = \frac{100}{338 + 111.11} = 0.222 \text{ A}$$

This battery current divides into two parallel paths [See Fig. 3.246 (*i*)] *viz*. branch *NEF* and branch *NCD*.

∴ Current in branch *NEF i.e.* in branch *EF*

$$= 0.222 \times \frac{644.44}{711.11 + 644.44} = \mathbf{0.1055\ A}$$

Example 3.99. *A square and its diagonals are made of a uniform covered wire. The resistance of each side is 1 Ω and that of each diagonal is 1·414 Ω. Determine the resistance between two opposite corners of the square.*

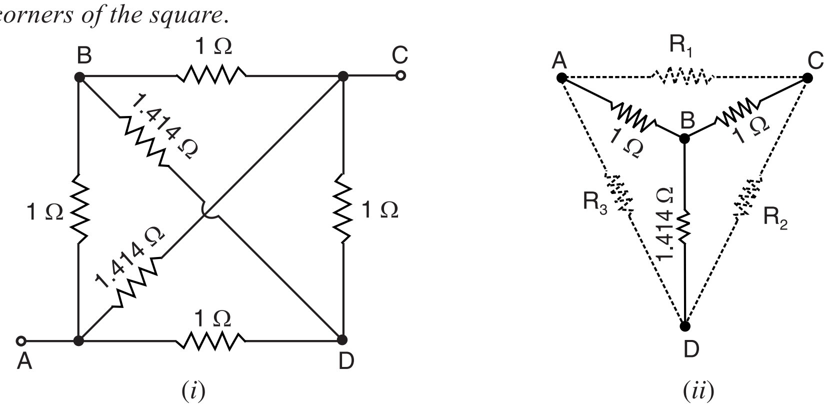


Fig. 3.247

Solution. Fig. 3.247 (*i*) shows the given square. It is desired to find the resistance between terminals *A* and *C*. The star-connected resistances 1 Ω, 1 Ω and 1·414 Ω (with star point at *B*) are shown separately in Fig. 3.247 (*ii*). These star-connected resistances can be converted into equivalent delta connected resistances R_1, R_2 and R_3 as shown in Fig. 3.247 (*ii*) where

$$R_1 = R_{AB} + R_{BC} + \frac{R_{AB} \cdot R_{BC}}{R_{BD}}$$

$$= 1 + 1 + \frac{1 \times 1}{1.414} = 2.7\ \Omega$$

$$R_2 = 1 + 1.414 + \frac{1 \times 1.414}{1} = 3{\cdot}83\ \Omega$$

$$R_3 = 1 + 1.414 + \frac{1 \times 1.414}{1} = 3{\cdot}83\ \Omega$$

The circuit shown in Fig. 3.247 (*i*) then reduces to the circuit shown in Fig. 3.248 (*i*). Note that R_1 comes in parallel with 1·414 Ω connected between *A* and *C*; R_2 comes in parallel with 1 Ω connected between *C* and *D* and R_3 comes in parallel with 1 Ω connected between *A* and *D*.

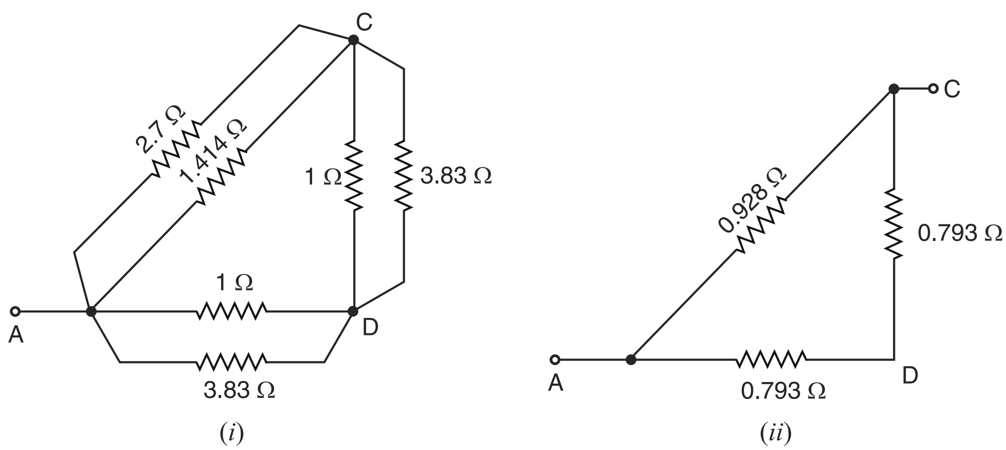


Fig. 3.248

In Fig. 3.248 (*i*), branch *AD* has 1 Ω and 3·83 Ω resistances in parallel.

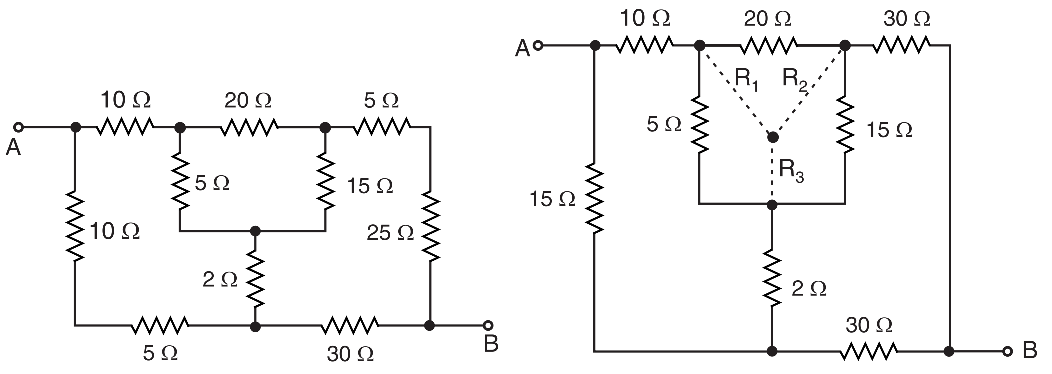
∴ $$R_{AD} = \frac{1 \times 3.83}{1 + 3.83} = 0.793\ \Omega\ ;\ R_{CD} = \frac{1 \times 3.83}{1 + 3.83} = 0.793\ \Omega\ ;$$

$$R_{AC} = \frac{2.7 \times 1.414}{2.7 + 1.414} = 0.928\ \Omega$$

∴ Resistance between terminals *A* and *C* [See Fig. 3.248 (*ii*)]

$$= 0{\cdot}928 \parallel (0{\cdot}793 + 0{\cdot}793) = 0{\cdot}928 \times 1{\cdot}586/2{\cdot}514 = \mathbf{0{\cdot}585\ \Omega}$$

Example 3.100. *Determine the resistance between the terminals A and B of the network shown in Fig. 3.249 (i).*



(*i*) (*ii*)

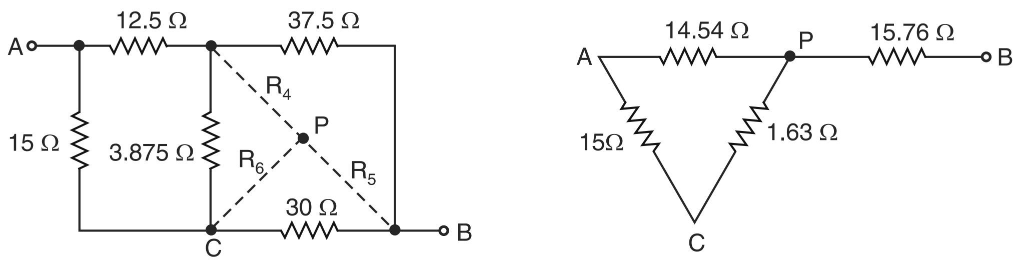
Fig. 3.249

Solution. We can combine series resistances on the right and left of Fig. 3.249 (*i*). The circuit then reduces to the one shown in Fig. 3.249 (*ii*). The resistances 5 Ω, 20 Ω and 15 Ω form a delta circuit and can be replaced by a star network where

$$R_1 = \frac{\text{Product of two adjacent arms of delta}}{\text{Sum of arms of delta}} = \frac{20 \times 5}{5 + 20 + 15} = \frac{100}{40} = 2.5\ \Omega\ ;$$

$$R_2 = \frac{20 \times 15}{40} = 7.5\ \Omega \quad ; \qquad R_3 = \frac{5 \times 15}{40} = 1.875\ \Omega$$

Referring to Fig. 3.249 (*ii*), R_1 is in series with 10 Ω resistor and their total resistance is $10 + R_1 = 10 + 2{\cdot}5 = 12{\cdot}5\ \Omega$. Similarly, we have $30 + R_2 = 30 + 7{\cdot}5 = 37{\cdot}5\ \Omega$ and $2 + R_3 = 2 + 1{\cdot}875 = 3{\cdot}875\ \Omega$. The circuit then reduces to the one shown in Fig. 3.249 (*iii*).



(*iii*) (*iv*)

Fig. 3.249

Referring to Fig. 3.249 (*iii*), 3·875 Ω, 37·5 Ω and 30 Ω form a delta network and can be reduced to star network where

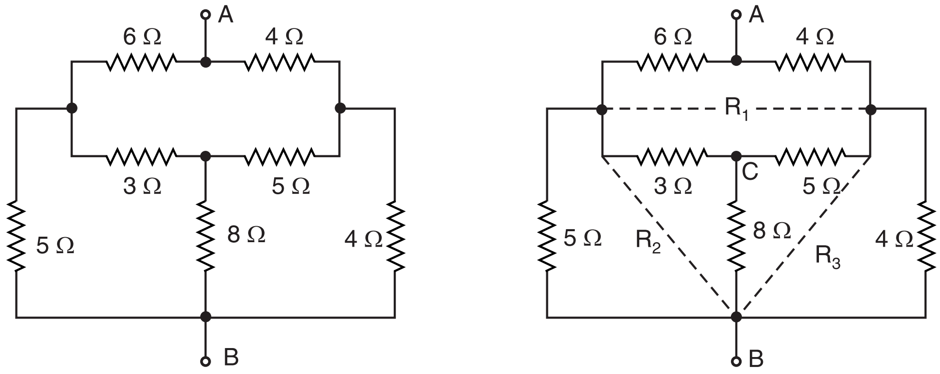
$$R_4 = \frac{3.875 \times 37.5}{3.875 + 37.5 + 30} = \frac{3.875 \times 37.5}{71.375} = 2.04\ \Omega\ ;$$

$$R_5 = \frac{37.5 \times 30}{71.375} = 15.76\ \Omega\ ;\ R_6 = \frac{3.875 \times 30}{71.375} = 1.63\ \Omega$$

Referring to Fig. 3.249 (*iii*), R_4 is in series with 12.5 Ω resistor and their combined resistance = $R_4 + 12{\cdot}5 = 2{\cdot}04 + 12{\cdot}5 = 14{\cdot}54\ \Omega$. The circuit then reduces to the one shown in Fig. 3.249 (*iv*). The resistance between terminals *A* and *B* is given by ;

$$R_{AB} = 15.76 + [14.54 \parallel (15 + 1.63)] = 15.76 + \frac{14.54 \times 16.63}{31.17} = \mathbf{23{\cdot}5\ \Omega}$$

Example 3.101. *Determine the resistance between points A and B in the network shown in Fig. 3.250 (i).*



(*i*) (*ii*)

Fig. 3.250

Solution. The 3 Ω, 5 Ω and 8 Ω form star network and can be replaced by delta network where

Resistance between two terminals of delta = Sum of star resistances connected to those terminals *plus* product of same two resistances divided by the third star resistance.

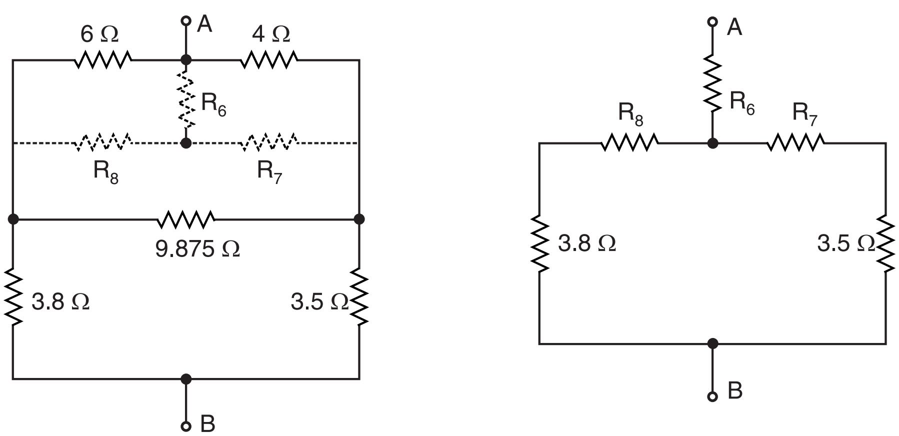
$$\therefore \qquad R_1 = 3 + 5 + \frac{3 \times 5}{8} = 9{\cdot}875\ \Omega$$

$$R_2 = 3 + 8 + \frac{3 \times 8}{5} = 15{\cdot}8\ \Omega$$

$$R_3 = 5 + 8 + \frac{5 \times 8}{3} = 26{\cdot}3\ \Omega$$

Referring to Fig. 3.250 (*ii*), 5 Ω resistor is in parallel with R_2 (= 15·8 Ω) and their combined resistance is 3·8 Ω. Similarly, 4 Ω resistor is in parallel with R_3 (= 26·3 Ω) and their combined resistance is 3·5 Ω. The circuit then reduces to the one shown in Fig. 3.250 (*iii*). Referring to Fig. 3.250 (*iii*), 6 Ω, 4 Ω and 9·875 Ω form a delta network and can be replaced by star network where

$$R_6 = \frac{6 \times 4}{6 + 4 + 9.875} = \frac{24}{19.875} = 1.2\ \Omega \ ; \ R_7 = \frac{9.875 \times 4}{19.875} = 1.99\ \Omega \ ; \ R_8 = \frac{9.875 \times 6}{19.875} = 2.98\ \Omega$$



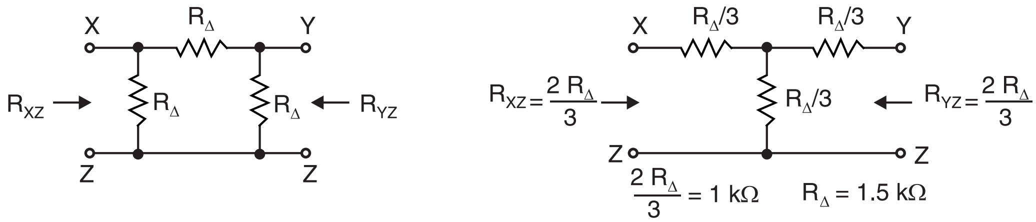
(*iii*) (*iv*)

Fig. 3.250

Therefore, the circuit shown in Fig. 3.250 (*iii*) reduces to the one shown in Fig. 3.250 (*iv*). It is clear that :

$$R_{AB} = (3{\cdot}8 + R_8) \parallel (R_7 + 3{\cdot}5) + R_6 = (3{\cdot}8 + 2{\cdot}98) \parallel (1{\cdot}99 + 3{\cdot}5) + 1{\cdot}2$$
$$= (6{\cdot}78 \parallel 5{\cdot}49) + 1{\cdot}2 = \mathbf{4{\cdot}23\ \Omega}$$

Example 3.102. *A π network is to be constructed as shown in Fig. 3.251 (i) so that the resistance R_{XZ} looking into the X – Z terminals (with Y – Z open) equals the resistance R_{YZ} looking into the Y – Z terminals (with X – Z open). If that resistance must equal 1 kΩ, find the value of R_Δ that should be used in the π network.*



(*i*) (*ii*)

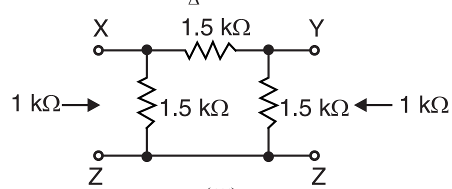
Fig. 3.251

Solution. The delta network shown in Fig. 3.251 (*i*) can be converted into star network as shown in Fig. 3.251 (*ii*). Note that the star network has equal-valued resistors $R_\Delta/3$. It is clear from this figure that :

$$R_{XZ} = R_{YZ} = \frac{R_\Delta}{3} + \frac{R_\Delta}{3} = \frac{2R_\Delta}{3}$$

or $$1\ \text{k}\Omega = \frac{2R_\Delta}{3} \quad \text{or} \quad R_\Delta = \mathbf{1.5\ k\Omega}$$

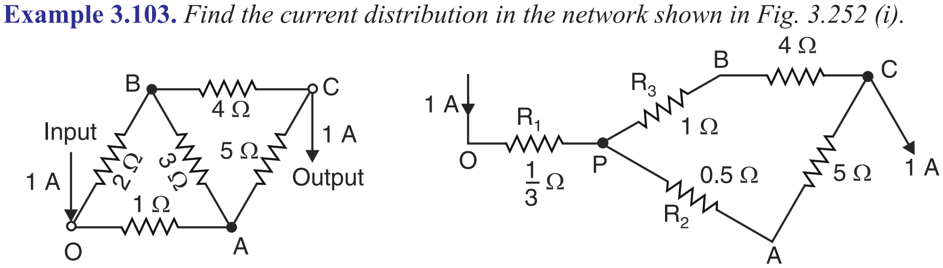
Therefore, the π network must have three 1.5 kΩ resistors as shown in Fig 3.251 (*iii*).



(*iii*)

Fig. 3.251

Example 3.103. *Find the current distribution in the network shown in Fig. 3.252 (i).*



(*i*) (*ii*)

Fig. 3.252

Solution. The network *OAB* forms a delta and can be replaced by star where :

$$R_1 = \frac{1 \times 2}{6} = \frac{1}{3}\Omega \quad ; \quad R_2 = \frac{1 \times 3}{6} = 0.5\ \Omega \quad ; \quad R_3 = \frac{2 \times 3}{6} = 1\ \Omega$$

The network then reduces to the one shown in Fig. 3.252 (*ii*). The current through *OP* is 1 A and divides between two parallel paths at point *P*. By current-divider rule :

$$\text{Current in } PA = \text{Current in } AC = 1 \times \frac{5}{1 + 4 + 0.5 + 5} = 1 \times \frac{5}{10.5} = \mathbf{0.477\ A}$$

$$\text{Current in } PB = \text{Current in } BC = 1 - 0.477 = \mathbf{0.523A}$$

$$\text{Voltage drop in } PB = 1 \times 0{\cdot}523 = 0.523 \text{ V}$$

$$\text{Voltage drop in } PA = 0{\cdot}5 \times 0{\cdot}477 = 0.238 \text{ V}$$

$$\therefore \quad V_{AB} = 0{\cdot}523 - 0{\cdot}238 = 0.285 \text{ V}$$

$$\therefore \quad I_{AB} = 0{\cdot}285/3 = \mathbf{0.095\ A}$$

$$\text{Current in } OB = \text{Current in } BC - \text{Current in } AB$$

$$= 0.523 - 0.095 = \mathbf{0.428\ A}$$

$$\text{Current in } OA = 1 - 0.428 = \mathbf{0.572\ A}$$

Example 3.104. *Find the current in 10 Ω resistor in the network shown in Fig. 3.253 (i) by star-delta transformation.*

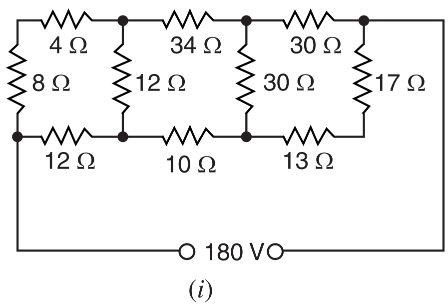


Fig. 3.253

Solution. In Fig. 3.253 (*i*), the 4 Ω and 8 Ω resistors are in series and their total resistance is 8 + 4 = 12 Ω. Similarly, at the right end of figure, 17 Ω and 13 Ω are in series so that their total resistance becomes 17 + 13 = 30 Ω. The circuit then reduces to the one shown in Fig. 3.253 (*ii*). Replacing the two deltas at the left end and right end in Fig. 3.253 (*ii*) by their equivalent star, we get the circuit shown in Fig. 3.253 (*iii*).

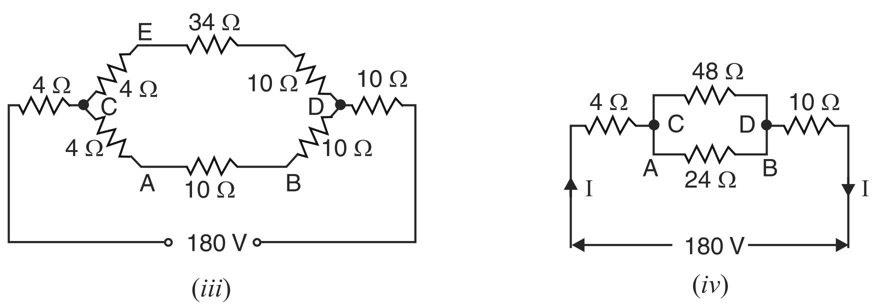


Fig. 3.253

Referring to Fig. 3.253 (*iii*), the path *CED* has resistance = 4 + 34 + 10 = 48 Ω and path *CABD* has resistance = 4 + 10 + 10 = 24 Ω. The circuit then reduces to the one shown in Fig. 3.253 (*iv*). The total resistance R_T presented to 180V source is

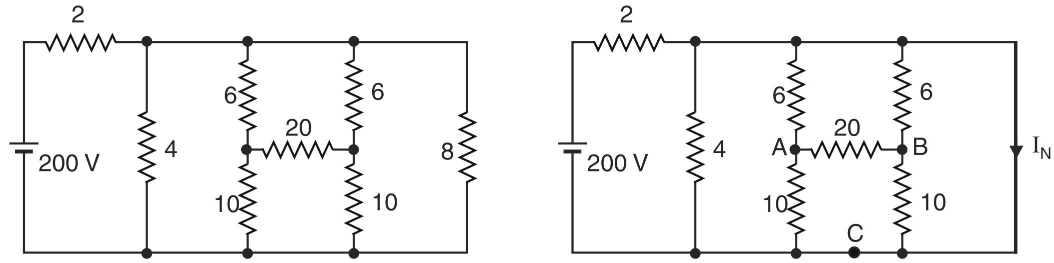
$$R_T = 4 + (48 \,||\, 24) + 10 = 30\ \Omega$$

$$\therefore \quad \text{Circuit current, } I = 180/30 = 6 \text{ A}$$

∴ Voltage across parallel combination = $I \times (48 \,||\, 24) = 6 \times 16 = 96$ V

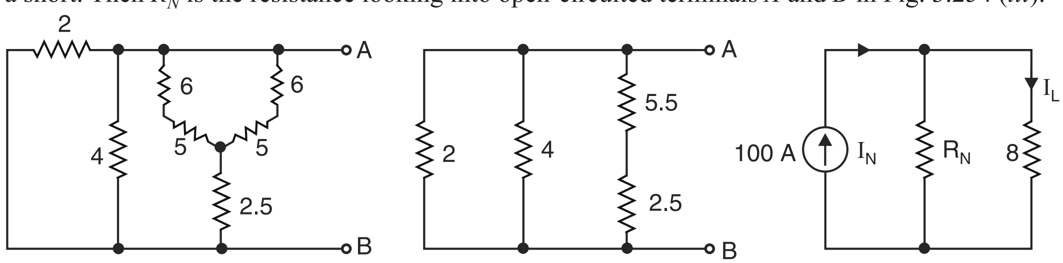
∴ Current in 10 Ω resistor [part of 24 Ω in Fig. 3.253 (*iv*)] = 96/24 = **4 A**

Example 3.105. *Using Norton's theorem, find the current through the 8 Ω resistor shown in Fig 3.254 (i). All resistance values are in ohms.*



(*i*) **Fig. 3.254** (*ii*)

Solution. In order to find Norton current I_N, place short circuit across the load of 8 Ω resistor. The circuit then becomes as shown in Fig. 3.254 (*ii*). The short circuit bypasses all the resistors except 2 Ω resistor. Therefore, $I_{SC} = I_N = 200/2 = 100$ A. In order to find R_N, replace 200 V source by a short. Then R_N is the resistance looking into open-circuited terminals *A* and *B* in Fig. 3.254 (*iii*).



(*iii*) (*iv*) (*v*)

Fig. 3.254

In Fig. 3.254 (*ii*), *ABC* network forms a delta and can be replaced by equivalent star network as shown in Fig. 3.254 (*iii*). This circuit reduces to the one shown in Fig. 3.254 (*iv*).

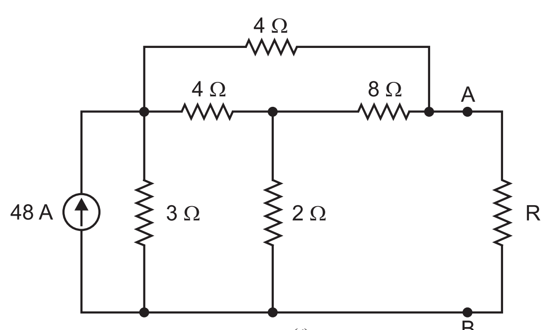
Norton's resistance, R_N = Resistance at the open-circuited terminals in Fig. 3.254 (*iv*)

$$= 2 \,\|\, 4 \,\|\, (5{\cdot}5 + 2{\cdot}5) = 8/7\ \Omega$$

Therefore, Norton equivalent circuit consists of 100A current source in parallel with a resistance of 8/7 Ω. When load R_L (= 8 Ω) is connected at the output terminals of Norton's equivalent circuit, the circuit becomes as shown in Fig 3.254 (*v*). By current-divider rule, the load current I_L through R_L (= 8 Ω) is given by ;

$$I_L = 100 \times \frac{8/7}{8 + (8/7)} = \mathbf{12{\cdot}5\ A}$$

Example 3.106. *In the network shown in Fig. 3.255 (i), find (i) Norton equivalent circuit at terminals AB (ii) the maximum power that can be provided to a resistor R connected between terminals A and B.*



(*i*)

Fig. 3.255

Solution. (*i*) The star- connected resistances 4 Ω, 8 Ω and 2 Ω in Fig. 3.255 (*i*) can be converted into equivalent delta-connected resistances R_{ab}, R_{bc} and R_{ca} as shown in Fig. 3.255 (*ii*).

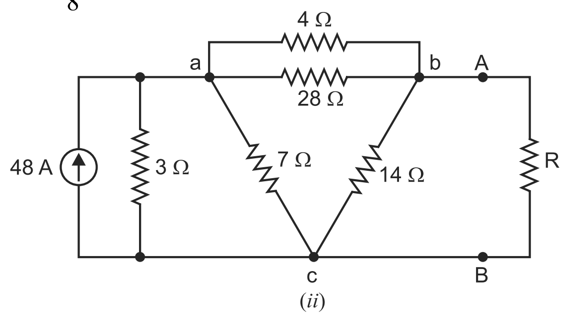
$$R_{ab} = 4+8+\frac{4\times 8}{2} = 28\ \Omega$$

$$R_{bc} = 8+2+\frac{8\times 2}{4} = 14\ \Omega$$

$$R_{ca} = 2+4+\frac{2\times 4}{8} = 7\ \Omega$$

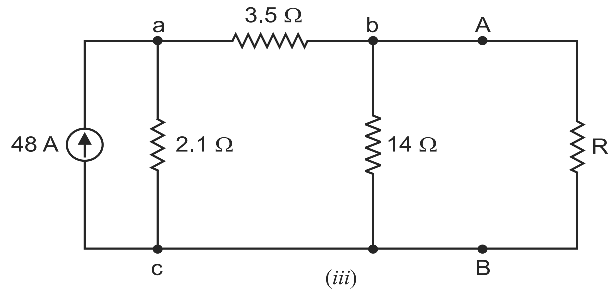
After above star-delta conversion, the circuit reduces to the one shown in Fig. 3.255 (*ii*). We can further simplify the circuit in Fig. 3.255 (*ii*) by combining the parallel resistances (4 Ω || 28 Ω = 3.5 Ω and 3 Ω || 7 Ω = 2.1 Ω). The circuit then becomes as shown in Fig. 3.255 (*iii*). We now convert 48A current source in parallel with 2.1Ω resistance in Fig. 3.255 (*iii*) into equivalent voltage source of 48 A × 2.1 Ω = 100.8 V in series with 2.1Ω resistance. The circuit then becomes as shown in Fig. 3.255 (*iv*). In order to find Norton current I_N, we short circuit terminals *A* and *B* in Fig. 3.255 (*iv*) and get the circuit of Fig. 3.255 (*v*). Then current in the short-circuit is I_N. Referring to Fig. 3.255 (*v*) and applying Ohm's law, the value of I_N is given by ;

$$I_N = \frac{100.8}{2.1+3.5} = 18\text{A}$$



(*ii*)

Fig. 3.255



(*iii*)

Fig. 3.255

Note that no current will pass through 14 Ω resistor in Fig. 3.255 (*v*). It is because there is a short across this resistor and the entire current (= I_N) will pass through the short.

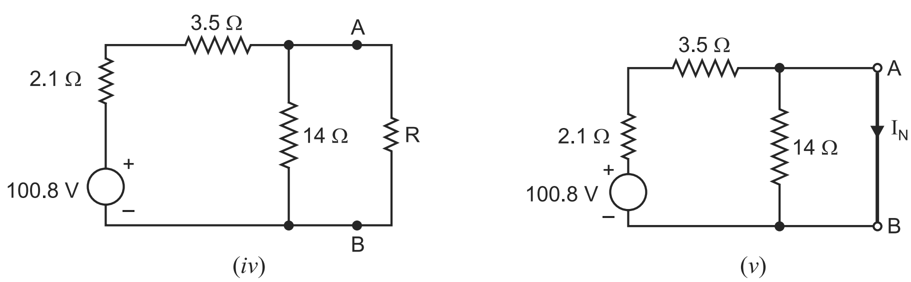
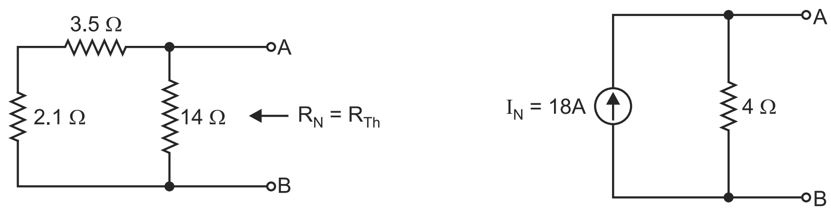


Fig. 3.255

In order to find Norton resistance $R_N(= R_{Th})$, we open circuit the terminals *AB* and replace the voltage source by a short in Fig. 3.255 (*iv*). The circuit then becomes as shown in Fig. 3.255 (*vi*).

∴ R_N = Resistance at terminals *AB* in Fig. 3.255 (*vi*)

$= (3.5 + 2.1)\ \Omega \parallel 14\ \Omega = 5.6\ \Omega \parallel 14\ \Omega = 4\ \Omega$



(*vi*) (*vii*)

Fig. 3.255

The Norton equivalent circuit at terminals *AB* is shown in Fig. 3.255 (*vii*).

(*ii*) Maximum power will be provided to resistance *R* connected between terminals *A* and *B* when resistance *R* is equal to Norton resistance R_N *i.e.*

$$R = R_N = 4\ \Omega$$

When $R(= 4\ \Omega)$ is connected across terminals *A* and *B* in Fig. 3.255 (*vii*), then by current-divider rule,

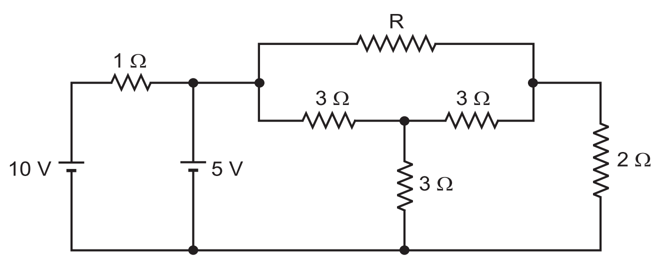
$$\text{Current in } R\ (= 4\ \Omega),\quad I = 18 \times \frac{4}{4+4} = 9\text{A}$$

∴ Maximum power (P_{max}) provided to *R* is

$$P_{max} = I^2R = (9)^2 \times 4 = \mathbf{324\ W}$$

Remember that under the condition of maximum power transfer, the circuit efficiency is *only* 50% and the remaining 50% is dissipated in the circuit.

Example 3.107. *Determine a non-negative value of R such that the power consumed by the 2 Ω resistor in Fig. 3.256 (i) is maximum.*

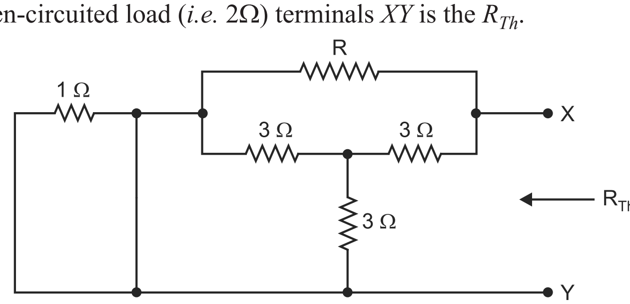


(*i*)

Fig. 3.256

Solution. In order to find maximum power consumed in 2 Ω resistor (*i.e.* load), we should find Thevenin resistance R_{Th} at 2 Ω terminals. For this purpose, we open circuit the load terminals (*i.e.* remove 2 Ω resistor) and short circuit the voltage sources as shown in Fig. 3.256 (*ii*). The resistance at the open-circuited load (*i.e.* 2Ω) terminals *XY* is the R_{Th}.

R_{Th} = Resistance at terminals *XY* in Fig. 3.256 (*ii*).



(*ii*)

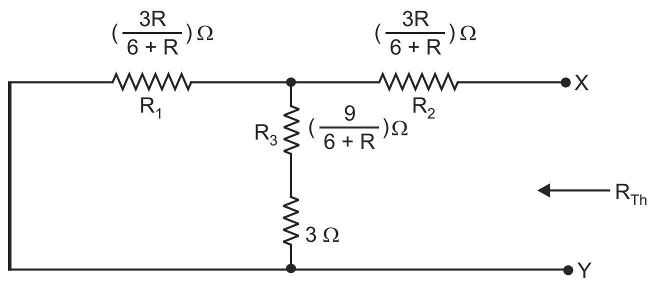
Fig. 3.256

In order to facilitate the determination of R_{Th}, we convert delta-connected resistances *R* Ω, 3 Ω and 3 Ω in Fig. 3.256 (*ii*) into equivalent star-connected resistances R_1, R_2 and R_3 as shown in Fig. 3.256 (*iii*). The values of R_1, R_2 and R_3 are given by ;

$$R_1 = \frac{3 \times R}{3+3+R} = \frac{3R}{6+R}$$

$$R_2 = \frac{3 \times R}{3+3+R} = \frac{3R}{6+R}$$

$$R_3 = \frac{3 \times 3}{3+3+R} = \frac{9}{6+R}$$



(*iii*)

Fig. 3.256

After above delta-star conversion, the circuit becomes as shown in Fig. 3.256 (*iii*). Then resistance at open-circuited terminals *XY* is R_{Th}.

Referring to Fig. 3.256 (*iii*),

$$R_{Th} = \left[\left(\frac{3R}{6+R}\right) \| \left(\frac{9}{6+R}+3\right)\right] + \frac{3R}{6+R}$$

$$= \left[\frac{3R}{6+R} \| \frac{27+3R}{6+R}\right] + \frac{3R}{6+R}$$

$$= \frac{3R \times (27+3R)}{(6+R)\,(27+3R+3R)} + \frac{3R}{6+R}$$

For maximum power in 2 Ω, the value of R_{Th} should be equal to 2 Ω.

$$\therefore \quad \frac{3R \times (27+3R)}{(6+R)(27+3R+3R)} + \frac{3R}{6+R} = 2$$

or
$$\frac{3R \times (27+3R)}{27+6R} + 3R = 2(6+R)$$

or
$$5R^2 + 12R - 108 = 0 \qquad \text{...after simplification}$$

$$\therefore \quad R = +3.6\,\Omega \quad \text{or} \quad -6\,\Omega$$

Accepting the positive value, R = **3.6 Ω**.

Tutorial Problems

1. Find the total current drawn from the voltage source and the current through R_1 (= 1 Ω) in the circuit shown in Fig. 3.257. **[4 A ; 2 A]**

2. Convert the delta network shown in Fig. 3.258 into equivalent wye network.

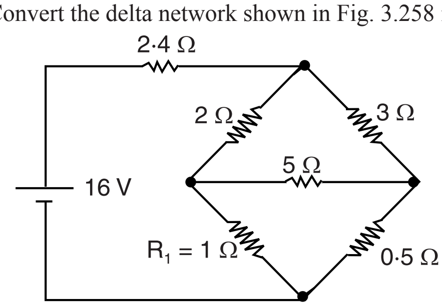


Fig. 3.257

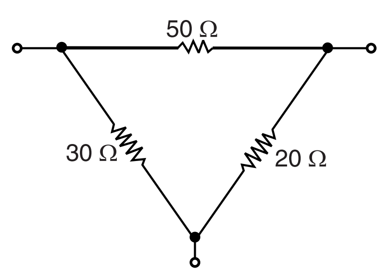


Fig. 3.258

3. Convert the wye network shown in Fig. 3.259 into equivalent delta network.

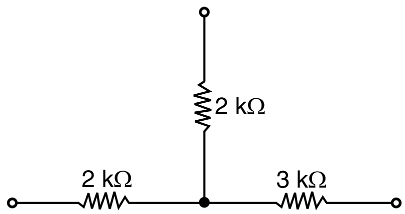


Fig. 3.259

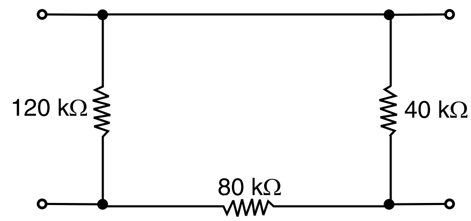


Fig. 3.260

4. Convert the delta network shown in Fig. 3.260 into the equivalent wye network.

5. In the network shown in Fig. 3.261, find the resistance between terminals *B* and *C* using star/delta transformation. **[17/12 Ω]**

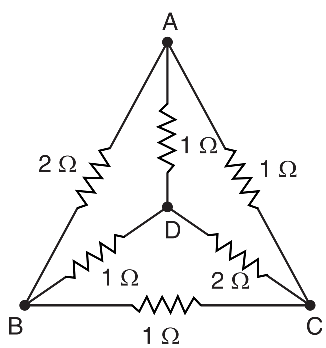


Fig. 3.261

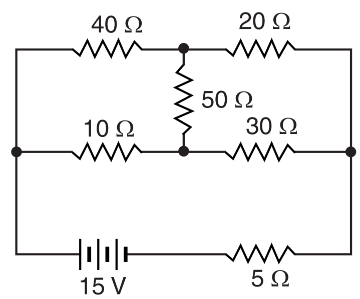


Fig. 3.262

6. In the network shown in Fig. 3.262, find the current supplied by the battery using star/delta transformation. **[0·452 A]**

7. What is the resistance between terminals *A* and *B* of the network shown in Fig. 3.263 ? **[274·2 Ω]**

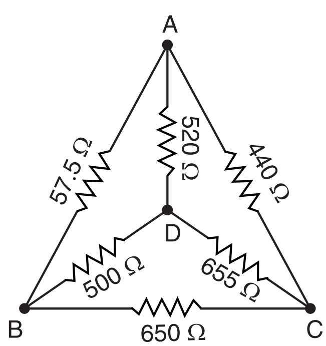


Fig. 3.263

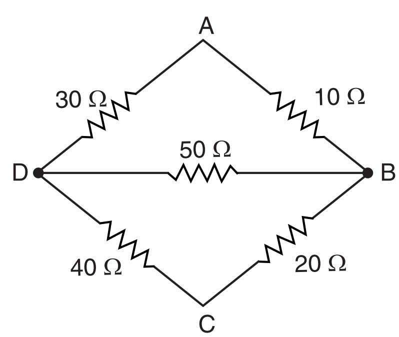


Fig. 3.264

8. Using delta/star transformation, find the resistance between terminals *A* and *C* of the network shown in Fig. 3.264.

9. Using star/delta transformation, determine the value of *R* for the network shown in Fig. 3.265 such that 4Ω resistor consumes the maximum power. **[*R* = 36Ω]**

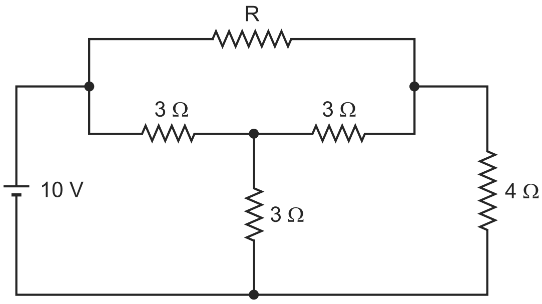


Fig. 3.265

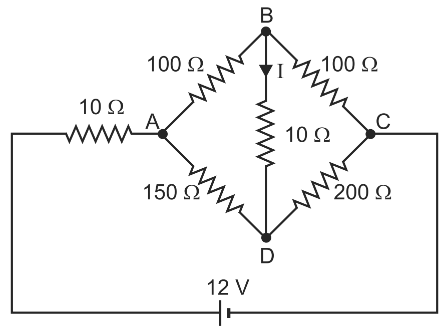
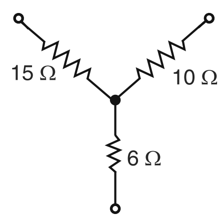


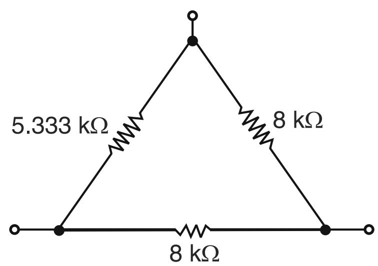
Fig. 3.266

10. Calculate the current I flowing through the 10 Ω resistor in the circuit shown in Fig. 3.266. Apply Thevenin's theorem and star/delta transformation. **[5.45 mA from *D* to *B*]**

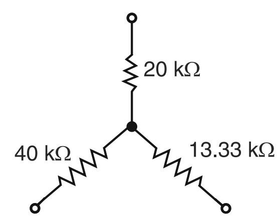
ANSWERS TO PROBLEMS 2 TO 4



Prob. 2



Prob. 3



Prob. 4

3.26. Tellegen's Theorem

This theorem has wide applications in electric networks and may be stated as under :

For a network consisting of n elements if i_1, i_2, i_3 .... i_n *are the instantaneous currents flowing through the elements satisfying KCL and* v_1, v_2, v_3 ... v_n *are the instantaneous voltages across these elements satisfying KVL, then,*

$$v_1 i_1 + v_2 i_2 + v_3 i_3 + ... + v_n i_n = 0$$

or

$$\sum_{n=1}^{n} v_n i_n = 0$$

Now vi is the instantaneous power. Therefore, Tellegen's theorem can also be stated as under :

*The sum of instantaneous powers for n branches in a network is always * zero.*

This theorem is valid for any lumped network that contains elements linear or non-linear, passive or active, time variant or time invariant.

Explanation. Let us explain Tellegen's theorem with a simple circuit shown in Fig. 3.267. The total resistance offered to the battery = 8 Ω + (4 Ω || 4 Ω) = 10 Ω. Therefore, current supplied by battery is I = 100/10 = 10A. This current divides equally at point A.

Voltage drop across 8 Ω = – (10 × 8) = – 80 V

Voltage drop across 4 Ω = – (5 × 4) = – 20 V

Voltage drop across 1 Ω = – (5 × 1) = – 5V

Voltage drop across 3 Ω = – (5 × 3) = – 15 V

* This is in accordance with the law of conservation of energy because power delivered by the battery is consumed in the circuit elements.

According to Tellegen's theorem,

Sum of instantaneous powers = 0

or $v_1 i_1 + v_2 i_2 + v_3 i_3 + v_4 i_4 + v_5 i_5 = 0$

or $(100 \times 10) + (-80 \times 10) + (-20 \times 5) + (-5 \times 5) + (-15 \times 5) = 0$

or $1000 - 800 - 100 - 25 - 75 = 0$

or $0 = 0$ which is true

Thus Tellegen's theorem stands proved.

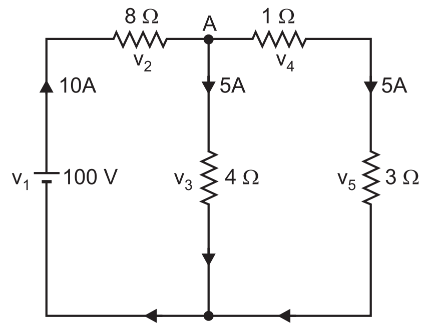


Fig. 3.267

Objective Questions

1. An active element in a circuit is one which ....

(*i*) receives energy
(*ii*) supplies energy
(*iii*) both receives and supplies energy
(*iv*) none of the above

2. A passive element in a circuit is one which ....

(*i*) supplies energy
(*ii*) receives energy
(*iii*) both supplies and receives energy
(*iv*) none of the above

3. An electric circuit contains ....

(*i*) active elements only
(*ii*) passive elements only
(*iii*) both active and passive elements
(*iv*) none of the above

4. A linear circuit is one whose parameters (*e.g.* resistances etc.) ....

(*i*) change with change in current
(*ii*) change with change in voltage
(*iii*) do not change with voltage and current
(*iv*) none of the above

5. In the circuit shown in Fig. 3.268, the number of nodes is ....

(*i*) one (*ii*) two
(*iii*) three (*iv*) four

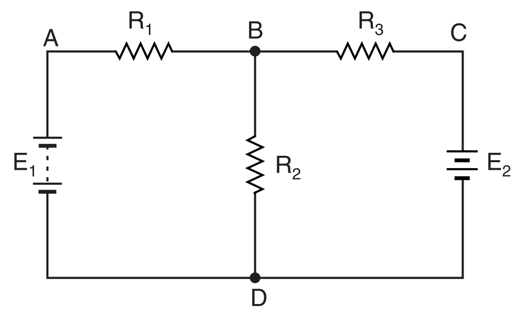


Fig. 3.268

6. In the circuit shown in Fig. 3.268, there are .... junctions.

(*i*) three (*ii*) four
(*iii*) two (*iv*) none of the above

7. The circuit shown in Fig. 3.268 has .... branches.

(*i*) two (*ii*) four
(*iii*) three (*iv*) none of these

8. The circuit shown in Fig. 3.268 has .... loops.

(*i*) two (*ii*) four
(*iii*) three (*iv*) none of the above

9. In the circuit shown in Fig. 3.268, there are .... meshes.

(*i*) two (*ii*) three
(*iii*) four (*iv*) five

10. To solve the circuit shown in Fig. 3.268 by Kirchhoff's laws, we require ....

(*i*) one equation (*ii*) two equations
(*iii*) three equations (*iv*) none of the above

11. To solve the circuit shown in Fig. 3.268 by nodal analysis, we require ....

(*i*) one equation (*ii*) two equations
(*iii*) three equations (*iv*) none of the above

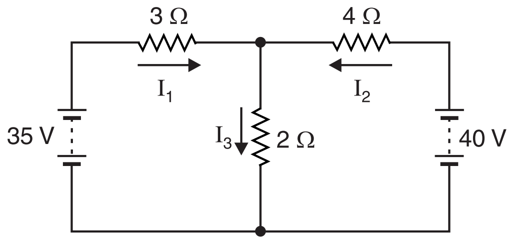


Fig. 3.269

12. To solve the circuit shown in Fig. 3.269 by superposition theorem, we require ....

(*i*) one circuit (*ii*) two circuits

(*iii*) three circuits (*iv*) none of the above

13. To solve the circuit shown in Fig. 3.269 by Maxwell's mesh current method, we require ....

(*i*) one equation (*ii*) three equations

(*iii*) two equations (*iv*) none of the above

14. In the circuit shown in Fig. 3.270, the voltage at node *B* *w.r.t.* *D* is calculated to be 15V. The current in 3 Ω resistor will be ....

(*i*) 2 A (*ii*) 5 A

(*iii*) 2·5 A (*iv*) none of the above

15. The current in 2 Ω horizontal resistor in Fig. 3.270 is ....

(*i*) 10 A (*ii*) 5 A

(*iii*) 2 A (*iv*) 2·5 A

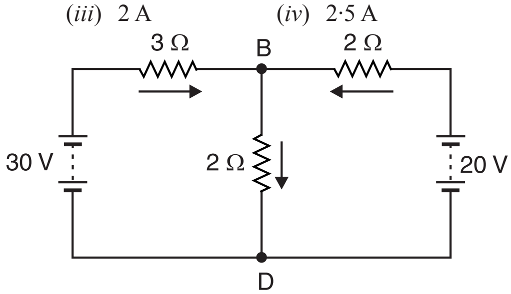


Fig. 3.270

16. In order to solve the circuit shown in Fig. 3.270 by nodal analysis, we require ....

(*i*) one equation (*ii*) two equations

(*iii*) three equations (*iv*) none of the above

17. The superposition theorem is used when the circuit contains ....

(*i*) a single voltage source

(*ii*) a number of voltage sources

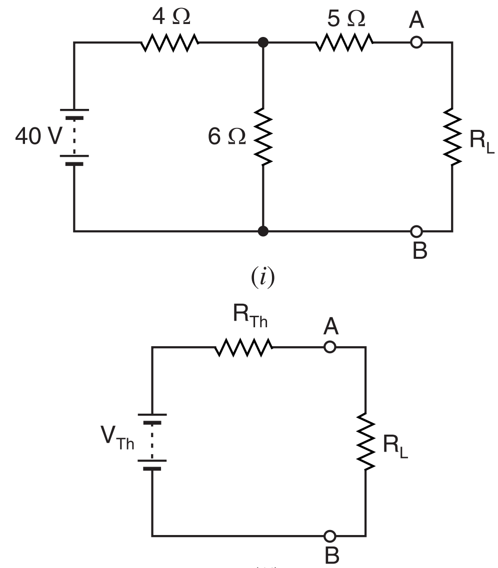
(*iii*) passive elements only

(*iv*) none of the above

18. Fig. 3.271 (*ii*) shows Thevenin's equivalent circuit of Fig. 3.271 (*i*). The value of Thevenin's voltage V_{Th} is ....

(*i*) 20 V (*ii*) 24 V

(*iii*) 12 V (*iv*) 36 V



(*i*)

(*ii*)

Fig. 3.271

19. The value of R_{Th} in Fig. 3.271 (*ii*) is ....

(*i*) 15 Ω (*ii*) 3·5 Ω

(*iii*) 6·4 Ω (*iv*) 7·4 Ω

20. The open-circuited voltage at terminals *AB* in Fig. 3.271 (*i*) is ....

(*i*) 12 V (*ii*) 20 V

(*iii*) 24 V (*iv*) 40 V

21. Find the value of R_L in Fig. 3.272 to obtain maximum power in R_L.

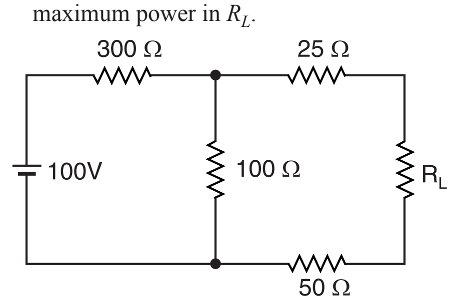


Fig. 3.272

(*i*) 100 Ω (*ii*) 75 Ω

(*iii*) 250 Ω (*iv*) 150 Ω

22. In Fig. 3.272, find the maximum power in R_L.

(*i*) 2 W (*ii*) 1·042 W

(*iii*) 2·34 W (*iv*) 4·52 W

23. What percent of the maximum power is delivered to R_L in Fig. 3.273 when $R_L = 2R_{Th}$?

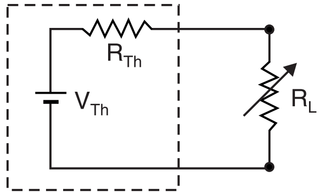


Fig. 3.273

(*i*) 79 % of P_L (max)
(*ii*) 65 % of P_L (max)
(*iii*) 88·89 % of P_L (max)
(*iv*) none of above

24. What percent of the maximum power is delivered to R_L in Fig. 3.273 when $R_L = R_{Th}/2$?

(*i*) 65 % (*ii*) 70 %
(*iii*) 88·89 % (*iv*) none of above

25. Find Millman's equivalent circuit *w.r.t.* terminals $x - y$ in Fig. 3.274.

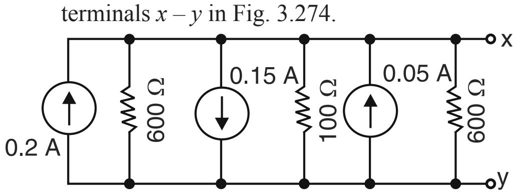


Fig. 3.274

(*i*) Single current source of 0·1A and resistance 75 Ω
(*ii*) Single current source of 2 A and resistance 50 Ω
(*iii*) Single current source of 1 A and resistance 25 Ω
(*iv*) none of above

26. Use superposition principle to find current through R_1 in Fig. 3.275.

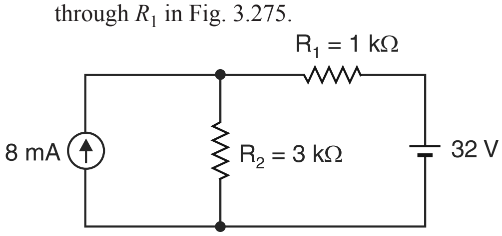


Fig. 3.275

(*i*) 1 mA ← (*ii*) 2 mA ←
(*iii*) 1·5 mA → (*iv*) 2·5 A ←

27. Use superposition principle to find current through R_1 in the circuit shown in Fig. 3.276.

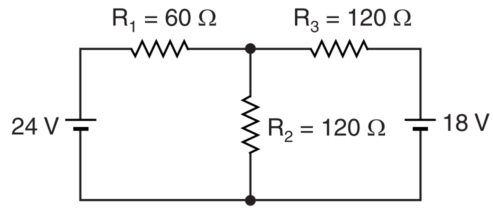


Fig. 3.276

(*i*) 0·2 A ← (*ii*) 0·25 A →
(*iii*) 0·125 A → (*iv*) 0·5 A →

28. Find Thevenin equivalent circuit to the left of terminals $x - y$ in Fig. 3.277.

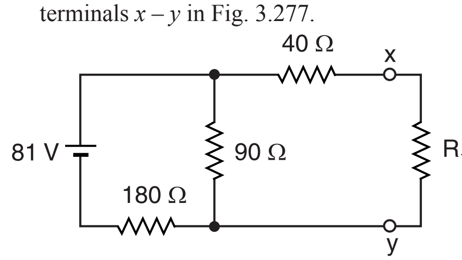


Fig. 3.277

(*i*) $V_{Th} = 5$ V ; $R_{Th} = 4{\cdot}5\ \Omega$
(*ii*) $V_{Th} = 6$ V ; $R_{Th} = 5\ \Omega$
(*iii*) $V_{Th} = 4{\cdot}5$ V ; $R_{Th} = 10\ \Omega$
(*iv*) $V_{Th} = 10$ V ; $R_{Th} = 9\ \Omega$

29. Convert delta network shown in Fig. 3.278 to equivalent Wye network.

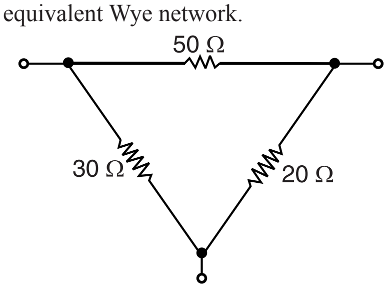
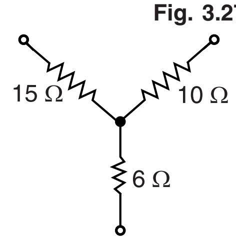
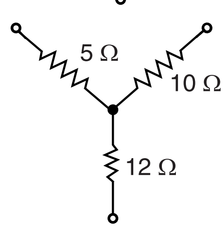


Fig. 3.278

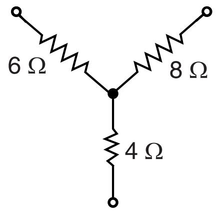
(*i*)



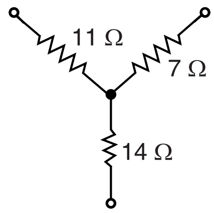
(*ii*)



(*iii*)



(*iv*)



30. What percentage of the maximum power is delivered to a load if load resistance is 10 times greater than the Thevenin resistance of the source to which it is connected ?

(*i*) 25 % (*ii*) 40 %
(*iii*) 35 % (*iv*) 33·06 %

Answers

1. *(ii)*	**2.** *(ii)*	**3.** *(iii)*	**4.** *(iii)*	**5.** *(iv)*
6. *(iii)*	**7.** *(iii)*	**8.** *(iii)*	**9.** *(i)*	**10.** *(ii)*
11. *(i)*	**12.** *(ii)*	**13.** *(iii)*	**14.** *(ii)*	**15.** *(iv)*
16. *(i)*	**17.** *(ii)*	**18.** *(ii)*	**19.** *(iv)*	**20.** *(iii)*
21. *(iv)*	**22.** *(ii)*	**23.** *(iii)*	**24.** *(iii)*	**25.** *(i)*
26. *(ii)*	**27.** *(iii)*	**28.** *(iv)*	**29.** *(i)*	**30.** *(iv)*

4

Units—Work, Power and Energy

Introduction

Engineering is an applied science dealing with a very large number of *physical quantities like distance, time, speed, temperature, force, voltage, resistance *etc*. Although it is possible to assign a standard unit for each quantity, it is rarely necessary to do so because many of the quantities are functionally related through experiment, derivation or definition. In the study of mechanics, for example, the units of only three quantities *viz. mass, length* and *time* need to be selected. All other quantities (*e.g*. area, volume, velocity, force *etc*.) can be expressed in terms of the units of these three quantities by means of experimental, derived and defined **relationship between the physical quantities. The units selected for these three quantities are called *fundamental units.* In order to cover the entire subject of engineering, three more fundamental quantities have been selected *viz*. †*electric current, temperature* and *luminous intensity*. *Thus there are in all six fundamental quantities (viz, mass, length, time, current, temperature and luminous intensity) which need to be assigned proper and standard units.* The units of all other physical quantities can be derived from the units of these six fundamental quantities. In this chapter, we shall focus our attention on the mechanical, electrical and thermal units of work, power and energy.

4.1. International System of Units

Although several systems were evolved to assign units to the above mentioned six fundamental quantities, only international system of units (abbreviated as SI) has been universally accepted. The units assigned to these six fundamental quantities in this system are given below.

Quantity	Symbol	Unit name	Unit symbol
Length	*l, L*	metre	m
Mass	*m*	kilogram	kg
Time	*t*	second	s
Electric Current	*I*	ampere	A
Temperature	*T*	degree kelvin	K
Luminous Intensity	*I*	candela	Cd

It may be noted that the units of all other physical quantities in science and engineering (*i.e*. other than six fundamental or basic quantities above) can be derived from the above basic units and are called *derived units*. Thus unit of velocity (= 1 m/s) results when the unit of length (= 1 m) is divided by the unit of time (= 1 s). Similarly, the unit of force (= 1 newton) results when unit of mass (= 1 kg) is multiplied by the unit of acceleration (= 1 m/s^2). Therefore, units of velocity and force are the derived units.

---

* A physical quantity is one which can be measured.

** For example, by definition, speed is the distance travelled per second. Therefore, speed is related to distance (*i.e.* length) and time.

† For practical reasons, electric current and *not* charge has been taken as the fundamental quantity, though one is derivable from the other. The important consideration which led to the selection of current as the fundamental quantity is that it serves as the link between electric, magnetic and mechanical quantities and can be readily measured.

4.2. Important Physical Quantities

It is profitable to give a brief description of the following physical quantities much used in science and engineering :

(*i*) **Mass.** It is the quantity of matter possessed by a body. The SI unit of mass is kilogram (kg). The mass of a body is a constant quantity and is independent of place and position of the body. Thus the mass of a body is the same whether it is on Earth's surface, the Moon's surface, on the top of a mountain or down a deep well.

$$1 \text{ quintal} = 100 \text{ kg} \ ; \quad 1 \text{ tonne} = 10 \text{ quintals} = 1000 \text{ kg}$$

(*ii*) **Force.** It is the product of mass (kg) and acceleration (m/s^2). The unit of force is newton (N) ; being the force required to accelerate a mass of 1 kg through an acceleration of 1 m/s^2.

$\therefore$ $F = m\,a$ newtons

where m = mass of the body in kg

a = acceleration in m/s^2

(*iii*) **Weight.** The force with which a body is attracted towards the centre of Earth is called the weight of the body. Now, force = mass × acceleration. If m is the mass of a body in kg and g is the acceleration due to gravity in m/s^2, then,

$$\text{Weight, } W = m\,g \text{ newtons}$$

As the value of g* varies from place to place on earth's surface, therefore, the weight of the body varies accordingly. However, for practical purposes, we take $g = 9{\cdot}81$ m/s^2 so that weight of the body = $9{\cdot}81\,m$ newtons. Thus if a mass of 1 kg rests on a table, the downward force on the table *i.e.*, weight of the body is $W = 9{\cdot}81 \times 1 = 9{\cdot}81$ newtons.

The following points may be noted carefully :

(*a*) *The mass of a body is a constant quantity whereas its weight depends upon the place or position of the body.* However, it is reasonably accurate to express weight $W = 9{\cdot}81\,m$ newtons where m is the mass of the body in kg.

(*b*) Sometimes weight is given in kg. wt. units. One kg-wt means weight of mass of 1 kg *i.e.* $9{\cdot}81 \times 1 = 9{\cdot}81$ newtons.

$\therefore$ 1 kg. wt. = $9{\cdot}81$ newtons

Thus, when we say that a body has a weight of 100 kg, it means that it has a mass of 100 kg and that it exerts a downward force of $100 \times 9{\cdot}81$ newtons.

4.3. Units of Work or Energy

Work is said to be done on a body when a force acts on it and the body moves through some distance. This work done is stored in the body in the form of energy. Therefore, work and energy are measured in the same units. The SI unit of work or energy is *joule* and is defined as under :

The work done on a body is **one joule** *if a force of one newton moves the body through 1 m in the direction of force.*

It may be noted that work done or energy possessed in an electrical circuit or mechanical system or thermal system is measured in the same units viz. joules. This is expected because mechanical, electrical and thermal energies are interchangeable. For example, when mechanical work is transferred into heat or heat into work, the quantity of work in joules is equal to the quantity of **heat in joules.

* The value of g is about 9.81 m/s^2 at sea level whereas at equator, it is about 9.78 m/s^2 and at each pole it is about 9.832 m/s^2.

** Although heat energy was assigned a separate unit *viz.* calorie but the reader remembers that 1 calorie = 4.186 joules. In fact, the thermal unit calorie is obsolete and now-a-days heat is expressed in joules.

Note. To gain some appreciation for the magnitude of a joule of heat energy, it would require about 90,000 J to heat a cup of water from room temperature to boiling.

4.4. Some Cases of Mechanical Work or Energy

It may be helpful to give a few important cases of work done or energy possessed in a mechanical system :

(*i*) When a force of F newtons is exerted on a body through a distance 'd' metres in the direction of force, then,
Work done = $F \times d$ joules or Nm

(*ii*) Suppose a force of F newtons in maintained tangentially at a radius r metres from O as shown in Fig. 4.1. In one revolution, the point of application of force travels through a distance of $2\pi r$ metres.

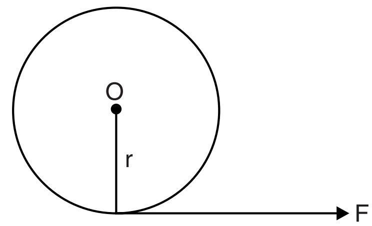


Fig. 4.1

∴ Work done in one revolution

$$= \text{Force} \times \text{Distance moved in 1 revolution}$$
$$= F \times 2\pi r$$
$$= 2\pi \times T \text{ joules or Nm}$$

where $T = F\,r$ is the torque. Clearly, the SI unit of torque will be joules or Nm. If the body makes N revolutions per minute, then,

$$\text{Work done/minute} = 2\pi N \text{T joules}$$

(*iii*) If a body of mass m kg is moving with a speed of v m/s, then kinetic energy possessed by the body is given by ;

$$\text{K.E. of the body} = \frac{1}{2}mv^2 \text{ joules}$$

(*iv*) If a body having a mass of m kg is lifted vertically through a height of h metres and g is acceleration due to gravity in m/s^2, then,

$$\text{Potential energy of body} = \text{Work done in lifting the body} = \text{Force required} \times \text{height}$$
$$= \text{Weight of body} \times \text{height} = m\,g \times h$$
$$= m\,g\,h \text{ joules}$$

4.5. Electrical Energy

The SI unit of electrical work done or electrical energy expended in a circuit is also joule—exactly the same as for mechanical energy. It is defined as under :

One joule *of energy is expended electrically when one coulomb is moved through a p.d. of 1 volt.*

Suppose a charge of Q coulomb moves through a p.d. of V volts in time t in part AB of a circuit as shown in Fig. 4.2. Then electrical energy expended is given by ;

Electrical energy expended

$$= V\,Q \text{ joules}$$
$$= V\,I\,t \text{ joules} \qquad (\because\ Q = I\,t)$$
$$= I^2R\,t \text{ joules} \qquad (\because\ V = I\,R)$$
$$= \frac{V^2t}{R} \text{ joules} \qquad \left(\because I = \frac{V}{R}\right)$$

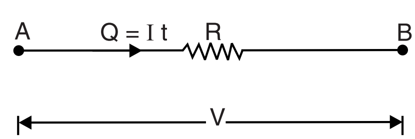


Fig. 4.2

It may be mentioned here that joule is also known as watt-second *i.e.* 1 joule = 1 watt-sec. When we are dealing with large amount of electrical energy, it is often convenient to express it in *kilowatt hours* (kWh).

$$1 \text{ kWh} = 1000 \text{ watt-hours} = 1000 \times 3600 \text{ watt-sec or joules}$$

∴ $1 \text{ kWh} = 36 \times 10^5$ joules or watt-sec

Although practical unit of electrical energy is kWh, yet it is easy to see that this unit is readily convertible to joules with the help of above relation.

The electricity bills are made on the basis of total electrical energy consumed by the consumer. *The unit for billing of electrical energy is 1 kWh*. Thus when we say that a consumer has consumed 100 units, it means the electrical energy consumption is 100 kWh. Note that 1 kWh is also called *Board of Trade Unit* (*B.O.T.U.*) or unit of electricity.

4.6. Thermal Energy

Heat is a particularly important form of energy in the study of electricity, not only because it affects the electrical properties of the materials but also because it is *liberated* whenever electric current flows. This liberation of heat is infact the conversion of electrical energy to heat energy.

The thermal energy was originally assigned the unit 'calorie'. One calorie is the amount of heat required to raise the temperature of 1 gm of water through 1ºC. If S is the specific heat of a body, then amount of heat required to raise the temperature of m gm of body through θºC is given by ;

$$\text{Heat gained} = (m\,S\,\theta) \text{ calories}$$

It has been found experimentally that 1 calorie = 4·186 joules so that heat energy in calories can be expressed in joules as under :

$$\text{Heat gained} = (m\,S\,\theta) \times 4{\cdot}186 \text{ joules}$$

The reader may note that SI unit of heat is also joule. In fact, the thermal unit calorie is obsolete and unit joule is preferred these days.

4.7. Units of Power

Power is the *rate* at which energy is expended or the rate at which work is performed. Since energy and work both have the units of joules, it follows that power, being rate, has the units joule/second. Now Joule/second is also called **watt.** In general,

$$\text{Power} = \frac{W}{t} \text{ watts}$$

where W is the total number of joules of work performed or total joules of energy expended in t seconds.

Suppose a charge of Q coulomb moves through a p.d. of V volts in time t in part AB of a circuit as shown in Fig. 4.2. Then,

$$\text{Electrical energy expended} = VQ = VIt = I^2Rt = \frac{V^2 t}{R}$$

$$\therefore \quad \text{Power of circuit, } P = \frac{VIt}{t} = \frac{I^2Rt}{t} = \frac{V^2t}{Rt}$$

$$\text{or} \quad P = VI = I^2R = \frac{V^2}{R}$$

In practice, watt is often found to be inconveniently small, consequently the unit kilowatt (kW) is used. One kW is equal to 1000 watts *i.e.*

$$1 \text{ kW} = 1000 \text{ watts}$$

For larger powers, the unit megawatt (MW) is used. One megawatt is equal to 1000 kW *i.e.*

$$1 \text{ MW} = 1000 \text{ kW} = 1000 \times 1000 \text{ watts}$$

$$\therefore \quad 1 \text{ MW} = 10^6 \text{ watts}$$

It may be noted that power of an electrical system or mechanical system or thermal system is measured in the same units viz joules/sec. or watts.

Important points. The following points are worth noting :

(*i*) Sometimes power is measured in *horse power (h.p.).

$$1 \text{ h.p.} = 746 \text{ watts}$$

(*ii*) If a body makes N r.p.m. and the torque acting is T newton-metre, then,

$$\text{Work done/minute} = 2\pi\, N\, T \text{ joules} \qquad \text{[See Art. 4·4]}$$

$$\text{Work done/sec} = \frac{2\pi NT}{60} \text{ joules/sec or watts}$$

i.e.,

$$\text{Power} = \frac{2\pi NT}{60} \text{ watts}$$

$$\text{Since 746 watts} = 1 \text{ h.p., we have,}$$

$$\text{Power} = \frac{2\pi NT}{60 \times 746} \text{ h.p.}$$

where T is in newton-m and N is in r.p.m.

(*iii*) Power can also be expressed in terms of force and velocity.

$$\text{Power} = \text{Work done/sec} = \text{Force} \times \text{distance/sec}$$

$$\therefore \quad \text{Power} = \text{Force} \times \text{velocity}$$

4.8. Efficiency of Electric Device

The efficiency of an electric device is the ratio of useful output power to the input power, i.e.

$$\text{Efficiency, } \eta = \frac{\text{Useful output power}}{\text{Input power}}$$

$$= \frac{\text{Useful output Energy}}{\text{Input Energy}}$$

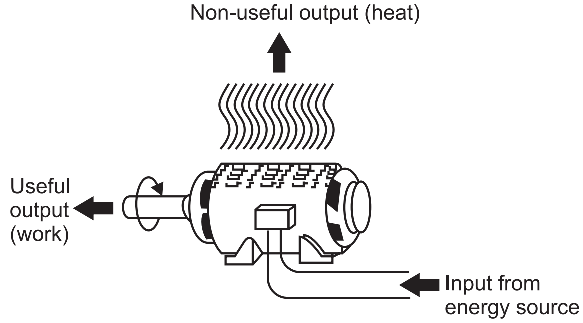


Fig. 4.3

The law of conservation of energy states that "energy cannot be created or destroyed but can be converted from one form to another." Some of the input energy to an electric device may be converted into a form that is not useful. For example, consider an electric motor shown in Fig. 4.3. The purpose of the motor is to convert electric energy into mechanical energy. It does this but it also converts a part of input energy into heat. The heat produced is not useful. Therefore, the useful output energy is less than the input energy. In other words, the efficiency of motor is less than 100%. While selecting an electric device, its efficiency is an important consideration because the operating cost of the device depends upon this factor.

Some electric devices are nearly 100% efficient. An electric heater is an example. In a heater, the heat is useful output energy and practically all the input electric energy is converted into heat energy.

4.9. Harmful Effects of Poor Efficiency

The poor (or low) efficiency of a device or of a circuit has the following harmful effects :

(*i*) Poor efficiency means waste of energy on non-useful output.

* This unit for power was conceived by James Watt, a Scottish scientist who invented the steam engine. In his experiments, he compared the output of his engine with the power a horse could put out. He found that an "average" horse could do work at the rate of 746 joules/sec. Although power can be expressed in watts or kW, the unit h.p. is still used.

(*ii*) Non-useful output of a device or circuit usually appears in the form of heat. Therefore, poor efficiency means a significant temperature rise. High temperature is one of the major limiting factors in producing reliable electric and electronic devices. Circuits and devices that run hot are more likely to fail.

(*iii*) The heat produced as a result of poor efficiency has to be dissipated *i.e.*, heat has to be transferred to the atmosphere or some other mass. Heat removal can become quite difficult in high power circuits and adds to the cost and size of the equipment.

Example 4.1. *An electrically driven pump lifts 80 m^3 of water per minute through a height of 12 m. Allowing an overall efficiency of 70% for the motor and pump, calculate the input power to motor. If the pump is in operation for an average of 2 hours per day for 30 days, calculate the energy consumption in kWh and the cost of energy at the rate of Rs 2 per kWh. Assume 1 m^3 of water has a mass of 1000 kg and $g = 9{\cdot}81\ m/s^2$.*

Solution. Mass of 80 m^3 of water, $m = 80 \times 1000 = 8 \times 10^4$ kg

Weight of water lifted, $W = m\,g = 8 \times 10^4 \times 9{\cdot}81$ N

Height through which water lifted, $h = 12\ m$

W.D. by motor/minute = $m\,g\,h = 8 \times 10^4 \times 9{\cdot}81 \times 12$ joules

W.D. by motor/second = $\dfrac{8\times10^4\times9.81\times12}{60}$ = 156960 watts

∴ Output power of motor = 156960 watts

Input power to motor = $\dfrac{\text{Motor output}}{\text{Efficiency}} = \dfrac{156960}{0.7}$ = 2,24,228 W = **224·228 kW**

Total energy consumption = Input power × Time of operation

= (224·228) × (2 × 30) kWh = **13453 kWh**

Total cost of energy =Rs 2 × 13453 = **Rs. 26906**

Example 4.2. *Fig. 4.4 shows an electric motor driving an electric generator. The 2 h.p. motor draws 14·6 A from a 120 V source and the generator supplies 56 A at 24 V.*

(*i*) *Find the motor efficiency and generator efficiency*

(*ii*) *Find the overall efficiency.*

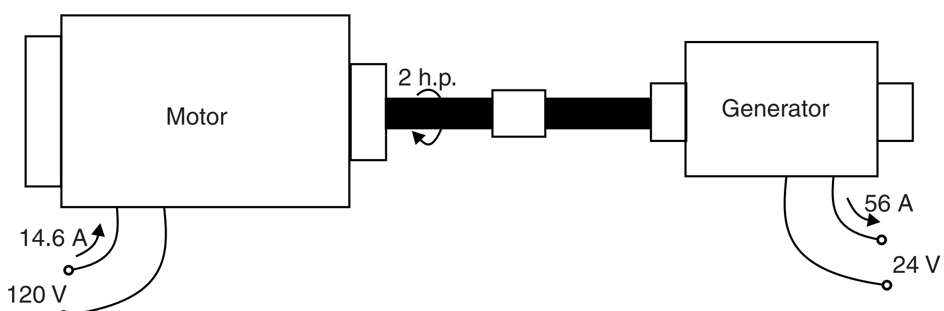


Fig. 4.4

Solution. Efficiency of a machine is output power (P_o) divided by input power (P_i).

(*i*) P_i (motor) = 120 × 14·6 = 1752 W

P_o (motor) = 2 h.p. = 2 × 746 = 1492 W

∴ η (motor) = $\dfrac{1492}{1752}$ = **0.8516** or **85·16%**

$$P_i \text{ (generator)} = 2 \text{ h.p.} = 1492 \text{ W}$$

$$P_o \text{ (generator)} = 24 \times 56 = 1344 \text{ W}$$

$$\therefore \quad \eta \text{ (generator)} = \frac{1344}{1492} = \mathbf{0{\cdot}90} \quad \text{or} \quad \mathbf{90\%}$$

(ii)

$$\eta \text{ (overall)} = \frac{P_o(\text{generator})}{P_i(\text{motor})} = \frac{1344}{1752} = \mathbf{0{\cdot}767} \text{ or } \mathbf{76{\cdot}7\%}$$

Note that overall η is the product of efficiencies of the individual machines.

$$\eta \text{ (overall)} = \eta \text{ (motor)} \times \eta \text{ (generator)} = 0{\cdot}8516 \times 0{\cdot}90 = 0{\cdot}767.$$

Example 4.3. *Neglecting losses, at what horse power rate could energy be obtained from Bhakra dam which has an average height of 225 m and water flows at a rate of 500,000 kg/minute ? If the overall efficiency of conversion were 25%, how many 100 watt light bulbs could Bhakra dam supply ?*

Solution. Wt. of water flowing/minute

$$= m\,g = 500{,}000 \times 9{\cdot}81 \text{ N}$$

$$\text{Work done/minute} = m\,g\,h = 500{,}000 \times 9{\cdot}81 \times 225 \text{ joules}$$

$$\text{Work done/second} = \frac{500{,}000 \times 9.81 \times 225}{60} = 18394 \times 10^3 \text{ watts}$$

$$\therefore \quad \text{Gross power obtained} = 18394 \times 10^3 \text{ watts} = 18394 \text{ kW}$$

$$\text{Useful output power} = 18394 \times 0{\cdot}25 = 4598{\cdot}5 \text{ kW}$$

$$= \frac{4598.5 \times 10^3}{746} \text{ h.p.} = \mathbf{6164 \text{ h.p.}}$$

No. of 100-watt bulbs that could be lighted

$$= \frac{4598.5 \times 10^3}{100} = \mathbf{45985}$$

Example 4.4. *A 100 MW hydro-electric station is supplying full-load for 10 hours a day. Calculate the volume of water which has been used. Assume effective head of station as 200 m and overall efficiency of the station as 80%.*

Solution. Energy supplied by the station in 10 hours

$$= (100 \times 10^3) \times 10 = 10^6 \text{ kWh}$$

$$= 36 \times 10^5 \times 10^6 = 36 \times 10^{11} \text{ joules}$$

$$\text{Energy input of station} = 36 \times 10^{11}/0{\cdot}8 = 45 \times 10^{11} \text{ joules}$$

Suppose m kg is the mass of water used in 10 hours.

Then, $$m\,g\,h = 45 \times 10^{11}$$

or $$m = \frac{45 \times 10^{11}}{9.81 \times 200} = 22.93 \times 10^8 \text{ kg}$$

Since 1 m^3 of water has a mass of 1000 kg,

$\therefore$ Volume of water used = $22{\cdot}93 \times 10^8/10^3$ = $\mathbf{22{\cdot}93 \times 10^5\ m^3}$

Example 4.5. *Two coils are connected in parallel and a voltage of 200 V is applied to the terminals. The total current taken is 15A and the power dissipated in one of the coils is 1500 W. What is the resistance of each coil ?*

Solution. Let R_1 and R_2 be the resistances of the coils and I_1 and I_2 be the current drawn from the supply. Since the coils are connected in parallel, voltage across each coil is the same *i.e.* 200 V.

$$VI_1 = W \quad \text{or} \quad I_1 = W/V = 1500/200 = 7{\cdot}5\text{A}$$

$$\therefore \quad R_1 = V/I_1 = 200/7{\cdot}5 = \mathbf{26.7\ \Omega}$$

$$I_1 + I_2 = 15 \quad \text{..... given} \quad \therefore \quad I_2 = 15 - I_1 = 15 - 7{\cdot}5 = 7{\cdot}5\text{A}$$

$$\therefore \quad R_2 = \frac{V}{I_2} = \frac{200}{7.5} = \mathbf{26.7\ \Omega}$$

Although not technically correct usage, it is convenient to say that resistance "dissipates power", meaning that it dissipates (liberates) heat at a certain rate.

Example 4.6. *A motor is being self-started against a resisting torque of 60 N-m and at each start, the engine is cranked at 75 r.p.m. for 8 seconds. For each start, energy is drawn from a lead-acid battery. If the battery has the capacity of 100 Wh, calculate the number of starts that can be made with such a battery. Assume an overall efficiency of the motor and gears as 25%.*

Solution. Angular speed, $\omega = 2\pi\text{ N}/60 \text{ rad/s} = 2\pi \times 75/60 = 7{\cdot}85 \text{ rad/s}$

$$\text{Power required per start, } P = \frac{\text{Torque} \times \text{Angular speed}}{\text{Efficiency of motor}} = \frac{60 \times 7.85}{0.25} = 1884 \text{ W}$$

$$\text{Energy required/start} = P \times \text{Time for start}$$
$$= 1884 \times 8 = 15072 \text{ Ws} = 15072 \text{ J}$$
$$= 15072/3600 = 4{\cdot}187 \text{ Wh}$$

$\therefore$ No. of starts with a fully-charged battery

$$= 100/4{\cdot}187 \simeq \mathbf{24}$$

Example 4.7. *A hydro-electric power station has a reservoir of area 2·4 square kilometres and capacity $5 \times 10^6\ m^3$. The effective head of water is 100 m. The penstock, turbine and generator efficiencies are 95%, 90% and 85% respectively.*

(*i*) *Calculate the total energy in kWh which can be generated from the power station.*

(*ii*) *If a load of 15,000 kW has been supplied for 3 hours, find the fall in reservoir level.*

Solution.

(***i***) Wt. of water available, $W = \text{Volume of reservoir} \times 1000 \times 9{\cdot}81 \text{ N}$

$$= (5 \times 10^6) \times (1000) \times (9{\cdot}81) = 49{\cdot}05 \times 10^9 \text{ N}$$

Overall efficiency, $\eta_{overall} = 0{\cdot}95 \times 0{\cdot}90 \times 0{\cdot}85 = 0{\cdot}726$

Electrical energy that can be generated from the station

$$= W \times \text{Effective head} \times \eta_{overall}$$
$$= (49{\cdot}05 \times 10^9) \times (100) \times (0{\cdot}726) = 35{\cdot}61 \times 10^{11} \text{ watt-sec.}$$
$$= \frac{35.61 \times 10^{11}}{1000 \times 3600} \text{ kWh} = \mathbf{9,89,116 \text{ kWh}}$$

(***ii***) $$\text{Level of reservoir} = \frac{\text{Volume of reservoir}}{\text{Area of reservoir}} = \frac{5 \times 10^6}{2.4 \times 10^6} = 2{\cdot}083 \text{ m}$$

$$\text{kWh generated in 3 hrs} = 15000 \times 3 = 45{,}000 \text{ kWh}$$

Using unitary method, we get,

$$\text{Fall in reservoir level} = \frac{2.083}{9{,}89{,}166} \times 45{,}000 = 0{\cdot}0947 \text{ m} = \mathbf{9{\cdot}47 \text{ cm}}$$

Example 4.8. *A large hydel power station has a head of 324 m and an average flow of 1370 m^3/sec. The reservoir is a lake covering an area of 6400 sq. km. Assuming an efficiency of 90% for the turbine and 95% for the generator, calculate (i) the available electric power and (ii) the number of days this power could be supplied for a drop in water level by 1 metre.*

Solution. Water discharge = 1370 m^3/sec ; Water head, h = 324 m ; $\eta_{overall} = 0.9 \times 0.95$

(*i*) As mass of 1 m^3 of water is 1000 kg,

$\therefore$ Mass of water flowing/sec, $m = 1370 \times 1000$ kg $= 137 \times 10^4$ kg

Weight of water flowing/sec, $W = mg = 137 \times 10^4 \times 9.81$ N

Energy or work available per second (*i.e.* power) is

$$\text{Power available, } P = Wh \times \eta_{overall}$$
$$= (137 \times 10^4 \times 9.81) \times 324 \times (0.9 \times 0.95)$$
$$= 3723 \times 10^6 \text{ W} = \mathbf{3723\ MW}$$

(*ii*) Area of reservoir, $A = 6400 \text{ km}^2 = 6400 \times 10^6 \text{ m}^2$

Rate of water discharge, $Q = 1370 \text{ m}^3\text{/sec}$

Fall of reservoir level, $h' = 1$ m

Volume of water used $= A \times h'$

$$\therefore \quad \text{Required time, } t = \frac{A \times h'}{Q} = \frac{6400 \times 10^6 \times 1}{1370}$$
$$= 4.67 \times 10^6 \text{ sec.} = \mathbf{54.07\ days}$$

Example 4.9. *Calculate the current required by a 500 V d.c. locomotive when drawing 100 tonne load at 25 km/hr with a tractive resistance of 7 kg/tonne along* (*i*) *level road and* (*ii*) *a gradient 1 in 100. Given that the efficiency of motor and gearing is 70%.*

Solution. Weight of locomotive, W = 100 tonne = 100,000 kg

Tractive resistance, F = 7 × 100 = 700 kg-wt = 700 × 9·81 = 6867 N

(*i*) Level Track. In this case, the force required is equal to the tractive resistance F [See Fig. 4.5 (*i*)].

$$\text{Distance travelled/sec} = \frac{25 \times 1000}{3600} = 6{\cdot}94 \text{ m}$$

Work done/sec = Force × Distance/sec

or Motor output = 6867 × 6·94 = 47,657 watts

Motor input = 47,657/0·7 = 68,081 watts

$\therefore$ Current drawn = 68,081/500 = **136·16A**

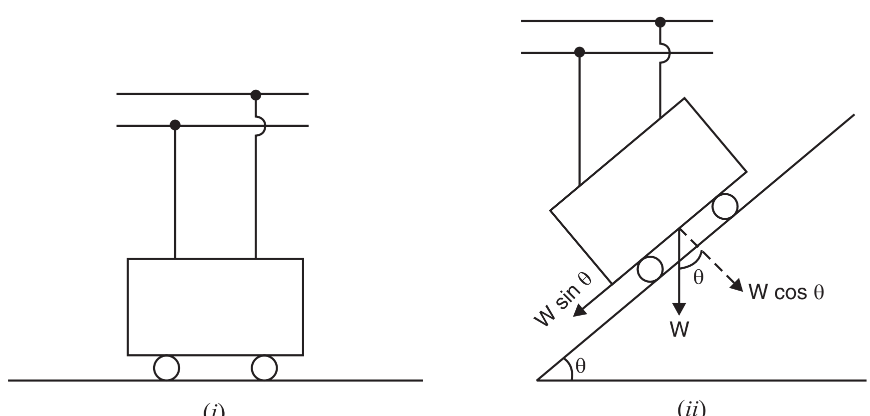


Fig. 4.5

***(ii)* Inclined plane.** In this case, the total force required is the sum of tractive resistance F and component $W \sin\theta$ of locomotive weight [See Fig. 4.5 (*ii*)]. Clearly, $\sin\theta = 1/100 = 0{\cdot}01$.

$$\therefore \quad \text{Force required} = W \sin\theta + F$$

$$= (100{,}000 \times 0{\cdot}01 + 700)\ 9{\cdot}81 \text{ N} = 16{,}677 \text{ N}$$

$$\text{Work done/sec} = \text{Force} \times \text{distance travelled/sec}$$

$$= 16{,}677 \times 6{\cdot}94 = 1{,}15{,}738 \text{ watts}$$

$$\therefore \quad \text{Motor output} = 1{,}15{,}738 \text{ watts}$$

$$\text{Motor input} = 1{,}15{,}738/0{\cdot}7 = 1{,}65{,}340 \text{ watts}$$

$$\therefore \quad \text{Current drawn} = 1{,}65{,}340/500 = \mathbf{330{\cdot}68A}$$

Example 4.10. *A diesel-electric generator set supplies an output of 25 kW. The calorific value of the fuel oil used is 12,500 kcal/kg. If the overall efficiency of the unit is 35%, calculate (i) the mass of oil required per hour (ii) the electric energy generated per tonne of the fuel.*

Solution. Output power of set = 25 kW ; $\eta_{overall} = 35\% = 0.35$

∴ Input power to set = 25/0.35 = 71.4 kW

(*i*) Input energy/hour = 71.4 kW × 1h = 71.4 kWh = 71.4 × 860 kcal

As 1 kg of fuel oil produces 12,500 kcal,

$$\therefore \quad \text{Mass of fuel oil required/hour} = \frac{71.4 \times 860}{12{,}500} = \mathbf{4.91\ kg}$$

(*ii*) Heat content in 1 tonne fuel oil (= 1000 kg) = 1000 × 12,500 = 12.5×10^6 kcal

$$= \frac{12.5 \times 10^6}{860} \text{ kWh} = 14{,}534 \text{ kWh}$$

$$\therefore \quad \text{Energy generated/tonne} = 14{,}534 \times 0.35 = \mathbf{5087\ kWh}$$

Example 4.11. *The reservoir for a hydro-electric station is 230 m above the turbine house. The annual replenishment of the reservoir is 45 × 10^{10} kg. What is the energy available at the generating station bus-bars if the loss of head in the hydraulic system is 30 m and the overall efficiency of the station is 85% ? Also, calculate the diameter of the steel pipes needed if a maximum demand of 45 MW is to be supplied using two pipes.*

Solution. Actual available head, $h = 230 - 30 = 200$ m

Energy available at turbine house is given by ;

$$E = mgh = 45 \times 10^{10} \times 9{\cdot}81 \times 200 = 8{\cdot}829 \times 10^{14} \text{ J}$$

$$= \frac{8.829 \times 10^{14}}{36 \times 10^5} \text{ kWh} = 24{\cdot}52 \times 10^7 \text{ kWh}$$

$$\text{Energy available at bus-bars} = E \times \eta = 24{\cdot}52 \times 10^7 \times 0.85 = \mathbf{20{\cdot}84 \times 10^7\ kWh}$$

$$\text{K.E. of water} = \text{Loss of potential energy of water}$$

$$\text{or} \quad \frac{1}{2}mv^2 = mgh \quad \therefore \quad v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 200} = 62{\cdot}65 \text{ m/s}$$

Power available from m kg of water is

$$P = \frac{1}{2}mv^2 = \frac{1}{2} \times m \times (62.65)^2 \text{ W}$$

This power is equal to 45 MW (= 45 × 10^6 W).

$\therefore \quad P = 45 \times 10^6$ W

or $\quad \frac{1}{2} \times m \times (62.65)^2 = 45 \times 10^6 \quad \therefore \quad m = 22930$ kg/s

If A is the total area of two pipes in m^2, then flow of water is Av m^3/s.

$\therefore$ Mass of water flowing/second = $Av \times 10^3$ kg ($\because$ 1 m^3 of water = 1000 kg)

$$\therefore \quad Av \times 10^3 = 22930 \quad \text{or} \quad A = \frac{22930}{62.65 \times 10^3} = 0{\cdot}366 \text{ m}^2$$

Area of each pipe = $0{\cdot}366/2 = 0{\cdot}183$ m^2

If d is the diameter of each pipe, then,

$$\frac{\pi}{4}d^2 = 0.183 \quad \text{or} \quad d = \sqrt{\frac{0.183 \times 4}{\pi}} = \mathbf{0{\cdot}4826 \text{ m}}$$

Example 4.12. *A proposed hydro-electric station has an available head of 30 m, catchment area of 50 × 10^6 m^2, the rainfall for which is 120 cm per annum. If 70% of the total rainfall can be collected, calculate the power that could be generated. Assume the following efficiencies: Penstock 95%, Turbine 80% and Generator 85%.*

Solution. Available head, h = 30 m ; $\eta_{overall} = 0.95 \times 0.8 \times 0.85 = 0.646$

Volume of water *available/annum = $0.7(50 \times 10^6 \times 1.2) = 4.2 \times 10^7$ m^3

Mass of water available/annum = $4.2 \times 10^7 \times 1000 = 4.2 \times 10^{10}$ kg

Mass of water available/sec; $m = \dfrac{4.2 \times 10^{10}}{365 \times 24 \times 3600} = 1.33 \times 10^3$ kg

Potential energy available/sec = $mgh = 1.33 \times 10^3 \times 9.8 \times 30 = 391 \times 10^3$ J/s

$\therefore$ Power that could be generated = $\eta_{overall} \times 391 \times 10^3$ W

$= 0.646 \times 391 \times 10^3 = 253 \times 10^3$ W = **253 kW**

Example 4·13. *A current of 20A flows for one hour in a resistance across which there is a voltage of 8V. Determine the velocity in metres per second with which a weight of one tonne must move in order that kinetic energy shall be equal in amount to the energy dissipated in the resistance.*

Solution. Energy dissipated in resistance

$= V\,It = 8 \times 20 \times 3600 = 576 \times 10^3$ J

Mass of body, m = 1 tonne = 1000 kg

Let v m/s be the required velocity of the weight.

$$\text{Kinetic energy} = \frac{1}{2}mv^2 \text{ joules}$$

In order that K.E. of weight is equal to energy dissipated in resistance,

$$\frac{1}{2}mv^2 = 576 \times 10^3 \quad \therefore v = \sqrt{\frac{2 \times 576 \times 10^3}{1000}} = \mathbf{33{\cdot}9 \text{ m/s}}$$

Example 4·14. *What must be the horse-power of an engine to drive by means of a belt a generator supplying 7000 lamps each taking 0·5 A at 250 V ? The line drop is 5V and the efficiency of the generator is 95%. There is a 2·5% loss in the belt drive.*

Solution. Total current supplied by generator, $I = 0{\cdot}5 \times 7000 = 3500$ A

Generated voltage, E = Load voltage + Line drop = 250 + 5 = 255 V

Generator output = $EI = 255 \times 3500$ W

* 0.7 × (Catchment area in m^2 × Rainfall in m)

$\therefore$ Engine output $= \dfrac{255 \times 3500}{0.95 \times 0.975} = 963562 \text{ W} = \dfrac{963562}{746}$ h.p. = **1292 h.p.**

Example 4.15. *Find the head in metres of a hydroelectric generating station in which the reservoir of area 4000 m^2 falls by 30 cm when 75 kWh is developed in the turbine. The efficiency of the turbine is 70%.*

Solution. Hydroelectric generating stations are generally built in hilly areas.

$$\text{Volume of water used, } V = 4000 \times 0{\cdot}3 = 1200 \text{ m}^3$$

$$\text{Mass of water used, } m = 1200 \times 10^3 = 1{\cdot}2 \times 10^6 \text{ kg}$$

$$\text{Useful energy developed in turbine} = mgh \times \eta = 1{\cdot}2 \times 10^6 \times 9.81 \times h \times 0{\cdot}7$$

$$\text{But useful energy developed in turbine} = 75 \text{ kWh} = 75 \times 3{\cdot}6 \times 10^6 \text{ J}$$

$$\therefore \quad 1{\cdot}2 \times 10^6 \times 9{\cdot}81 \times h \times 0{\cdot}7 = 75 \times 3{\cdot}6 \times 10^6$$

or $h = \mathbf{32{\cdot}76 \text{ m}}$

Example 4.16. *A room measures 3m × 4m × 4·75m and air in it has to be always kept 10°C higher than that of the incoming air. The air inside has to be renewed every 30 minutes. Neglecting radiation losses, find the necessary rating of electric heater for this purpose. Take specific heat of air as 0·24 and density as 1·28 kg/m^3.*

Solution. It is desired to find the power of the electric heater.

$$\text{Volume of air to be changed/second} = \frac{3 \times 4 \times 4.75}{30 \times 60} = 0{\cdot}032 \text{ m}^3$$

$$\text{Mass of air to be changed/second} = 0{\cdot}032 \times 1{\cdot}28 = 0{\cdot}041 \text{ kg}$$

$$\text{Heat required/second} = \text{Mass/second} \times \text{Specific heat} \times \text{Rise in temp.}$$

$$= 0{\cdot}041 \times 0{\cdot}24 \times 10 \text{ kcal}$$

$$= 0{\cdot}041 \times 0{\cdot}24 \times 10 \times 4186 \text{ W} = \mathbf{411 \text{ W}}$$

Here, we have neglected radiation losses. However, in practice, radiation losses do occur so that heater power required would be greater than the calculated value. $\left(\because \dfrac{1 \text{ kcal}}{\text{sec.}} = 4186 \text{ W}\right)$

Example 4.17. *An electric lift is required to raise a load of 5 tonne through a height of 30 m. One quarter of electrical energy supplied to the lift is lost in the motor and gearing. Calculate the energy in kWh supplied. If the time required to raise the load is 27 minutes, find the kW rating of the motor and the current taken by the motor, the supply voltage being 230V d.c. Assume the efficiency of the motor at 90%.*

Solution. Work done by lift = $mgh = (5 \times 10^3) \times 9.8 \times 30 = 1.47 \times 10^6$ J

$$\text{Input energy to lift} = \frac{1.47 \times 10^6}{\eta_{lift}^*} = \frac{1.47 \times 10^6}{0.75} = 1.96 \times 10^6 \text{ J}$$

$$= \frac{1.96 \times 10^6}{36 \times 10^5} \text{ kWh} = \mathbf{0.545 \text{ kWh}}$$

$$\text{Motor energy output} = \text{Input energy to lift} = 1.96 \times 10^6 \text{ J}$$

$$\text{Motor energy input} = \frac{1.96 \times 10^6}{\eta_{motor}} = \frac{1.96 \times 10^6}{0.9} = 2.18 \times 10^6 \text{ J}$$

* Since 25% energy is *wasted* in the motor and gearing, the efficiency of the lift is 75%.

$$\text{Power rating of motor} = \frac{\text{Work done}}{\text{Time taken}} = \frac{2.18 \times 10^6}{27 \times 60} = \mathbf{1346\ W}$$

$$\text{Current taken by motor} = \frac{1346}{230} = \mathbf{5.85\ A}$$

Example 4.18. *An electric hoist makes 10 double journeys per hour. In each journey, a load of 6000 kg is raised to a height of 60 m in 90 seconds and the hoist returns empty in 75 seconds. The hoist cage weighs 500 kg and has a balance weight of 3000 kg. The efficiency of the hoist is 80% and that of the driving motor 88%. Calculate* (*i*) *the electrical energy absorbed per double journey* (*ii*) *the hourly consumption in kWh* (*iii*) *the horse-power of the motor (iv) the cost of electric energy if hoist works for 4 hours per day for 30 days. Cost per kWh is Rs 4.50.*

Solution. When the hoist cage goes up, the balance weight goes down and when the cage goes down, the balance weight goes up.

Total mass lifted on upward journey = Load + mass of cage − mass of balance weight
= 6000 + 500 − 3000 = 3500 kg

Work done during upward journey = $mgh = 3500 \times 9{\cdot}8 \times 60$ J

Total mass moved on downward journey = Mass of balance wt. − Mass of cage
= 3000 − 500 = 2500 kg

Work done during downward journey = $mgh = 250 \times 9{\cdot}8 \times 60$ J

Work done during each double journey = $9{\cdot}8 \times 60\,(3500 + 2500)\text{J} = 353 \times 10^4$ J

Overall η = $0{\cdot}8 \times 0{\cdot}88 = 0{\cdot}704$

(*i*) Input energy per double journey = $353 \times 10^4/0{\cdot}704 = 501 \times 10^4$ J

$$= \frac{501 \times 10^4}{3.6 \times 10^6}\ \text{kWh} = \mathbf{1{\cdot}4\ kWh}$$

(*ii*) Hourly consumption = 1·4 × No. of double journeys/hr
= 1·4 × 10 = **14 kWh**

(*iii*) The maximum rate of working is during upward journey.

$$\therefore \quad \text{h.p. rating of motor} = \frac{\text{Work done in upward journey}}{\text{Hoist efficiency} \times \text{time for up journey} \times 746}$$

$$= \frac{3500 \times 9.8 \times 60}{0.8 \times 90 \times 746} = \mathbf{38{\cdot}4\ h.p.}$$

(*iv*) Energy consumption for 30 days = Hourly consumption × 4 × 30 = 14 × 4 × 30 = 1680 kWh

Total cost of energy = Rs. 1680 × 4·5 = **Rs. 7560**

Example 4.19. *A generator supplies power to a factory through cables of total resistance 20 ohms. The potential difference at the generator is 5000 V and power output is 50 kW. Calculate* (*i*) *power suppled by the generator,* (*ii*) *potential difference at the factory.*

Solution. Fig. 4.6 shows the conditions of the problem.

Output power of generator is given by ;

$$P = 50\ \text{kW} = 50 \times 10^3\ \text{W}$$

P.D. at the generator, E = 5000 V

$\therefore$ Current in cables is given by ;

$$I = \frac{P}{E} = \frac{50 \times 10^3}{5000} = 10\ \text{A}$$

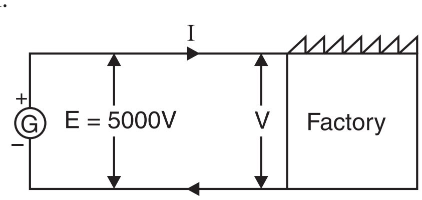


Fig. 4.6

(*i*) Power loss in cables $= I^2 R = (10)^2 \times 20 = 2000$ W

$\therefore$ Power supplied at the factory $= 50 \times 10^3 - 2000 =$ **48,000 W**

(*ii*) Voltage drop in cables $= I R = 10 \times 20 = 200$ V

$\therefore$ P.D. at the factory, $V = E - I R = 5000 - 200 =$ **4800 V**

Tutorial Problems

1. The power required to drive a certain machine at 350 r.p.m. is 600 kW. Calculate the driving torque. **[16370 Nm]**
2. An electrically driven pump lifts 1500 litres of water per minute through a height of 25 m. Allowing an overall efficiency of 75%, calculate the input power to the motor. If the pump is in operation for an average of 8 hours per day for 30 days, calculate the energy consumed in kWh and the cost of energy at the rate of 50 P/kWh. Assume 1 litre of water has a mass of 1000 kg and $g = 9{\cdot}81$ m/s^2. **[8·167 kW, 1960 kWh, Rs. 980]**
3. A 440-volt motor is used to drive an irrigation pump. The efficiency of motor is 85% and the efficiency of pump is 66%. The pump is required to lift 240 tonne of water per hour to a height of 30 metres. Calculate the current taken by the motor. **[79·48 A]**
4. A hydro-electric generating plant is supplied from a reservoir of capacity $2 \times 10^7\ m^3$ with a head of 200 m. The hydraulic efficiency of the plant is 0·8 and electric efficiency is 0·9. What is the total available energy ? **[7·85 × 10^9 watt-hours]**
5. A 460-V d.c. motor drives a hoist which raises a load of 100 kg with a velocity of 15 m/s. Calculate :
   (*i*) The power output of the motor assuming the hoist gearing to have an efficiency of 0·8.
   (*ii*) The motor current, assuming the motor efficiency to be 0·75. **[(*i*) 18·4 kW (*ii*) 53·2 A]**
6. When a certain electric motor is operated for 30 minutes, it consumes 0·75 kWh of energy. During that time, its total energy loss is 3×10^5 J.
   (*i*) What is the efficiency of the motor ?
   (*ii*) How many joules of work does it perform in 30 minutes ? **[(*i*) 88·8% (*ii*) 2·4 × 10^{6}J]**
7. The total power supplied to an engine that drives an electric generator is 40·25 kW. If the generator delivers 15A to a 100 Ω load, what is the efficiency of the system ? **[55·9%]**
8. A certain system consists of three identical devices in cascade, each having efficiency 0·85. The first device draws 3A from a 20V source. How much current does the third device deliver to a 50Ω load ? **[0·027 A]**

4.10. Heating Effect of Electric Current

When electric current is passed through a conductor, heat is produced in the conductor. This effect is called **heating effect of electric current.**

It is a matter of common experience that when electric current is passed through the element of an electric heater, the element becomes red hot. It is because electrical energy is converted into heat energy. This is called heating effect of electric current and is utilised in the manufacture of many heating appliances, *e.g.,* electric iron, electric kettle, etc. The basic principle of all these devices is the same. Electric current is passed through a high resistance (called *heating element*), thus producing the required heat.

Cause. Let us discuss the cause of heating effect of electric current. When potential difference is applied across the ends of a conductor, the free electrons move with drift velocity and current is established in the conductor. As the free electrons move through the conductor, they collide with positive ions of the conductor. On collision, the kinetic energy of an electron is transferred to the ion with which it has collided. As a result, the kinetic energy of vibration of the positive ion increases, *i.e.,* temperature of the conductor increases. Therefore, as current flows through a conductor, the free electrons lose energy which is converted into heat. Since the source of e.m.f. (*e.g.,* a battery) is maintaining current in the conductor, it is clear that electrical energy supplied by the battery is converted into heat in the conductor.

Applications. The heating effect of electric current is utilised in the manufacture of many heating appliances such as electric heater, electric toaster, electric kettle, soldering iron etc. The basic principle of all these appliances is the same. Electric current is passed through a high resistance (called heating element), thus producing the required heat. There are a number of substances used for making a heating element. One that is commonly used is an alloy of nickel and chromium, called **nichrome.** This alloy has a resistance more than 50 times that of copper. The heating element may be either nichrome wire or ribbon wound on some insulating material that is able to withstand heat.

4.11. Heat Produced in a Conductor by Electric Current

On the basis of his experimental results, Joule found that the amount of heat produced (H) when current I amperes flows through a conductor of resistance R ohms for time t seconds is $H = I^2Rt$ joules. This equation is known as Joule's law of heating.

Suppose a battery maintains a potential difference of V volts across the ends of a conductor AB of resistance R ohms as shown in Fig. 4.7. Let the steady current that passes from A to B be I amperes. If this current flows for t seconds, then charge transferred from A to B in t seconds is

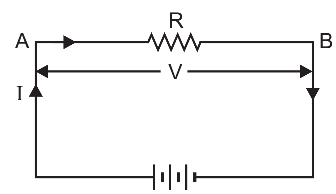


Fig. 4.7

$$q = It$$

The electric potential energy lost (W) by the charge q as it moves from A to B is given by ;

$$W = \text{Charge} \times \text{P.D. between } A \text{ and } B$$

$$= qV = (It)\,V = I^2Rt \quad (\because V = IR)$$

or $$W = I^2Rt$$

This loss of electric potential energy of charge is converted into heat (H) because the conductor AB has resistance only.

$$\therefore \quad H = W = I^2Rt \text{ joules} = \frac{I^2Rt}{4.18} \text{ calories} \quad ...(i)$$

It is found experimentally that 1 cal = 4.18 J.

Eq. (*i*) is known as **Joule's law of heating.** It is because Joule was the first scientist who studied the heating effect of electric current through a resistor. Thus according to Joule, heat produced in a conductor is directly proportional to

(*i*) square of current through the conductor

(*ii*) resistance of the conductor

(*iii*) time for which current is passed through the conductor.

Note. $$H = VIt = I^2Rt = \frac{V^2}{R}t \text{ joules}$$

$$= \frac{VIt}{4.18} = \frac{I^2Rt}{4.18} = \frac{V^2t}{R \times 4.18} \text{ calories}$$

Important points. While dealing with problems on heating effect of electric current, the following points may be kept in mind :

(*i*) The electrical energy in kWh can be converted into joules by the following relation :

$$1 \text{ kWh} = 36 \times 10^5 \text{ joules}$$

(*ii*) The heat energy in calories can be converted into joules by the following relation :

$$1 \text{ calorie} = 4{\cdot}186 \text{ joules}$$
$$1 \text{ kcal} = 4186 \text{ joules}$$

(*iii*) The electrical energy in kWh can be converted into calories (or kilocalories) by the following relation :

$$1 \text{ kWh} = 36 \times 10^5 \text{ joules} = \frac{36 \times 10^5}{4.186} \text{ calories} = 860 \times 10^3 \text{ calories}$$

$$\therefore \quad 1 \text{ kWh} = 860 \text{ kcal}$$

(*iv*) The electrical energy supplied to the heating appliance forms the *input energy.* The heat obtained from the device is the *output energy.* The difference between the two, if any, represents the loss of energy during conversion from electrical into heat energy.

4.12. Mechanical Equivalent of Heat (J)

Joule performed a series of experiments to establish the relationship between the mechanical work done and heat produced. He found that heat produced (H) is directly proportional to the amount of mechanical work done (W) *i.e.*,

$$H \propto W \quad \text{or} \quad W = JH$$

where J is a constant of proportionality and is called *mechanical equivalent of heat*. The experimentally found value of J is

$$J = 4{\cdot}2 \text{ J/cal}$$

Note that J is a numerical factor relating mechanical units to heat units. Let us interpret the meaning of J. It takes 4·2 J of mechanical work to raise the temperature of 1g of water by 1°C. In other words, 4·2J of mechanical energy is equivalent to 1 calorie of heat energy.

Example 4.20. *In Fig. 4.8, the heat produced in 5 Ω resistor due to current flowing through it is 10 calories per second. Calculate the heat generated in 4 Ω resistor.*

Solution. Let I_1 and I_2 be the currents in the two parallel branches as shown in Fig. 4.8. The p.d. across the parallel branches is the same *i.e.*

$$I_1(4+6) = 5\,I_2 \quad \therefore \quad I_2 = 2\,I_1$$

Heat produced per second in 5Ω resistor is

$$H_1 = \frac{I_2^2 \times 5}{4.2}$$

or

$$10 = \frac{(2I_1)^2 \times 5}{4.2}$$

$$\therefore \quad I_1^2 = 2{\cdot}1$$

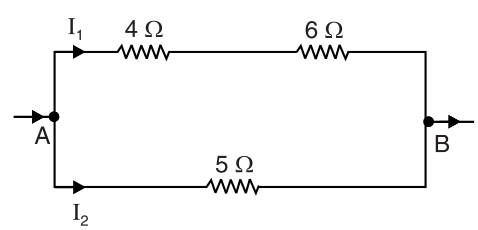


Fig. 4.8

Heat produced in 4Ω resistor per second

$$= \frac{I_1^2 \times 4}{4.2} = \frac{2.1 \times 4}{4.2} = \textbf{2 cal/sec}$$

Example 4.21. *An electric heater contains 4 litres of water initially at a mean temperature of 15ºC. 0·25 kWh is supplied to the water by the heater. Assuming no heat losses, what is the final temperature of the water ?*

Solution. Let tºC be the final temperature of water.

Heat received by water (*i.e.* output energy)

$$= \text{mass} \times \text{sp. heat} \times \text{rise in temp.} = 4 \times 1 \times (t - 15) \text{ kcal}$$

Electrical energy supplied to heater (*i.e.* input energy)

$$= 0{\cdot}25 \text{ kWh} = 0{\cdot}25 \times 860 \text{ kcal} \qquad (\because\ 1 \text{ kWh} = 860 \text{ kcal})$$

As there are no losses, output energy is equal to the input energy *i.e.*

$$4 \times 1 \times (t - 15) = 0{\cdot}25 \times 860 \quad \text{or} \quad t = \mathbf{68{\cdot}8^\circ C}$$

Example 4.22. *An immersion heater takes 1 hour to heat 50 kg of water from 20ºC to boiling point. Calculate the power rating of the heater, assuming the heating equipment to have an efficiency of 90%.*

Solution. Heat received by water (*i.e.* output energy)

$$= \text{mass} \times \text{specific heat} \times \text{rise in temperature}$$

$$= 50 \times 1 \times 80 = 4000 \text{ kcal} = 4000/860 = 4{\cdot}65 \text{ kWh}$$

Electrical energy supplied to heater (*i.e.* input energy)

$$= 4{\cdot}65/0{\cdot}9 = 5{\cdot}167 \text{ kWh}$$

$$\therefore \quad \text{Power rating} = \frac{\text{Energy}}{\text{Time}} = \frac{5.167}{1 \text{ hour}} = \mathbf{5.167 \text{ kW}}$$

Example 4.23. *The cost of boiling 2 kg of water in an electric kettle is 12 paise. The kettle takes 6 minutes to boil water from an ambient temperature of 20ºC. Calculate (i) the efficiency of kettle and (ii) the wattage of kettle if cost of 1 kWh is 40 paise.*

Solution. (*i*) Heat received by water (*i.e.* output energy)

$$= 2 \times 1 \times 80 = 160 \text{ kcal}$$

Electrical energy supplied (*i.e.* input energy)

$$= 12/40 \text{ kWh} = 860 \times 12/40 = 258 \text{ kcal}$$

$$\therefore \quad \text{Kettle efficiency} = \frac{160}{258} \times 100 = \mathbf{62\%}$$

(*ii*) Let *W* kilowatt be the power rating of the kettle.

$$\text{Input energy} = W \times \text{time in hours}$$

$$\text{or} \quad 12/40 = W \times 6/60$$

$$\therefore \quad \text{Wattage of kettle, } W = \frac{12}{40} \times \frac{60}{6} = \mathbf{3 \text{ kW}}$$

Example 4.24. *How long will it take to raise the temperature of 880 gm of water from 16ºC to boiling point ? The heater takes 2 amperes at 220 V and its efficiency is 90%.*

Solution. Heat received by water (*i.e.* output energy)

$$= 0{\cdot}88 \times 1 \times (100 - 16) = 73{\cdot}92 \text{ kcal} = 73{\cdot}92/860 = 0{\cdot}086 \text{ kWh}$$

Electrical energy supplied to the heater (*i.e.* input energy)

$$= 0{\cdot}086/0{\cdot}9 = 0{\cdot}096 \text{ kWh}$$

The heater is supplying a power of 220 × 2 = 440 watts = 0·44 kW. Let *t* hours be the required time.

$$\text{Input energy} = \text{wattage} \times \text{time} \quad \text{or} \quad 0{\cdot}096 = 0{\cdot}44 \times t$$

$$\therefore \quad t = 0{\cdot}096/0{\cdot}44 = 0{\cdot}218 \text{ hours} = 0{\cdot}218 \times 60 = \mathbf{13{\cdot}08 \text{ minutes}}$$

Example 4.25. *An electric kettle is required to raise the temperature of 2 kg of water from 20ºC to 100ºC in 15 minutes. Calculate the resistance of the heating element if the kettle is to be used on a 240 volts supply. Assume the efficiency of the kettle to be 80%.*

Solution. Heat received by water (*i.e.* output energy)

$= 2 \times 1 \times (100 - 20) = 160$ kcal $= 160/860 = 0{\cdot}186$ kWh

Electrical energy supplied to the kettle

$= 0{\cdot}186/0{\cdot}8 = 0{\cdot}232$ kWh

The electrical energy of 0·232 kWh is supplied in 15/60 = 0·25 hours.

∴ Power rating of kettle $= 0{\cdot}232/0{\cdot}25 = 0{\cdot}928$ kW $= 928$ watts

Let R ohms be the resistance of the heating element.

$$\therefore \quad V^2/R = 928 \quad \text{or} \quad R = \frac{240 \times 240}{928} = \mathbf{62\ \Omega}$$

Example 4.26. *The heater element of an electric kettle has a constant resistance of 100 Ω and the applied voltage is 250 V. Calculate the time taken to raise the temperature of one litre of water from 15°C to 90°C assuming that 85% of the power input to the kettle is usefully employed. If the water equivalent of the kettle is 100g, find how long will it take to raise a second litre of water through the same temperature range immediately after the first.*

Solution. Mass of water, $m = 1$ litre $= 1$ kg ; $\theta = 90 - 15 = 75$°C ; $S = 1$

Heat taken by water $= mS\theta = 1 \times 1 \times 75 = 75$ kcal

Heat taken by kettle = water equivalent of kettle × θ = 0.1 × 75 = 7.5 kcal

Heat taken by both = 75 + 7.5 = 82.5 kcal

Now, $I = \frac{250}{100} = 2.5$ A ; $J = 4200$ J/kcal

Heat produced electrically $= \frac{I^2Rt}{J}$ kcal ... t in seconds

Heat available for heating $= 0.85 \times \frac{I^2Rt}{J}$ kcal

$$\text{or} \quad 0.85 \times \frac{I^2Rt}{J} = 82.5$$

$$\text{or} \quad 0.85 \times \frac{(2.5)^2 \times 100 \times t}{4200} = 82.5$$

$$\therefore \quad t = 652 \text{ s} = \mathbf{10\ min.\ 52\ seconds}$$

In the second case, heat would be required to heat water only because kettle would be already hot.

$$\therefore \quad \frac{0.85 \times (2.5)^2 \times 100 \times t}{4200} = 75 \quad \text{or} \quad t = \mathbf{9\ min.\ 53\ seconds}$$

As expected, the time required for heating in the second case is less than the first case.

Example 4.27. *The heaters A and B are in parallel across the supply voltage V. Heater A produces 500 kcal in 20 minutes and B produces 1000 kcal in 10 minutes. The resistance of heater A is 10 Ω. What is the resistance of heater B ? If the same heaters are connected in series, how much heat will be produced in 5 minutes ?*

Solution. Heat produced $= \frac{V^2t}{R \times J}$ kcal

$$\text{For heater } A,\ 500 = \frac{V^2 \times (20 \times 60)}{10 \times J} \qquad ...(i)$$

$$\text{For heater } B,\ 1000 = \frac{V^2 \times (10 \times 60)}{R \times J} \qquad ....(ii)$$

Dividing eq. (*i*) by eq. (*ii*), we get,

$$\frac{500}{1000} = \frac{20\times 60}{10\times 60}\times\frac{R}{10} \quad \therefore \quad R = \mathbf{2{\cdot}5\ \Omega}$$

When the heaters are connected in series, the total resistance becomes $R_T = 10 + 2{\cdot}5 = 12{\cdot}5\ \Omega$.

$\therefore$ Heat produced in 5 minutes

$$= \frac{V^2 t}{R_T \times J} = \frac{V^2}{J}\times\frac{t}{R_T}$$

$$= \frac{5{,}000}{20\times 60}\times\frac{5\times 60}{12.5} = \mathbf{100\ kcal} \qquad \left\{\begin{matrix}\text{From eq. } (i) \\ \frac{V^2}{J} = \frac{5000}{20\times 60}\end{matrix}\right\}$$

Example 4.28. *A soldering iron is rated at 50 watts when connected to a 250 V supply. If the soldering iron takes 5 minutes to heat to a working temperature of 190°C from 20°C, find its mass, assuming it to be made of copper. Given specific heat capacity of copper is 390 J/kg°C.*

Solution. Let m kg be the mass of soldering iron.

Heat gained by the soldering iron = $mS\theta = m \times 390 \times (190 - 20) = 66{,}300\ m$ joules

Heat released by the heating element = power × time = (50) × (5 × 60) = 15,000 joules

Assuming all the heat released by the element is absorbed by the copper *i.e.* soldering iron is 100% efficient,

$$15{,}000 = 66{,}300\ m \quad \therefore \quad m = 15{,}000/66{,}300 = \mathbf{0.226\ kg}$$

Example 4.29. *A cubic water tank has surface area of 6 m² and is filled to 90% capacity 6 times daily. The water is heated from 20°C to 65°C. The losses per square metre of tank surface per 1°C temperature difference are 6.3 W. Find the loading in kW and the efficiency of the tank. Assume specific heat of water = 4200 J/kg/°C and 1 kWh = 3.6 MJ.*

Solution. Rise in temp, $\theta = 65 - 20 = 45$°C ; $S = 4200$ J/kg/°C. If l metres is one side of the tank, then surface area of the tank is $6l^2$.

$$\therefore \quad 6l^2 = 6\text{m}^2 \quad \text{or} \quad l = 1\text{m}$$

Volume of tank = $l^3 = (1)^3 = 1\text{m}^3$

Volume of water to be heated daily = 6 × 0.9 = 5.4 m³. As the mass of 1 m³ of water is 1000 kg,

$\therefore$ Mass of water to be heated daily, $m = 5.4 \times 1000 = 5400$ kg

Heat required to heat water to the desired temperature is

$$H_1 = mS\theta = 5400 \times 4200 \times 45 = 1020.6 \times 10^6 \text{ J}$$

$$= \frac{1020.6\times 10^6}{36\times 10^5} \text{ kWh} = 283.5 \text{ kWh}$$

$$\text{Heat losses, } H_2 = \frac{6.3\times 6\times \theta\times 24}{1000} \text{ kWh}$$

$$= \frac{6.3\times 6\times 45\times 24}{1000} = 40.82 \text{ kWh}$$

$$\text{Total energy supplied, } H = H_1 + H_2 = 283.5 + 40.82 = 324.32 \text{ kWh}$$

$$\text{Loading in kW} = \frac{H}{24 \text{ hr}} = \frac{324.32 \text{ kWh}}{24 \text{ hr}} = \mathbf{13.5\ kW}$$

$$\text{Efficiency of tank} = \frac{H_1}{H}\times 100 = \frac{283.5}{324.32}\times 100 = \mathbf{87.4\%}$$

Example 4.30. *An electric furnace is being used to melt 10 kg of aluminium. The initial temperature of aluminium is 20°C. Assume the melting point of aluminium to be 660°C, its specific heat capacity to be 950 J/kg°C and its specific latent heat of fusion to be 387000 J/kg. Calculate the power required to accomplish the conversion in 20 minutes, assuming the efficiency of conversion to be 75%. What is the cost of energy consumed if tariff is 50 paise per kWh ?*

Solution. Heat used to melt aluminium (*i.e.* output energy)

$$= 10 \times 950 \times (660 - 20) + 10 \times 387000 = 995 \times 10^4 \text{ joules}$$

$$= \frac{995 \times 10^4}{36 \times 10^5} = 2 \cdot 76 \text{ kWh}$$

Electrical energy supplied to the heating element

$$= 2 \cdot 76/0 \cdot 75 = 3 \cdot 68 \text{ kWh}$$

This much energy (*i.e.* 3·68 kWh) is to be supplied in 20/60 = 1/3 hour.

$$\therefore \quad \text{Power required} = \frac{3.68}{1/3} = 3 \cdot 68 \times 3 = \mathbf{11 \cdot 04\ kW}$$

$$\text{Cost of energy} = \text{Rs. } 0 \cdot 5 \times 3 \cdot 68 = \mathbf{Rs.\ 1 \cdot 84}$$

Example 4.31. *A transmitting valve is cooled by water circulating through its hollow electrodes. The water enters the valve at 25°C and leaves it at 85°C. Calculate the rate of flow in kg/second needed per kW of cooling. The temperature of 1 kg of water is raised to 1°C by 4178 joules.*

Solution. Heat to be taken away/sec = 1 kW × 1 sec = 1000 × 1 = 1000 joules. Let the required flow of water be m kg per second.

$$\text{Heat produced/sec} = \text{mass} \times \text{Sp. heat} \times \text{rise in temp.}$$

$$= m \times 4178 \times (85 - 25) = 250{,}680\, m \text{ joules}$$

$$\therefore \quad 250{,}680\, m = 1000 \quad \text{or} \quad m = \frac{1000}{250{,}680} = \mathbf{0 \cdot 004\ kg/sec}$$

Tutorial Problems

1. An electric kettle marked 1 kW, 230 V, takes 7·5 minutes to bring 1 kg of water at 15ºC to boiling point (100ºC). Find the efficiency of the kettle. **[79·07%]**
2. An electric kettle contains 1·5 kg of water at 15ºC. It takes 2·5 hours to raise the temperature to 90ºC. Assuming the heat losses due to radiation and heating the kettle to be 15 kcal, find (*i*) wattage of the kettle and (*ii*) current taken if supply voltage is 230 V. **[(*i*) 59·2 W (*ii*) 0·257 A]**
3. A soldering iron is rated at 50 watts when connected to a 250 *V* supply. If the soldering iron takes 5 minutes to heat to a working temperature of 190ºC from 20ºC, find its mass, assuming it to be made of copper. Given specific heat capacity of copper is 390 J/kgºC. **[0·226 kg]**
4. Find the amount of electrical energy expended in raising the temperature of 45 litres of water by 75ºC. To what height could a weight of 5 tonnes be raised with the expenditure of the same energy ? Assume efficiencies of heating and lifting equipment to be 90% and 70% respectively **[4·36 kWh, 224 m]**
5. Calculate the time taken for a 25 kW furnace, having an overall efficiency of 80% to melt 20 kg of aluminium. Take the specific heat capacity, melting point and latent heat of fusion of aluminium as 896 J/kgºC, 657ºC and 402 kJ/kg respectively. **[16 min 13 sec]**
6. An electric boiler has two heating elements each of 230 V, 3·5 kW rating and containing 8 litres of water at 30ºC. Assuming 10% loss of heat from the boiler, find how long after switching on the heater circuit will the water boil at atmospheric pressure
   (*i*) if the two elements are in parallel
   (*ii*) if the two elements are in series ? The supply voltage is 230 V. **[(*i*) 373·3 s (*ii*) 1493·2 s]**
7. A coil of resistance 100 Ω is immersed in a vessel containing 0·5 kg of water at 16ºC and is connected to a 220 V electric supply. Calculate the time required to boil away all the water. Given J = 4200 J/kcal; latent heat of steam = 536 kcal/kg. **[44 min 50 sec]**

Objective Questions

1. A 25W, 220 V bulb and a 100 W, 220 V bulb are joined in parallel and connected to 220 V supply. Which bulb will glow more brightly ?

(*i*) 25 W bulb
(*ii*) 100 W bulb
(*iii*) both will glow with same brightness
(*iv*) neither bulb will glow

2. A 25 W, 220 V bulb and a 100 W, 220 V bulb are joined in series and connected to 220 V supply. Which bulb will glow brighter ?

(*i*) 25 W bulb
(*ii*) 100 W bulb
(*iii*) both will glow with same brightness
(*iv*) neither bulb will glow

3. You are given three bulbs of 25 W, 40 W and 60 W. Which of them has the lowest resistance ?

(*i*) 25 W bulb
(*ii*) 40 W bulb
(*iii*) 60 W bulb
(*iv*) information incomplete

4. You have the following electric appliances :

(*a*) 1 kW, 250 V electric heater
(*b*) 1 kW, 250 V electric kettle
(*c*) 1 kW, 250 V electric bulb

Which of these has the highest resistance ?

(*i*) heater
(*ii*) kettle
(*iii*) all have equal resistances
(*iv*) electric bulb

5. The time required for 1 kW electric heater to raise the temperature of 10 litres of water through 10ºC is

(*i*) 210 sec (*ii*) 420 sec
(*iii*) 42 sec (*iv*) 840 sec

6. Two electric bulbs rated at P_1 watt, V volt and P_2 watt, V volt are connected in series across V volt. The total power consumed is

(*i*) $P_1 + P_2$ (*ii*) $\sqrt{P_1 P_2}$
(*iii*) $\frac{P_1 + P_2}{2}$ (*iv*) $\frac{P_1 P_2}{P_1 + P_2}$

7. A tap supplies water at 22ºC. A man takes 1 litre of water per minute at 37ºC from the geyser. The power of geyser is

(*i*) 1050 W (*ii*) 1575 W
(*iii*) 525 W (*iv*) 2100 W

8. A 3ºC rise in temperature is observed in a conductor by passing a certain amount of current. When the current is doubled, the rise in temperature is

(*i*) 15ºC (*ii*) 12ºC
(*iii*) 9ºC (*iv*) 3ºC

9. How much electrical energy in kWh is consumed in operating ten 50 W bulbs for 10 hours in a day in a month of 30 days ?

(*i*) 500 (*ii*) 15000
(*iii*) 150 (*iv*) 15

10. Two heater wires of equal length are first connected in series and then in parallel. The ratio of heat produced in the two cases will be

(*i*) 2 : 1 (*ii*) 1 : 2
(*iii*) 4 : 1 (*iv*) 1 : 4

11. Two identical heaters each marked 1000 W, 250 V are placed in series and connected to 250 V supply. Their combined rate of heating is

(*i*) 500 W (*ii*) 2000 W
(*iii*) 1000 W (*iv*) 250 W

12. A constant voltage is applied between the ends of a uniform metallic wire. Some heat is developed in it. If both length and radius of the wire are halved, the heat developed during the same duration will become

(*i*) half (*ii*) twice
(*iii*) one fourth (*iv*) same

13. What is immaterial for a fuse ?

(*i*) its specific resistance
(*ii*) its radius
(*iii*) its length
(*iv*) current flowing through it

14. If the current in an electric bulb drops by 2%, then power decreases by

(*i*) 1% (*ii*) 2%
(*iii*) 4 % (*iv*) 16%

15. The fuse wire is made of

(*i*) tin-lead alloy (*ii*) copper
(*iii*) tungsten (*iv*) nichrome

Answers

1. (*ii*)	**2.** (*i*)	**3.** (*iii*)	**4.** (*iii*)	**5.** (*ii*)
6. (*iv*)	**7.** (*i*)	**8.** (*ii*)	**9.** (*iii*)	**10.** (*iv*)
11. (*i*)	**12.** (*i*)	**13.** (*iii*)	**14.** (*iii*)	**15.** (*i*)

5
Electrostatics

Introduction

So far we have discussed that if two oppositely charged bodies are connected through a conductor, electrons will flow from the negative charge (excess of electrons) to the positive charge (deficiency of electrons). This directed flow of electrons is called electric current. The electric current will continue to flow so long as the 'excess' and 'deficiency' of electrons exist in the bodies. In other words, electric current will continue to flow so long as we maintain the potential difference between the bodies. The branch of engineering which deals with the flow of electrons (*i.e.* electric current) is called **current electricity** and is important in many ways. For example, it is the electric current by means of which electrical energy can be transferred from one point to another for utilisation.

There can be another situation where charges (*i.e.* electrons) do not move but remain static or stationary on the bodies. Such a situation will arise when the charged bodies are separated by some insulating medium, disallowing the movement of electrons. This is called **static electricity** and the branch of engineering which deals with static electricity is called **electrostatics**. Although current electricity is of greater practical use, yet the importance of static electricity cannot be ignored. Many of the advancements made in the field of electricity owe their developments to the knowledge scientists obtained from electrostatics. *The most useful outcomes of static electricity are the development of* **lightning rod** *and the* **capacitor.** In this chapter, we shall confine our attention to the behaviour and applications of static electricity.

5.1. Electrostatics

The branch of engineering which deals with charges at rest is called **electrostatics**.

When a glass rod is rubbed with silk and then separated, the former becomes positively charged and the latter attains equal negative charge. It is because during rubbing, some electrons are transferred from glass to silk. Since glass rod and silk are separated by an insulating medium (*i.e.*, air), they retain the charges. In other words, the charges on them are static or stationary. Note that the word 'electrostatic' means electricity at rest.

5.2. Importance of Electrostatics

During the past century, there was considerable increase in the practical importance of electrostatics. A few important applications of electrostatics are given below :

(*i*) Electrostatic generators can produce voltages as high as 10^6 volts. Such high voltages are required for X-ray work and nuclear bombardment.

(*ii*) We use principles of electrostatics for spray of paints, powder, etc.

(*iii*) The principles of electrostatics are used to prevent pollution.

(*iv*) The problems of preventing sparks and breakdown of insulators in high voltage engineering are essentially electrostatic.

(*v*) *The development of lightning rod and capacitor are the outcomes of electrostatics.*

5.3. Methods of Charging a Conductor

An uncharged conductor can be charged by the following two methods :

(*i*) By conduction (*ii*) By induction

(*i*) **By conduction.** *In this method, a charged body is brought in contact with the uncharged conductor.* Fig. 5.1 (*i*) shows the uncharged conductor *B* kept on an insulating stand. When the positively charged conductor *A* provided with insulating handle is touched with uncharged conductor *B* [See Fig. 5.1 (*ii*)], free electrons from conductor *B* move to conductor *A*. As a result, there occurs a deficit of electrons in conductor *B* and it becomes positively charged. Similarly, if the conductor *A* is negatively charged, the conductor *B* will also get negatively charged.

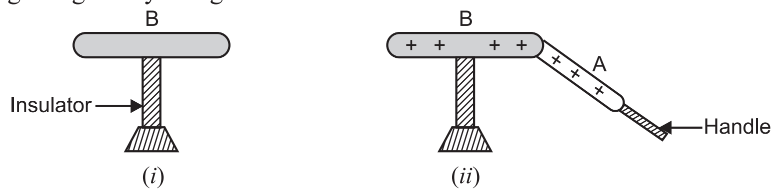


Fig. 5.1

It may be noted that conductor *A* is provided with an insulting handle so that its charge does not escape to the ground through our body. For the same reason, the conductor *B* is kept on the insulating stand.

(*ii*) **By Induction.** *In this method, a charged body is brought close to the uncharged conductor but does not touch it.* Fig. 5.2 (*i*) shows a negatively charged plastic rod (provided with insulating handle) kept near an uncharged metal sphere. The free electrons of the sphere near the rod are repelled to the farther end. As a result, the region of the sphere near the rod becomes positively charged and the farthest end of sphere becomes equally negatively charged. If now the sphere is connected to the ground through a wire as shown in Fig. 5.2 (*ii*), its free electrons at the farther end flow to the ground. On removing the wire to the ground [See Fig. 5.2 (*iii*)], the positive charge at the near end of sphere remains held there due to the attractive force of external negative charge. Finally, when the plastic rod is removed [See Fig. 5.2 (*iv*)], the positive charge spreads uniformly on the sphere. Thus, the sphere is positively charged by induction. Note that in the process, the negatively charged plastic rod loses none of its negative charge. Similarly, the metal sphere can be negatively charged by bringing a positively charged rod near it.

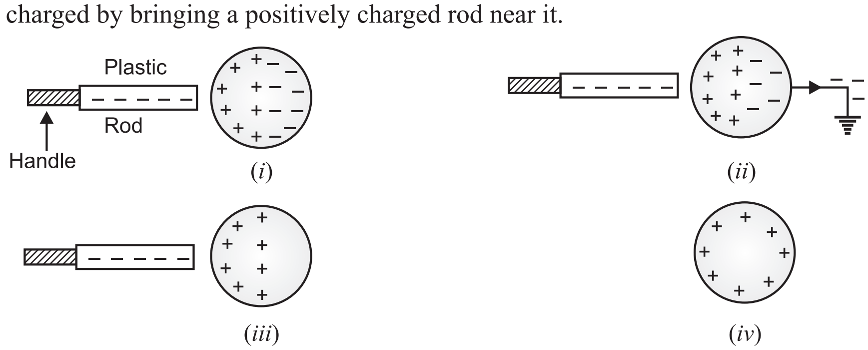


Fig. 5.2

Note that charging a body by induction requires no contact with the body inducing the charge. This is in contrast to charging a body by conduction which does require contact between the two bodies.

5.4. Coulomb's Laws of Electrostatics

Charles Coulomb, a French scientist, observed that when two charges are placed near each other, they experience a force. He performed a number of experiments to study the nature and magnitude of the force between the charged bodies. He summed up his conclusions into two laws, known as Coulomb's laws of electrostatics.

First law. This law relates to the nature of force between two charged bodies and may be stated as under :

Like charges repel each other while unlike charges attract each other.

In other words, if two charges are of the same nature (*i.e.* both positive or both negative), the force between them is repulsion. On the other hand, if one charge is positive and the other negative, the force between them is an attraction.

Second law. This law tells about the magnitude of force between two charged bodies and may be stated as under :

*The force between two *point charges is directly proportional to the product of their magnitudes and inversely proportional to the square of distance between their centres.*

Mathematically, $$F \propto \frac{Q_1 Q_2}{d^2}$$

or $$F = k\frac{Q_1 Q_2}{d^2} \quad ...(i)$$

where k is a constant whose value depends upon the medium in which the charges are placed and the system of units employed. In SI units, force is measured in newtons, charge in coulombs, distance in metres and the value of k is given by ;

$$k = \frac{1}{4\pi\varepsilon_0\varepsilon_r}$$

where ε_0 = Absolute permittivity of vacuum or air.

ε_r = Relative permittivity of the medium in which the charges are placed. For vacuum or air, its value is 1.

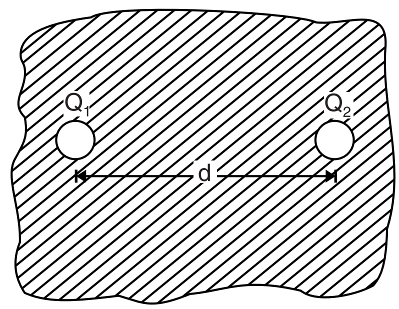


Fig. 5.3

The value of $\varepsilon_0 = 8{\cdot}854 \times 10^{-12}$ F/m and the value of ε_r is different for different media.

$$\therefore \quad F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r d^2} \quad ...(ii)$$

Now $$\frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi \times 8.854 \times 10^{-12}} = 9 \times 10^9$$

$$\therefore \quad F = 9\times10^9 \frac{Q_1 Q_2}{\varepsilon_r d^2} \quad \textit{...in a medium}$$

$$= 9\times10^9 \frac{Q_1 Q_2}{d^2} \quad \textit{...in air}$$

Unit of charge. The unit of charge (*i.e.* 1 coulomb) can also be defined from Coulomb's second law of electrostatics. Suppose two equal charges placed 1 m apart in *air* exert a force of 9×10^9 newtons *i.e.*

$$Q_1 = Q_2 = Q \; ; \; d = 1\text{m} \; ; \; F = 9 \times 10^9 \text{ N}$$

$$\therefore \quad F = 9\times10^9 \frac{Q_1 Q_2}{d^2}$$

or $$9 \times 10^9 = 9\times10^9 \frac{Q^2}{(1)^2}$$

* Charged bodies approximate to point charges if they are small compared to the distance between them.

or $Q^2 = 1$

or $Q = \pm 1 = 1\text{coulomb}$

Hence **one coulomb** *is that charge which when placed in air at a distance of one metre from an equal and similar charge repels it with a force of* 9×10^9 *N.*

Note that coulomb is very large unit of charge in the study of electrostatics. In practice, charges produced experimentally range between pico-coulomb (pC) and micro-coulomb (μC).

$$1\text{pC} = 10^{-12}\text{C} \quad ; \quad 1\mu\text{C} = 10^{-6}\text{C}$$

Note. One disadvantage of SI units is that coulomb is an inconveniently large unit. This is clear from the fact that the force exerted by a charge of 1C on another equal charge at a distance of 1m is 9×10^9N. Could you hold two one-coulomb charges a metre apart ?

5.5. Absolute and Relative Permittivity

Permittivity is the property of a medium and affects the magnitude of force between two point charges. The greater the permittivity of a medium, the lesser the force between the charged bodies placed in it and *vice-versa*. Air or vacuum has a minimum value of permittivity. The absolute (or actual) permittivity ε_0 (Greek letter 'epsilon') of air or vacuum is $8{\cdot}854 \times 10^{-12}$ F/m. The absolute (or actual) permittivity ε of all other insulating materials is greater than ε_0. The ratio $\varepsilon/\varepsilon_0$ is called the **relative permittivity* of the material and is denoted by ε_r *i.e.*

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

where ε = absolute (or actual) permittivity of the material

ε_0 = absolute (actual) permittivity of air or vacuum ($8{\cdot}854 \times 10^{-12}$ F/m)

ε_r = relative permittivity of the material.

Obviously, ε_r for air would be $\varepsilon_0/\varepsilon_0 = 1$.

Permittivity of a medium plays an important role in electrostatics. For instance, the relative permittivity of insulating oil is 3. It means that for the same charges (Q_1 and Q_2) and distance (d), the force between the two charges in insulating oil will be one-third of that in air [See eq. (*ii*) in Art.5·4].

5.6. Coulomb's Law in Vector Form

Consider two like point charges Q_1 and Q_2 separated by distance d in vacuum. Clearly, charges will repel each other [See Fig. 5.4].

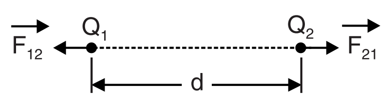


Fig. 5.4

Let $\vec{F}_{21}$ = force on Q_2 due to Q_1

$\vec{F}_{12}$ = force on Q_1 due to Q_2

$\hat{d}_{12}$ = unit vector pointing from Q_1 to Q_2

$\hat{d}_{21}$ = unit vector pointing from Q_2 to Q_1

According to Coulomb's law,

$$\vec{F}_{21} = k\frac{Q_1 Q_2}{d^2}\hat{d}_{12}$$

or
$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{d^2}\hat{d}_{12} \quad ...(i)$$

Similarly,
$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{d^2}\hat{d}_{21} \quad ...(ii)$$

Eqs. (*i*) and (*ii*) express Coulomb's law in vector form.

* Thus when we say that relative permittivity of a material is 10, it means that its absolute or acutal permittivity $\varepsilon = \varepsilon_0\, \varepsilon_r = 8.854 \times 10^{-12} \times 10 = 8.854 \times 10^{-11}$ F/m.

Importance of vector form. The reader may wonder about the utility of Coulomb's law in vector form over the scalar form. The answer will be readily available from the following discussion :

(*i*) The vector form shows at a glance that forces $\vec{F}_{21}$ and $\vec{F}_{12}$ are equal and opposite.

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{d^2}\hat{d}_{12}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{d^2}\hat{d}_{21}$$

As $$\hat{d}_{12} = -\hat{d}_{21}$$

$$\therefore \quad \vec{F}_{21} = -\vec{F}_{12}$$

That is $\vec{F}_{21}$ is equal in magnitude to $\vec{F}_{12}$ but opposite in direction. The scalar form does not show this fact. This is a distinct advantage over the scalar form.

(*ii*) $$\vec{F}_{21} = -\vec{F}_{12}$$

This means that $\vec{F}_{21}$ and $\vec{F}_{12}$ act along the same line *i.e.* along the line joining charges Q_1 and Q_2. In other words, the electrostatic force between two charges is a central force *i.e.* it acts along the line joining the centres of the two charges. However, scalar form does not show such a nature of electrostatic force between two charges.

5.7. The Superposition Principle

If we are given two charges, the electrostatic force between them can be found by using Coulomb's laws. However, if a number of charges are present, the force on any charge due to the other charges can be found by superposition principle stated below :

When a number of charges are present, the total force on a given charge is equal to the vector sum of the forces due to the remaining other charges on the given charge.

This simply means that we first find the force on the given charge (by Coulomb's laws) due to each of the other charges in turn. We then determine the total or net force on the given charge by finding the vector sum of all the forces.

Notes. (*i*) Consider two charges Q_1 and Q_2 located in air. If a third charge Q_3 is brought nearby, it has been found experimentally that presence of the third charge (Q_3) has no effect on the force between Q_1 and Q_2. This fact permits us to use superposition principle for electric forces.

(*ii*) The superposition principle holds good for electric forces and electric fields. This fact has made the mathematical description of electrostatic phenomena simpler than it otherwise would be.

(*iii*) We can use superposition principle to find (*a*) net force (*b*) net field (*c*) net flux (*d*) net potential and (*e*) net potential energy due to a number of charges.

Example 5.1. *A small sphere is given a charge of + 20μC and a second sphere of equal diameter is given a charge of −5 μC. The two spheres are allowed to touch each other and are then spaced 10 cm apart. What force exists between them ? Assume air as the medium.*

Solution. When the two spheres touch each other, the resultant charge = (20) + (−5) = 15 μC. When the spheres are separated, charge on each sphere, $Q_1 = Q_2 = 15/2 = 7{\cdot}5$ μC.

$$\therefore \quad \text{Force, } F = 9\times10^9\times\frac{Q_1Q_2}{d^2}$$

$$= 9\times10^9\times\frac{(7.5\times10^{-6})\,(7.5\times10^{-6})}{(0.1)^2} = \textbf{50.62 N } \textit{repulsive}$$

Example 5.2. *A charge q is divided into two parts in such a way that they repel each other with a maximum force when held at a certain distance apart. Find the distribution of the charge.*

Solution. Let the two parts be q' and $(q - q')$. Therefore, force F between them is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q'(q-q')}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{qq'-q'^2}{d^2}$$

For maximum value of F, $\frac{dF}{dq'} = 0 \quad \therefore \quad \frac{dF}{dq'} = \frac{1}{4\pi\varepsilon_0 d^2}(q-2q') = 0$

or $\qquad q - 2q' = 0 \quad \therefore \quad q' = \frac{q}{2}$

Hence in order to have maximum force, q should be divided into two equal parts.

Example 5.3. *Three point charges of + 5µC, + 5µC and + 5µC are placed at the vertices of an equilateral triangle which has sides 10 cm long. Find the force on each charge.*

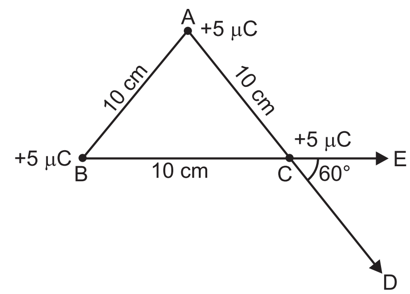


Fig. 5.5

Solution. The conditions of the problem are represented in Fig. 5.5. Consider + 5µC placed at the corner C. It is being repelled by the charges at A and B along ACD and BCE respectively. These two forces are equal, each being given by ;

$$F = 9\times10^9 \frac{(5\times10^{-6})(5\times10^{-6})}{(0.1)^2} = 22.5 \text{ N}$$

Resultant force at $C = 2F \cos 30° = 2 \times 22.5 \times \frac{\sqrt{3}}{2}$ = **38.97 N**

The forces acting on the charges placed at A and B will also be the same (*i.e.,* 38.97 N)

Example 5.4. *Two small spheres, each having a mass of 0·1g are suspended from a point by threads 20 cm long. They are equally charged and they repel each other to a distance of 24cm. What is the charge on each sphere ?*

Solution. Fig. 5.6 shows the conditions of the problem. Let B and C be the spheres, each carrying a charge q. The force of repulsion between the spheres is given by ;

$$F = 9\times10^9 \frac{q^2}{(0.24)^2}$$
$$= 156.25 \times 10^9 q^2$$

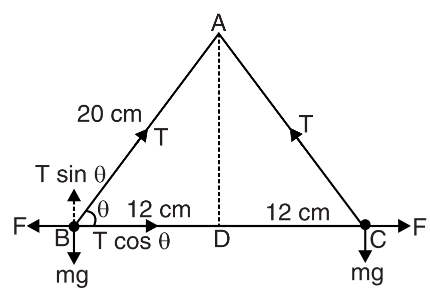


Fig. 5.6

Each sphere is under the action of three forces :

(i) weight $m\,g$ acting vertically downward, ***(ii)*** tension T, and ***(iii)*** electrostatic force F. Considering the sphere B and resolving T into rectangular components, we have,

$$m\,g = T\sin\theta \;;\; F = T\cos\theta$$

$\therefore \qquad \tan\theta = mg/F$

Now, $\qquad AD = \sqrt{AB^2 - BD^2} = \sqrt{(20)^2-(12)^2} = 16 \text{ cm}$

$\therefore \qquad \tan\theta = \frac{AD}{BD} = \frac{16}{12} \quad \therefore \quad \frac{16}{12} = \frac{mg}{F}$

or $$F = \frac{12}{16} mg = 0.75\ mg = 0.75 \times 10^{-4} \times 9.8 = 7.4 \times 10^{-4}\ \text{N}$$

But $$F = 156{\cdot}25 \times 10^9\ q^2$$

$$\therefore \quad 156{\cdot}25 \times 10^9\ q^2 = 7{\cdot}4 \times 10^{-4} \quad \text{or} \quad q^2 = \frac{7.4 \times 10^{-4}}{156.25 \times 10^9} = 4.8 \times 10^{-15}$$

$$\therefore \quad q = \mathbf{6{\cdot}9 \times 10^{-8}\ C}$$

Example 5.5. *Two point charges +Q and +4Q are placed at a distance 'a' apart on a horizontal plane. Where should the third charge be placed for it to be in equilibrium ?*

Solution. Let the point charge $+q$ be placed at a distance x from the charge $+4Q$ [See Fig. 5.7].

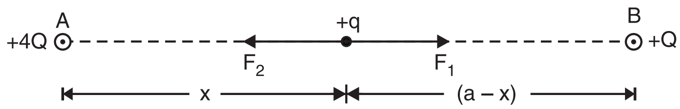


Fig. 5.7

Force on charge $+q$ due to charge $+4Q$ is

$$F_1 = \frac{q(4Q)}{4\pi\varepsilon_0 x^2} \quad \text{from } A \text{ to } B$$

Force on charge $+q$ due to charge $+Q$ is

$$F_2 = \frac{q(Q)}{4\pi\varepsilon_0 (a-x)^2} \quad \text{from } B \text{ to } A$$

In order that charge $+q$ is in equilibrium, $F_1 = F_2$.

$$\therefore \quad \frac{q(4Q)}{4\pi\varepsilon_0 x^2} = \frac{q(Q)}{4\pi\varepsilon_0 (a-x)^2} \quad \text{or} \quad x = \mathbf{2a/3}$$

Example 5.6. *Two point charges of +16 µC and −9 µC are 8 cm apart in air. Where can a third charge be located so that no net electrostatic force acts on it ?*

Solution. Let the third charge $+Q$ be located at P at a distance x from the charge −9µC as shown in Fig. 5.8.

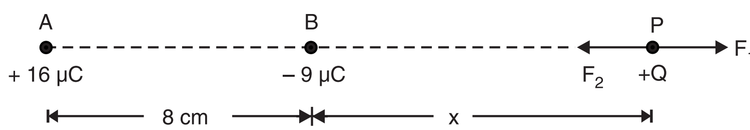


Fig. 5.8

Force at P due to charge + 16 µC at A is

$$F_1 = k\frac{16 \times 10^{-6} \times Q}{(x + 0.08)^2} \quad \text{along } AP$$

Force at P due to charge −9 µC at B is

$$F_2 = k\frac{9 \times 10^{-6} \times Q}{x^2} \quad \text{along } PB$$

For zero electrostatic force at P, $F_1 = F_2$.

$$\therefore \quad k\frac{16 \times 10^{-6} \times Q}{(x + 0.08)^2} = k\frac{9 \times 10^{-6} \times Q}{x^2}$$

or $$\frac{16}{(x + 0.08)^2} = \frac{9}{x^2} \quad \text{or} \quad \frac{4}{x + 0.08} = \frac{3}{x}$$

$$\therefore \quad x = 0.24\ \text{m} = \mathbf{24\ cm}$$

Example 5.7. *Two small balls are having equal charge Q (coulomb). The balls are suspended by two insulating strings of equal length L (metre) from a hook fixed to a stand. The whole set up is taken in a satellite into space where there is no gravity.*

(*i*) *What is the angle between the two strings ?*

(*ii*) *What is the tension in each string ?*

Solution. ***(i)*** In the absence of gravity, the tension in the strings is only due to Coulomb's repulsive force. Therefore, the strings become horizontal due to the electric force between the charges. Consequently, the angle between the strings is **180°**.

(ii) $$F = 9 \times 10^9 \times \frac{Q_1 Q_2}{d^2}$$

Here $$Q_1 = Q_2 = Q \; ; \quad d = 2L$$

$$\therefore \quad F = \mathbf{9 \times 10^9 \frac{Q^2}{4L^2}}$$

Example 5.8. *Two identical charged spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density 800 kg m^{-3}, the angle remains the same. What is the dielectric constant of the liquid ? The density of the material of the sphere is 1600 kg m^{-3}.*

Solution. Fig. 5.9 shows the conditions of the problem. Suppose the mass of each sphere is m kg, the charge on each q coulomb and in equilibrium, the distance between them is r. Each sphere is in equilibrium under the action of three forces as shown. Considering the sphere A,

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{r^2}$$

Now $$T \cos 15° = mg \; ; T \sin 15° = F = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{r^2}$$

$$\therefore \quad \tan 15° = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{mg\, r^2} \qquad ...(i)$$

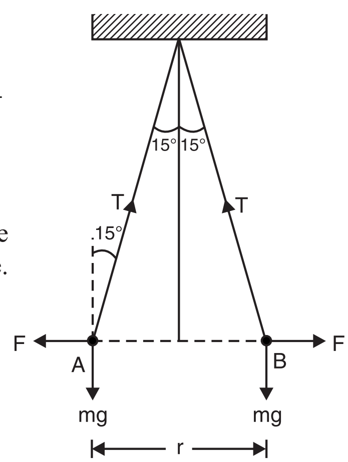


Fig. 5.9

When the spheres are immersed in the liquid, the effective weight of each sphere and the force of repulsion both decrease. Consequently, tension also decreases.

$$\text{Weight of sphere in liquid} = mg^* \left(1 - \frac{800}{1600}\right) = \frac{mg}{2}$$

$$\text{Electric force in liquid, } F' = \frac{1}{4\pi\varepsilon_0 K} \times \frac{q^2}{r^2}$$

Here K is the dielectric constant of the liquid. If the reduced tension is T', then for the equilibrium of sphere A, we have,

$$T' \cos 15° = \frac{mg}{2} \quad \text{and} \quad T' \sin 15° = \frac{1}{4\pi\varepsilon_0 K} \times \frac{q^2}{r^2}$$

$$\therefore \quad \tan 15° = \frac{1}{4\pi\varepsilon_0 K} \frac{2q^2}{mg\, r^2} \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), we have,

* Weight of sphere in liquid, W' = Weight in air – Weight of liquid displaced.

Now, Weight in air = mg

$$\text{Also, weight of liquid displaced} = m\left(\frac{\sigma}{\rho}\right)g = mg\left(\frac{\sigma}{\rho}\right) = mg\left(\frac{800}{1600}\right)$$

$$\therefore \quad W' = mg - mg\left(\frac{800}{1600}\right) = mg\left(1 - \frac{800}{1600}\right) = \frac{mg}{2}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{mg\,r^2} = \frac{1}{4\pi\varepsilon_0 K}\frac{2q^2}{mg\,r^2} \qquad \therefore K = \mathbf{2}$$

Tutorial Problems

1. Two copper spheres A and B have their centres separated by 50 cm. If charge on each sphere is $6{\cdot}5 \times 10^{-7}$C, what is the mutual force of repulsion between them ? The radii of the spheres are negligible compared to the distance of separation. What will be the magnitude of force if the two spheres are placed in water ? (Dielectric constant of water = 80). **[$1{\cdot}52 \times 10^{-2}$ N; $1{\cdot}9 \times 10^{-4}$ N]**
2. Charges q_1 and q_2 lie on the x-axis at points $x = -4$ cm and $x = +4$ cm respectively. How must q_1 and q_2 be related so that net electrostatic force on a charge placed at $x = +2$ cm is zero ? **[$q_1 = 9q_2$]**
3. Two small spheres of equal size are 10 cm apart in air and carry charges +1 μC and − 3 μC. Where should a third charge be located so that no net electrostatic force acts on it ? **[24 cm from −3 μC]**
4. Two identical spheres, having unequal and opposite charges are placed at a distance of 90 cm apart. After touching them mutually, they are again separated by same distance. Now they repel each other with a force of 0·025N. Find the final charge on each of them. **[1·5 μC on each]**
5. Two small spheres, each of mass 0·05 g are suspended by silk threads from the same point. When given equal charges, they separate the threads making an angle of 10° with each other. What is the force of repulsion acting on each sphere ? **[$4{\cdot}3 \times 10^{-5}$ N]**
6. Point charges of 2×10^{-9}C lie at each of the three corners of a square of side 20cm. Find the magnitude of force on a charge of -1×10^{-9}C placed at the centre of square. **[9×10^{7} N]**
7. The electrostatic force of repulsion between two positively charged ions carrying equal charge is $3{\cdot}7 \times 10^{-9}$ N. If their separation is 5 Å, how many electrons are missing from each ion ? **[2]**

5.8. Electric Field

The region surrounding a charged body is always under stress and strain because of the electrostatic charge. If a small charge is placed in this region, it will experience a force according to Coulomb's laws. This stressed region around a charged body is called electric field. Theoretically, electric field due to a charge extends upto infinity but its effect practically dies away very quickly as the distance from the charge increases.

The space (*or field*) *in which a charge experiences a force is called an* **electric field** or **electrostatic field.**

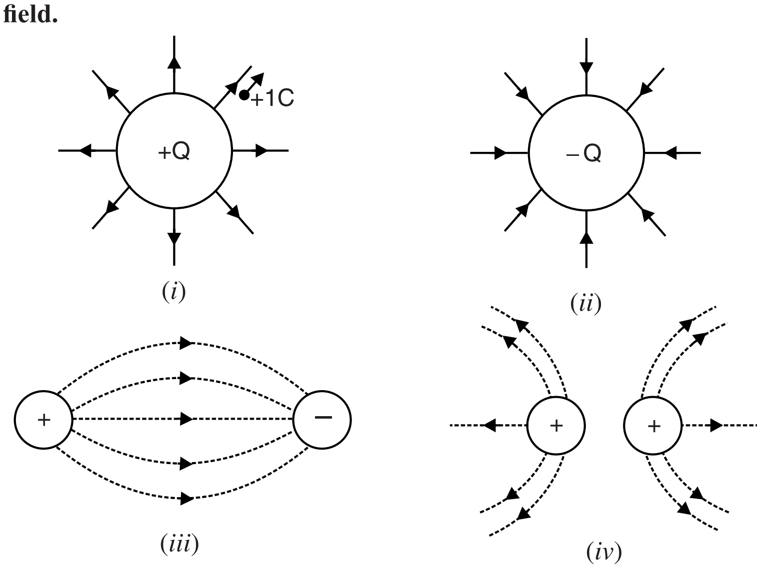


Fig. 5.10

The electric field around a charged body is represented by imaginary lines, called *electric lines of *force*. By convention, the direction of these lines of force at any point is the direction along which a unit positive charge (*i.e.,* positive charge of 1C) placed at that point would move or tend to move. The unit positive charge is sometimes called a *test charge* because it is used as an indicator to find the direction of electric field. Following this convention, it is clear that electric lines of force would always originate from a positive charge and end on a negative charge. The electric lines of force leave or enter the charged surface **normally.

Fig. 5.10 shows typical field distribution. Fig. 5.10 (*i*) shows electric field due to an isolated positively charged sphere. A unit positive charge placed near it will experience a force directed radially away from the sphere. Therefore, the direction of electric field will be radially outward as shown in Fig. 5.10 (*i*). For the negatively charged sphere [See Fig. 5.10 (*ii*)], the force acting on the unit positive charge would be directed radially towards the sphere. Fig. 5.10 (*iii*) shows the electric field between a positive charge and a negative charge while Fig. 5.10 (*iv*) shows electric field between two similarly charged (*i.e.* + vely charged) bodies.

5.9. Properties of Electric Lines of Force

(*i*) The electric field lines are directed away from a positive charge and towards a negative charge so that at any point, the tangent to a field line gives the direction of electric field at that point.

(*ii*) Electric lines of force start from a positive charge and end on a negative charge.

(*iii*) Electric lines of force leave or enter the charged surface normally.

(*iv*) Electric lines of force cannot pass through a ***conductor. This means that electric field inside a conductor is zero.

(*v*) Electric lines of force can never intersect each other. In case the two electric lines of force intersect each other at a point, then two tangents can be drawn at that point. This would mean two directions of electric field at that point which is impossible.

(*vi*) Electric lines of force have the tendency to contract in length. This explains attraction between oppositely charged bodies.

(*vii*) Electric lines of force have the tendency to expand laterally *i.e.* they tend to separate from each other in the direction perpendicular to their lengths. This explains repulsion between two like charges.

5.10. Electric Intensity or Field Strength (E)

To describe an electric field, we must specify its intensity or strength. The intensity of electric field at any point is determined by the force acting on a unit positive charge placed at that point.

Electric intensity (*or* **field strength**) *at a point in an electric field is the force acting on a unit positive charge placed at that point. Its direction is the direction along which the force acts.*

Electric intensity at a point, $E = \dfrac{F}{+Q}$ N/C

where Q = Charge in coulombs placed at that point

F = Force in newtons acting on Q coulombs

* So called because forces are experienced by charges in this region.

** If a line of force is at an angle other than 90°, it will have a tangential component. This tangential component would cause redistribution (*i.e.* movement) of charge. By definition, electrostatic charge is static and hence tangential component cannot exist.

*** However, electric lines of force can pass through an insulator.

Thus, if a charge of 2 coulombs placed at a point in an electric field experiences a force of 10N, then electric intensity at that point will be 10/2 = 5N/C. The following points may be noted carefully:

(*i*) Since electric intensity is a force, it is a vector quantity possessing both magnitude and direction.

(*ii*) Electric intensity can also be *described in terms of lines of force. Where the lines of force are close together, the intensity is high and where the lines of force are widely separated, intensity will be low.

(*iii*) Electric intensity can also be expressed in V/m.

1 V/m = 1 N/C (See foot note on page 284)

Electric intensity due to a point charge. The value of electric intensity at any point in an electric field due to a point charge can be calculated by Coulomb's laws. Suppose it is required to find the electric intensity at point P situated at a distance d metres from a charge of + Q coulomb (See Fig. 5.11). Imagine a unit positive charge (*i.e.* + 1C) is placed at point P. Then, by definition, electric intensity at P is the force acting on + 1C placed at P *i.e.*

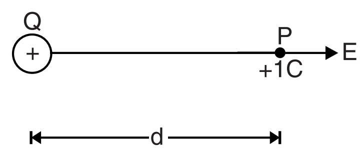


Fig. 5.11

$$\text{Electric intensity at } P, E = \text{Force on + 1C placed at } P$$

$$= 9\times10^9 \frac{Q\times1}{\varepsilon_r d^2}$$

$$\therefore \quad E = 9\times10^9 \frac{Q}{\varepsilon_r d^2} \quad \textit{...in a medium}$$

$$= 9\times10^9 \frac{Q}{d^2} \quad \textit{...in air}$$

Note the direction of electric intensity. It is acting radially away from + Q. For a negative charge (*i.e.* − Q), its direction would have been radially towards the charge.

The electric field intensity in vector form is given as :

$$\vec{E} = 9\times10^9 \frac{Q}{d^2}\hat{d} \quad \textit{... in air}$$

$$= 9\times10^9 \frac{Q}{\varepsilon_r d^2}\hat{d} \quad \textit{... in a medium}$$

where $\hat{d}$ is a unit vector directed from + Q to + 1C.

Electric field intensity due to a group of point charges. The resultant (or net) electric field intensity at a point due to a group of point charges can be found by applying **superposition principle. Thus electric field intensity at a point P due to n point charges ($q_1, q_2, q_3 ... q_n$) is equal to the vector sum of electric field intensities due to $q_1, q_2, q_3 .... q_n$ at point P *i.e.*

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... + \vec{E}_n$$

where $\vec{E}$ = Net or resultant electric field intensity at P

$\vec{E}_1$ = Electric field intensity at P due to q_1

$\vec{E}_2$ = Electric field intensity at P due to q_2 and so on.

* It may be noted that electric lines of force do not actually exist. It is only a way of representing an electric field. However, it is a useful method of representation. It is a usual practice to indicate high field strength by drawing lines of force close together and low field strength by widely spaced lines.

** Since the electric force obeys the superposition principle, so does the electric field intensity—the force per unit charge.

Example 5.9. *Two equal and opposite charges of magnitude 2×10^{-7} C are placed 15cm apart. (i) What is the magnitude and direction of electric intensity (E) at a point mid-way between the charges? (ii) What force would act on a proton (charge = + 1·6 × 10^{-19} C) placed there ?*

Solution. Fig. 5.12 shows two equal and opposite charges separated by a distance of 15cm *i.e.* 0·15 m. Let M be the mid-point *i.e.* $AM = MB = 0·075$ m.

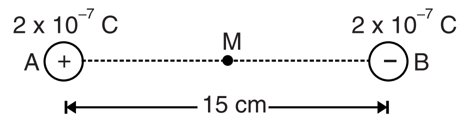


Fig. 5.12

(*i*) Imagine a charge of + 1 C placed at M.

∴ Electric intensity at M due to charge $+ 2 \times 10^{-7}$ C is

$$E_1 = 9 \times 10^9 \times \frac{2 \times 10^{-7}}{(0.075)^2} = 0.32 \times 10^6 \text{ N/C along } AM$$

Electric intensity at M due to charge -2×10^{-7}C is

$$E_2 = 9 \times 10^9 \times \frac{2 \times 10^{-7}}{(0.075)^2} = 0.32 \times 10^6 \text{ N/C along } MB$$

Since electric intensities are acting in the same direction, the resultant intensity E is the sum of E_1 and E_2.

∴ Resultant intensity at point M is

$$E = 0·32 \times 10^6 + 0·32 \times 10^6 = \mathbf{0·64 \times 10^6 \text{ N/C}} \textit{ along AB}$$

(*ii*) Electric intensity E at M is $0·64 \times 10^6$ N/C. Therefore, force F acting on a proton (charge, $Q = +1·6 \times 10^{-19}$ C) placed at M is

$$F = E\,Q = (0·64 \times 10^6) \times (1·6 \times 10^{-19}) = \mathbf{1·024 \times 10^{-13} \text{ N}} \textit{ along AB}$$

Example 5.10. *A charged oil drop remains stationary when situated between two parallel plates 25mm apart. A p.d. of 1000V is applied to the plates. If the mass of the drop is 5 ×10^{-15} kg, find the charge on the drop (take g = 10ms^{-2}).*

Solution. Let Q coulomb be the charge on the oil drop. Since the drop is stationary,

Upward force on drop = Weight of drop [See Fig. 5.13]

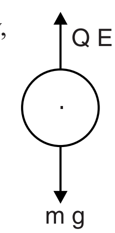


Fig. 5.13

or $$Q\,E = m\,g$$

Here $$E = \frac{V}{d} = \frac{1000}{25 \times 10^{-3}} = 4 \times 10^4 \text{ V/m}$$

∴ $$Q = \frac{mg}{E} = \frac{(5 \times 10^{-15}) \times 10}{4 \times 10^4} = \mathbf{1.25 \times 10^{-18} \text{ C}}$$

Example 5.11. *The diameter of a hollow metallic sphere is 60cm and the sphere carries a charge of 500µC. Find the electric field intensity (i) at a distance of 100cm from the centre of the sphere and (ii) at the surface of sphere.*

Solution. The electric field due to a charged sphere has spherical symmetry. Therefore, a charged sphere behaves for external points as if the whole charge is placed at its centre. [See Fig. 5.14]

(*i*) $d = OP = 100\text{cm} = 1\text{m}$; $Q = 500\ \mu\text{C} = 500 \times 10^{-6}$C

∴ $$E = 9 \times 10^9 \frac{Q}{d^2} = 9 \times 10^9 \times \frac{500 \times 10^{-6}}{1} = \mathbf{4.5 \times 10^6 \text{ N/C}}$$

(*ii*) $d = OP' = 30$ cm = 0·3 m ; $Q = 500\mu\text{C} = 500 \times 10^{-6}$C

∴ $$E = 9 \times 10^9 \frac{Q}{d^2} = 9 \times 10^9 \times \frac{500 \times 10^{-6}}{(0.3)^2} = \mathbf{5 \times 10^7 \text{ N/C}}$$

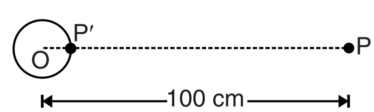


Fig. 5.14

Example 5.12. *Three point charges of +8 × 10^{-9} C, +32 × 10^{-9} C and +24 × 10^{-9} C are placed at the corners A, B and C of a square ABCD having each side 4 cm. Find the electric field intensity at the corner D. Assume that the medium is air.*

Solution. The conditions of the problem are represented in Fig. 5.15. It is clear that $BD = \sqrt{2} \times 0.04\,\text{m}$.

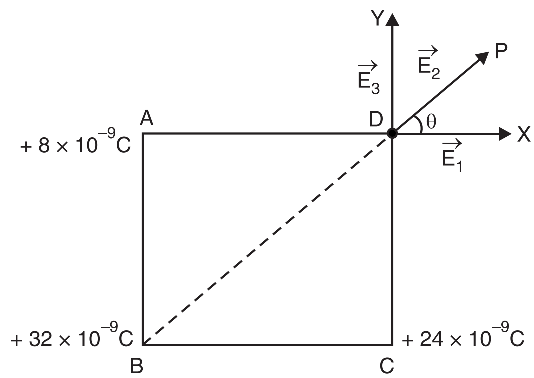


Fig. 5.15

Magnitude of electric field intensity at D due to charge +8 × 10^{-9} C is

$$E_1 = 9 \times 10^9 \times \frac{8 \times 10^{-9}}{(0.04)^2} = 4{\cdot}5 \times 10^4 \text{ N/C} \quad \text{along } DX$$

Magnitude of electric field intensity at D due to charge +32 × 10^{-9} C is

$$E_2 = 9 \times 10^9 \times \frac{32 \times 10^{-9}}{\left(\sqrt{2} \times 0.04\right)^2} = 9 \times 10^4 \text{ N/C along } DP$$

Magnitude of electric field intensity at D due to charge +24 × 10^{-9} C is

$$E_3 = 9 \times 10^9 \times \frac{24 \times 10^{-9}}{(0.04)^2} = 13{\cdot}5 \times 10^4 \text{ N/C along } DY$$

It is easy to see that $\theta = 45°$.

Resolving electric field intensities along X-axis and Y-axis, we have,

$$\text{Total } X\text{-component} = E_1 + E_2 \cos\theta + 0$$
$$= 4{\cdot}5 \times 10^4 + 9 \times 10^4 \times \cos 45° = 10{\cdot}86 \times 10^4 \text{ N/C}$$
$$\text{Total } Y\text{-component} = 0 + E_2 \sin 45° + E_3$$
$$= 0 + 9 \times 10^4 \sin 45° + 13{\cdot}5 \times 10^4 = 19{\cdot}86 \times 10^4 \text{ N/C}$$

∴ Magnitude of resultant electric intensity at D

$$= \sqrt{(10.86 \times 10^4)^2 + (19.86 \times 10^4)^2} = \mathbf{22{\cdot}63 \times 10^4 \text{ N/C}}$$

Let the resultant intensity make an angle ϕ with DX.

$$\therefore \quad \tan\phi = \frac{Y - \text{component}}{X - \text{component}} = \frac{19.86 \times 10^4}{10.86 \times 10^4} = 1{\cdot}828$$

or $$\phi = \tan^{-1} 1{\cdot}828 = \mathbf{61{\cdot}32°}$$

Tutorial Problems

1. What is the magnitude of a point charge chosen so that electric field 20 cm away from it has a magnitude of 18 × 10^6 N/C ? **[80μC]**

2. Two point charge s of 0·12 μC and −0·06 μC are situated 3m apart in air. Calculate the electric field strength at a point midway between them on the line joining their centres.

[720 N/C towards −ve charge]

3. An oil drop of 12 excess electrons is held stationary in a uniform electric field of $2{\cdot}55 \times 10^4$ N/C. If the density of oil is 12600 kg/m^3, find (*i*) mass of the drop (*ii*) radius of the drop.

[(*i*) $1{\cdot}5 \times 10^{-15}$ kg (*ii*) $9{\cdot}8 \times 10^{-7}$ m]

4. A point charge of $0{\cdot}33 \times 10^{-8}$C is placed in a medium of relative permittivity of 5. Calculate electric field intensity at a point 10cm from the charge. **[525 N/C]**

5. Three point charges of $+0{\cdot}33 \times 10^{-8}$C, $+ 0{\cdot}33 \times 10^{-8}$C and $0{\cdot}165 \times 10^{-8}$C are at the points *A*, *B* and *C* respectively of a square *ABCD*. Find the electric field intensity at the corner *D*. **[$1{\cdot}63 \times 10^4$ N/C]**

5.11. Electric Flux (ψ)

Fig. 5.16 shows electric field between two equal and oppositely charged parallel plates. The electric field is considered to be filled with electric flux and each unit of charge is assumed to give rise to one unit of electric flux. The symbol for electric flux is the Greek letter ψ(psi) and it is measured in coulombs. Thus in Fig. 5.16, the charge on each plate is *Q* coulombs so that electric flux between the plates is

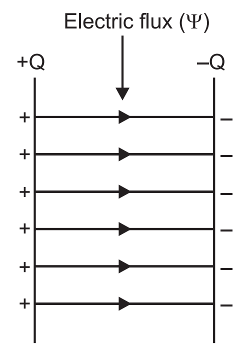


Fig. 5.16

$$\text{Electric flux, } \psi = Q \text{ coulombs}$$

Electric flux is a measure of electric lines of force. The greater the electric flux passing through an area, the greater is the number of electric lines of force passing through that area and *vice-versa*. Suppose there is a charge of *Q* coulombs in a medium of absolute permittivity ε ($= \varepsilon_0 \varepsilon_r$) where ε_r is the relative permittivity of the medium. Then number of electric lines of force *N* produced by this charge is

$$N = \frac{Q}{\varepsilon} = \frac{Q}{\varepsilon_0 \varepsilon_r}$$

(*i*) The electric flux through a surface area has maximum value when the surface is perpendicular to the electric field.

(*ii*) The electric flux through the surface is zero when the surface is parallel to the electric field.

5.12. Electric Flux Density (D)

The **electric flux density** *at any section in an electric field is the electric flux crossing normally per unit area of that section i.e.*

$$\text{Electric flux density, } D = \frac{\psi}{A}$$

The SI unit of electric flux density is *C/m^2.

For example, when we say that electric flux density in an electric field is 4C/m^2, it means that 4C of electric flux passes normally through an area of 1m^2. Electric flux density is a vector quantity; possessing both magnitude and direction. Its direction is the same as the direction of electric intensity.

Relation between D and E. Consider a charge of $+Q$ coulombs placed in a medium of relative permittivity ε_r as shown in Fig. 5.17. The electric flux density at *P* at a distance *d* metres from the charge can be found as follows. With centre at the charge and radius *d* metres, an imaginary sphere can be considered. The electric flux of *Q* coulombs will pass normally through this imaginary sphere. Now area of sphere = $4\pi\, d^2$.

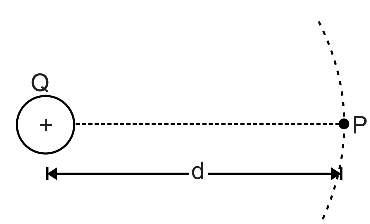


Fig. 5.17

* $D = \varepsilon_0 \varepsilon_r E = [C^2\ N^{-1}\ m^{-2}]\ [N/C] = Cm^{-2} = C/m^2$

$\therefore$ Flux density at P, $D = \dfrac{\text{Flux}}{\text{Area}} = \dfrac{Q}{4\pi d^2}$

Also, Electric intensity at P, $E = \dfrac{Q}{4\pi\varepsilon_0\varepsilon_r d^2} = \dfrac{Q}{4\pi d^2} \times \dfrac{1}{\varepsilon_0\,\varepsilon_r}$

$$= \frac{D}{\varepsilon_r\varepsilon_0} \qquad \left[\because D = \frac{Q}{4\pi d^2}\right]$$

$\therefore$ $D = \varepsilon_0\,\varepsilon_r\,E$

Hence flux density at any point in an electric field is $\varepsilon_0\,\varepsilon_r$ times the electric intensity at that point.

The electric flux density (D) is also called **electric displacement.**

It may be noted that D and E are vector quantities having magnitude and direction. Therefore, in vector form,

$$\vec{D} = \varepsilon_0\varepsilon_r\,\vec{E}$$

Also
$$\vec{D} = \frac{Q}{4\pi d^2}\hat{d}$$

The direction of $\vec{D}$ at every point is the same as that of $\vec{E}$ but its magnitude is $D = \varepsilon_0\varepsilon_r E$.

(*i*) *The value of E depends upon the permittivity $\varepsilon(=\varepsilon_0\varepsilon_r)$ of the surrounding medium, that of D is independent of it.*

(*ii*) Electric flux density (D) is directly related to electric field intensity (E); permittivity $\varepsilon(=\varepsilon_0\varepsilon_r)$ of the medium being the factor by which one quantity differs from the other.

(*iii*) The importance of relation $D = \varepsilon_0\varepsilon_r E$ lies in the fact that it relates density concept to intensity concept.

(*iv*) Electric intensity at a point is also defined as equal to the electric lines of force passing normally through a unit cross-sectional area at that point. If Q coulombs is the charge, then number of electric lines of force produced by it is Q/ε. If these lines fall normally on area A m^2 surrounding the point, then electric intensity E at the point is

$$E = \frac{Q/\varepsilon}{A} = \frac{Q}{\varepsilon A}$$

But $\dfrac{Q}{A} = D =$ Electric flux density over the area.

$$\therefore \quad E = \frac{D}{\varepsilon} = \frac{D}{\varepsilon_0\varepsilon_r} \quad \textit{... in a medium}$$

$$= \frac{D}{\varepsilon_0} \quad \textit{... in air}$$

Example 5.13. *Calculate the dielectric flux between two parallel flat metal plates each 35 cm square with an air gap of 1·5 mm between; the potential difference being 3000 V. A sheet of insulating material 1·5 mm thick is inserted between the plates and the potential difference raised to 7400V. What is the relative permittivity of this material if the charge is now 32 μC ?*

Solution. $E = V/d \;;\; D = \varepsilon_0\varepsilon_r E = \dfrac{\varepsilon_0\varepsilon_r V}{d} \;;\; \psi = DA$

$$\therefore \quad \psi = \left(\frac{\varepsilon_0\varepsilon_r V}{d}\right) \times A$$

When medium is air ($\varepsilon_r = 1$)

$$\psi = \frac{\varepsilon_0 V}{d} \times A = \frac{(8.85 \times 10^{-12}) \times 3000 \times (35 \times 35 \times 10^{-4})}{1.5 \times 10^{-3}}$$

$$= 21{\cdot}6 \times 10^{-7}\text{C} = \mathbf{2{\cdot}16\ \mu C}$$

When medium is insulating material

$$\psi = \frac{\varepsilon_0 \varepsilon_r V}{d} \times A$$

Here $\psi = Q = 32\ \mu\text{C} = 32 \times 10^{-6}$ C ; $V = 7400$ volts ; $d = 1{\cdot}5 \times 10^{-3}$ m

$$\therefore \qquad \varepsilon_r = \frac{\psi \times d}{\varepsilon_0 VA} = \frac{32 \times 10^{-6} \times 1.5 \times 10^{-3}}{8.85 \times 10^{-12} \times 7400 \times (35)^2 \times 10^{-4}} = \mathbf{6}$$

Tutorial Problems

1. What is the total flux passing through a 10 cm × 6 cm surface in a region where the electric flux density is 2700 μC/m^2 ? **[1·62 × 10^{-5} C]**
2. At a certain point in a material, the flux density is 0·09 C/m^2 and electric field intensity is 1·2 × 10^9 V/m. What is the absolute permittivity of the material ? **[7·5 × 10^{-11} C^2 N^{-1} m^{-2}]**

5.13. Gauss's Theorem

This theorem was first expressed by a German scientist Karl Fredrich Gauss (1777–1855) and may be stated as under :

The electric flux passing through a closed surface surrounding a number of charges is equal to the algebraic sum of the charges inside the closed surface.

To illustrate Gauss's theorem, consider Fig. 5.18 where charges Q_1, Q_2, Q_3 and $-Q_4$ coulombs are placed inside a closed surface. According to Gauss, the total electric flux ψ passing through this closed surface is given by the algebraic sum of the charges inside the closed surface *i.e.*

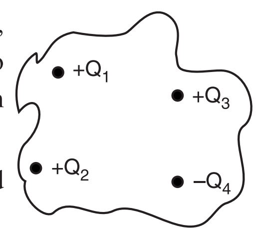


Fig. 5.18

ψ = Algebraic sum of the charges inside the closed surface

$$= (Q_1) + (Q_2) + (Q_3) + (-Q_4)$$

$$= Q_1 + Q_2 + Q_3 - Q_4 \text{ coulombs}$$

The following points may be noted :

(a) The location of charge/charges inside the closed surface does not matter.

(b) The shape of the surface does not matter provided it is a closed surface enclosing the charge/charges.

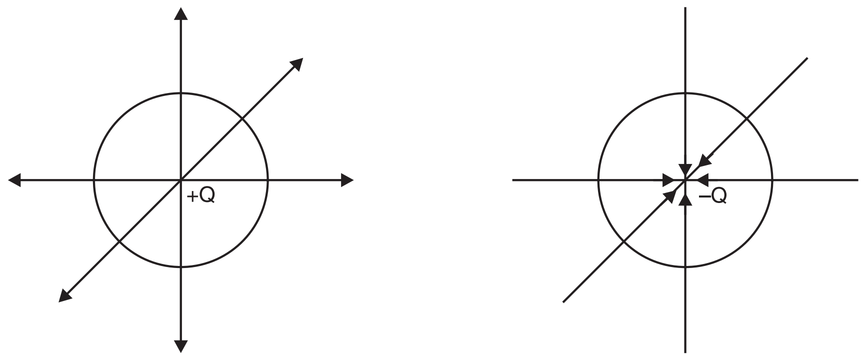
Explanation. (*i*) Consider a charge of $+Q$ coulomb placed at the centre of sphere of radius r as shown in Fig. 5.19 (*i*). Since the charge is at the centre of the sphere, electric flux density (D) is uniform over all the surface and perpendicular to the surface at every point.

$$D = \frac{\text{Charge}}{\text{Area of sphere}} = \frac{Q}{4\pi r^2}$$

Therefore, the electric flux ψ passing outward through the sphere is

$$\psi = D \times \text{Area} = \frac{Q}{4\pi r^2} \times 4\pi r^2 = Q \text{ coulomb}$$

The number of electric lines of force passing through the closed surface normally is Q/ε_0.



(*i*) (*ii*)

Fig. 5.19

Thus the electric flux passing through the surface of sphere is equal to Q, the charge enclosed in the sphere. This establishes Gauss's theorem.

If the sphere were enclosing a charge $-Q$ placed at the centre [See Fig. 5.19 (*ii*)], then electric flux $\psi = Q$ coulomb would pass inward through the surface and terminate at the charge.

(*ii*) Now consider that the charge $+Q$ coulomb is placed at any other point (other than centre O) inside the sphere as shown in Fig. 5.20. The electric lines of force flow outward but not normal to the surface. However, at any point on the sphere (such as point P), electric flux can be resolved into two rectangular components *viz*

(*a*) Component normal to the surface *i.e.*, cos θ component.

(*b*) Component perpendicular to the normal to the surface *i.e.* sin θ component.

If we add all the sin θ components of electric flux over the whole surface, the result will be zero. It is because various sin θ components cancel each other. However, all cos θ components of flux are normal to the sphere surface and meet at the centre if produced backward. Hence the resultant of all cos θ components over the surface of sphere is equal to Q coulomb *i.e.*

$$\psi = Q \text{ coulomb}$$

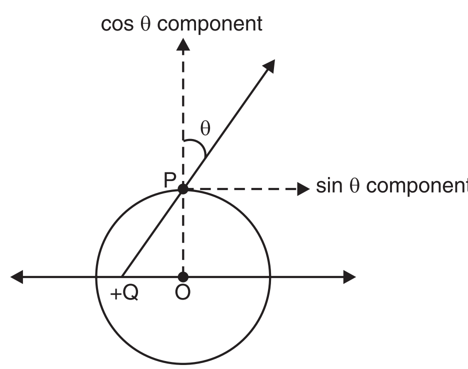


Fig. 5.20

The number of electric lines of force passing through the closed surface normally is Q/ε_0.

Thus irrespective of the position of charge Q within the sphere, the flux passing through the sphere surface is Q coulomb. This establishes Gauss's theorem. Similarly, it can be shown that if a surface encloses a number of charges, the electric flux passing through the surface is equal to the algebraic sum of charges inside the closed surface.

Gauss's law can also be expressed *mathematically*.

We know that : $\psi = \oint \vec{E} \cdot \vec{dS}$

where $\oint \vec{E} \cdot \vec{dS}$ is the surface integral of electric field $(\vec{E})$ over the entire closed surface enclosing the charge Q.

$$\therefore \qquad \psi = \oint \vec{E} \cdot \vec{dS} = \frac{Q}{\varepsilon_0}$$

Hence, Gauss's law may be stated as under :

If a closed surface encloses a net charge (Q), then surface integral of electric field $(\vec{E})$ *over the closed surface is equal to* $1/\varepsilon_0$ *times the charge enclosed.*

5.14. Proof of Gauss's Law

Consider a positive charge $+Q$ located at point O as shown in Fig. 5.21. We draw a sphere of radius r with charge $+Q$ as its centre. We now show that total electric flux (*i.e.* total number of electric lines of force) passing through the closed surface is Q/ε_0. The magnitude of electric field at any point on the spherical surface is given by ;

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

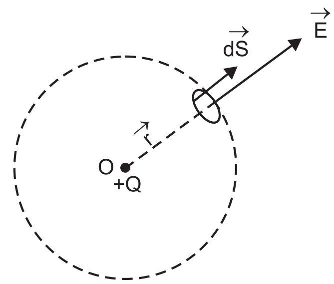


Fig. 5.21

The electric field is directed radially outward from $+Q$. The spherical surface is only imaginary and is called *Gaussian surface*.

Consider a small elementary area $\vec{dS}$ on the surface of sphere as shown in Fig. 5.21. It is clear that $\vec{E}$ is * parallel to $\vec{dS}$ *i.e.* angle between $\vec{E}$ and $\vec{dS}$ is zero. Therefore, electric flux through the entire closed spherical surface is

$$\psi = \oint \vec{E}\cdot\vec{dS} = \oint E\,dS\cos 0° = \oint E\,dS$$

Since E (magnitude of $\vec{E}$) is constant over the considered closed surface, it can be taken out of integral.

$$\therefore \quad \psi = E\oint dS$$

Now $\quad E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ and $\oint dS$ = Surface area of sphere = $4\pi r^2$

$$\therefore \quad \psi = \frac{Q}{4\pi\varepsilon_0 r^2}\times 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

Hence, $\quad \psi = \oint \vec{E}.\vec{dS} = \dfrac{Q}{\varepsilon_0}$

Note. We know : $\quad \psi = \oint \vec{E}.\vec{dS} = \dfrac{Q}{\varepsilon_0}$

$$= \oint \varepsilon_0 \vec{E}.\vec{dS} = Q$$

$$\therefore \quad \psi = \oint \vec{D}.\vec{dS} = Q \qquad (\because\ \varepsilon_0\vec{E} = \vec{D})$$

Note that ψ can be expressed in Q or Q/ε_0.

Hence Gauss's law may be stated in terms of flux density $(\vec{D})$ as under :

If a closed surface encloses a net charge (Q), then surface integral of $\vec{D}$ *(electric flux density) over the closed surface is equal to the charge enclosed by the closed surface.*

Example 5.14. *A spherical surface 50 cm in diameter is penetrated by an inward flux uniformly distributed over the surface, the electric flux density being* $2{\cdot}5 \times 10^{-7}$ C/m^2. *What is the magnitude and sign of the charge enclosed by this surface ?*

Solution. Area of spherical surface is

$$A = 4\pi r^2 = 4\pi \times (25 \times 10^{-2})^2 = 0{\cdot}785 \text{ m}^2$$

* This is true for every elementary area on the surface.

$$\text{Electric flux, } \psi = D \times A = (2{\cdot}5 \times 10^{-7}) \times (0{\cdot}785) = 0{\cdot}1962 \times 10^{-6} \text{ C}$$

$$\therefore \quad \text{Charge enclosed} = 0{\cdot}1962 \times 10^{-6} \text{ C} = \mathbf{0{\cdot}1962\ \mu C}$$

Since the electric flux is passing inward through the sphere, the charge enclosed is **negative**.

5.15. Electric Potential Energy

We know that earth has gravitational field which attracts the bodies towards earth. When a body is raised above the ground level, it possesses mechanical potential energy which is equal to the amount of work done in raising the body to that point. The greater the height to which the body is raised, the greater will be its potential energy. Thus, the potential energy of the body depends upon its position in the gravitational field; being zero on earth's surface. Strictly speaking, sea level is chosen as the place of zero potential energy.

Like earth's gravitational field, every charge ($+ Q$) has electric field which theoretically extends upto infinity. If a small positive test charge $+ q_0$ is placed in this electric field, the test charge will experience a force of repulsion. If test charge $+ q_0$ is moved towards $+ Q$, work will have to be done against the force of repulsion. This work done is stored in $+ q_0$ in the form of potential energy. We say the charge $+ q_0$ has electric potential energy. The electric potential energy of $+ q_0$ depends upon its position in the electric field ; being zero if q_0 is situated at infinity.

From the above discussion, it follows that just as a mass has mechanical potential energy in the gravitational field, similarly a charge has electric potential energy in the electric field. The electric potential energy of a charge is positive or negative depending upon the kind of charge.

5.16. Electric Potential

Just as we define electric field intensity as the force per unit charge, similarly *electric potential is defined as the electric potential energy per unit charge*.

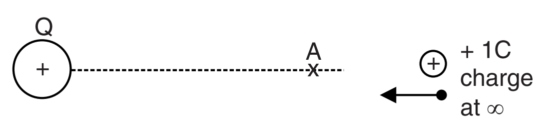


Fig. 5.22

Consider an isolated charge $+Q$ fixed in space as shown in Fig. 5.22. If a unit positive charge (*i.e.* + 1C) is placed at infinity, the force on it due to charge $+Q$ is *zero. If the unit positive charge at infinity is moved towards $+Q$, a force of repulsion acts on it (like charges repel) and hence work is required to be done to bring it to a point like A. Hence when the unit positive charge is at A, it has some amount of electric potential energy which is a measure of electric potential. The closer the point to the charge, the higher will be the electric potential energy and hence the electric potential at that point. Therefore, electric potential at a point due to a charge depends upon the position of the point; being zero if the point is situated at infinity. Obviously, in electric field, infinity is chosen as the point of **zero potential.

Hence **electric potential** *at a point in an electric field is the amount of work done in bringing a unit positive charge* (*i.e.* +1 C) *from infinity to that point i.e.*

$$\text{Electric potential} = \frac{\text{Work}}{\text{Charge}} = \frac{W}{Q}$$

where W is the work done to bring a charge of Q coulombs from infinity to the point under consideration.

---

* $F = 9 \times 10^9 \times \dfrac{Q \times 1}{d^2}$; As $d \to \infty$, $F \to 0$

** In practice, earth is chosen to be at zero electric potential. It is because earth is such a huge conductor that its electric potential practically remains constant.

Unit. The *SI* unit of electric potential is *volt and may be defined as under :

The electric potential at a point in an electric field is **1 volt** *if 1 joule of work is done in bringing a unit positive charge* (*i.e.* + *1* C) *from infinity to that point* ***against the electric field.*

Thus when we say that potential at a point in an electric field is +5V, it simply means that 5 joules of work has been done in bringing a unit positive charge from infinity to that point.

5.17. Electric Potential Difference

In practice, we are more concerned with potential difference between two points rather than their †absolute potentials. The potential difference (p.d.) between two points may be defined as under :

The potential difference between two points is the amount of work done in moving a unit positive charge (*i.e.* + *1*C) *from the point of lower potential to the point of higher potential.*

Consider two points A and B in the electric field of a charge $+Q$ as shown in Fig. 5.23. Let V_2 and V_1 be the absolute potentials at A and B respectively. Clearly, $V_2 > V_1$. The potential V_1 at B means that V_1 joules of work has been done in bringing a unit positive charge from infinity to point B. Let the extra work done to bring the unit positive charge from B to A be W joules.

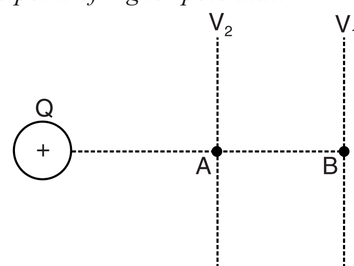


Fig. 5.23

$\therefore$ Potential at $A = V_1 + W$

$\therefore$ P.D. between A and $B = (V_1 + W) - V_1$

or $V_2 - V_1 = W = W.D.$ to move + 1C from B to A

The SI unit of potential difference is volt and may be defined as under :

The p.d. between two points is **1 V** *if 1 joule of work is done in bringing a unit positive charge* (*i.e.* + *1 C*) *from the point of lower potential to the point of higher potential.*

Thus when we say that p.d. between two points is 5 volts, it simply means that 5 joules of work will have to be done to bring +1C of charge from the point of lower potential to the point of higher potential. Conversely, 5 joules of work or energy will be released if + 1 C charge moves from the point of higher potential to the point of lower potential.

5.18. Potential at a Point Due to a Point Charge

Consider an isolated positive charge of Q coulombs placed in a medium of relative permittivity ε_r. It is desired to find the electric potential at point P due to this charge. Let P be at a distance d metres from the charge. Imagine a unit positive charge (*i.e.* + 1 C) placed at A and situated x metres from the charge. Then the force acting on this unit charge (*i.e.* electric intensity) is given by [See Fig. 5.24] ;

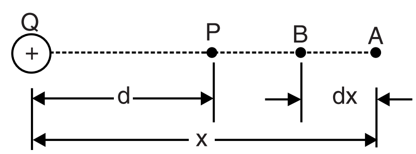


Fig. 5.24

$$F = E = \frac{Q}{4\pi\varepsilon_0\varepsilon_r x^2}$$

If this unit positive charge at A is moved through a small distance dx towards the charge $+Q$, then work done is given by ;

$$dW = \frac{Q}{4\pi\varepsilon_0\varepsilon_r x^2} \times (-\dagger\dagger\, dx) = -\frac{Q}{4\pi\varepsilon_0\varepsilon_r x^2}dx$$

* Electric potential = W/Q = joules/coulomb. Now joule/coulomb has been given a special name *viz* volt.

** Note if the field is due to a positive charge (as is in this case), work will be done against the electric field. However, if the field is due to a negative charge, work is done by the electric field.

† The potential at a point with infinity as reference is termed as absolute potential.

†† The negative sign is taken because dx is considered in the negative direction of distance (x).

Total work done in bringing a unit positive charge from infinity to point P is

$$\text{Total work done, } W = \int_{\infty}^{d} -\frac{Q}{4\pi\varepsilon_0\varepsilon_r x^2}dx = -\frac{Q}{4\pi\varepsilon_0\varepsilon_r}\int_{\infty}^{d}\frac{1}{x^2}dx$$

$$= -\frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left[-\frac{1}{x}\right]_{\infty}^{d} = \frac{-Q}{4\pi\varepsilon_0\varepsilon_r}\left[-\frac{1}{d}-\left(-\frac{1}{\infty}\right)\right]$$

$$= \frac{Q}{4\pi\varepsilon_0\varepsilon_r d}$$

$$= 9\times10^9\frac{Q}{\varepsilon_r d} \text{ joules} \qquad \left[\because \frac{1}{4\pi\varepsilon_0} = 9\times10^9\right]$$

By definition, the work done in joules to bring a unit positive charge from infinity to point P is equal to potential at P in volts.

$$\therefore \quad V_P = 9\times10^9\frac{Q}{\varepsilon_r d} \text{ volts} \qquad \text{...in a medium}$$

$$= 9\times10^9\frac{Q}{d} \text{ volts} \qquad \text{...in air}$$

The following points may be noted carefully :

(i) The potential varies inversely with the distance d from the point charge Q. If the distance is increased three times, the potential is reduced one-third of its value and so on.

(ii) Electric potential is a scalar quantity.

(iii) At $d = \infty$ in air/vacuum, $V_P = 9\times10^9\frac{q}{\infty} = 0$.

(iv) If Q is positive, then potential at P is *positive. On the other hand, if Q is negative, then potential at P is negative.

5.19. Potential at a Point Due to Group of Point Charges

Electric potential obeys superposition principle. Therefore, electric potential at any point P due to a group of point charges Q_1, Q_2, Q_3 .... Q_n is equal to the algebraic sum of potentials due to Q_1, Q_2, Q_3 ... Q_n at point P. Note that an algebraic sum is one in which sign of the physical quantity (potential in this case) is taken into account.

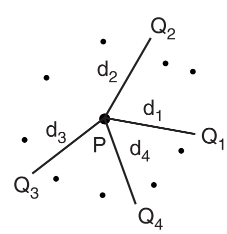


Fig. 5.25

Let the distances of Q_1, Q_2, Q_3, ... Q_n be d_1, d_2, d_3 ... d_n respectively from point P as shown in Fig. 5.25. Further, let V_1, V_2, V_3 ... V_n be the potentials at P due to Q_1, Q_2, Q_3 .... Q_n respectively. Assuming the medium to be free space/air,

$$\text{Total potential at } P,\ V_P = V_1 + V_2 + V_3 + ... + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{d_1} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{d_2} + \frac{1}{4\pi\varepsilon_0}\frac{Q_3}{d_3} + ... + \frac{1}{4\pi\varepsilon_0}\frac{Q_n}{d_n}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{d_1}+\frac{Q_2}{d_2}+\frac{Q_3}{d_3}+...+\frac{Q_n}{d_n}\right]$$

$$\therefore \quad V_P = 9\times10^9\left[\frac{Q_1}{d_1}+\frac{Q_2}{d_2}+\frac{Q_3}{d_3}+...+\frac{Q_n}{d_n}\right]$$

* The potential near an isolated positive charge is positive because work is done by an external agency to push a test charge (positive) from infinity to that point. The potential near an isolated negative charge is negative because outside agent must exert a restraining force as test charge comes in from infinity.

If the system of charges is placed in a medium of relative permittivity ε_r, then,

$$V_P = \frac{9\times10^9}{\varepsilon_r}\left[\frac{Q_1}{d_1}+\frac{Q_2}{d_2}+\frac{Q_3}{d_3}+...+\frac{Q_n}{d_n}\right]$$

5.20. Behaviour of Metallic Conductors in Electric Field

When a metallic conductor (solid or hollow) is placed in an electric field, there is a momentary flow of charges (*i.e.*, free electrons). Once the flow of charges ceases, the conductor is said to be in *electrostatic equilibrium*. It has been seen experimentally that under the conditions of electrostatic equilibrium, a conductor (solid or hollow) shows the following properties [See Fig. 5.26] :

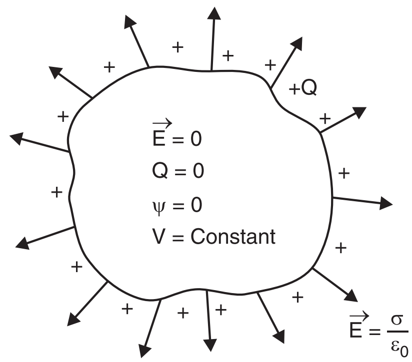


Fig. 5.26

(*i*) *The net electric field inside a charged conductor is zero i.e., no electric lines of force exist inside the conductor.*

(*ii*) *The net charge inside a charged conductor is zero.*

(*iii*) *The electric field (i.e., electric lines of force) on the surface of a charged conductor is perpendicular to the surface of the conductor at every point.*

(*iv*) *The magnitude of electric field just outside a charged conductor is* σ/ε_0 *where* σ *is the surface charge density.*

(*v*) *The electric potential is the same (i.e., constant) at the surface and inside a charged conductor.*

Inside a charged conductor, $E = 0$

Now $E = -\frac{dV}{dS}$ or $0 = -\frac{dV}{dS}$

This means that V is constant.

5.21. Potential of a Charged Conducting Sphere

Consider an isolated conducting sphere of radius r metres placed in air and charged uniformly with Q coulombs. The field has spherical symmetry *i.e.* lines of force spread out normally from the surface and meet at the centre of the sphere if produced backward. *Outside the sphere,* the field is exactly the same as though the charge Q on sphere were concentrated at its centre.

(*i*) Potential at the sphere surface. Due to spherical symmetry of the field, we can imagine the charge Q on the sphere as concentrated at its centre O [See Fig. 5.27 (*i*)]. The problem then reduces to find the potential at a point r metres from a charge Q.

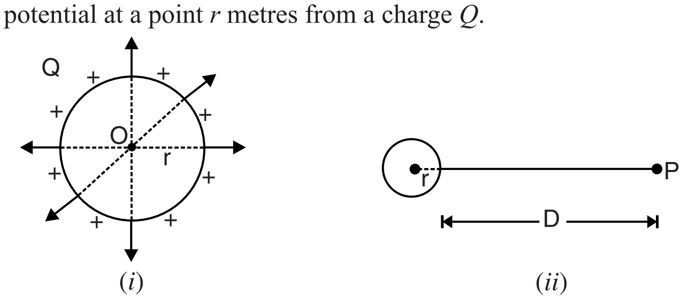


Fig. 5.27

∴ Potential at the surface of sphere

$$= \frac{Q}{4\pi\varepsilon_0 r} \text{ volts}$$ [See Art. 5·18]

$$= 9 \times 10^9 \frac{Q^*}{r} \text{ volts}$$

(*ii*) Potential outside the sphere. Consider a point P outside the sphere as shown in Fig. 5.27 (*ii*). Let this point be at a distance of D metres from the surface of sphere.

$$\text{Then potential at } P = 9 \times 10^9 \frac{Q}{(D+r)} \text{ volts}$$

(*iii*) Potential inside the sphere. Since there is no electric flux inside the sphere, electric intensity inside the sphere is zero.

$$\text{Now, electric intensity} = \frac{\text{Change in potential}}{r}$$

or $$0 = \text{Change in potential}$$

Hence, all the points inside the sphere are at the same potential as the points on the surface.

Example 5.15. *Two positive point charges of 16 × 10^{-10} C and 12 × 10^{-10} C are placed 10 cm apart. Find the work done in bringing the two charges 4 cm closer.*

Solution. Suppose the charge 16 × 10^{-10} C to be fixed.

Potential of a point 10 cm from the charge 16 × 10^{-10} C

$$= 9 \times 10^9 \frac{16 \times 10^{-10}}{0.1} = 144 \text{ V}$$

Potential of a point 6 cm from the charge 16 × 10^{-10} C

$$= 9 \times 10^9 \frac{16 \times 10^{-10}}{0.06} = 240 \text{ V}$$

$\therefore$ Potential difference = 240 − 144 = 96 V

Work done = Charge × p.d. = 12 × 10^{-10} × 96 = **11·52 × 10^{-8} joules**

Example 5.16. *A square ABCD has each side of 1 m. Four point charges of +0·01 μC, − 0·02 μC, +0·03 μC and +0·02 μC are placed at A, B, C and D respectively. Find the potential at the centre of the square.*

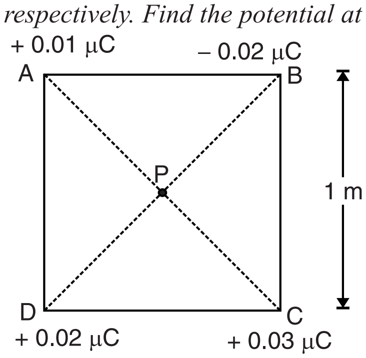


Fig. 5.28

Solution. Fig. 5.28 shows the square *ABCD* with charges placed at its corners. The diagonals of the square intersect at point P. Clearly, point P is the centre of the square. The distance of each charge from point P (*i.e.* centre of square) is

$$= \frac{1}{2}\sqrt{1^2 + 1^2} = 0{\cdot}707 \text{ m}$$

The potential at point P due to all charges is equal to the algebraic sum of potentials due to each charge.

$\therefore$ Potential at P due to all charges

$$= 9 \times 10^9 \left[\frac{Q_1}{0.707} + \frac{Q_2}{0.707} + \frac{Q_3}{0.707} + \frac{Q_4}{0.707}\right]$$

$$= \frac{9 \times 10^9}{0.707}\left[(0.01 - 0.02 + 0.03 + 0.02)10^{-6}\right]$$

$$= \frac{9 \times 10^9}{0.707} \times 0.04 \times 10^{-6} = \mathbf{509.2V}$$

* If the sphere is placed in a medium (ε_r), then potential is

$$= 9 \times 10^9 \frac{Q}{\varepsilon_r r}$$

Example 5.17. *A hollow sphere is charged to 12μC. Find the potential (i) at its surface (ii) inside the sphere (iii) at a distance of 0·3m from the surface. The radius of the sphere is 0·1m.*

Solution. (*i*) The potential at the surface of the sphere in air is

$$V = \frac{Q}{4\pi\varepsilon_0 d} = 9\times10^9 \times \frac{Q}{d}$$

Here $Q = 12\ \mu C = 12 \times 10^{-6}\ C$; $d = 0{\cdot}1m$

$$\therefore \quad V = 9\times10^9 \times \frac{12\times10^{-6}}{0.1} = \mathbf{108 \times 10^4\ volts}$$

(*ii*) Potential inside the sphere is the same as at the surface *i.e.* **108 × 10⁴ volts.**

(*iii*) Distance of the point from the centre = 0·3 + 0·1 = 0·4m

$$\therefore \quad \text{Potential} = 9\times10^9 \times \frac{12\times10^{-6}}{0.4} = \mathbf{27 \times 10^4\ volts}$$

Example 5.18. *If 300 J of work is done in carrying a charge of 3 C from a place where the potential is −10 V to another place where potential is V, calculate the value of V.*

Solution. $$V_B - V_A = \frac{W}{Q}$$

Here $V_B = V$; $V_A = -10V$; $W = 300\ J$; $Q = 3\ C$

$$\therefore \quad V - (-10) = 300/3 \quad \text{or} \quad V + 10 = 100$$

$$\therefore \quad V = 100 - 10 = \mathbf{90\ volts}$$

Example 5.19. *The electric field at a point due to a point charge is 30 N/C and the electric potential at that point is 15 J/C. Calculate the distance of the point from the charge and magnitude of charge.*

Solution. Suppose q coulomb is the magnitude of charge and its distance from the point is r metres.

Now, $$E = \frac{kq}{r^2} = 30 \ ; \quad V = \frac{kq}{r} = 15$$

$$\therefore \quad \frac{E}{V} = \frac{1}{r} \quad \text{or} \quad r = \frac{V}{E} = \frac{15}{30} = \mathbf{0{\cdot}5\ m}$$

Now $$kq = 15\,r = 15 \times 0{\cdot}5 = 7{\cdot}5$$

$$\therefore \quad q = \frac{7.5}{k} = \frac{7.5}{9\times10^9} = \mathbf{0{\cdot}83 \times 10^{-9}\ C}$$

Example 5.20. *Two point charges of +4 μC and −6 μC are separated by a distance of 20 cm in air. At what point on the line joining the two charges is the electric potential zero ?*

Solution. Fig. 5.29 shows the conditions of the problem. Suppose C is the point of zero potential. Potential at point C is given by ;

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{4\times10^{-6}}{d_1} - \frac{6\times10^{-6}}{d_2}\right]$$

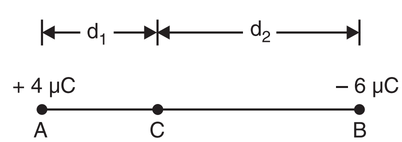


Fig. 5.29

or $$0 = \frac{10^{-6}}{4\pi\varepsilon_0}\left[\frac{4}{d_1} - \frac{6}{d_2}\right]$$

or $$\frac{4}{d_1} - \frac{6}{d_2} = 0 \quad \text{or} \quad d_1 = \frac{2}{3}d_2 \qquad ...(i)$$

Also $$d_1 + d_2 = 20\ \text{cm} \qquad ...(ii)$$

Solving eqs. (*i*) and (*ii*), we get, $d_1 = 8$ cm ; $d_2 = 12$ cm.

Therefore, the point of zero potential lies **8 cm from the charge of +4 μC** or at 12 cm from the charge of −6 μC.

Tutorial Problems

1. A charge of $-4{\cdot}5 \times 10^{-7}$ C is carried from a distant point upto a charged metal sphere. What is the electrical potential of the body if the work done is $1{\cdot}8 \times 10^{-3}$ joule ? **[4×10^3 V]**
2. The difference of potentials between two points in an electric field is 6 volts. How much work is required to move a charge of 300 μ*C* between these points ? **[$1{\cdot}8 \times 10^{-3}$ joule]**
3. A force of 0·032 N is required to move a charge of 42 μC in an electric field between two points 25 cm apart. What potential difference exists between the two points ? **[$1{\cdot}9 \times 10^2$ V]**
4. What is the magnitude of an isolated positive charge to give an electric potential of 100V at 10 cm from the charge ? **[$1{\cdot}11 \times 10^{-9}$ C]**
5. A square *ABCD* has each side of 1m. Four charges of +0·02 μC, +0·04 μC, +0·06 μC and +0·02 μC are placed at *A*, *B*, *C* and *D* respectively. Find the potential at the centre of the square. **[1000V]**
6. A sphere of radius 0·1 m has a charge of 5×10^{-8} C. Determine the potential (*i*) at the surface of sphere, (*ii*) inside the sphere and (*iii*) at a distance of 1m from the surface of the sphere. Assume air as the medium. [(*i*) **4500 V** (*ii*) **4500 V** (*iii*) **409 V**]

5.22. Potential Gradient

The change of potential per unit distance is called **potential gradient** *i.e.*

$$\text{Potential gradient} = \frac{V_2 - V_1}{S}$$

where $V_2 - V_1$ is the change in potential (or p.d.) between two points *S* metres apart. Obviously, the unit of potential gradient will be volts/m.

Consider a charge $+Q$ and let there be two points *A* and *B* situated *S* metres apart in its electric field as shown in Fig. 5.30. Clearly, potential at point *A* is more than the potential at point *B*. If distance *S* is small, then the electric intensity will be approximately the same in this small distance. Let it be *E* newtons/coulomb. It means that a force of *E* newtons will act on a unit positive charge (*i.e.* + 1C) placed anywhere between *A* and *B*. If a unit positive charge is moved from *B* to *A*, then work done to do so is given by ;

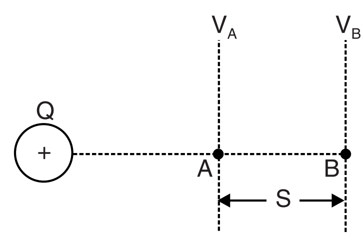


Fig. 5.30

$$\text{Work done} = E \times S \text{ joules}$$

But work done in bringing a unit positive charge from *B* to *A* is the potential difference $(V_A - V_B)$ between *A* and *B*.

$$\therefore \quad E \times S = V_A - V_B$$

or
$$E = \frac{V_A - V_B}{S} = \text{Potential gradient}$$

In differential form,
$$E = -\frac{{}^*dV}{dS}$$

Hence electric intensity at a point is numerically equal to the potential gradient at that point.

Since electric intensity is numerically equal to potential gradient at any point, both must be measured in the same units. Clearly, electric intensity can also be measured in V/m. For example, when we say that potential gradient at a point is 1000 V/m, it means that electric intensity at that point is also **1000 V/m or 1000 N/C.

* Since work done in moving +1C from *B* to *A* is against electric field, a negative sign must be used to make the equation technically correct.

** It can be shown that 1 V/m = 1 N/C.

$$1 \text{ V/m} = \frac{\text{joule/coulomb}}{\text{metre}} = \frac{\text{newton} \times \text{metre}}{\text{metre} \times \text{coulomb}} = 1 \text{ N/C}$$

5.23. Breakdown Voltage or Dielectric Strength

In an insulator or dielectric, the valence electrons are tightly bound so that no free electrons are available for current conduction. However, when voltage applied to a dielectric is gradually increased, a point is reached when these electrons are torn away, a large current (much larger than the usual leakage current) flows through the dielectric and the material loses its insulating properties. Usually, a *spark or arc occurs which burns up the material. The minimum voltage required to break down a dielectric is called breakdown voltage or dielectric strength.

The maximum voltage which a unit thickness of a dielectric can withstand without being punctured by a spark discharge is called ****dielectric strength*** *of the material.*

The dielectric strength (or breakdown voltage) is generally measured in kV/cm or kV/mm. For example, air has a dielectric strength of 30kV/cm. It means that maximum p.d. which 1 cm thickness of air can withstand across it without breaking down is 30kV. If p.d. exceeds this value, the breakdown of air insulation will occur; allowing a large current to flow through it. Below is given the table showing dielectric constant and dielectric strength of some common insulators or dielectrics :

S.No.	Dielectric	Dielectric Constant (ε_r)	Dielectric strength (kV/cm)
1	Air	1	30
2	Paper (oiled)	2	400
3	Paraffin	2·25	350
4	Mica	6	500
5	Glass	8	1000

The following points may be noted :

(*i*) The value of dielectric strength of an insulator (or dielectric) depends upon temperature, moisture content, shape *etc*.

(*ii*) The electric intensity, potential gradient and dielectric strength are numerically equal *i.e.*

Electric intensity = Potential gradient = Dielectric strength

(*iii*) The breakdown of solid insulating material (dielectric) usually renders it unfit for further use by puncturing, burning, cracking or otherwise damaging it. Gaseous and liquid dielectrics are self-healing and may be used repeatedly following breakdown.

(*iv*) *For reasons of safety, electric field applied to a dielectric is only 10% of the dielectric strength of the dielectric material.*

Note. To avoid electric breakdown of dielectric, capacitors are rated according to their *working voltage*, meaning the maximum safe voltage that can be applied to the capacitor.

5.24. Uses of Dielectrics

The insulating materials (or dielectrics) are widely used to provide electrical insulation to electrical and electronics apparatus. The choice of a dielectric for a particular situation will depend upon service requirements. A few cases are given below by way of illustration :

(*i*) If the dielectric is to be subjected to a great heat, as in soldering irons or toasters; mica should be used.

(*ii*) If space, flexibility and a fair dielectric strength are the deciding factors, as in the dielectric for small fixed capacitors, cellulose and animal tissue materials are used.

* This spark may burn a path through such dielectrics as paper, cloth, wood or mica. Hard materials such as porcelain or glass will crack or allow a small path to be melted through them.

** Dielectric strength should not be confused with dielectric constant (relative permittivity).

(*iii*) If a high dielectric strength is desired, as in case of high voltage transformers, glass and porcelain should be used.

(*iv*) If the insulation must remain liquid, like that used in large switches and circuit breakers to quench the arc when the circuit is opened, then various oils are used.

Example 5.21. *A parallel plate capacitor has plates 1 mm apart and a dielectric with relative permittivity of 3·39. Find (i) electric intensity and (ii) the voltage between plates if the surface charge density is* 3×10^{-4} C/m^2.

Solution. (*i*) The surface charge density is equal to electric flux density D.

Now, $D = \varepsilon_0 \varepsilon_r E$

$$\therefore \quad \text{Electric intensity, } E = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{3\times10^{-4}}{8.854\times10^{-12}\times3.39} = \mathbf{10^7\ V/m}$$

(*ii*) P.D. between plates, $V = E \times dx = 10^7 \times (1 \times 10^{-3}) = \mathbf{10^4 V}$

Example 5.22. *The electric potential difference between the parallel deflection plates in an oscilloscope is 300V. If the potential drops uniformly when going from one plate to the other and if distance between the plates is 0·75 cm, what is the magnitude of the electric field between them and in which direction does it point?*

Solution. Let us choose the positive direction of ΔS to be in the direction of increasing potential.

$$\therefore \quad E = -\frac{\Delta V}{\Delta S}$$

Here $\Delta V = +300\text{V}\ ; \quad \Delta S = +0{\cdot}75 \text{ cm} = 0{\cdot}75 \times 10^{-2}$ m

$$\therefore \quad E = -\frac{300}{0.75\times10^{-2}} = \mathbf{-40{,}000\ V/m}$$

The negative value of E tells us that E is directed opposite to ΔS. Thus E is directed from the higher-voltage plate towards the lower-voltage one.

Example 5.23. *A uniform electric field is acting from left to right. If a + 2C charge moves from a to b, a distance of 4m, [See Fig. 5.31], find (i) electric field strength and (ii) potential energy of charge at b w.r.t. a. Given that p.d. between a and b is 50 volts.*

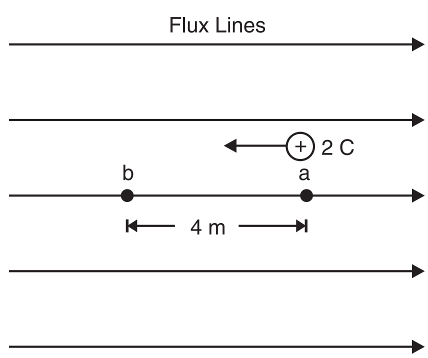


Fig. 5.31

Solution. Referring to Fig. 5.31, we have,

(*i*) Electric intenstity = Potential gradient = 50/4

= **12·5 V/m**

(*ii*) Potential energy of charge (*i.e.,* +2C) at b w.r.t. a

= Work per unit charge × Charge

= Voltage between a and b × Charge

= 50 joules/C × (2C) = **100 joules**

Example 5.24. *A sheet of glass 1·5 cm thick and of relative permittivty 7 is introduced between two parallel brass plates 2 cm apart. The remainder of the space between the plates is occupied by air. If a p.d. of 10,000 V is applied between the plates, calculate (i) electric intensity in air film between glass and plate and (ii) in the glass sheet.*

Solution. Fig. 5.32 shows the arrangement. Let V_1 and V_2 be the p.d. across air and glass respectively and E_1 and E_2 the corresponding electric intensities.

Now, $V_1 = E_1 x_1 = E_1 \times (0{\cdot}5 \times 10^{-2})$

and $V_2 = E_2 x_2 = E_2 \times (1{\cdot}5 \times 10^{-2})$

Now $V = V_1 + V_2$

or $10{,}000 = (0{\cdot}5\,E_1 + 1{\cdot}5\,E_2)10^{-2}$

or $E_1 + 3E_2 = 2 \times 10^6$ ...(*i*)

Now electric flux density D (= $\varepsilon_0\,\varepsilon_r\,E$) is the same in the two media because it is independent of the surrounding medium.

$\therefore$ $\varepsilon_0\,\varepsilon_{r2}\,E_1 = \varepsilon_0\,\varepsilon_{r2}\,E_2$

or $E_1 = 7\,E_2$ ...(*ii*)

From exps. (*i*) and (*ii*), we get,

(*i*) Electric intensity in air = **$1{\cdot}4 \times 10^6$ V/m**

(*ii*) Electric intensity in glass = **$0{\cdot}2 \times 10^6$ V/m**

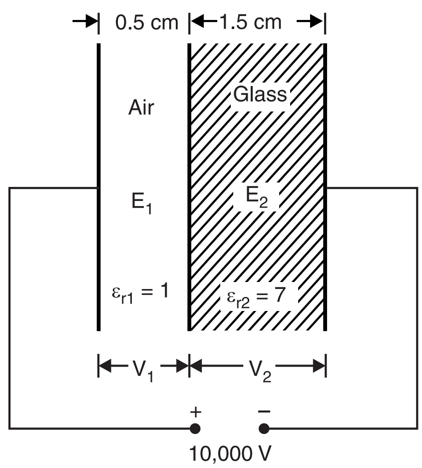


Fig. 5.32

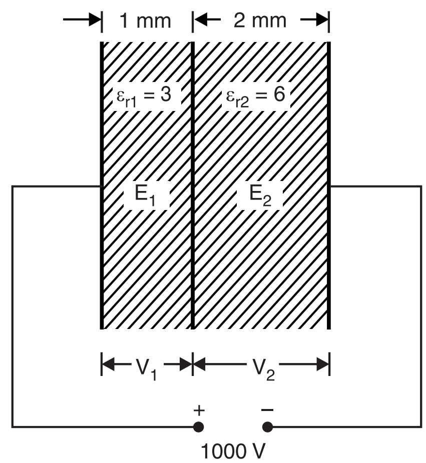


Fig. 5.33

Example 5.25. *A capacitor has two dielectrics 1 mm and 2 mm thick. The relative permittivities of these dielectrics are 3 and 6 respectively. Calculate the potential gradient along the dielectrics if a p.d. of 1000 V is applied between the plates.*

Solution. Fig. 5.33 shows the arrangement. Finding the potential gradient means to find the electric intensity (or electric stress).

$$V_1 = E_1\,x_1 = E_1 \times (1 \times 10^{-3})$$

$$V_2 = E_2\,x_2 = E_2 \times (2 \times 10^{-3})$$

Now $V = V_1 + V_2$

or $1000 = (E_1 + 2E_2)\,10^{-3}$

or $E_1 + 2E_2 = 10^6$ ...(*i*)

Since flux density D (= $\varepsilon_0\,\varepsilon_r\,E$) is the same in the two media,

$\therefore$ $\varepsilon_0\,\varepsilon_{r1}\,E_1 = \varepsilon_0\,\varepsilon_{r2}\,E_2$

or $3\,E_1 = 6\,E_2$ ...(*ii*)

From exps. (*i*) and (*ii*), we get, E_1 = **$0{\cdot}5 \times 10^6$ V/m** ; E_2 = **$0{\cdot}25 \times 10^6$ V/m**

Example 5.26. *Two series connected parallel plate capacitors have plate areas of $0{\cdot}2\ m^2$ and $0{\cdot}04\ m^2$, plate separation of 0·5 mm and 0·125 mm and relative permittivities of 1 and 6 respectively. Calculate the total voltage across the capacitors that will produce a potential gradient of 100 kV/cm between the plates of first capacitor.*

Solution. We shall use suffix 1 for the first capacitor and suffix 2 for the second capacitor. Suppose for a potential gradient of 100 kV/cm between the plates of first capacitor, the voltages across first and second capacitors are V_1 and V_2 respectively. Then,

Total voltage V across capacitors is

$$V = V_1 + V_2$$

For the first capacitor $E_1 = 100 \text{ kV/cm} = 100 \times 10^3 \times 10^2 = 10^7 \text{ V/m}$

$$\therefore \quad V_1 = E_1 d_1 = (10^7) \times (0{\cdot}5 \times 10^{-3}) = 5 \times 10^3 \text{ V} = 5\text{kV}$$

$$D_1 = \varepsilon_0 \varepsilon_{r1} E_1 = \varepsilon_0 \times 10^7 \qquad (\because \; \varepsilon_{r1} = 1)$$

$$Q_1 = A_1 D_1 = (0{\cdot}2) \times \varepsilon_0 \times 10^7 \text{ C}$$

For the second capacitor. Since the capacitors are connected in series, the charge on them is the same *i.e.*

$$Q_1 = Q_2 = 0{\cdot}2 \times \varepsilon_0 \times 10^7 \text{ C}$$

$$\therefore \quad D_2 = \frac{Q_2}{A_2} = \frac{0.2 \times \varepsilon_0 \times 10^7}{0.04} = 0{\cdot}5 \times 10^8 \times \varepsilon_0 \text{ C/m}^2$$

$$\therefore \quad E_2 = \frac{D_2}{\varepsilon_0 \varepsilon_{r2}} = \frac{0.5 \times 10^8 \times \varepsilon_0}{\varepsilon_0 \times 6} = \frac{10^8}{12} \text{V/m} \qquad (\because \; \varepsilon_{r2} = 6)$$

$$\therefore \quad V_2 = E_2 d_2 = \frac{10^8}{12}(0.125 \times 10^{-3}) = 1{\cdot}04 \times 10^3 \text{ V} = 1{\cdot}04 \text{ kV}$$

$\therefore$ Total voltage across the capacitors is

$$V = V_1 + V_2 = 5 + 1{\cdot}04 = \mathbf{6{\cdot}04 \text{ kV}}$$

Example 5.27. *A parallel plate capacitor consists of two square metal plates 500 mm on a side separated by 10 mm. A slab of Teflon (ε_r = 2) 6 mm thick is placed on the lower plate leaving an air gap 4 mm thick between it and the upper plate. If 100V is applied across the capacitor, find the electric field E_a in the air, electric field E_t in Teflon, flux density D_a in air, flux density D_t in Teflon and potential difference V_t across Teflon slab.*

Solution. Electric flux density (D) in the two media is the same. However, electric field intensity (E) is inversely proportional to the relative permittivity (ε_r) of the medium. If E_a is the electric intensity in air, then electric intensity in Teflon is $E_t = E_a/2$ ($\because$ relative permittivity of Teflon = 2).

Thickness of air, t_a = 4 mm ; Thickness of Teflon, t_t = 6 mm

Voltage between two plates, $V = E_a t_a + E_t t_t$

$$\text{or} \quad 100 = E_a \times 4 + \frac{E_a}{2} \times 6 \qquad \left[\because E_t = \frac{E_a}{2}\right]$$

$$\therefore \quad E_a = \frac{100}{7} \text{ volts/mm} = \mathbf{14.286 \text{ kV/m}}$$

$$\text{Electric field in Teflon, } E_t = \frac{14.286}{2} = \mathbf{7.143 \text{ kV/m}}$$

As electric flux density is the same in the two media,

$$\therefore \quad D_a = D_t = \varepsilon_0 \varepsilon_r E_a = 8.854 \times 10^{-12} \times 1 \times 14.286 \times 1000$$

$$= \mathbf{1.265 \times 10^{-7} \text{ C/m}^2}$$

P.D. across Teflon slab, $V_t = E_t \times t_t = 7.143 \times 1000 \times 6 \times 10^{-3} = \mathbf{42.86 \text{ V}}$

Tutorial Problems

1. An electron (charge = $1{\cdot}6 \times 10^{-19}$ C; mass = $9{\cdot}1 \times 10^{-31}$ kg) is released in a vacuum between two flat, parallel metal plates that are 10cm apart and are maintained at a constant electric potential difference of 750V. If the electron is released at the negative plate, what is the speed just before it strikes the positive plate ? **[$1{\cdot}6 \times 10^7$ ms^{-1}]**

2. To move a charged particle through an electric potential difference of 10^{-3}V requires 2×10^{-6} J of work. What is the magnitude of charge ? **[2×10^{-3} C]**

3. A proton of mass $1{\cdot}67 \times 10^{-27}$ kg and charge = $1{\cdot}6 \times 10^{-19}$ C is accelerated from rest through an electric potential of 400 kV. What is its final speed ? **[$8{\cdot}8 \times 10^{6}$ ms^{-1}]**

5.25. Refraction of Electric Flux

When electric flux passes from one uniform dielectric medium to another of different permittivities, the electric flux gets refracted at the boundary of the two dielectric media. Under this condition, the following two conditions exist at the boundary (called **boundary conditions**) :

(*i*) The normal components of electric flux density are equal *i.e.*

$$D_{1n} = D_{2n}$$

(*ii*) The tangential components of electric field intensities are equal *i.e.*

$$E_{1t} = E_{2t}$$

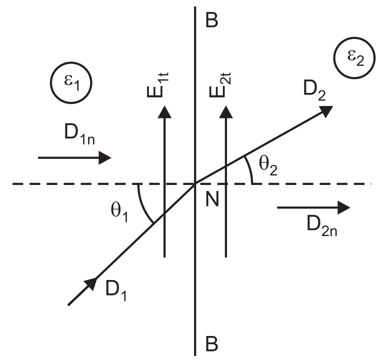


Fig. 5.34

Fig. 5.34 shows the refraction of electric flux at the boundary *BB* of two dielectric media of permittivities ε_1 and ε_2. As shown, the electric flux in the first medium (ε_1) approaches the boundary *BB* at an angle θ_1 and leaves it at θ_2. D_{1n} and D_{2n} are the normal components of D_1 and D_2 while E_{1t} and E_{2t} are the tangential components of E_1 and E_2. Referring to Fig. 5.34,

$$D_{1n} = D_1 \cos\theta_1 \text{ and } D_{2n} = D_2\cos\theta_2$$

Also $\quad E_1 = D_1/\varepsilon_1$ and $E_{1t} = D_1\sin\theta_1/\varepsilon_1$

Similarly, $\quad E_2 = D_2/\varepsilon_2$ and $E_{2t} = D_2\sin\theta_2/\varepsilon_2$

$$\therefore \quad \frac{D_{1n}}{E_{1t}} = \frac{\varepsilon_1}{\tan\theta_1} \text{ and } \frac{D_{2n}}{E_{2t}} = \frac{\varepsilon_2}{\tan\theta_2}$$

Since $D_{1n} = D_{2n}$ and $E_{1t} = E_{2t}$,

$$\therefore \quad \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} \quad \text{... (}i\text{)}$$

Eq. (*i*) gives the law of refraction of electric flux at the boundary of two dielectric media whose permittivities are different.

It is clear that if $\varepsilon_2 > \varepsilon_1$, then $\theta_2 > \theta_1$.

Note. When electric flux passes from one of the commonly used dielectrics (ε being 2 to 8) into another or air, there is hardly any refraction of electric flux.

Example 5.28. *An electric field in a medium with relative permittivity 7 passes into a medium of relative permittivity 2. If E makes an angle of 60° with the normal to the boundary in the first dielectric, what angle does the field make with the normal in the second dielectric ?*

Solution. As proved in Art 5.25,

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

Here $\theta_1 = 60°$; $\varepsilon_1 = 7$; $\varepsilon_2 = 2$; $\theta_2 = ?$

$$\therefore \quad \frac{\tan 60°}{\tan\theta_2} = \frac{7}{2} \quad \text{or} \quad \tan\theta_2 = \sqrt{3} \times \frac{2}{7} = 0.495$$

$$\therefore \quad \theta_2 = \tan^{-1} 0.495 = \mathbf{26.33°}$$

5.26. Equipotential Surface

Any surface over which the potential is constant is called an **equipotential surface.**

In other words, the potential difference between any two points on an equipotential surface is zero. For example, consider two points A and B on an equipotential surface as shown in Fig. 5.35.

$$V_B - V_A = 0 \quad \therefore V_B = V_A$$

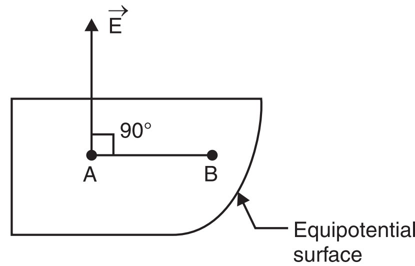


Fig. 5.35

The two important properties of equipotential surfaces are :

(a) *Work done in moving a charge over an equipotential surface is zero.*

Work done = Charge × P.D.

Since potential difference (P.D.) over an equipotential surface is zero, work done is zero.

(b) *The electric field (or electric lines of force) are *perpendicular to an equipotential surface.*

Some cases of Equipotential surfaces. The fact that the electric field lines and equipotential surfaces are mutually perpendicular helps us to locate the equipotential surfaces when the electric field lines are known.

(*i*) **Isolated point charge.** The potential at a point P at a distance r from a point charge $+q$ is given by ;

$$V_P = k\frac{q}{r} \quad \text{where} \quad k = \frac{1}{4\pi\varepsilon_0}$$

It is clear that potential at various points equidistant from the point charge is the same. Hence, in case of an isolated point charge, the spheres concentric with the charge will be the equipotential surfaces as shown in Fig. 5.36. Note that in drawing the equipotential surfaces, the potential difference is kept the same, *i.e.,* 10 V in this case. It may be seen that distance between charge and equipotential surface I is small so that E (= dV/dr = $10/dr$) is high. However, the distance between charge and equipotential surfaces II and III is large so that E (= dV/dr = $10/dr$) is small. It follows, therefore, that equipotential surfaces near the charge are crowded (*i.e.,* more E) and become widely spaced as we move away from the charge.

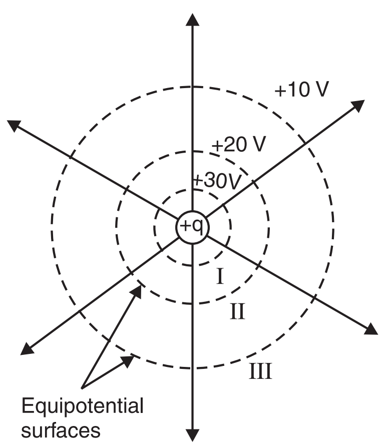


Fig. 5.36

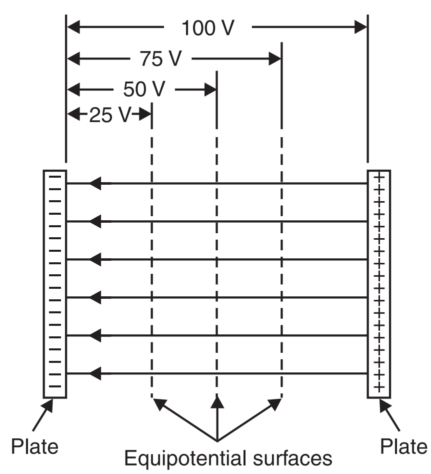


Fig. 5.37

* If this were not so that is if there were a component of $\vec{E}$ parallel to the surface — it would require work to move the charge along the surface against this component of $\vec{E}$; and this would contradict that it is an equipotential surface.

***(ii)* Uniform electric field.** In case of uniform electric field (*e.g.*, electric field between the plates of a charged parallel-plate capacitor), the field lines are straight and equally spaced. Therefore, equipotential surfaces will be parallel planes at right angles to the field lines as shown in Fig. 5.37.

5.27. Motion of a Charged Particle in Uniform Electric Field

Consider that a charged particle of charge $+q$ and mass m enters at right angles to a uniform electric field of strength E with velocity v along OX-axis as shown in Fig. 5.38. The electric field is along OY-axis and acts over a horizontal distance x.

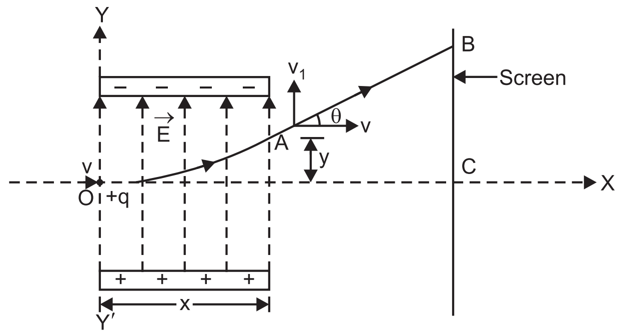


Fig. 5.38

Since the electric field is along OY-axis, no horizontal force acts on the charged particle entering the field. Therefore, the horizontal velocity v of the charged particle remains the same throughout the journey. *The electric field accelerates the charged particle along OY-axis only*.

Force on the charged particle, $F = qE$ ... along OY

Acceleration of the charged particle, $a = \dfrac{qE}{m}$ ...along OY

Time taken to traverse the field, $t = \dfrac{x}{v}$

If y is the displacement of the charged particle along OY direction in the electric field during the time t, then,

$$y = {}^*(0)t + \frac{1}{2}at^2$$

or $$y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{v}\right)^2$$

or $$y = \frac{qE}{2mv^2}x^2$$

or $$y = kx^2 \qquad \left(\because \frac{qE}{2mv^2} = \text{Constant} = k\right)$$

This is the equation of a parabola. *Therefore, inside the electric field, the charged particle follows a parabolic path OA*. As the charged particle leaves the electric field at A, it follows a straight line path AB tangent to path OA at A.

Note. When an electron (or a charged particle) at rest is accelerated through a potential difference (P.D.) of V volts, then,

Energy imparted to electron = Charge × P.D. = $e \times V$

K.E. gained by electron = $\dfrac{1}{2}mv^2$

$$\therefore \quad \frac{1}{2}mv^2 = eV \quad \text{or} \quad v = \sqrt{\frac{2eV}{m}}$$

Here e is the charge on electron and m is the mass of electron. The velocity acquired by the electron is v.

* At the time the charged particle enters the electric field, its velocity along OY-axis is zero.

Example 5.29. *An electron moving with a velocity of 10^7 ms^{-1} enters mid-way between two horizontal plates P, Q in a direction parallel to the plates as shown in Fig. 5.39. The length of the plates is 5 cm and their separation is 2 cm. If a p.d. of 90 V is applied between the plates, calculate the transverse deflection produced by the electric field when the electron just passes the field. Assume $e/m = 1.8 \times 10^{11}$ C kg^{-1}.*

Solution. Fig. 5.39 shows the conditions of the problem.

$$\text{Electric field, } E = \frac{V}{d} = \frac{90}{2\times10^{-2}} = 45 \times 10^2 \text{ Vm}^{-1}$$

Downward force on the electron = eE

Downward acceleration of the electron is

$$a = \frac{eE}{m} = (1.8 \times 10^{11}) \times (45 \times 10^2) = 81 \times 10^{13} \text{ ms}^{-2}$$

Time taken to cross the field, $t = \dfrac{x}{v} = \dfrac{5\times10^{-2}}{10^7} = 5 \times 10^{-9}$ s

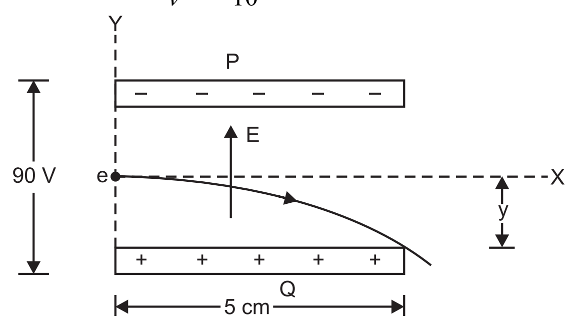


Fig. 5.39

∴ Transverse deflection, $y = \dfrac{1}{2}at^2 = \dfrac{1}{2}(81 \times 10^{13}) \times (5 \times 10^{-9})^2 = 0.01$ m = **1cm**

Example 5.30. *A potential gradient of 3×10^6 V/m is maintained between two horizontal parallel plates 1 cm apart. An electron starts from rest at the negative plate, travels under the influence of potential gradient to the positive plate. Given the mass of electron = 9.1×10^{-31} kg and the charge on electron = 1.603×10^{-19} C. Calculate (i) the force acting on the electron (ii) the ratio of electric force to gravitational force (iii) acceleration (iv) time taken to reach the positive plate.*

Solution. $E = 3 \times 10^6$ V/m ; $e = 1.603 \times 10^{-19}$ C ; $m = 9.1 \times 10^{-31}$ kg ; $S = 1 \times 10^{-2}$ m

(i) Force on electron, $F = eE = 1.603 \times 10^{-19} \times 3 \times 10^6 =$ **4.81×10^{-13} N**

(ii) Ratio of electric force to gravitational force

$$= \frac{F}{mg} = \frac{4.81\times10^{-13}}{9.1\times10^{-31}\times9.81} = \mathbf{5.39 \times 10^{16}}$$

Note that electric force is very large compared to the gravitational force.

(iii) Acceleration of electron, $a = \dfrac{F}{m} = \dfrac{4.81\times10^{-13}}{9.1\times10^{-31}} =$ **51.66×10^{16} m/s²**

(iv) Distance travelled, $S = \dfrac{1}{2}at^2$

∴ Time taken to reach + *ve* plate, $t = \sqrt{\dfrac{2S}{a}} = \sqrt{\dfrac{2\times1\times10^{-2}}{51.66\times10^{16}}} =$ **1.968×10^{-10} s**

Example 5.31. *An electron of charge 1.6 × 10^{-19} C can move freely for a distance of 2 cm in a field of 1000 V/cm. The mass of the electron is 9.1 × 10^{-28} g. If the electron starts with an initial velocity of zero, what velocity will it attain, what will be the time taken and what will be its kinetic energy?*

Solution. $e = 1.6 \times 10^{-19}$ C ; $m = 9.1 \times 10^{-31}$ kg ; $E = 1000$ V/cm = 10^5 V/m

Distance of free movement, $d = 2$ cm = 0.02 m

∴ Potential difference applied, $V = E \times d = 10^5 \times 0.02$ volts

Energy imparted to electron = Charge × P.D. = $e \times V$

$$= 1.6 \times 10^{-19} \times 10^5 \times 0.02 = 3.2 \times 10^{-16} \text{ J}$$

Now, Energy imparted = K.E. of electron = **3.2 × 10^{-16} J**

Also $$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 3.2 \times 10^{-16}}{9.1 \times 10^{-31}}} = \mathbf{2.652 \times 10^7 \text{ m/s}}$$

Force on electron, $F = eE = 1.6 \times 10^{-19} \times 10^5 = 1.6 \times 10^{-14}$ N

Acceleration of electron, $$a = \frac{F}{m} = \frac{1.6 \times 10^{-14}}{9.1 \times 10^{-31}} = 1.758 \times 10^{16} \text{ m/s}^2$$

Distance travelled, $$d = \frac{1}{2}at^2$$

∴ Time taken, $$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2 \times 0.02}{1.758 \times 10^{16}}} = \mathbf{1.51 \times 10^{-9} \text{ s}}$$

Objective Questions

1. The force between two electrons separated by a distance r varies as
   (*i*) r^2 (*ii*) r
   (*iii*) r^{-1} (*iv*) r^{-2}
2. Two charges are placed at a certain distance apart. A brass sheet is placed between them. The force between them will
   (*i*) increase (*ii*) decrease
   (*iii*) remain unchanged
   (*iv*) none of the above
3. Which of the following appliance will be studied under electrostatics ?
   (*i*) incandescent lamp
   (*ii*) electric iron
   (*iii*) lightning rod (*iv*) electric motor
4. The relative permittivity of air is
   (*i*) 0 (*ii*) 1
   (*iii*) 8·854 × 10^{-12} (*iv*) none of the above
5. The relative permittivity of a material is 10. Its absolute permittivity will be
   (*i*) 8·854 × 10^{-11} F/m
   (*ii*) 9 × 10^8 F/m
   (*iii*) 5 × 10^{-5} F/m (*iv*) 9 × 10^5 F/m
6. Another name for relative permittivity is
   (*i*) dielectric constant
   (*ii*) dielectric strength
   (*iii*) potential gradient
   (*iv*) none of the above
7. The relative permittivity of most materials lies between
   (*i*) 20 and 100 (*ii*) 10 and 20
   (*iii*) 100 and 200 (*iv*) 1 and 10
8. When the relative permittivity of the medium is increased, the force between two charges placed at a given distance apart
   (*i*) increases (*ii*) decreases
   (*iii*) remains the same
   (*iv*) none of the above
9. Two charges are placed at a distance apart. If a glass slab is placed between them, the force between the charges will
   (*i*) be zero (*ii*) increase
   (*iii*) decrease (*iv*) remain the same

10. There are two charges of +1 μC and +5 μC. The ratio of the forces acting on them will be

(*i*) 1 : 1 (*ii*) 1 : 5
(*iii*) 5 : 1 (*iv*) 1 : 25

11. A soap bubble is given a negative charge. Its radius

(*i*) decreases (*ii*) increases
(*iii*) remains unchanged
(*iv*) information is incomplete to say anything

12. The ratio of force between two small spheres with constant charge in air and in a medium of relative permittivity K is

(*i*) $K^2 : 1$ (*ii*) $1 : K$
(*iii*) $1 : K^2$ (*iv*) $K : 1$

13. An electric field can deflect

(*i*) x-rays (*ii*) neutrons
(*iii*) α-particles (*iv*) γ-rays

14. Electric lines of force enter or leave a charged surface at an angle

(*i*) of 90º (*ii*) of 30º
(*iii*) of 60º
(*iv*) depending upon surface conditions

15. The relation between absolute permittivity of vacuum (ε_0), absolute permeability of vacuum (μ_0) and velocity of light (c) in vacuum is

(*i*) $\mu_0\varepsilon_0 = c^2$ (*ii*) $\mu_0/\varepsilon_0 = c$
(*iii*) $\varepsilon_0/\mu_0 = c$ (*iv*) $\frac{1}{\mu_0\varepsilon_0} = c^2$

16. As one penetrates a uniformly charged sphere, the electric field strength E

(*i*) increases (*ii*) decreases
(*iii*) is zero at all points
(*iv*) remains the same as at the surface

17. If the relative permittivity of the medium increases, the electric intensity at a point due to a given charge

(*i*) decreases (*ii*) increases
(*iii*) remains the same
(*iv*) none of the above

18. Electric lines of force about a negative point charge are

(*i*) circular, anticlockwise
(*ii*) circular, clockwise
(*iii*) radial, inward (*iv*) radial, outward

19. A hollow sphere of charge does not produce an electric field at any

(*i*) outer point (*ii*) interior point
(*iii*) beyond 2 m (*iv*) beyond 10 m

20. Two charged spheres of radii 10 cm and 15 cm are connected by a thin wire. No current will flow if they have

(*i*) the same charge (*ii*) the same energy
(*iii*) the same field on their surface
(*iv*) the same potential

21. Electric potential is a

(*i*) scalar quantity (*ii*) vector quantity
(*iii*) dimensionless
(*iv*) nothing can be said

22. A charge Q_1 exerts some force on a second charge Q_2. A third charge Q_3 is brought near. The force of Q_1 exerted on Q_2

(*i*) decreases (*ii*) increases
(*iii*) remains unchanged
(*iv*) increases if Q_3 is of the same sign as Q_1 and decreases if Q_3 is of opposite sign

23. The potential at a point due to a charge is 9 V. If the distance is increased three times, the potential at that point will be

(*i*) 27 V (*ii*) 3 V
(*iii*) 12 V (*iv*) 18 V

24. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at the centre of the sphere is

(*i*) 10 V (*ii*) 0 V
(*iii*) same as at point 5 cm away from the surface
(*iv*) same as at point 25 cm away from the surface

25. If a unit charge is taken from one point to another over an equipotential surface, then,

(*i*) work is done on the charge
(*ii*) work is done by the charge
(*iii*) work on the charge is constant
(*iv*) no work is done

Answers

1. (*iv*)	**2.** (*ii*)	**3.** (*iii*)	**4.** (*ii*)	**5.** (*i*)
6. (*i*)	**7.** (*iv*)	**8.** (*ii*)	**9.** (*iii*)	**10.** (*i*)
11. (*ii*)	**12.** (*iv*)	**13.** (*iii*)	**14.** (*i*)	**15.** (*iv*)
16. (*iii*)	**17.** (*i*)	**18.** (*iii*)	**19.** (*ii*)	**20.** (*iv*)
21. (*i*)	**22.** (*iii*)	**23.** (*ii*)	**24.** (*i*)	**25.** (*iv*)

6
Capacitance and Capacitors

Introduction

It is well known that different bodies hold different charge when given the same potential. This charge holding property of a body is called *capacitance* or *capacity* of the body. In order to store sufficient charge, a device called capacitor is purposely constructed. A capacitor essentially consists of two conducting surfaces (say metal plates) separated by an insulating material (*e.g.*, air, mica, paper etc.). It has the property to store electrical energy in the form of electrostatic charge. The capacitor can be connected in a circuit so that this stored energy can be made to flow in a desired circuit to perform a useful function. Capacitance plays an important role in d.c. as well as a.c. circuits. In many circuits (*e.g.*, radio and television circuits), capacitors are intentionally inserted to introduce the desired capacitance. In this chapter, we shall confine our attention to the role of capacitance in d.c. circuits only.

6.1. Capacitor

Any two conducting surfaces separated by an insulating material is called a ***capacitor** *or* **condenser.** Its purpose is to store charge in a small space.

The conducting surfaces are called the *plates* of the capacitor and the insulating material is called the ***dielectric.* The most commonly used dielectrics are air, mica, waxed paper, ceramics *etc.* The following points may be noted carefully :

(i) The ability of a capacitor to store charge (*i.e.* its capacitance) depends upon the area of plates, distance between plates and the nature of insulating material (or dielectric).

(ii) A capacitor is generally named after the dielectric used *e.g.* air capacitor, paper capacitor, mica capacitor *etc.*

(iii) The capacitor may be in the form of parallel plates, concentric cylinder or other arrangement.

6.2. How does a Capacitor Store Charge ?

Fig. 6.1 shows how a capacitor stores charge when connected to a d.c. supply. The parallel plate capacitor having plates *A* and *B* is connected across a battery of *V* volts as shown in Fig. 6.1 (*i*). When the switch *S* is open as shown in Fig. 6.1 (*i*), the capacitor plates are neutral *i.e.* there is no charge on the plates. When the switch is closed as shown in Fig. 6.1 (*ii*), the electrons from plate *A* will be attracted by the +ve terminal of the battery and these electrons start ***accumulating on plate *B*. The result is that plate *A* attains more and more positive charge and plate *B* gets more and more negative charge. This action is referred to as charging a capacitor because the capacitor plates are becoming charged. This process of electron flow or charging (*i.e.* detaching electrons from plate *A* and accumulating on *B*) continues till p.d. across capacitor plates becomes equal to battery voltage *V*. When the capacitor is charged to battery voltage *V*, the current flow ceases as shown in Fig. 6.1

* The name is derived from the fact that this arrangement has the capacity to store charge. The name condenser is given to the device due to the fact that when p.d. is applied across it, the electric lines of force are condensed in the small space between the plates.

** A steady current cannot pass through an insulator but an electric field can. For this reason, an insulator is often referred to as a dielectric.

*** The electrons cannot flow from plate *B* to *A* as there is insulating material between the plates. Hence electrons detached from plate *A* start piling up on plate *B*.

(*iii*). If now the switch is opened as shown in Fig. 6.1 (*iv*), the capacitor plates will retain the charges. Thus the capacitor plates which were neutral to start with now have charges on them. This shows that a capacitor stores charge. The following points may be noted about the action of a capacitor :

(*i*) When a d.c. potential difference is applied across a capacitor, a charging current will flow until the capacitor is fully charged when the current will cease. This whole charging process takes place in a very short time, a fraction of a second. *Thus a capacitor once charged, prevents the flow of direct current*.

(*ii*) *The current does not flow through the capacitor i.e. between the plates*. There is only transference of electrons from one plate to the other.

(*iii*) When a capacitor is charged, the two plates carry equal and opposite charges (say + Q and –Q). This is expected because one plate loses as many electrons as the other plate gains. *Thus charge on a capacitor means charge on either plate.*

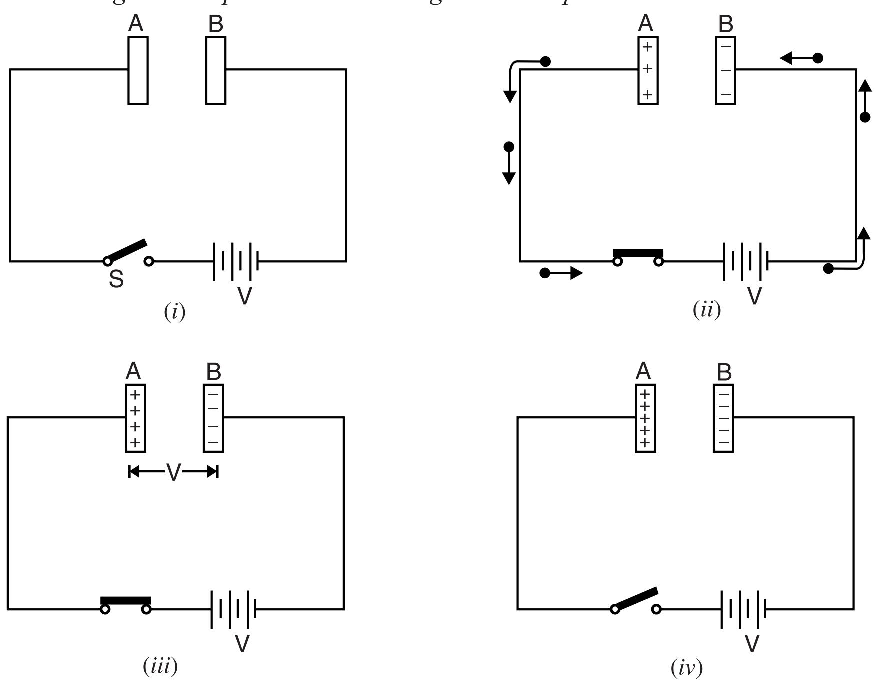


Fig. 6.1

(*iv*) The energy required to charge the capacitor (*i.e.* transfer of electrons from one plate to the other) is supplied by the battery.

6.3. Capacitance

The ability of a capacitor to store charge is known as its capacitance. It has been found experimentally that charge Q stored in a capacitor is directly proportional to the p.d. V across it *i.e.*

$$Q \propto V$$

or

$$\frac{Q}{V} = \text{Constant} = C$$

The constant C is called the capacitance of the capacitor. Hence capacitance of a capacitor can be defined as under :

The ratio of charge on capacitor plates to the p.d. across the plates is called **capacitance** *of the capacitor*.

Unit of capacitance

We know that : $C = Q/V$

The SI unit of charge is 1 coulomb and that of voltage is 1 volt. Therefore, the SI unit of capacitance is one coulomb/volt which is also called *farad* (Symbol F) in honour of Michael Faraday.

$$1 \text{ farad} = 1 \text{ coulomb/volt}$$

A capacitor is said to have a capacitance of **1 farad** *if a charge of 1 coulomb accumulates on each plate when a p.d. of 1 volt is applied across the plates.*

Thus if a charge of 0·1C accumulates on each plate of a capacitor when a p.d. of 10V is applied across its plates, then capacitance of the capacitor = 0·1/10 = 0·01 F. The farad is an extremely large unit of capacitance. Practical capacitors have capacitances of the order of microfarad (μF) and micro-microfarad (μμF) or picofarad (pF).

$$1 \ \mu F = 10^{-6} F \quad ; \quad 1 \mu\mu F \text{ (or 1 pF)} = 10^{-12} F$$

6.4. Factors Affecting Capacitance

The ability of a capacitor to store charge (*i.e.* its capacitance) depends upon the following factors :

(*i*) **Area of plate.** The greater the area of capacitor plates, the larger is the capacitance of the capacitor and *vice-versa.* It is because larger the plates, the greater the charge they can hold for a given p.d. and hence greater will be the capacitance.

(*ii*) **Thickness of dielectric.** The capacitance of a capacitor is inversely proportional to the thickness (*i.e.* distance between plates) of the dielectric. The smaller the thickness of dielectric, the greater the capacitance and *vice-versa.* When the plates are brought closer, the electrostatic field is intensified and hence capacitance increases.

(*iii*) **Relative permittivity of dielectric.** The greater the relative permittivity of the insulating material (*i.e.*, dielectric), the greater will be the capacitance of the capacitor and *vice-versa.* It is because the nature of dielectric affects the electrostatic field between the plates and hence the charge that accumulates on the plates.

6.5. Dielectric Constant or Relative Permittivity

The insulating material between the plates of a capacitor is called dielectric. When the capacitor is charged, the electrostatic field extends across the dielectric. The presence of dielectric* increases the concentration of electric lines of force between the plates and hence the charge on each plate. The degree of concentration of electric lines of force between the plates depends upon the nature of dielectric.

The ability of a dielectric material to concentrate electric lines of force between the plates of a capacitor is called **dielectric constant** *or* **relative permittivity** *of the material.*

Air has been assigned a reference value of dielectric constant (or relative permittivity) as 1. The dielectric constant of all other insulating materials is greater than unity. The dielectric constants of materials commonly used in capacitors range from 1 to 10. For example, dielectric constant of mica is 6. It means that if mica is used as a dielectric between the plates of a capacitor, the charge on each plate will be 6 times the value when air is used; other things remaining equal. In other words, with mica as dielectric, the capacitance of the capacitor becomes 6 times as great as when air is used.

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* Normally, the electrons of the atoms of the dielectric revolve around their nuclei in their regular orbits. When the capacitor is charged, electrostatic field causes distortion of the orbits of the electrons of the dielectric. This distortion of orbits causes an additional electrostatic field within the dielectric which causes more electrons to be transferred from one plate to the other. Hence, the presence of dielectric increases the charge on the capacitor plates and hence the capacitance.

Let V = Potential difference between capacitor plates

Q = Charge on capacitor when air is dielectric

Then, $C_{air} = Q/V$

When mica is used as a dielectric in the capacitor and the same p.d. is applied, the capacitor will now hold a charge of $6Q$.

$$\therefore \quad C_{mica} = \frac{6Q}{V} = 6\frac{Q}{V} = 6\,C_{air}$$

or $$\frac{C_{mica}}{C_{air}} = 6 = \text{Dielectric constant of mica}$$

Hence **dielectric constant** (*or* **relative permittivity**) *of a dielectric material is the ratio of capacitance of a capacitor with that material as a dielectric to the capacitance of the same capacitor with air as dielectric.*

6.6. Capacitance of an Isolated Conducting Sphere

We can find the capacitance of an isolated spherical conductor by assuming that "missing" plate is earth (zero potential). Suppose an isolated conducting sphere of radius r is placed in a medium of relative permittivity ε_r as shown in Fig. 6.2. Let charge $+Q$ be given to this spherical conductor. The charge is spread *uniformly over the surface of the sphere. Therefore, in order to find the potential at any point on the surface of sphere (or outside the sphere), we can assume that entire charge $+Q$ is concentrated at the centre O of the sphere.

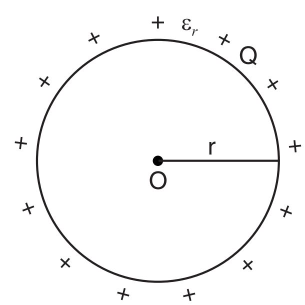


Fig. 6.2

Potential at the surface of the sphere, $V = \dfrac{Q}{4\pi\varepsilon_0\varepsilon_r r}$

$\therefore$ Capacitance of isolated sphere, $C = \dfrac{Q}{V} = 4\pi\varepsilon_0\varepsilon_r r$

$$\therefore \quad {}^{**}C = 4\pi\,\varepsilon_0\,\varepsilon_r\,r \quad \textit{... in a medium}$$

$$= 4\pi\,\varepsilon_0\,r \quad \textit{... in air}$$

The following points may be noted :

(*i*) *The capacitance of an isolated spherical conductor is directly proportional to its radius.* Therefore, for a given potential, a large spherical conductor (more r) will hold more charge Q (= CV) than the smaller one.

(*ii*) Unit of $\varepsilon_0 = C/4\pi r$ = F/m. Hence, the SI unit of ε_0 is F/m.

Example 6.1. *Twenty seven spherical drops, each of radius 3 mm and carrying 10^{-12} C of charge are combined to form a single drop. Find the capacitance and potential of the bigger drop.*

Solution. Let r and R be the radii of smaller and bigger drops respectively.

Volume of bigger drop = 27 × Volume of smaller drop

or $$\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$$

or $$R = 3r = 3 \times 3 = 9 \text{ mm} = 9 \times 10^{-3} \text{ m}$$

Capacitance of bigger drop, $C = 4\pi\,\varepsilon_0\,R = \dfrac{1}{9\times10^9}\times 9\times10^{-3} = 10^{-12}$ F = **1 pF**

* Note that a charged conductor is an equipotential surface. Therefore, electric lines of force emerging from the sphere are everywhere normal to the sphere.

** Note that values of Q and V do not occur in the expression for capacitance. This again reminds us that capacitance is a property of physical construction of a capacitor.

Since charge is conserved, the charge on the bigger drop is 27×10^{-12} C.

$\therefore$ Potential of bigger drop, $V = \dfrac{Q}{C} = \dfrac{27\times10^{-12}}{10^{-12}} = \mathbf{27\ V}$

6.7. Capacitance of Spherical Capacitor

We shall discuss two cases.

(*i*) When outer sphere is earthed. A spherical capacitor consists of two concentric hollow metallic spheres A and B which do not touch each other as shown in Fig. 6.3. The outer sphere B is earthed while charge is given to the inner sphere A. Suppose the medium between the two spheres has relative permittivity ε_r.

Let r_A = radius of inner sphere A

r_B = radius of outer sphere B

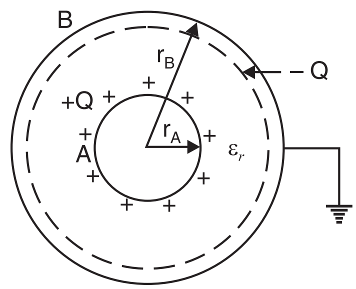


Fig. 6.3

When a charge $+Q$ is given to the inner sphere A, it induces a charge $-Q$ on the inner surface of outer sphere B and $+Q$ on the outer surface of B. Since sphere B is earthed, $+Q$ charge on its outer surface is neutralised by earth.

*Potential at inner sphere A, $V_A = \left(\dfrac{Q}{4\pi\varepsilon_0\varepsilon_r r_A}\right) + \left(\dfrac{-Q}{4\pi\varepsilon_0\varepsilon_r r_B}\right)$

$$= \frac{Q}{4\pi\,\varepsilon_0\varepsilon_r}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) = \frac{Q(r_B - r_A)}{4\pi\,\varepsilon_0\varepsilon_r\, r_A\, r_B}$$

Since sphere B is earthed, its potential is zero (*i.e.,* $V_B = 0$).

$\therefore$ P.D. between A and B, $V_{AB} = V_A - V_B = V_A - 0 = V_A$

$\therefore$ Capacitance of spherical capacitor, $C = \dfrac{Q}{V_A} = \dfrac{4\pi\,\varepsilon_0\varepsilon_r\, r_A\, r_B}{(r_B - r_A)}$

$$\therefore \quad C = \frac{4\pi\,\varepsilon_0\varepsilon_r\, r_A\, r_B}{(r_B - r_A)} \quad \text{... in a medium}$$

$$= \frac{4\pi\,\varepsilon_0\, r_A\, r_B}{(r_B - r_A)} \quad \text{... in air}$$

(*ii*) When inner sphere is earthed. Fig. 6.4 shows the situation. The system constitutes two capacitors in parallel.

(*a*) One capacitor (C_{BA}) consists of the inner surface of B and outer surface of A. Its capacitance as found above is

$$C_{BA} = \frac{4\pi\,\varepsilon_0\,\varepsilon_r\, r_A\, r_B}{r_B - r_A}$$

(*b*) The second capacitor (C_{BG}) consists of outer surface of B and earth. Its capacitance is that of an isolated sphere.

$\therefore \quad C_{BG} = 4\pi\,\varepsilon_0\, r_B$ ... if surrounding medium is air

$\therefore$ Total capacitance $= C_{BA} + C_{BG}$

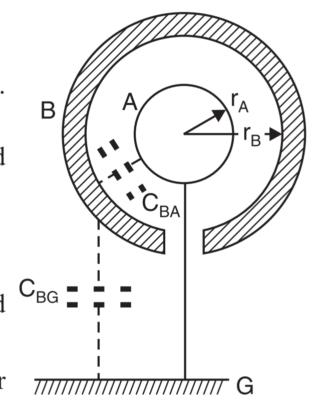


Fig. 6.4

Note. Unless stated otherwise, the outer sphere of a spherical capacitor is assumed to be earthed.

* Potential on sphere A = (Potential on sphere A due to its own charge $+Q$) + (Potential on sphere A due to charge $-Q$ on sphere B) = $\left(\dfrac{Q}{4\pi\,\varepsilon_0\varepsilon_r\, r_A}\right) + \left(\dfrac{-Q}{4\pi\,\varepsilon_0\varepsilon_r\, r_B}\right)$

Example 6.2. *The thickness of air layer between two coatings of a spherical capacitor is 2 cm. The capacitor has the same capacitance as the capacitance of sphere of 1.2 m diameter. Find the radii of its surfaces.*

Solution. Given : $\dfrac{4\pi\,\varepsilon_0\, r_A\, r_B}{r_B - r_A} = 4\pi\,\varepsilon_0\, R \quad \therefore \quad \dfrac{r_A\, r_B}{r_B - r_A} = R$

Here, $r_B - r_A = 2$ cm and $R = 1{\cdot}2/2 = 0{\cdot}6$ m $= 60$ cm

$\therefore \quad \dfrac{r_A\, r_B}{2} = 60$ or $r_A\, r_B = 120$

Now $(r_B + r_A)^2 = (r_B - r_A)^2 + 4r_A\, r_B = (2)^2 + 4 \times 120 = 484$

$\therefore \quad r_B + r_A = \sqrt{484} = 22$ cm

Since $r_B - r_A = 2$ cm and $r_B + r_A = 22$ cm, $r_B =$ **12 cm** ; $r_A =$ **10 cm**

Example 6.3. *A capacitor has two concentric thin spherical shells of radii 8 cm and 10 cm. The outer shell is earthed and a charge is given to the inner shell. Calculate (i) the capacitance of this capacitor and (ii) the final potential acquired by the inner shell if the outer shell is removed after the inner shell has acquired a potential of 200 V.*

Solution. It is assumed that medium between the two spherical shells is air so that $\varepsilon_r = 1$.

(*i*) Radius of inner sphere, $r_A = 8$ cm $= 0.08$ m; Radius of outer sphere, $r_B = 10$ cm $= 0.1$ m

The capacitance C of the spherical capacitor is

$$C = \frac{4\pi\varepsilon_0\varepsilon_r r_A r_B}{r_B - r_A} = \frac{4\pi \times 8.854 \times 10^{-12} \times 0.08 \times 0.1}{0.1 - 0.08} = \mathbf{44.44 \times 10^{-12}\ F}$$

(*ii*) Charge on the capacitor when the inner sphere acquires a potential of 200 V is

$$Q = CV = 44.44 \times 10^{-12} \times 200 = 8888 \times 10^{-12}\text{ C}$$

When the outer shell is removed, the capacitance C' of the resulting isolated sphere is

$$C' = 4\pi\varepsilon_0\varepsilon_r r_A = \frac{1}{9 \times 10^9} \times 1 \times 0.08 = 8.88 \times 10^{-12}\text{ F}$$

$\therefore$ Potential V' acquired by the inner shell when outer shell is removed is

$$V' = \frac{Q}{C'} = \frac{8888 \times 10^{-12}}{8.88 \times 10^{-12}} = \mathbf{1000\ V}$$

Tutorial Problems

1. Calculate the capacitance of a conducting sphere of radius 10 cm situated in air. How much charge is required to raise it to a potential of 1000 V? **[11 pF; 1.1 × 10^{-8} C]**
2. When 1.0×10^{12} electrons are transferred from one conductor to another of a capacitor, a potential difference of 10V develops between the two conductors. Calculate the capacitance of the capacitor. **[1.6 × 10^{-8} F]**
3. Calculate the capacitance of a spherical capacitor if the diameter of inner sphere is 0.2 m and that of the outer sphere is 0.3 m, the space between them being filled with a liquid having dielectric constant 12. **[4 × 10^{-10} F]**
4. The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of the earth, then calculate the capacitance of the spherical capacitor formed between stratosphere and earth's surface. Take radius of earth as 6400 km. **[0.092 F]**
5. A spherical capacitor has an outer sphere of radius 0.15 m and the inner sphere of radius 0.1m. The outer sphere is earthed and inner sphere is given a charge of 6μC. The space between the concentric spheres is filled with a material of dielectric constant 18. Calculate the capacitance and potential of the inner sphere. **[6 × 10^{-10} F; 10^4 V]**

6.8. Capacitance of Parallel-Plate Capacitor with Uniform Medium

We have seen that the capacitance of a capacitor can be determined from its electrical properties using the relation $C = Q/V$. However, it is often desirable to determine the capacitance of a capacitor in terms of its dimensions and relative permittivity of the dielectric. Although there are many forms of capacitors, the most important arrangement is the parallel-plate capacitor.

Consider a parallel plate capacitor consisting of two plates, each of area A square metres and separated by a *uniform dielectric* of thickness d metres and relative permittivity ε_r as shown in Fig. 6.5. Let a p.d. of V volts applied between the plates place a charge of $+Q$ and $-Q$ on the plates. With reasonable accuracy, it can be assumed that electric field between the plates is uniform.

Electric flux density between plates is

$$D = Q/A \text{ coulomb/m}^2$$

Electric intensity between plates is

$$E = V/d$$

But $\quad D = \varepsilon_0 \varepsilon_r E \quad$ ...See Art. 5.12

or $\quad \dfrac{Q}{A} = \varepsilon_0 \varepsilon_r \dfrac{V}{d}$

or $\quad \dfrac{Q}{V} = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$

The ratio Q/V is the capacitance C of the capacitor.

$\therefore \quad C = \dfrac{\varepsilon_0 \varepsilon_r A}{d} \quad$ *...in a medium*

$\quad = \dfrac{\varepsilon_0 A}{d} \quad$ *...in air*

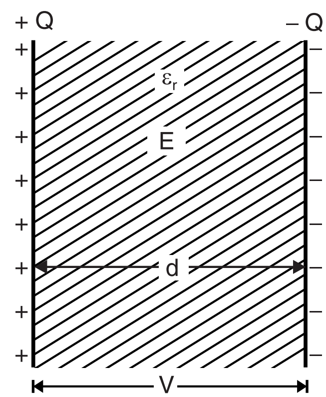


Fig. 6.5

The following points may be noted carefully :

(*i*) Capacitance is directly proportional to ε_r and A and inversely proportional to d.

(*ii*) $\dfrac{C_{med}}{C_{air}} = \varepsilon_r$ = Relative permittivity of medium

(*iii*) Re-arranging the relation for C in air

$$\varepsilon_0 = \frac{Cd}{A} = \frac{\text{farad} \times \text{m}}{\text{m}^2} = \text{F/m}$$

Obviously, permittivity can also be measured in F/m.

6.9. Parallel-Plate Capacitor with Composite Medium

Suppose the space between the plates is occupied by three dielectrics of thicknesses d_1, d_2 and d_3 metres and relative permittivities ε_{r1}, ε_{r2} and ε_{r3} respectively as shown in Fig. 6.6. *The electric flux density D in the dielectrics remains the *same and is equal to Q/A*. However, the electric intensities in the three dielectrics will be different and are given by ;

$$E_1 = \frac{D}{\varepsilon_0 \varepsilon_{r1}} \; ; \quad E_2 = \frac{D}{\varepsilon_0 \varepsilon_{r2}} \; ; \quad E_3 = \frac{D}{\varepsilon_0 \varepsilon_{r3}}$$

If V is the total p.d. across the capacitor and V_1, V_2 and V_3 the p.d.s. across the three dielectrics respectively, then,

$$V = V_1 + V_2 + V_3$$
$$= E_1 d_1 + E_2 d_2 + E_3 d_3$$

* The total charge on each plate is Q. Hence Q coulombs is also the total electric flux through each dielectric.

$$= \frac{D}{\varepsilon_0\,\varepsilon_{r1}} d_1 + \frac{D}{\varepsilon_0\,\varepsilon_{r2}} d_2 + \frac{D}{\varepsilon_0\,\varepsilon_{r3}} d_3$$

$$= \frac{D}{\varepsilon_0}\left[\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right]$$

$$= \frac{Q}{\varepsilon_0\,A}\left[\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right] \qquad \left(\because D = \frac{Q}{A}\right)$$

or $$\frac{Q}{V} = \frac{\varepsilon_0\,A}{\left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right)}$$

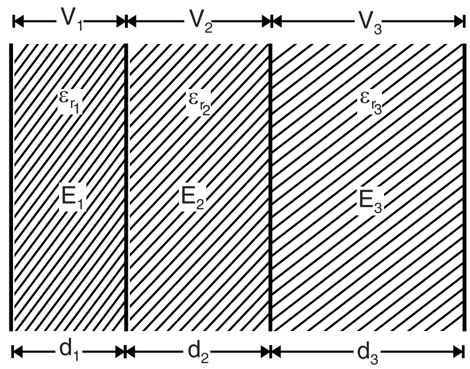


Fig. 6.6

But Q/V is the capacitance C of the capacitor.

$$\therefore \quad C = \frac{\varepsilon_0\,A}{\left(\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}\right)} \text{ farad}$$

In general,

$$C = \frac{\varepsilon_0 A}{\sum \frac{d}{\varepsilon_r}} \text{ farad} \qquad ...(i)$$

Different cases. We shall discuss the following two cases :

(*i*) Medium partly air. Fig. 6.7 shows a parallel plate capacitor having plates d metres apart. Suppose the medium between the plates consists partly of air and partly of dielectric of thickness t metres and relative permittivity ε_{r2}. Then thickness of air is $d - t$. Using the relation (*i*) above, we have,

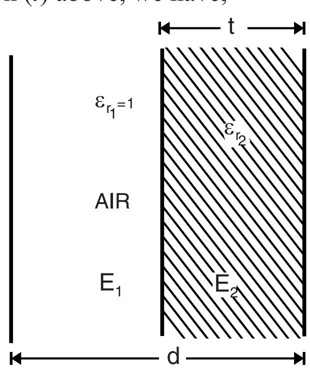


Fig. 6.7

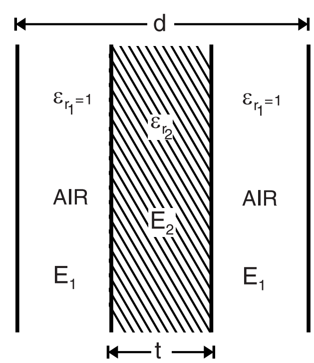


Fig. 6.8

$$C = \frac{\varepsilon_0\,A}{\frac{d-t}{1} + \frac{t}{\varepsilon_{r2}}} = \frac{\varepsilon_0\,A}{d - \left(t - \frac{t}{\varepsilon_{r2}}\right)} \text{ farad}$$

(*ii*) When dielectric slab introduced. Fig. 6.8 shows a parallel-plate air capacitor having plates d metres apart. Suppose a dielectric slab of thickness t metres and relative permittivity ε_{r2} is introduced between the plates of the capacitor.

Using the relation (*i*) above, we have,

$$C = \frac{\varepsilon_0\,A}{\frac{d-t}{1} + \frac{t}{\varepsilon_{r2}}} = \frac{\varepsilon_0\,A}{d - \left(t - \frac{t}{\varepsilon_{r2}}\right)} \text{ farad}$$

6.10. Special Cases of Parallel-Plate Capacitor

We have seen that capacitance of a capacitor depends upon plate area, thickness of dielectric and value of relative permittivity of the dielectric.

We consider two cases by way of illustration.

(*i*) Fig. 6.9 shows that dielectric thickness is d but plate area is divided into two parts; area A_1 having air as the dielectric and area A_2 having dielectric of relative permittivity ε_r. The arrangement is equivalent to two capacitors in parallel. Their capacitances are :

$$C_1 = \frac{\varepsilon_0 A_1}{d} \quad ; \quad C_2 = \frac{\varepsilon_0 \varepsilon_r A_2}{d}$$

The total capacitance C of this parallel-plate capacitor is

$$C = C_1 + C_2$$

(*ii*) Fig. 6.10 shows that plate area is divided into two parts ; area A_1 has dielectric (air) of thickness d and area A_2 has a dielectric (ε_r) of thickness t and the remaining thickness is occupied by air. The arrangement is equivalent to two capacitors connected in parallel. Their capacitances are :

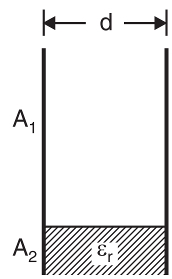


Fig. 6.9

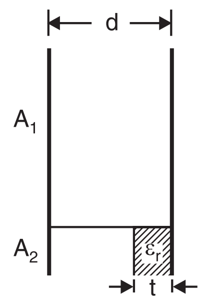


Fig. 6.10

$$C_1 = \frac{\varepsilon_0 A_1}{d} \quad ; \quad C_2 = \frac{\varepsilon_0 A_2}{[d-(t-t/\varepsilon_r)]}$$

The total capacitance C of this parallel plate capacitor is

$$C = C_1 + C_2$$

6.11. Multiplate Capacitor

The most *convenient way of achieving large capacitance is by using large plate area. Increasing the plate area may increase the physical size of the capacitor enormously. In order to obtain a large area of plate surface without using too bulky a capacitor, multiplate construction is employed. In this construction, the capacitor is built up of alternate sheets of metal foil (*i.e.* plates) and thin sheets of dielectric. The odd-numbered metal sheets are connected together to form one terminal T_1 and even-numbered metal sheets are connected together to form the second terminal T_2.

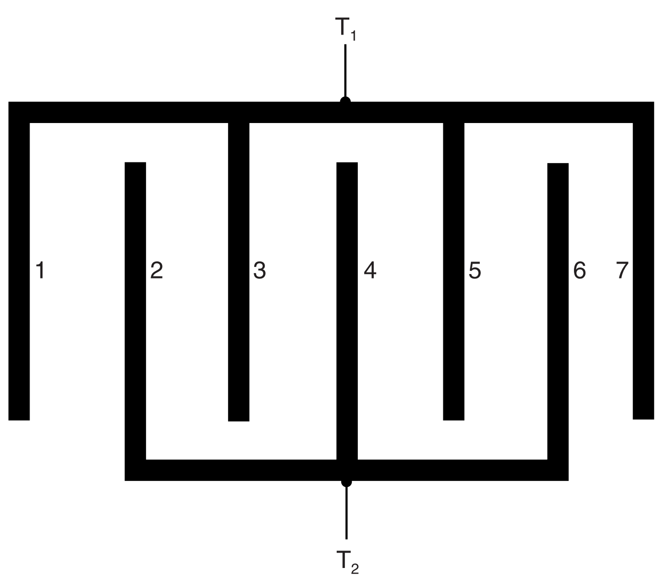


Fig. 6.11

Fig. 6.11 shows a multiplate capacitor with seven plates. A little

* The capacitance of a capacitor can also be increased by (*i*) using a dielectric of high ε_r and (*ii*) decreasing the distance between plates. High cost limits the choice of dielectric and dielectric strength of the insulating material limits the reduction in spacing between the plates..

reflection shows that this arrangement is equivalent to 6 capacitors in parallel. The total capacitance will, therefore, be 6 times the capacitance of a single capacitor (formed by say plates 1 and 2). If there are n plates, each of area A, then $(n - 1)$ capacitors will be in parallel.

∴ Capacitance of n plate capacitor is

$$C = (n-1)\frac{\varepsilon_0\, \varepsilon_r\, A}{d}$$

where d is the distance between any two adjacent plates and ε_r is the relative permittivity of the medium. It may be seen that plate area is increased from A to $A\,(n-1)$.

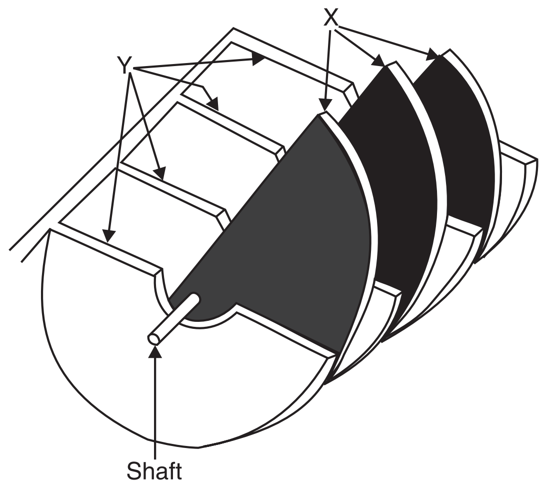


Fig. 6.12

Variable Air capacitor. It is a multiplate air capacitor whose capacitance can be varied by changing the plate area. Fig. 6.12 shows a variable air capacitor commonly used to "tune in" radio stations in the radio receiver. It consists of a set of stationary metal plates Y fixed to the frame and another set of movable metal plates X fixed to the central shaft. The two sets of plates are electrically insulated from each other. Rotation of the shaft moves the plates X into the spaces between plates Y, thus changing the *common (or effective) plate area and hence the capacitance. The capacitance of such a capacitor is given by ;

$$C = (n-1)\frac{\varepsilon_0\, A}{d} \qquad (\because\ \varepsilon_r = 1)$$

When the movable plates X are completely rotated in (*i.e.* the two sets of plates completely overlap each other), the common plate area 'A' is maximum and so is the capacitance of the capacitor. Minimum capacitance is obtained when the movable plates X are completely rotated out of stationary plates Y. The capacitance of such variable capacitors is from zero to about 4000 pF.

Note. In all the formulas derived for capacitance, capacitance will be in farad if area is in m^2 and the distance between plates is in m.

Example 6.4. *A p.d. of 10 kV is applied to the terminals of a capacitor consisting of two parallel plates, each having an area of 0·01 m^2 separated by a dielectric 1 mm thick. The resulting capacitance of the arrangement is 300 pF. Calculate (i) total electric flux (ii) electric flux density (iii) potential gradient and (iv) relative permittivity of the dielectric.*

Solution. $C = 300 \times 10^{-12}$ F ; $V = 10 \times 10^3 = 10^4$ volts

(*i*) Total electric flux, $Q = C\,V = (300 \times 10^{-12}) \times 10^4 = 3 \times 10^{-6}$ C = **3μC**

(*ii*) Electric flux density, $D = \dfrac{Q}{A} = \dfrac{3\times10^{-6}}{0\cdot01}$ = **3×10^{-4} C/m²**

(*iii*) Potential gradient $= \dfrac{V}{d} = \dfrac{10^4}{1\times10^{-3}}$ = **10^7 V/m**

(*iv*) Now, $E = 10^7$ V/m

Since $D = \varepsilon_0 \varepsilon_r E$

$$\therefore \quad \varepsilon_r = \frac{D}{\varepsilon_0 E} = \frac{3\times10^{-4}}{(8.854\times10^{-12})\times10^7} = \mathbf{3{\cdot}39}$$

* Remember in the formula for capacitance, A is the common plate area *i.e.* plate area facing the opposite polarity plate area.

Example. 6.5. *A capacitor is composed of two plates separated by 3mm of dielectric of permittivity 4. An additional piece of insulation 5mm thick is now inserted between the plates. If the capacitor now has capacitance one-third of its original capacitance, find the relative permittivity of the additional dielectric.*

Solution. Figs. 6.13 (*i*) and 6.13 (*ii*) respectively show the two cases.

For the first case, $$C = \frac{\varepsilon_0 \varepsilon_{r1} A}{d} = \frac{\varepsilon_0 \times 4 \times A}{3 \times 10^{-3}} \quad ...(i)$$

For the second case, $$\frac{C}{3} = \frac{\varepsilon_0 A}{\dfrac{d_1}{\varepsilon_{r1}} + \dfrac{d_2}{\varepsilon_{r2}}}$$

$$= \frac{\varepsilon_0 A}{\dfrac{3 \times 10^{-3}}{4} + \dfrac{5 \times 10^{-3}}{\varepsilon_{r2}}} \quad ...(ii)$$

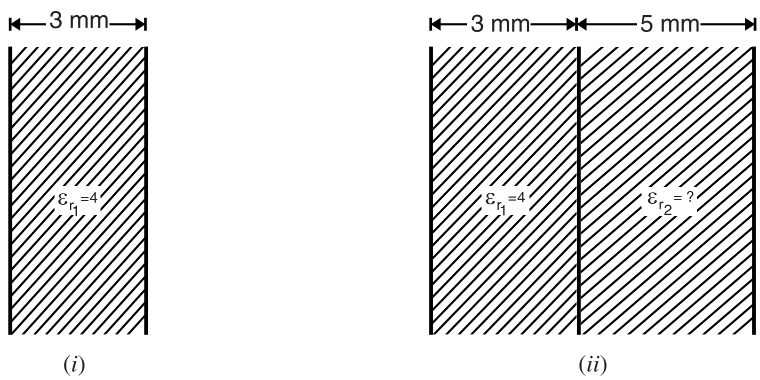


Fig. 6.13

Dividing eq. (*i*) by eq. (*ii*), we get,

$$3 = \frac{4}{3}\left(\frac{3}{4} + \frac{5}{\varepsilon_{r2}}\right)$$

or $$9 = 3 + 20/\varepsilon_{r2} \quad \therefore \ \varepsilon_{r2} = 20/6 = \mathbf{3 \cdot 33}$$

Example 6.6. *Determine the dielectric flux in microcoulombs between two parallel plates each 0.35 metre square with an air gap of 1.5 mm between them, the p. d. being 3000 V. A sheet of insulating material 1 mm thick is inserted between the plates, the relative permittivity of the insulating material being 6. Find out the potential gradient in the insulating material and also in air if the voltage across the plates is raised to 7500 V.*

Solution. $A = 0.35 \times 0.35 = 0.1225$ m^2 ; $d = 1.5$ mm $= 1.5 \times 10^{-3}$ m ; $\varepsilon_r = 1$(air).

Capacitance C of the parallel-plate air capacitor is

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.854 \times 10^{-12} \times 1 \times 0.1225}{1.5 \times 10^{-3}} = 723 \times 10^{-12} \text{ F}$$

Dielectric flux, $\psi = Q = CV = 723 \times 10^{-12} \times 3000 = 2.17 \times 10^{-6}$ C = **2.17 μC**

Suppose the potential gradient in air is g_a. Then potential gradient in the insulating material is $g_i = g_a/\varepsilon_r = g_a/6$.

Thickness of air ; $t_a = 1.5 - 1 = 0.5$ mm $= 0.5 \times 10^{-3}$ m ; Thickness of insulating material, $t_i =$ 1mm $= 10^{-3}$ m.

$\therefore$ Applied voltage, $V = g_a t_a + g_i t_i$

or $$7500 = g_a \times 0.5 \times 10^{-3} + \frac{g_a}{6} \times 10^{-3}$$

∴ $$g_a = \mathbf{11.25 \times 10^6\ V/m}$$

and $$g_i = \frac{g_a}{6} = \frac{11.25\times10^6}{6} = \mathbf{1.875 \times 10^6\ V/m}$$

Example 6.7. *An air capacitor has two parallel plates of 1500 cm² in area and 5 mm apart. If a dielectric slab of area 1500 cm², thickness 2 mm and relative permittivity 3 is now introduced between the plates, what must be the new separation between the plates to bring the capacitance to the original value?*

Solution. This is a case of introduction of dielectric slab into an air capacitor. As proved in Art. 6.9, the capacitance under this condition becomes :

$$C = \frac{\varepsilon_0 A}{d-(t-t/\varepsilon_r)} \quad ...(i)$$

If the medium were totally air, capacitance would have been

$$C_{air} = \frac{\varepsilon_0 A}{d} \quad ...(ii)$$

Inspection of eqs. (*i*) and (*ii*) shows that with the introduction of dielectric slab between the plates of air capacitor, its capacitance increases. The distance between the plates is effectively reduced by $t - (t/\varepsilon_r)$. In order to bring the capacitance to the original value, the plates must be separated by this much distance in air.

∴ New separation between the plates

$$= d + (t - t/\varepsilon_r) = 5 + (2 - 2/3) = \mathbf{6{\cdot}33\ mm}$$

Example 6.8. *A variable air capacitor has 11 movable plates and 12 stationary plates. The area of each plate is 0·0015 m² and separation between opposite plates is 0·001 m. Determine the maximum capacitance of this variable capacitor.*

Solution. The capacitance will be maximum when the movable plates are completely rotated in *i.e.* when the two sets of plates completely overlap each other. Under this condition, the common (or effective) area is equal to the physical area of each plate.

$$C = (n-1)\frac{\varepsilon_0 \varepsilon_r A}{d}$$

Here $n = 11 + 12 = 23\ ;\quad \varepsilon_r = 1\ ;\quad A = 0{\cdot}0015\ \text{m}^2\ ;\quad d = 0{\cdot}001\ \text{m}$

∴ $$C = (23-1)\times\frac{8.854\times10^{-12}\times1\times0.0015}{0.001} = 292 \times 10^{-12}\text{F} = \mathbf{292\ pF}$$

Example 6.9. *The capacitance of a variable radio capacitor can be changed from 50 pF to 950 pF by turning the dial from 0º to 180º. With dial set at 180º, the capacitor is connected to 400 V battery. After charging, the capacitor is disconnected from the battery and the dial is tuned at 0º. What is the potential difference across the capacitor when the dial reads 0º ?*

Solution. With dial at 0º, the capacitance of the capacitor is

$$C_1 = 50\ \text{pF} = 50 \times 10^{-12}\ \text{F}$$

With dial at 180º, the capacitance of the capacitor is

$$C_2 = 950\ \text{pF} = 950 \times 10^{-12}\ \text{F}$$

P.D. across C_2, $V_2 = 400$ V

∴ Charge on C_2, $Q = C_2V_2 = (950 \times 10^{-12}) \times 400 = 380 \times 10^{-9}$ C

When the battery is disconnected, charge Q remains the same. Suppose V_1 is the potential difference across the capacitor when the dial reads 0º.

∴ $$Q = C_1V_1$$

or $$V_1 = \frac{Q}{C_1} = \frac{380\times10^{-9}}{50\times10^{-12}} = \mathbf{7600\ V}$$

Example 6.10. *A parallel plate capacitor has plates of area 2 m^2 spaced by three layers of different dielectric materials. The relative permittivities are 2, 4, 6 and thicknesses are 0.5, 1.5 and 0.3 mm respectively. Calculate the combined capacitance and the electric stress (potential gradient) in each material when applied voltage is 1000 V.*

Solution. Capacitance, $$C = \frac{\varepsilon_0 A}{\frac{d_1}{\varepsilon_{r1}} + \frac{d_2}{\varepsilon_{r2}} + \frac{d_3}{\varepsilon_{r3}}}$$

$$= \frac{8\cdot854\times10^{-12}\times2}{\frac{0\cdot5\times10^{-3}}{2} + \frac{1\cdot5\times10^{-3}}{4} + \frac{0\cdot3\times10^{-3}}{6}} = \mathbf{0\cdot0262\times10^{-6}\ F}$$

Charge on each plate, $Q = CV = (0\cdot0262\times10^{-6})\times1000 = 26\cdot2\times10^{-6}$ C

Electric flux density, $$D = \frac{Q}{A} = \frac{26\cdot2\times10^{-6}}{2} = 13\cdot1\times10^{-6}\ \text{C/m}^2$$

Electric stress in the material with $\varepsilon_{r1} = 2$ is

$$E_1 = \frac{D}{\varepsilon_0\,\varepsilon_{r1}} = \frac{13\cdot1\times10^{-6}}{8\cdot854\times10^{-12}\times2} = \mathbf{74\times10^4\ V/m}$$

Electric stress in the material with $\varepsilon_{r2} = 4$ is

$$E_2 = \frac{13\cdot1\times10^{-6}}{8.854\times10^{-12}\times4} = \mathbf{37\times10^4\ V/m}$$

Electric stress in the material with $\varepsilon_{r3} = 6$ is

$$E_3 = \frac{13\cdot1\times10^{-6}}{8\,854\times10^{-12}\times6} = \mathbf{24.67\times10^4\ V/m}$$

It is clear from the above example that electric stress is greatest in the material having the least relative permittivity. Since air has the lowest relative permittivity, efforts should be made to avoid air pockets in the dielectric materials.

Example 6.11. *A parallel plate capacitor is maintained at a certain potential difference. When a 3 mm slab is introduced between the plates in order to maintain the same potential difference, the distance between the plates is increased by 2·4 mm. Find the dielectric constant of the slab.*

Solution. The capacitance of parallel-plate capacitor in air is

$$C = \frac{\varepsilon_0 A}{d} \quad ...(i)$$

With the introduction of slab of thickness t, the new capacitance is

$$C' = \frac{\varepsilon_0 A}{d' - t(1 - 1/\varepsilon_r)} \quad ...(ii)$$

Now the charge ($Q = CV$) remains the same in the two cases.

$$\therefore \quad \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t(1-1/\varepsilon_r)}$$

or $$d = d' - t(1 - 1/\varepsilon_r)$$

Here, $d' = d + 2\cdot4\times10^{-3}$ m ; $t = 3$ mm $= 3\times10^{-3}$ m

$$\therefore \quad d = d + 2\cdot4\times10^{-3} - 3\times10^{-3}\left(1 - \frac{1}{\varepsilon_r}\right)$$

or $$2\cdot4\times10^{-3} = 3\times10^{-3}\left(1 - \frac{1}{\varepsilon_r}\right)$$

$$\therefore \quad \varepsilon_r = \mathbf{5}$$

Example 6.12. *A parallel plate capacitor has three similar parallel plates. Find the ratio of capacitance when the inner plate is mid-way between the outers to the capacitance when inner plate is three times as near one plate as the other.*

Solution. Fig. 6.14 (*i*) shows the condition when the inner plate is mid-way between the outer plates. This arrangement is equivalent to two capacitors in parallel.

$\therefore$ Capacitance of the capacitor $C_1 = \dfrac{\varepsilon_0\,\varepsilon_r\,A}{d/2} + \dfrac{\varepsilon_0\,\varepsilon_r\,A}{d/2} = \dfrac{4\varepsilon_0\,\varepsilon_r\,A}{d}$

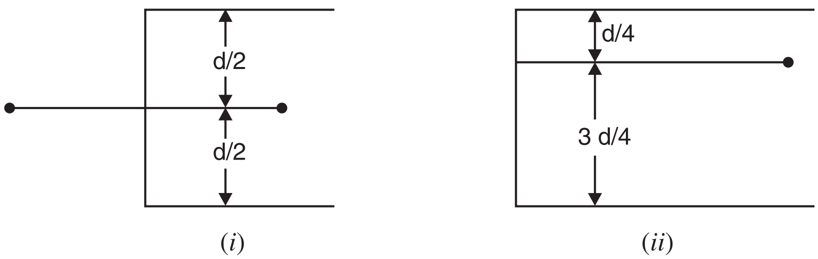


Fig. 6.14

Fig. 6.14 (*ii*) shows the condition when inner plate is three times as near as one plate as the other.

$\therefore$ Capacitance of the capacitor $C_2 = \dfrac{\varepsilon_0\,\varepsilon_r\,A}{d/4} + \dfrac{\varepsilon_0\,\varepsilon_r\,A}{3d/4} = \dfrac{16\varepsilon_0\,\varepsilon_r\,A}{3d}$

$\therefore$ $C_1/C_2 = \mathbf{0.75}$

Example 6.13. *The permittivity of the dielectric material between the plates of a parallel-plate capacitor varies uniformly from ε_1 at one plate to ε_2 at other plate. Show that the capacitance is given by ;*

$$C = \frac{A}{d}\,\frac{\varepsilon_2 - \varepsilon_1}{\log_e \varepsilon_2/\varepsilon_1}$$

where A and d are the area of each plate and separation between the plates respectively.

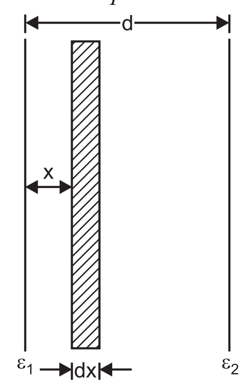


Fig. 6.15

Solution. Fig. 6.15 shows the conditions of the problem. The permittivity of the dielectric material at a distance x from the left plate is

$$\varepsilon_x = \varepsilon_1 + \frac{x}{d}(\varepsilon_2 - \varepsilon_1)$$

Consider an elementary strip of width dx at a distance x from the left plate. The capacitance C of this strip is

$$C = \frac{\varepsilon_x A}{dx}$$

or
$$\frac{1}{C} = \frac{dx}{\varepsilon_x A} = \frac{dx}{A\left[\varepsilon_1 + \frac{x}{d}(\varepsilon_2 - \varepsilon_1)\right]} = \frac{d}{A}\,\frac{dx}{\varepsilon_1 d + x(\varepsilon_2 - \varepsilon_1)}$$

$\therefore$ Total capacitance C_T between the plates is

$$\frac{*1}{C_T} = \int_{x=0}^{x=d} \frac{1}{C} = \frac{d}{A}\int_0^d \frac{dx}{\varepsilon_1 d + x(\varepsilon_2 - \varepsilon_1)}$$

$$= \frac{d}{A}\,{}^{**}\left[\frac{\log_e\{\varepsilon_1 d + (\varepsilon_2 - \varepsilon_1)x\}}{\varepsilon_2 - \varepsilon_1}\right]_0^d$$

* The arrangement constitutes capacitors in series.

** $\displaystyle\int \frac{dx}{a+bx} = \frac{\log_e(a+bx)}{b}$

$$= \frac{d}{A(\varepsilon_2 - \varepsilon_1)}[\log_e(\varepsilon_1 d + \varepsilon_2 d - \varepsilon_1 d) - \log_e \varepsilon_1 d]$$

$$= \frac{d}{A(\varepsilon_2 - \varepsilon_1)}\log_e \frac{\varepsilon_2 d}{\varepsilon_1 d} = \frac{d}{A(\varepsilon_2 - \varepsilon_1)}\log_e \frac{\varepsilon_2}{\varepsilon_1}$$

$$\therefore \quad C_T = \frac{A}{d}\frac{\varepsilon_2 - \varepsilon_1}{\log_e \dfrac{\varepsilon_2}{\varepsilon_1}}$$

Tutorial Problems

1. A capacitor consisting of two parallel plates 0·5 mm apart in air and each of effective area 500 cm^2 is connected to a 100V battery. Calculate (*i*) the capacitance and (*ii*) the charge. **[(*i*) 885 pF (*ii*) 0·0885 μC]**
2. A capacitor consisting of two parallel plates in air, each of effective area 50 cm^2 and 1 mm apart, carries a charge of 1770 × 10^{-12} *C*. Calculate the p.d. between the plates. If the distance between the plates is increased to 5mm, what will be the electrical effect ? **[40 V ; p.d. across plates is increased to 200 V]**
3. Two insulated parallel plates each of 600 cm^2 effective area and 5 mm apart in air are charged to a p.d. of 1000 V. Calculate (*i*) the capacitance and (*ii*) the charge on each plate.

   The source of supply is now disconnected, the plates remaining insulated. Calculate (*iii*) the p.d. between the plates when their spacing is increased to 10 mm and (*iv*) the p.d. when the plates, still 10 mm apart, are immersed in oil of relative permittivity 5. **[(*i*) 106·2 pF (*ii*) 106·2 × 10^{-12} C (*iii*) 2000 V (*iv*) 400 V]**
4. A p.d. of 500 V is applied across a parallel plate capacitor with a plate area of 0·025 m^2. The plates are separated by a dielectric of relative permittivity 2·5. If the capacitance of the capacitor is 500 μF, find (*i*) the electric flux (*ii*) electric flux density and (*iii*) the electric intensity.

   [(*i*) 0·25 μC (*ii*) 0·01 mC/m^2 (*iii*) 45.3 × 10^6 V/m]
5. A capacitor consists of two parallel metal plates, each of area 2000 cm^2 and 5 mm apart. The space between the plates is filled with a layer of paper 2 mm thick and a sheet of glass 3 mm thick. The relative permittivities of paper and glass are 2 and 8 respectively. A p.d. of 5 kV is applied across the plates. Calculate (*i*) the capacitance of the capacitor and (*ii*) the potential gradient in each dielectric.

   [(*i*) 1290 pF (*ii*) 1820 V/mm (paper); **453 V/mm** (glass)**]**
6. A parallel plate capacitor has a plate area of 20 cm^2 and the plates are separated by three dielectric layers each 1 mm thick and of relative permittivity 2, 4 and 5 respectively. Find the capacitance of the capacitor and the electric stress in each dielectric if applied voltage is 1000 V.

   [18·6 pF ; 5·26 × 10^5 V/m; 2·63 × 10^5 V/m; 2·11 × 10^5 V/m]
7. A 1μF parallel plate capacitor that can just withstand a p.d. of 6000 V uses a dielectric having a relative permittivity 5, which breaks down if the electric intensity exceeds 30 × 10^6 V/m. Find (*i*) the thickness of dielectric required and (*ii*) the effective area of each plate. **[(*i*) 0·2 mm (*ii*) 4·5 m^2]**
8. An air capacitor has two parallel plates 10 cm^2 in area and 5 mm apart. When a dielectric slab of area 10 cm^2 and thickness 5 mm was inserted between the plates, one of the plates has to be moved by 0·4 cm to restore the capacitance. What is the dielectric constant of the slab ? **[5]**
9. A multiplate parallel capacitor has 6 fixed plates connected in parallel, interleaved with 5 similar plates; each plate has effective area of 120 cm^2. The gap between the adjacent plates is 1 mm. The capacitor is immersed in oil of relative permittivity 5. Calculate the capacitance. **[5·31 pF]**
10. Calculate the number of sheets of tin foil and mica for a capacitor of 0·33 μF capacitance if area of each sheet of tin foil is 82 cm^2, the mica sheets are 0·2 mm thick and have relative permittivity 5.

    [182 sheets of mica; **183** sheets of tin foil**]**

6.12. Cylindrical Capacitor

A cylindrical capacitor consists of two co-axial cylinders separated by an insulating medium. This is an important practical case since *a single core cable is in effect a capacitor of this kind*. The conductor (or core) of the cable is the inner cylinder while the outer cylinder is represented by lead sheath which is at earth potential. The two co-axial cylinders have insulation between them.

Consider a single core cable with conductor diameter d metres and inner sheath diameter D metres (See Fig. 6.16). Let the charge per metre axial length of the cable be Q coulombs and ε_r be the relative permittivity of the insulating material. Consider a cylinder of radius x metres. According to Gauss's theorem, electric flux passing through this cylinder is Q coulombs. The surface area of this cylinder is

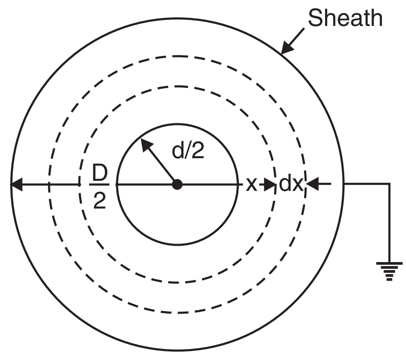


Fig. 6.16

$$= 2\pi x \times 1 = 2\pi x \text{ m}^2$$

$\therefore$ Electric flux density at any point P on the considered cylinder is given by ;

$$D_x = \frac{Q}{2\pi x} \text{ C/m}^2$$

Electric intensity at point P is given by;

$$E_x = \frac{D_x}{\varepsilon_0 \varepsilon_r} = \frac{Q}{2\pi x \varepsilon_0 \varepsilon_r} \text{ V/m}$$

The work done in moving a unit positive charge from point P through a distance dx in the direction of electric field is $E_x\, dx$. Hence the work done in moving a unit positive charge from conductor to sheath, which is the p.d. V between the conductor and sheath, is given by ;

$$V = \int_{d/2}^{D/2} E_x\, dx = \int_{d/2}^{D/2} \frac{Q}{2\pi x \varepsilon_0 \varepsilon_r} dx = \frac{Q}{2\pi \varepsilon_0 \varepsilon_r} \log_e \frac{D}{d}$$

$\therefore$ Capacitance of cable, $C = \dfrac{Q}{V} = \dfrac{Q}{\dfrac{Q}{2\pi\varepsilon_0\varepsilon_r}\log_e \dfrac{D}{d}} \text{ F/m} = \dfrac{2\pi\varepsilon_0\varepsilon_r}{\log_e (D/d)} \text{ F/m}$

$$= \frac{2\pi \times 8\cdot854 \times 10^{-12} \times \varepsilon_r}{2\cdot303 \log_{10} (D/d)} \text{ F/m} = \frac{\varepsilon_r}{41\cdot4 \log_{10} (D/d)} \times 10^{-9} \text{ F/m}$$

If the cable has a length of l metres, then capacitance of the cable is

$$= \frac{\varepsilon_r l}{41\cdot4 \log_{10} (D/d)} \times 10^{-9} \text{ F} = \frac{24\varepsilon_r l}{\log_{10} (D/d)} \text{pF}$$

Example 6.14. *In a concentric cable 20 cm long, the diameter of inner and outer cylinders are 15 cm and 15·4 cm respectively. The relative permittivity of the insulation is 5. If a p.d. of 5000 V is maintained between the two cylinders, calculate :*

(*i*) *capacitance of cylindrical capacitor*

(*ii*) *the charge*

(*iii*) *the electric flux density and electric intensity in the dielectric.*

Solution. (*i*) Capacitance of the cylindrical capacitor is

$$C = \frac{\varepsilon_r l}{41\cdot4 \log_{10} (D/d)} \times 10^{-9} = \frac{5 \times 0\cdot2}{41\cdot4 \log_{10} (15\cdot4/15)} \times 10^{-9} \text{ F} = \mathbf{2.11 \times 10^{-9} \text{ F}}$$

(*ii*) Charge on capacitor, $Q = CV = (2\cdot11 \times 10^{-9}) \times 5000 = 10\cdot55 \times 10^{-6}$ C = **10·55 μC**

(*iii*) To determine D and E in the dielectric, we shall consider the average radius of dielectric, *i.e.,*

Average radius of dielectric, $x = \dfrac{1}{2}\left[\dfrac{15}{2} + \dfrac{15\cdot4}{2}\right] = 7\cdot6 \text{ cm} = 0\cdot076 \text{ m}$

Flux density in dielectric, $D = \frac{Q}{2\pi x l} \text{C/m}^2 = \frac{10 \cdot 55 \times 10^{-6}}{2\pi \times 0 \cdot 076 \times 0 \cdot 2}$ = **$110 \cdot 47 \times 10^{-6}$ C/m²**

Electric intensity in dielectric, $E = \frac{D}{\varepsilon_0 \varepsilon_r} = \frac{110 \cdot 47 \times 10^{-6}}{8 \cdot 854 \times 10^{-12} \times 5}$ = **2.5×10^6 V/m**

Example 6.15. *A 33 kV, 50 Hz, 3-phase underground cable, 4 km long uses three single core cables. Each of the conductor has a diameter of 2·5 cm and the radial thickness of insulation is 0·5 cm. Determine (i) capacitance of the cable/phase (ii) charging current/phase (iii) total charging kVAR. The relative permittivity of insulation is 3.*

Solution. (*i*) Capacitance of cable/phase, $C = \frac{\varepsilon_r \; l}{41 \cdot 4 \log_{10}(D/d)} \times 10^{-9}$ F

Here $\varepsilon_r = 3 \;;\; l = 4 \text{ km} = 4000 \text{ m}$

$d = 2 \cdot 5 \text{ cm} \;;\; D = 2.5 + 2 \times 0.5 = 3.5 \text{ cm}$

Putting these values in the above expression, we get,

$$C = \frac{3 \times 4000 \times 10^{-9}}{41 \cdot 4 \times \log_{10}(3 \cdot 5 / 2 \cdot 5)} = \mathbf{1984 \times 10^{-9} \ F}$$

(*ii*) Voltage/phase, $V_{ph} = \frac{33 \times 10^3}{\sqrt{3}} = 19 \cdot 05 \times 10^3$ V

Charging current/phase, $I_C = \frac{V_{ph}}{X_C} = 2\pi f C V_{ph}$

$= 2\pi \times 50 \times 1984 \times 10^{-9} \times 19 \cdot 05 \times 10^3 = \mathbf{11 \cdot 87 \ A}$

(*iii*) Total charging kVAR $= 3 V_{ph} I_C = 3 \times 19 \cdot 05 \times 10^3 \times 11 \cdot 87 = \mathbf{678 \cdot 5 \times 10^3 \ kVAR}$

6.13. Potential Gradient in a Cylindrical Capacitor

Under operating conditions, the insulation of a cable is subjected to electrostatic forces. This is known as dielectric stress. The dielectric stress at any point in a cable is infact the potential gradient (or *electric intensity) at that point.

Consider a single core cable with core diameter *d* and internal sheath diameter *D*. As proved in Art. 6.12, the electric intensity at a point *x* metres from the centre of the cable is

$$E_x = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x} \text{ volts/m}$$

By definition, electric intensity is equal to potential gradient. Therefore, potential gradient *g* at a point *x* metres from the centre of the cable is

$$g = E_x$$

or $$g = \frac{Q}{2\pi\varepsilon_0\varepsilon_r x} \text{ volts/m} \quad ...(i)$$

As proved in Art. 6.12, potential difference *V* between conductor and sheath is

$$V = \frac{Q}{2\pi\varepsilon_0\varepsilon_r} \log_e \frac{D}{d} \text{ volts}$$

or $$Q = \frac{2\pi\varepsilon_0\varepsilon_r V}{\log_e \frac{D}{d}} \quad ...(ii)$$

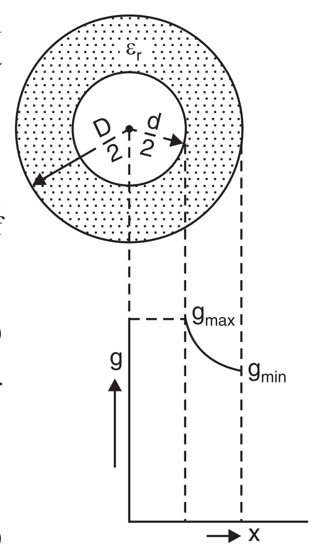


Fig. 6.17

* It may be recalled that potential gradient at any point is equal to the electric intensity at that point.

Substituting the value of Q from exp. (*ii*) in exp. (*i*), we get,

$$g = \frac{\dfrac{2\pi\varepsilon_0\varepsilon_r V}{\log_e D/d}}{2\pi\varepsilon_0\varepsilon_r x} = \frac{V}{x\log_e \dfrac{D}{d}} \text{ volts/m} \qquad ...(iii)$$

It is clear from exp. (*iii*) that potential gradient varies inversely as the distance x. Therefore, potential gradient will be maximum when x is minimum *i.e.,* when $x = d/2$ or at the surface of the conductor. On the other hand, potential gradient will be minimum at $x = D/2$ or at sheath surface.

$\therefore$ Maximum potential gradient, $g_{max} = \dfrac{2V}{d\log_e \dfrac{D}{d}}$ volts/m [Putting $x = d/2$ in exp. (*iii*)]

Minimum potential gradient, $g_{min} = \dfrac{2V}{D\log_e \dfrac{D}{d}}$ volts/m [Putting $x = D/2$ in exp. (*iii*)]

$$\therefore \qquad \frac{g_{max}}{g_{min}} = \frac{\dfrac{2V}{d\log_e D/d}}{\dfrac{2V}{D\log_e D/d}} = \frac{D}{d}$$

The variation of stress in the dielectric is shown in Fig. 6.17. *It is clear that dielectric stress is maximum at the conductor surface and its value goes on decreasing as we move away from the conductor*. It may be noted that maximum stress is an important consideration in the design of a cable. For instance, if a cable is to be operated at such a voltage that *maximum stress is 5 kV/mm, then the insulation used must have a dielectric strength of atleast 5 kV/mm, otherwise breakdown of the cable will become inevitable.

6.14. Most Economical Conductor Size in a Cable

It has already been shown that maximum stress in a cable occurs at the surface of the conductor. For safe working of the cable, dielectric strength of the insulation should be more than the maximum stress. Rewriting the expression for maximum stress, we get,

$$g_{max} = \frac{2V}{d\log_e \dfrac{D}{d}} \text{ volts/m} \qquad ...(i)$$

The values of working voltage V and internal sheath diameter D have to be kept fixed at certain values due to design considerations. This leaves conductor diameter d to be the only variable in exp. (*i*). For given values of V and D, the most economical conductor diameter will be one for which g_{max} has a minimum value. The value of g_{max} will be minimum when $d \log_e D/d$ is maximum *i.e.*

$$\frac{d}{dd}\left[d\log_e \frac{D}{d}\right] = 0 \quad \text{or} \quad \log_e \frac{D}{d} + d.\frac{d}{D}.\frac{-D}{d^2} = 0$$

$$\therefore \qquad \log_e (D/d) - 1 = 0$$

$$\text{or} \qquad \log_e (D/d) = 1 \quad \text{or} \quad (D/d) = e = 2{\cdot}718$$

$\therefore$ Most economical conductor diameter, $d = \dfrac{D}{2.718}$

and the value of g_{max} under this condition is

$$g_{max} = \frac{2V}{d} \text{ volts/m} \qquad [\text{Putting } \log_e D/d = 1 \text{ in exp. } (i)]$$

* Of course, it will occur at the conductor surface.

For low and medium voltage cables, the value of conductor diameter arrived at by this method (*i.e.,* $d = 2V/g_{max}$) is often too small from the point of view of current density. Therefore, the conductor diameter of such cables is determined from the consideration of safe current density. For high voltage cables, designs based on this theory give a very high value of *d,* much too large from the point of view of current carrying capacity and it is, therefore, advantageous to increase the conductor diameter to this value. There are three ways of doing this without using excessive copper :

(*i*) Using aluminium instead of copper because for the same current, diameter of aluminium will be more than that of copper.

(*ii*) Using copper wires stranded around a central core of hemp.

(*iii*) Using a central lead tube instead of hemp.

Example 6.16. *The maximum and minimum stresses in the dielectric of a single core cable are 40 kV/cm* (*r.m.s.*) *and 10 kV/cm* (*r.m.s.*) *respectively. If the conductor diameter is 2 cm, find :*

(*i*) *thickness of insulation* (*ii*) *operating voltage*

Solution. Here, g_{max} = 40 kV/cm ; g_{min} = 10 kV/cm ; d = 2 cm ; D = ?

(*i*) As proved in Art. 6.13,

$$\frac{g_{max}}{g_{min}} = \frac{D}{d} \quad \text{or} \quad D = \frac{g_{max}}{g_{min}} \times d = \frac{40}{10} \times 2 = 8 \text{ cm}$$

$$\therefore \quad \text{Insulation thickness} = \frac{D-d}{2} = \frac{8-2}{2} = \mathbf{3\ cm}$$

(*ii*)

$$g_{max} = \frac{2V}{d \log_e \frac{D}{d}}$$

$$\therefore \quad V = \frac{g_{max}\, d \log_e \frac{D}{d}}{2} = \frac{40 \times 2 \log_e 4}{2} \text{ kV} = \mathbf{55.45\ kV}\ r.m.s.$$

Example 6.17. *A single core cable for use on 11 kV, 50 Hz system has conductor area of 0·645 cm*^{2} *and internal diameter of sheath is 2·18 cm. The permittivity of the dielectric used in the cable is 3·5. Find (i) the maximum electrostatic stress in the cable (ii) minimum electrostatic stress in the cable (iii) capacitance of the cable per km length (iv) charging current.*

Solution. Area of cross-section of conductor, $a = 0.645 \text{ cm}^2$

$$\text{Diameter of the conductor, } d = \sqrt{\frac{4a}{\pi}} = \sqrt{\frac{4 \times 0 \cdot 645}{\pi}} = 0 \cdot 906 \text{ cm}$$

Internal diameter of sheath, D = 2·18 cm

(*i*) Maximum electrostatic stress in the cable is

$$g_{max} = \frac{2V}{d \log_e \frac{D}{d}} = \frac{2 \times 11}{0.906 \log_e \frac{2.18}{0.906}} \text{ kV/cm} = \mathbf{27.65\ kV/cm}\ r.m.s.$$

(*ii*) Minimum electrostatic stress in the cable is

$$g_{min} = \frac{2\,V}{D \log_e \frac{D}{d}} = \frac{2 \times 11}{2.18 \log_e \frac{2.18}{0.906}} \text{ kV/cm} = \mathbf{11.5\ kV/cm\ r.m.s.}$$

(*iii*) Capacitance of cable, $C = \dfrac{\varepsilon_r\ l}{41.4 \log_{10} \frac{D}{d}} \times 10^{-9}$ F

Here ε_r = 3.5 ; l = 1 km = 1000 m

$\therefore \quad C = \dfrac{3\cdot5\times1000}{41\cdot4\,\log_{10}\dfrac{2\cdot18}{0\cdot906}}\times10^{-9}$ = **$0{\cdot}22 \times 10^{-6}$ F**

(*iv*) Charging current, $I_C = \dfrac{V}{X_C} = 2\pi f C V = 2\pi \times 50 \times 0{\cdot}22 \times 10^{-6} \times 11000$ = **0·76 A**

Example 6.18. *Find the most economical size of a single-core cable working on a 132 kV, 3-phase system, if a dielectric stress of 60 kV/cm can be allowed.*

Solution. Phase voltage of cable = $132/\sqrt{3} = 76{\cdot}21$ kV

Peak value of phase voltage, $V = 76{\cdot}21 \times \sqrt{2} = 107{\cdot}78$ kV

Max. permissible stress, g_{max} = 60 kV/cm

∴ Most economical conductor diameter is

$$d = \frac{2V}{g_{max}} = \frac{2\times107\cdot78}{60} = \mathbf{3{\cdot}6\ cm}$$

Internal diameter of sheath, $D = 2{\cdot}718\, d = 2{\cdot}718 \times 3{\cdot}6$ = **9·78 cm**

Therefore, the cable should have a conductor diameter of 3.6 cm and internal sheath diameter of 9·78 cm.

Example 6.19. *The radius of the copper core of a single-core rubber-insulated cable is 2.25 mm. Calculate the radius of the lead sheath which covers the rubber insulation and the cable capacitance per metre. A voltage of 10 kV may be applied between the core and the lead sheath with a safety factor of 3. The rubber insulation has a relative permittivity of 4 and breakdown field strength of 18 × 10⁶ V/m.*

Solution. As proved in Art 6.13,

$$g_{max} = \frac{2V}{d \log_e \dfrac{D}{d}}$$

Here, $g_{max} = E_{max} = 18 \times 10^6$ V/m ; V = Breakdown voltage × Safety factor

$= 10^4 \times 3 = 30{,}000$ volts ; $d = 2.25 \times 2 = 4.5$ mm

$$\therefore \quad 18 \times 10^6 = \frac{2\times30{,}000}{4.5\times10^{-3}\times\log_e\dfrac{D}{d}}$$

or $\dfrac{D}{d} = 2.1 \quad \therefore D = 2.1 \times d = 2.1 \times 4.5 = 9.45$ mm

∴ Radius of sheath = $\dfrac{D}{2} = \dfrac{9.45}{2}$ = **4.72 mm**

Capacitance, $C = \dfrac{\varepsilon_r l}{41.4\log_{10}\dfrac{D}{d}}\times10^{-9}\text{ F} = \dfrac{4\times1}{41.4\log_{10}\dfrac{9.45}{4.5}}\times10^{-9}$ = **0.3×10^{-9} F**

6.15. Capacitance Between Parallel Wires

This case is of practical importance in overhead transmission lines. The simplest system for power transmission is 2-wire d.c. or a.c. system. Consider 2-wire transmission line consisting of two parallel conductors A and B spaced d metres apart in air. Suppose that radius of each conductor is r metres. Let their respective charges be $+Q$ and $-Q$ coulombs per metre length [See Fig. 6.18].

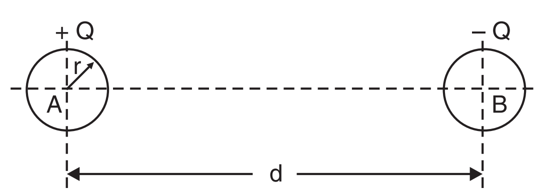


Fig. 6.18

The total p.d. between conductor A and neutral "infinite" plane is

$$V_A^* = \int_r^\infty \frac{Q}{2\pi x \varepsilon_0} dx + \int_d^\infty \frac{-Q}{2\pi x \varepsilon_0} dx$$

$$= \frac{Q}{2\pi \varepsilon_0}\left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d}\right] \text{volts} = \frac{Q}{2\pi \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

Similarly, p.d. between conductor B and neutral "infinite" plane is

$$V_B = \int_r^\infty \frac{-Q}{2\pi x \varepsilon_0} dx + \int_d^\infty \frac{Q}{2\pi x \varepsilon_0} dx$$

$$= \frac{-Q}{2\pi \varepsilon_0}\left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d}\right] = \frac{-Q}{2\pi \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

Both these potentials are *w.r.t.* the same neutral plane. Since the unlike charges attract each other, the potential difference between the conductors is

$$V_{AB} = 2V_A = \frac{2Q}{2\pi \varepsilon_0} \log_e \frac{d}{r} \text{ volts}$$

$\therefore$ Capacitance, $C_{AB} = Q/V_{AB} = \dfrac{Q}{\dfrac{2Q}{2\pi \varepsilon_0} \log_e \dfrac{d}{r}}$ F/m

$$\therefore \quad C_{AB} = \frac{\pi \varepsilon_0}{\log_e \dfrac{d}{r}} \text{ F/m} \quad ...(i)$$

The capacitance for a length l is given by ;

$$C_{AB} = \frac{\pi \varepsilon_0 l}{\log_e \dfrac{d}{r}} \text{F} \quad \text{... in air}$$

$$= \frac{\pi \varepsilon_0 \varepsilon_r l}{\log_e \dfrac{d}{r}} \text{F} \quad \text{... in a medium}$$

Example 6.20. *A 3-phase overhead transmission line has its conductors arranged at the corners of an equilateral triangle of 2 m side. Calculate the capacitance of each line conductor per km. Given that diameter of each conductor is 1·25 cm.*

Solution. Conductor radius, r = 1·25/2 = 0·625 cm ; Spacing of conductors, d = 2 m = 200 cm

$$\text{Capacitance of each line conductor} = \frac{2\pi \varepsilon_0}{\log_e d/r} \text{F/m} = \frac{2\pi \times 8 \cdot 854 \times 10^{-12}}{\log_e 200/0.625} \text{F/m}$$

$= 0·0096 \times 10^{-9}$ F/m $= 0·0096 \times 10^{-6}$ F/km = **0·0096 μF/km**

* The electric intensity E at a distance x from the centre of the conductor in air is given by ;

$$E = \frac{Q}{2\pi x \varepsilon_0} \text{ volts/m}$$

Here, Q = charge per metre length ; ε_0 = permittivity of air

As x approaches infinity, the value of E approaches zero. Therefore, the potential difference between the conductors and infinity distant neutral plane is

$$V_A = \int_r^\infty \frac{Q}{2\pi x \varepsilon_0} dx$$

6.16. Insulation Resistance of a Cable Capacitor

The cable conductor is provided with a suitable thickness of insulating material in order to prevent leakage current. The path for leakage current is radial through the insulation. The opposition offered by insulation to leakage current is known as insulation resistance of the cable. For satisfactory operation, the insulation resistance of the cable should be very high.

Consider a single-core cable of conductor radius r_1 and internal sheath radius r_2 as shown in Fig. 6.19. Let l be the length of the cable and ρ be the resistivity of the insulation.

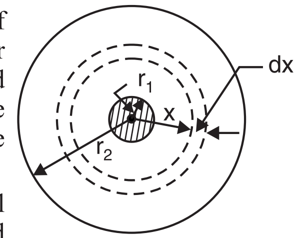


Fig. 6.19

Consider a very small layer of insulation of thickness dx at a radius x. The length through which leakage current tends to flow is dx and the area of X-section offered to this flow is $2\pi\, x\, l$.

∴ Insulation resistance of considered layer

$$= \rho \frac{dx}{2\pi\, x\, l}$$

Insulation resistance of the whole cable is

$$R = \int_{r_1}^{r_2} \rho \frac{dx}{2\pi\, x\, l} = \frac{\rho}{2\pi l}\int_{r_1}^{r_2} \frac{1}{x}dx$$

$$\therefore \quad R = \frac{\rho}{2\pi\, l}\log_e \frac{r_2}{r_1}$$

This shows that insulation resistance of a cable is inversely proportional to its length. In other words, if the cable length increases, its insulation resistance decreases and *vice-versa.*

Example 6.21. *Two underground cables having conductor resistances of 0.7Ω and 0.5Ω and insulation resistances of 300 MΩ and 600 MΩ respectively are joined (i) in series (ii) in parallel. Find the resultant conductor and insulation resistance.*

Solution. (*i*) Series connection. In this case, conductor resistances are added like resistances in series. However, insulation resistances are given by reciprocal relation.

∴ Total conductor resistance = 0.7 + 0.5 = **1.2Ω**

The total insulation resistance R is given by ;

$$\frac{1}{R} = \frac{1}{300} + \frac{1}{600} \quad \therefore R = \mathbf{200\ M\Omega}$$

(*ii*) Parallel connection. In this case, conductor resistances are governed by reciprocal relation while insulation resistances are added.

$$\therefore \quad \text{Total conductor resistance} = \frac{0.7 \times 0.5}{0.7 + 0.5} = \mathbf{0.3\ \Omega}$$

Total insulation resistance = 300 + 600 = **900 MΩ**

Example 6.22. *The insulation resistance of a single-core cable is 495 MΩ per km. If the core diameter is 2·5 cm and resistivity of insulation is 4·5 × 10*^{14} *Ω-cm, find the insulation thickness.*

Solution. Length of cable, l = 1 km = 1000 m

Cable insulation resistance, R = 495 MΩ = 495 × 10^6Ω

Conductor radius, r_1 = 2·5/2 = 1·25 cm

Resistivity of insulation, ρ = 4·5 × 10^{14} Ω-cm = 4·5 × 10^{12}Ωm

Let r_2 cm be the internal sheath radius.

Now, $$R = \frac{\rho}{2\pi\, l}\log_e \frac{r_2}{r_1}$$

or $$\log_e \frac{r_2}{r_1} = \frac{2\pi l R}{\rho} = \frac{2\pi \times 1000 \times 495 \times 10^6}{4\cdot5 \times 10^{12}} = 0\cdot69$$

or $$2\cdot3 \log_{10} r_2/r_1 = 0\cdot69$$

or $$r_2/r_1 = \text{Antilog } 0\cdot69/2\cdot3 = 2$$

or $$r_2 = 2\, r_1 = 2 \times 1\cdot25 = 2\cdot5 \text{ cm}$$

$\therefore$ Insulation thickness $= r_2 - r_1 = 2\cdot5 - 1\cdot25 =$ **1·25 cm**

Example 6.23. *The insulation resistance of a kilometre of the cable having a conductor diameter of 1.5 cm and an insulation thickness of 1.5 cm is 500 MΩ. What would be the insulation resistance if the thickness of the insulation were increased to 2.5 cm?*

Solution. $R_1 = 500$ MΩ ; $l = 100$ m ; $R_2 = ?$

For first case : $$R_1 = \frac{\rho}{2\pi l} \log_e \frac{r_2}{r_1}$$

For second case: $$R_2 = \frac{\rho}{2\pi l} \log_e \frac{r_2'}{r_1'}$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{\log_e (r_2'/r_1')}{\log_e (r_2/r_1)}$$

Now, $r_1 = 1.5/2 = 0.75$ cm ; $r_2 = 0.75 + 1.5 = 2.25$ cm $\therefore r_2/r_1 = 3$

$r'_1 = 0.75$ cm ; $r'_2 = 0.75 + 2.5 = 3.25$ cm ; $\therefore r'_2/r'_1 = 4.333$

$$\therefore \quad \frac{R_2}{500} = \frac{\log_e (4.333)}{\log_e (3)} = 1.334$$

or $$R_2 = 500 \times 1.334 = \mathbf{667.3\ M\Omega}$$

Tutorial Problems

1. A single-core cable has a conductor diameter of 2·5 cm and insulation thickness of 1·2 cm. If the specific resistance of insulation is $4\cdot5 \times 10^{14}$ Ω cm, calculate the insulation resistance per kilometre length of the cable. **[305·5 MΩ]**
2. A single core cable 3 km long has an insulation resistance of 1820 MΩ. If the conductor diameter is 1·5 cm and sheath diameter is 5 cm, calculate the resistivity of the dielectric in the cable. **[$28\cdot57 \times 10^{12}$ Ωm]**
3. Determine the insulation resistance of a single-core cable of length 3 km and having conductor radius 12·5 mm, insulation thickness 10 mm and specific resistance of insulation of 5×10^{12}Ωm. **[156 MΩ]**

6.17. Leakage Resistance of a Capacitor

The resistance of the dielectric of the capacitor is called **leakage resistance**. The dielectric in an *ideal* capacitor is a perfect insulator (*i.e.,* it has infinite resistance) and zero current flows through it when a voltage is applied across its terminals. The dielectric in a *real* capacitor has a large but finite resistance so a very small current flows between the capacitor plates when a voltage is applied.

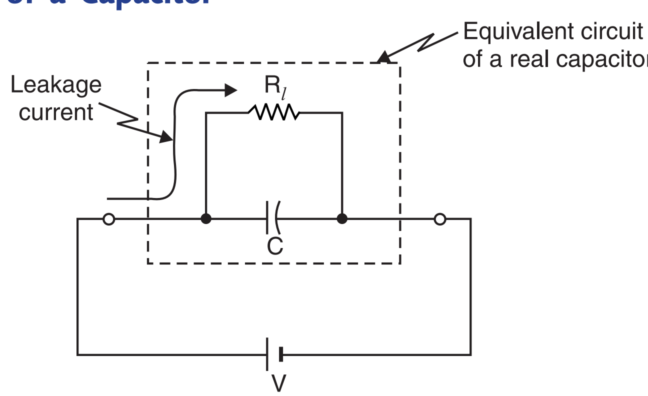


Fig. 6.20

Fig. 6.20 shows the equivalent circuit of a real capacitor consisting of an ideal capacitor in parallel with leakage resistance R_l. Typical values of leakage resistance may range from about 1 MΩ (considered a very "leaky" capacitor) to greater than 100,000 MΩ. A well designed capacitor has very high leakage resistance (> 10^4 MΩ) so that very little power is dissipated even when high voltage is applied across it.

6.18. Voltage Rating of a Capacitor

The maximum voltage that may be safely applied to a capacitor is usually expressed in terms of its d.c. working voltage.

The maximum d.c. voltage that can be applied to a capacitor without breakdown of its dielectric is called **voltage rating** *of the capacitor.*

If the voltage rating of a capacitor is exceeded, the dielectric may break down and conduct current, causing permanent damage to the capacitor. Both capacitance and voltage rating must be taken into consideration before a capacitor is used in a circuit application.

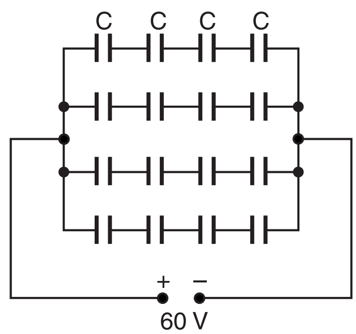


Fig. 6.21

Example 6.24. *Given some capacitors of 0·1 μF capable of withstanding 15 V. Calculate the number of capacitors needed if it is desired to obtain a capacitance of 0·1 μF for use in a circuit involving 60 V.*

Solution. Fig. 6.21 shows the conditions of the problem.

Capacitance of each capacitor, $C = 0.1$ μF

Voltage rating of each capacitor, $V_C = 15$ V

Supply voltage, $V = 60$ V

Since each capacitor can withstand 15 V, the number of capacitors to be connected in series = 60/15 = 4.

Capacitance of 4 series-connected capacitors, $C_T = C/4 = 0{\cdot}1/4 = 0{\cdot}025$ μF. Since it is desired to have a total capacitance of 0·1 μF, number of such rows in parallel = C/C_T = 0·1/0·025 = 4.

∴ Total number of capacitors = 4 × 4 = **16**

Fig. 6.21 shows the arrangement of capacitors.

Example 6.25. *A capacitor of capacitance C_1 = 1 μF withstands the maximum voltage V_1 = 6 kV while another capacitance C_2 = 2 μF withstands the maximum voltage V_2 = 4 kV. What maximum voltage will the system of these two capacitors withstand if they are connected in series ?*

Solution. The maximum charges Q_1 and Q_2 that can be placed on C_1 and C_2 are :

$$Q_1 = C_1V_1 = (1 \times 10^{-6}) \times (6 \times 10^3) = 6 \times 10^{-3} \text{ C}$$

$$Q_2 = C_2V_2 = (2 \times 10^{-6}) \times (4 \times 10^3) = 8 \times 10^{-3} \text{ C}$$

The charge on capacitor C_1 should not exceed 6×10^{-3} C. Therefore, when capacitors are connected in series, the maximum charge that can be placed on the capacitors is 6×10^{-3} C (= Q_1).

$$\therefore \quad V_{max} = \frac{Q_1}{C_1} + \frac{Q_1}{C_2} = \frac{6\times10^{-3}}{1\times10^{-6}} + \frac{6\times10^{-3}}{2\times10^{-6}}$$

$$= 6 \times 10^3 + 3 \times 10^3 = 10^3\,(6 + 3) = 9 \times 10^3 \text{ V} = \mathbf{9\ kV}$$

Example 6.26. *A parallel plate capacitor has plates of dimensions 2 cm × 3 cm. The plates are separated by a 1 mm thickness of paper.*

(*i*) *Find the capacitance of the paper capacitor. The dielectric constant of paper is 3·7.*

(*ii*) *What is the maximum charge that can be placed on the capacitor ? The dielectric strength of paper is 16×16^6 V/m.*

Solution. **(*i*)** $C = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$

Here $\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$ F/m ; $\varepsilon_r = 3{\cdot}7$; $A = 6 \times 10^{-4}$ m^2; $d = 1 \times 10^{-3}$ m

$\therefore \quad C = \dfrac{(8{\cdot}85\times10^{-12})\times(3{\cdot}7)\times(6\times10^{-4})}{1\times10^{-3}} = \mathbf{19{\cdot}6 \times 10^{-12}\ F}$

(*ii*) Since the thickness of the paper is 1 mm, the maximum voltage that can be applied before breakdown occurs is

$$V_{max} = E_{max} \times d$$

Here $E_{max} = 16 \times 10^6$ V/m ; $d = 1$ mm $= 1 \times 10^{-3}$ m

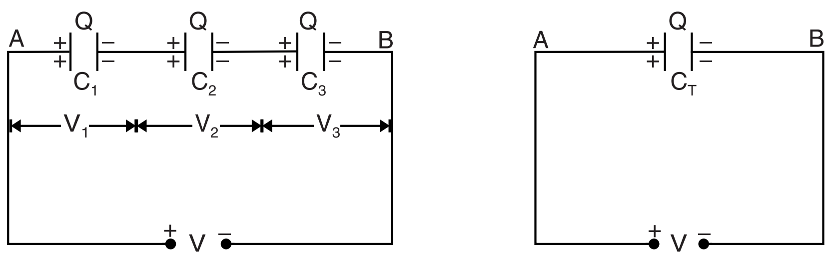
$\therefore \quad V_{max} = (16 \times 10^6) \times (1 \times 10^{-3}) = 16 \times 10^3$ V

$\therefore$ Maximum charge that can be placed on capacitor is

$$Q_{max} = CV_{max} = (19{\cdot}6 \times 10^{-12}) \times (16 \times 10^3) = 0.31 \times 10^{-6}\ \text{C} = \mathbf{0{\cdot}31\ \mu C}$$

6.19. Capacitors in Series

Consider three capacitors, having capacitances C_1, C_2 and C_3 farad respectively, connected in series across a p.d. of V volts [See Fig. 6.22 (*i*)]. In series connection, charge on each capacitor is the *same (*i.e.* $+Q$ on one plate and $-Q$ on the other) but p.d. across each is different.



(*i*) (*ii*)

Fig. 6.22

Now, $V = V_1 + V_2 + V_3 = \dfrac{Q}{C_1} + \dfrac{Q}{C_2} + \dfrac{Q}{C_3}$

$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

or $\dfrac{V}{Q} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$

But Q/V is the **total capacitance C_T between points A and B so that $V/Q = 1/C_T$ [See Fig. 6.22 (*ii*)].

$\therefore \quad \dfrac{1}{C_T} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$

Thus capacitors in series are treated in the same manner as are resistors in parallel.

Special Case. Frequently we come across two capacitors in series. The total capacitance in such a case is given by ;

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

* When voltage V is applied, a similar electron movement occurs on each plate. Hence the same charge is stored by each capacitor. Alternatively, current (charging) in a series circuit is the same. Since $Q = It$ and both I and t are the same for each capacitor, the charge on each capacitor is the same.

** Total or equivalent capacitance is the single capacitance which if substituted for the series capacitances, would provide the same charge for the same applied voltage.

or $$C_T = \frac{C_1 C_2}{C_1 + C_2} \quad i.e. \quad \frac{\text{Product}}{\text{Sum}}$$

Note. The capacitors are connected in series when the circuit voltage exceeds the voltage rating of individual units. In using the series connection, it is important to keep in mind that the voltages across capacitors in series are not the same unless the capacitances are equal. The greater voltage will be across the smaller capacitance which may result in its failure if the capacitances differ very much.

6.20. Capacitors in Parallel

Consider three capacitors, having capacitances C_1, C_2 and C_3 farad respectively, connected in parallel across a p.d. of V volts [See Fig. 6.23 (*i*)]. In parallel connection, p.d. across each capacitor is the same but charge on each is different.

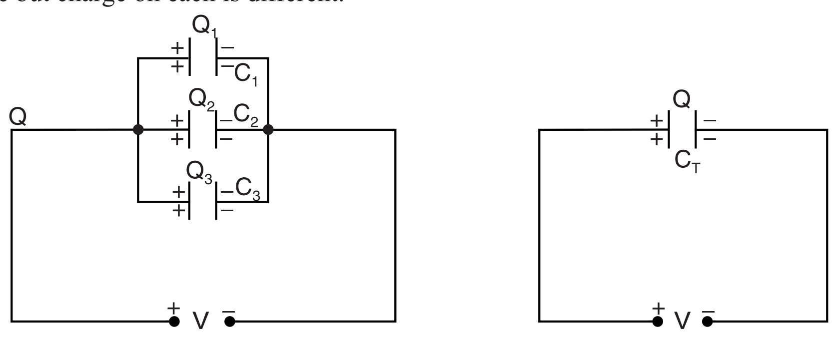


Fig. 6.23

Now, $$Q = Q_1 + Q_2 + Q_3 = C_1 V + C_2 V + C_3 V$$
$$= V(C_1 + C_2 + C_3)$$

or $$Q/V = C_1 + C_2 + C_3$$

But Q/V is the total capacitance C_T of the parallel combination [See Fig. 6.23 (*ii*)].

∴ $$C_T = C_1 + C_2 + C_3$$

Thus capacitors in parallel are treated in the same manner as are resistors in series.

Note. Capacitors may be connected in parallel to obtain larger values of capacitance than are available from individual units.

Example 6.27. *In the circuit shown in Fig. 6.24, the total charge is 750 μC. Determine the values of V_1, V and C_2.*

Solution. $$V_1 = \frac{Q}{C_1} = \frac{750 \times 10^{-6}}{15 \times 10^{-6}} = \mathbf{50\ V}$$

$$V = V_1 + V_2 = 50 + 20 = \mathbf{70\ V}$$

Charge on $C_3 = C_3 \times V_2$
$= (8 \times 10^{-6}) \times 20$
$= 160 \times 10^{-6}\ \text{C} = 160\ \mu\text{C}$

∴ Charge on $C_2 = 750 - 160 = 590\ \mu\text{C}$

∴ Capacitance of $C_2 = \dfrac{590 \times 10^{-6}}{20}$
$= 29{\cdot}5 \times 10^{-6}\ \text{F} = \mathbf{29{\cdot}5\ \mu F}$

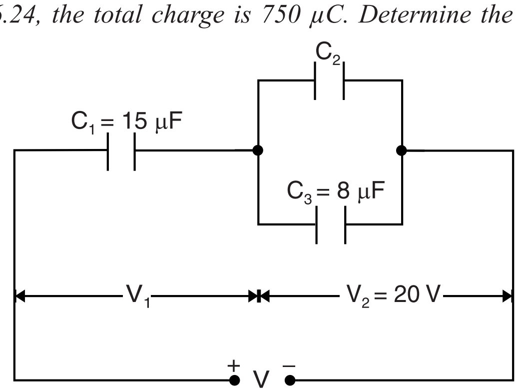
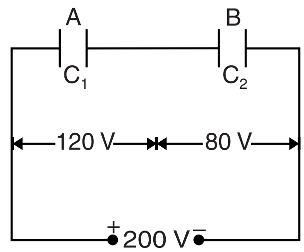


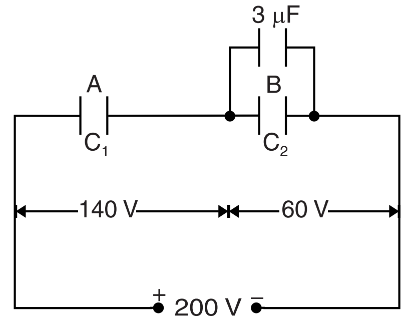
Fig. 6.24

Example 6.28. *Two capacitors A and B are connected in series across a 200 V d.c. supply. The p.d. across A is 120 V. This p.d. is increased to 140 V when a 3μF capacitor is connected in parallel with B. Calculate the capacitances of A and B.*

Solution. Let C_1 and C_2 μF be the capacitances of capacitors *A* and *B* respectively. When the capacitors are connected in series [See Fig. 6.25 (*i*)], charge on each capacitor is the same.



(*i*)



(*ii*)

Fig. 6.25

$$\therefore \qquad C_1 \times 120 = C_2 \times 80 \quad \text{or} \quad C_2 = 1{\cdot}5\, C_1 \qquad ...(i)$$

When a 3μF capacitor is connected in parallel with *B* [See Fig. 6.25 (*ii*)], the combined capacitance of this parallel branch is $(C_2 + 3)$. Thus the circuit shown in Fig. 6.25 (*ii*) can be thought as a series circuit consisting of capacitances C_1 and $(C_2 + 3)$ connected in series.

$$\therefore \qquad C_1 \times 140 = (C_2 + 3)\, 60$$

$$\text{or} \qquad 7C_1 - 3\,C_2 = 9 \qquad \ldots(ii)$$

Solving eqs. (*i*) and (*ii*), we get, C_1 = **3.6 μF ;** C_2 = **5·4 μF**

Example 6.29. *Obtain the equivalent capacitance for the network shown in Fig. 6.26. For 300 V d.c. supply, determine the charge and voltage across each capacitor.*

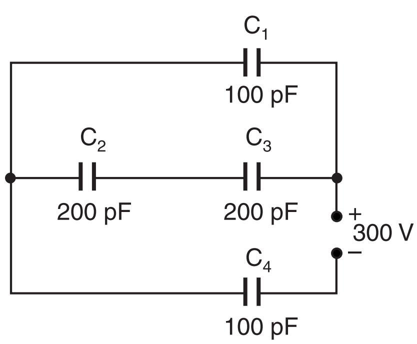


Fig. 6.26

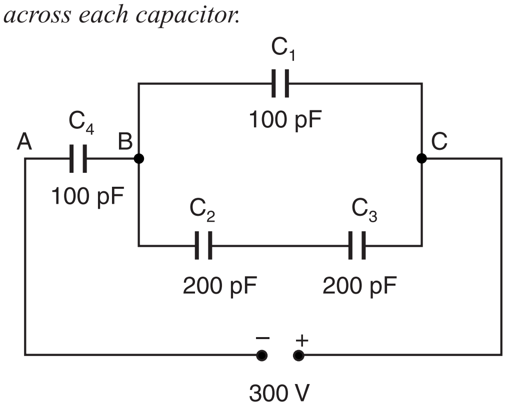


Fig. 6.27

Solution. Equivalent Capacitance. The above network can be redrawn as shown in Fig. 6.27. The equivalent capacitance C' of series-connected capacitors C_2 and C_3 is

$$C' = \frac{C_2 \times C_3}{C_2 + C_3} = \frac{200 \times 200}{200 + 200} = 100 \text{ pF}$$

The equivalent capacitance of parallel combination C' (= 100 pF) and C_1 is

$$C_{BC} = C' + C_1 = 100 + 100 = 200 \text{ pF}$$

The entire circuit now reduces to two capacitors C_4 and C_{BC} (= 200 pF) in series.

∴ Equivalent capacitance of the network is

$$C = \frac{C_4 \times C_{BC}}{C_4 + C_{BC}} = \frac{100 \times 200}{100 + 200} = \mathbf{\frac{200}{3}\ pF}$$

Charges and p.d. on various capacitors

$$\text{Total charge, } Q = CV = \left(\frac{200}{3}\times 10^{-12}\right)\times 300 = 2\times 10^{-8}\text{ C}$$

$$\therefore \quad \text{Charge on } C_4 = \mathbf{2\times 10^{-8}\ C}$$

$$\therefore \quad \text{P.D. across } C_4, V_4 = \frac{Q}{C_4} = \frac{2\times 10^{-8}}{100\times 10^{-12}} = \mathbf{200\ V}$$

$$\text{P.D. between } B \text{ and } C, V_{BC} = 300 - 200 = 100\text{ V}$$

$$\text{Charge on } C_1, Q_1 = C_1V_{BC} = (100\times 10^{-12})\times 100 = \mathbf{10^{-8}\ C}$$

$$\text{P.D. across } C_1, V_1 = V_{BC} = \mathbf{100\ V}$$

$$\text{P.D. across } C_2 = \text{P.D. across } C_3 = 100/2 = \mathbf{50\ V}$$

$$\text{Charge on } C_2 = \text{Charge on } C_3 = \text{Total charge} - \text{Charge on } C_1$$

$$= (2\times 10^{-8}) - (10^{-8}) = \mathbf{10^{-8}\ C}$$

Example 6.30. *Two perfect insulated capacitors are connected in series. One is an air capacitor with a plate area of 0.01 m², the plates being 1 mm apart, the other has a plate area of 0.001 m², the plates separated by a solid dielectric of 0.1 mm thickness with a dielectric constant of 5. Determine the voltage across the combination if the potential gradient in the air capacitor is 200 V/mm.*

Solution. Capacitance C_1 of air capacitor is

$$C_1 = \frac{\varepsilon_0\varepsilon_{r1}A_1}{t_1} = \frac{8.854\times 10^{-12}\times 1\times 0.01}{1\times 10^{-3}} = 88.54\times 10^{-12}\text{ F}$$

Capacitance C_2 of the capacitor with dielectric of $\varepsilon_{r_2} = 5$ is

$$C_2 = \frac{\varepsilon_0\varepsilon_{r2}A_2}{t_2} = \frac{8.854\times 10^{-12}\times 5\times 0.001}{0.1\times 10^{-3}} = 442.7\times 10^{-12}\text{ F}$$

$$\text{Voltage across } C_1, V_1 = g_1\times t_1 = 200\text{V/mm}\times 1\text{ mm} = 200\text{ V}$$

$$\text{Charge on } C_1, Q_1 = C_1V_1 = 88.54\times 10^{-12}\times 200 = 177.08\times 10^{-10}\text{ C}$$

As the capacitors are in series, the charge on each capacitor is the same *i.e.* $Q_2 = Q_1 = 177.08\times 10^{-10}$ C.

$$\therefore \quad \text{Voltage across } C_2, V_2 = \frac{Q_2}{C_2} = \frac{177.08\times 10^{-10}}{442.7\times 10^{-12}} = 40\text{ V}$$

$\therefore$ Voltage across combination, $V = V_1 + V_2 = 200 + 40 =$ **240 volts**

Example 6.31. *In the network shown in Fig. 6.28 (i), $C_1 = C_2 = C_3 = C_4 = 8\ \mu F$ and $C_5 = 10\ \mu F$. Find the equivalent capacitance between points A and B.*

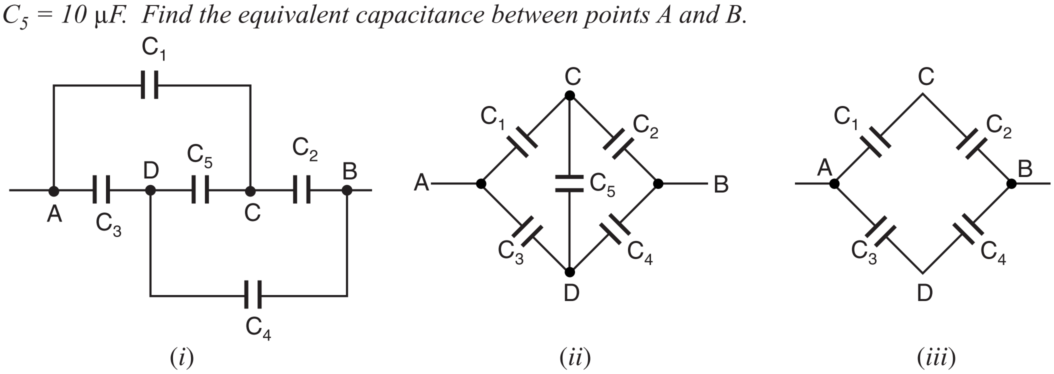


Fig. 6.28

Solution. A little reflection shows that circuit of Fig. 6.28 (*i*) can be redrawn as shown in Fig. 6.28 (*ii*). We find that the circuit is a Wheatstone bridge. Since the product of opposite arms of the bridge are equal ($C_1C_4 = C_2C_3$ because $C_1 = C_2 = C_3 = C_4$), the bridge is balanced. It means that points C and D are at the same potential. Therefore, there will be no charge on capacitor C_5. Hence, this capacitor is ineffective and can be removed from the circuit as shown in Fig. 6.28 (*iii*). Referring to Fig. 6.28 (*iii*), the equivalent capacitance C' of the series connected capacitors C_1 and C_2 is

$$C' = \frac{C_1C_2}{C_1+C_2} = \frac{8\times 8}{8+8} = 4\ \mu F$$

The equivalent capacitance C'' of series connected capacitors C_3 and C_4 [See Fig. 6.28 (*iii*)] is

$$C'' = \frac{C_3C_4}{C_3+C_4} = \frac{8\times 8}{8+8} = 4\ \mu F$$

Now $\quad C_{AB} = C' \parallel C'' = 4 \parallel 4 = 4 + 4 =$ **8 μF**

Example 6.32. *Find the charge on 5 μF capacitor in the circuit shown in Fig. 6.29.*

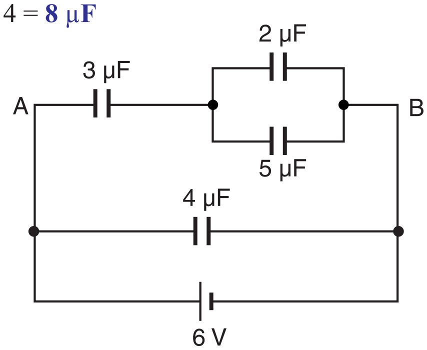


Fig. 6.29

Solution. The p.d. between A and B is 6 V. Considering the branch AB, the capacitors 2 μF and 5 μF are in parallel and their equivalent capacitance = 2 + 5 = 7 μF. The branch AB then has 7 μF and 3 μF in series. Therefore, the effective capacitance of branch AB is

$$C_{AB} = \frac{7\times 3}{7+3} = \frac{21}{10}\ \mu F$$

Total charge in branch AB is

$$Q = C_{AB}V = \frac{21}{10} \times 6 = \frac{63}{5}\ \mu C$$

P.D. across 3 μF capacitor $= \frac{Q}{3} = \frac{63}{5}\times\frac{1}{3} = \frac{21}{5}$ volts

∴ P.D. across parallel combination $= 6 - \frac{21}{5} = \frac{9}{5}$ volts

Charge on 5 μF capacitor $= (5 \times 10^{-6}) \times \frac{9}{5} = 9 \times 10^{-6}$ C = **9 μC**

Example 6.33. *Two parallel plate capacitors A and B having capacitances of 1 μF and 5 μF are charged separately to the same potential of 100 V. Now positive plate of A is connected to the negative plate of B and the negative plate of A is connected to the positive plate of B. Find the final charge on each capacitor.*

Solution. Initial charge on A, $Q_1 = C_1V = (1 \times 10^{-6}) \times 100 = 100\ \mu C$

Initial charge on B, $Q_2 = C_2V = (5 \times 10^{-6}) \times 100 = 500\ \mu C$

When the oppositely charged plates of A and B are connected together, the net charge is

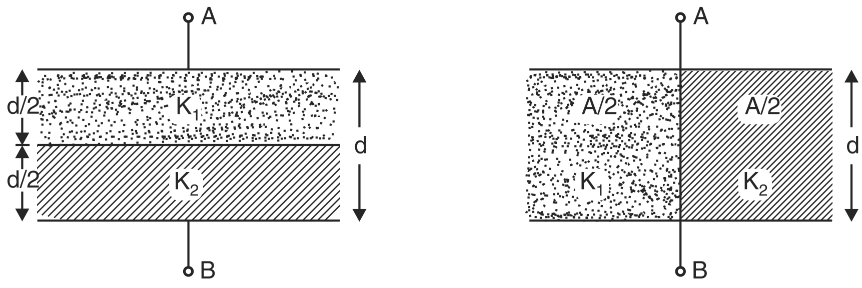
$$Q = Q_2 - Q_1 = 500 - 100 = 400\ \mu C$$

$$\text{Final potential difference} = \frac{\text{Net charge}}{\text{Net capacitance}} = \frac{400\times 10^{-6}}{(1+5)10^{-6}} = \frac{200}{3}\ V$$

$$\text{Final charge on } A = C_1 \times \frac{200}{3} = (1\times 10^{-6})\times\frac{200}{3} = \mathbf{\frac{200}{3}\ \mu C}$$

$$\text{Final charge on } B = C_2 \times \frac{200}{3} = (5\times 10^{-6})\times\frac{200}{3} = \mathbf{\frac{1000}{3}\ \mu C}$$

Example 6.34. *A capacitor is filled with two dielectrics of the same dimensions but of dielectric constants K_1 and K_2 respectively. Find the capacitances in two possible arrangements.*



(*i*) (*ii*)

Fig. 6.30

Solution. The two possible arrangements are shown in Fig. 6.30.

(*i*) The arrangement shown in Fig. 6.30 (*i*) is equivalent to two capacitors in series, each with plate area *A* and plate separation *d*/2 *i.e.,*

$$C_1 = \frac{K_1 \varepsilon_0 A}{d/2} = \frac{2K_1 \varepsilon_0 A}{d} \quad ; \quad C_2 = \frac{K_2 \varepsilon_0 A}{d/2} = \frac{2K_2 \varepsilon_0 A}{d}$$

The equivalent capacitance C' is given by ;

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1 \varepsilon_0 A} + \frac{d}{2K_2 \varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$$

$$\therefore \quad C' = \mathbf{\frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)}$$

(*ii*) The arrangement shown in Fig. 6.30 (*ii*) is equivalent to two capacitors in parallel, each with plate area *A*/2 and plate separation *d i.e.,*

$$C_1 = \frac{K_1 \varepsilon_0 (A/2)}{d} = \frac{K_1 \varepsilon_0 A}{2d} \quad ; \quad C_2 = \frac{K_2 \varepsilon_0 (A/2)}{d} = \frac{K_2 \varepsilon_0 A}{2d}$$

The equivalent capacitance C'' is given by ;

$$C'' = C_1 + C_2 = \frac{K_1 \varepsilon_0 A}{2d} + \frac{K_2 \varepsilon_0 A}{2d} = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

$$\therefore \quad C'' = \mathbf{\frac{\varepsilon_0 A}{2d}(K_1 + K_2)}$$

Example 6.35. *Determine the capacitance between terminals A and B of the network shown in Fig. 6.31. The values shown are capacitances in μF.*

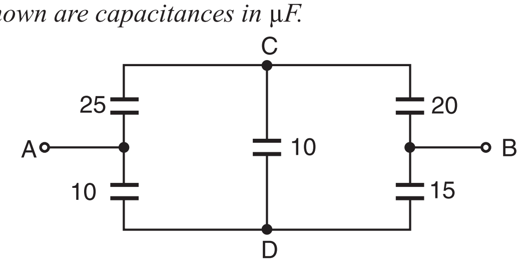


Fig. 6.31

Solution. The circuit shown in Fig. 6.31 is equivalent to the circuit shown in Fig. 6.32.

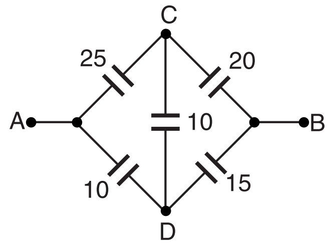


Fig. 6.32

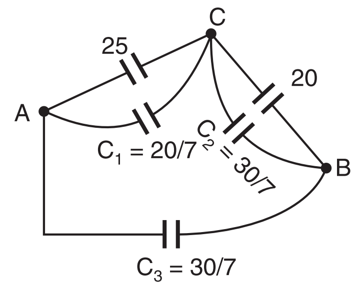


Fig. 6.33

Replacing the star network at D (consisting of capacitances 10, 10 and 15) by equivalent delta, we have,

$$C_1 = \frac{10\times10}{10+10+15} = \frac{20}{7} \quad \text{(between } A \text{ and } C\text{)}$$

$$C_2 = \frac{10\times15}{10+10+15} = \frac{30}{7} \quad \text{(between } B \text{ and } C\text{)}$$

$$C_3 = \frac{10\times15}{10+10+15} = \frac{30}{7} \quad \text{(between } A \text{ and } B\text{)}$$

The circuit then reduces to the circuit shown in Fig. 6.33. Referring to Fig. 6.33,

$$C_{AC} = 25+\frac{20}{7} = \frac{195}{7} = 27{\cdot}86 \;;\; C_{BC} = 20+\frac{30}{7} = \frac{170}{7} = 24{\cdot}29$$

The circuit then reduces to the circuit shown in Fig. 6.34.

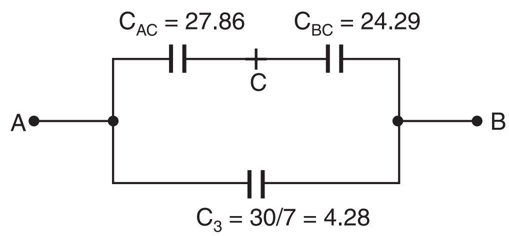


Fig. 6.34

$$\therefore \quad C_{AB} = \frac{C_{AC}\times C_{BC}}{C_{AC}+C_{BC}} + C_3$$

$$= \frac{27{\cdot}86\times24{\cdot}29}{27{\cdot}86+24{\cdot}29} + 4{\cdot}28$$

$$= 12{\cdot}98 + 4{\cdot}28 = \mathbf{17{\cdot}3\ \mu F}$$

Example 6.36. *In the network shown in Fig. 6.35, the capacitances are in* μF*. If the capacitance between terminals P and Q is 5* μF*, find the value of C.*

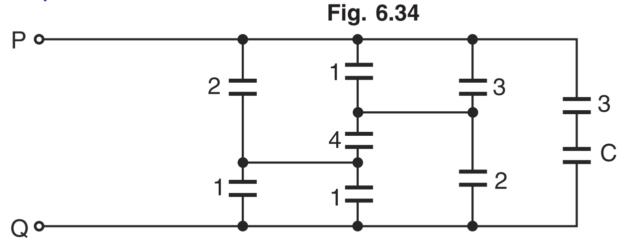


Fig. 6.35

Solution. The capacitances 1 and 1 are in parallel and their equivalent capacitance = 1 + 1 = 2. Likewise, the capacitances 1 and 3 are in parallel and their equivalent capacitance = 1 + 3 = 4. Therefore, the original circuit reduces to the circuit shown in Fig. 6.36.

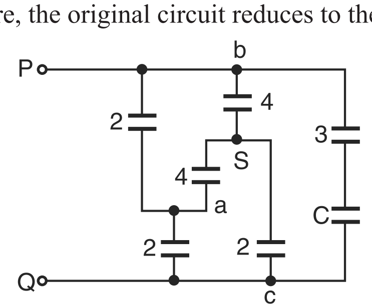


Fig. 6.36

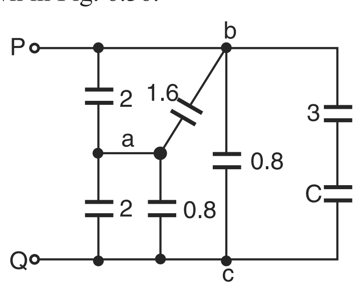


Fig. 6.37

Replacing the star network at *S* (consisting of capacitances 4, 4 and 2) in Fig. 6.36 by its equivalent delta network,

$$C_{ab} = \frac{4\times4}{4+4+2} = 1\cdot6 \;;\quad C_{bc} = \frac{4\times2}{4+4+2} = 0\cdot8 \;;\quad C_{ca} = \frac{4\times2}{4+4+2} = 0\cdot8$$

The circuit in Fig. 6.36 then reduces to the one shown in Fig. 6.37. Referring to Fig. 6.37, capacitances 2 and 1·6 are in parallel and their equivalent capacitance = 2 + 1·6 = 3·6. Likewise, the capacitances 2 and 0·8 are in parallel and their equivalent capacitance = 2 + 0·8 = 2·8. Therefore, the circuit shown in Fig. 6.37 reduces to that shown in Fig. 6.38.

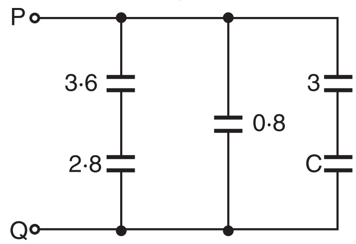


Fig. 6.38

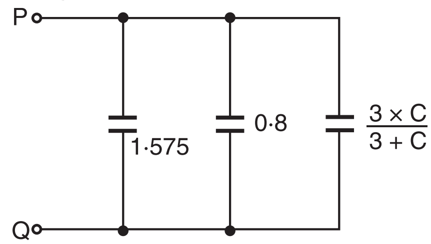


Fig. 6.39

Referring to Fig. 6.38, capacitances 3·6 and 2·8 are in series and their equivalent capacitance = 3·6 × 2·8/(3·6 + 2·8) = 1·575. Likewise, capacitances 3 and *C* are in series and their equivalent capacitance = 3 × *C*/(3 + *C*). The circuit shown in Fig. 6.38 reduces to that shown in Fig. 6.39. Referring to Fig. 6.39,

$$C_{PQ} = 1\cdot575 + 0\cdot8 + \frac{3\times C}{3+C}$$

or $$5 = 1\cdot575 + 0\cdot8 + \frac{3C}{3+C} \qquad [\text{Given } C_{PQ} = 5\ \mu\text{F}]$$

$\therefore$ $$C = \mathbf{21\ \mu F}$$

Tutorial Problems

1. Three capacitors have capacitances of 2, 3 and 4μF respectively. Calculate the total capacitance when they are connected (*i*) in series (*ii*) in parallel. **[(*i*) 0·923μF (*ii*) 9μF]**
2. Three capacitors of values 8μF, 12 μF and 16μF respectively are connected in series across a 240 V d.c. supply. Calculate (*i*) the resultant capacitance and (*ii*) p.d. across each capacitor. **[(*i*) 3·7μF (*ii*) V_1 = 111V, V_2 = 74 V, V_3 = 55 V]**
3. How can three capacitors of capacitances 3μF, 6μF and 9μF respectively be arranged to give a capacitance of 11μF ? **[3μF and 6μF in series, with 9μF in parallel with both]**
4. Two capacitors of capacitances 0·5μF and 0·3μF are joined in series. What value of capacitance joined in parallel with this combination would give a capacitance of 0·5μF ? **[0·31μF]**
5. Three capacitors *A*, *B* and *C* are connected in series across a 200 V d.c. supply. The p.d.s. across the capacitors are 40 V, 70V and 90V respectively. If the capacitance of *A* is 8μF, what are the capacitances of *B* and *C* ? **[4·57 μF, 3·56 μF]**
6. A capacitor of 4μF capacitance is charged to a p.d. of 400V and then connected in parallel with an uncharged capacitor of 2μF capacitance. Calculate the p.d. across the parallel capacitors. **[267 V]**
7. Circuit *ABC* is made up as follows : *AB* consists of a 3μF capacitor, *BC* consists of a 3μF capacitor in parallel with 5μF capacitor. If a d.c. supply of 100 V is connected between *A* and *C*, determine the charge on each capacitor. **[160 μC (AB); 60 μC (3μF in BC); 100 μC]**
8. Two capacitors, *A* and *B*, having capacitances of 20μF and 30μF respectively, are connected in series to a 600 V d.c. supply. If a third capacitor *C* is connected in parallel with *A*, it is found that p.d. across *B* is 400 V. Determine the capacitance of capacitor *C*. **[40μF]**

6.21. Joining Two Charged Capacitors

Consider two charged capacitors of capacitances C_1 and C_2 charged to potentials V_1 and V_2 respectively as shown in Fig. 6.40. With switch S open,

$$Q_1 = C_1V_1 \quad \text{and} \quad Q_2 = C_2V_2$$

When switch S is closed, positive charge will flow from the capacitor of higher potential to the capacitor of lower potential. This flow of charge will continue till p.d. across each capacitor is the same. This is called *common potential* (V).

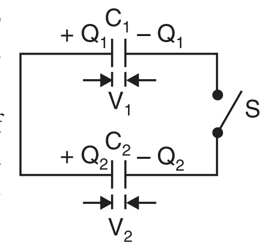


Fig. 6.40

$$\text{Common potential, } V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{Q_1 + Q_2}{C_1 + C_2}$$

$$\therefore \quad V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \quad ...(i)$$

The following points may be noted :

(*i*) Although there is a redistribution of charge on connecting the capacitors (*i.e.,* closing switch S), the total charge before and after the connection remains the same (Remember charge is a conserved quantity). This means that charge lost by one capacitor is *equal to the charge gained by the other capacitor.

(*ii*) When switch S is closed, the capacitors are in parallel.

(*iii*) Since the two capacitors acquire the same common potential V,

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \quad \therefore \quad \frac{Q_1}{Q_2} = \frac{C_1}{C_2}$$

Therefore, the charges acquired by the capacitors are in the ratio of their capacitances.

(*iv*) In this process of charge sharing, the total stored energy of the capacitors decreases. It is because energy is dissipated as heat in the connecting wires when charge flows from one capacitor to the other.

Example 6.37. *Two capacitors of capacitances 4 μF and 6 μF respectively are connected in series across a p.d. of 250 V. The capacitors are disconnected from the supply and are reconnected in parallel with each other. Calculate the new p.d. and charge on each capacitor.*

Solution. In series-connected capacitors, p.d.s across the capacitors are in the inverse ratio of their capacitances.

$$\therefore \text{ P.D. across 4 μF capacitor} = 250 \times \frac{6}{4+6} = 150 \text{ V}$$

$$\text{Charge on 4 μF capacitor} = (4 \times 10^{-6}) \times 150 = 0{\cdot}0006 \text{ C}$$

Since the capacitors are connected in series, charge on each capacitor is the same.

$$\therefore \text{ Charge on both capacitors} = 2 \times 0{\cdot}0006 = 0{\cdot}0012 \text{ C}$$

Parallel connection. When the capacitors are connected in parallel, the total capacitance C_T = 4 + 6 = 10 μF. The total charge 0·0012 C is distributed between the capacitors to have a common p.d.

$$\therefore \quad \text{P.D. across capacitors} = \frac{\text{Total charge}}{C_T} = \frac{0{\cdot}0012}{10 \times 10^{-6}} = \mathbf{120\ V}$$

$$\text{Charge on 4 μF capacitor} = (4 \times 10^{-6}) \times 120 = 480 \times 10^{-6}\text{C} = \mathbf{480\ μC}$$

$$\text{Charge on 6 μF capacitor} = (6 \times 10^{-6}) \times 120 = 720 \times 10^{-6}\text{C} = \mathbf{720\ μC}$$

* Thus referring to exp. (*i*), $V(C_1 + C_2) = C_1V_1 + C_2V_2$ or $C_1V_1 - C_1V = C_2V - C_2V_2$

∴ Charge lost by one = Charge gained by the other

6.22. Energy Stored in a Capacitor

Charging a capacitor means transferring electrons from one plate of the capacitor to the other. This involves expenditure of energy because electrons have to be moved against the *opposing forces. This energy is stored in the electrostatic field set up in the dielectric medium. On discharging the capacitor, the field collapses and the stored energy is released.

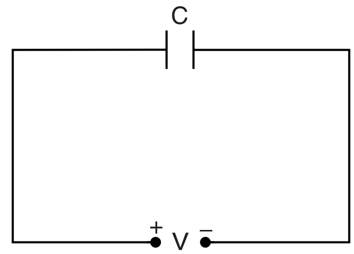


Fig. 6.41

Consider a capacitor of C farad being charged from a d.c. source of V volts as shown in Fig. 6.41. Suppose at any stage of charging, the charge on the capacitor is q coulomb and p.d. across the plates is v volts.

Then, $$C = \frac{q}{v}$$

At this instant, v joules (by definition of v) of work will be done in transferring 1 C of charge from one plate to the other. If further small charge dq is transferred, then work done is

$$dW = v\,dq$$
$$= C\,v\,dv \qquad \left[\begin{array}{l}\because q = C\,v \\ \therefore dq = C\,dv\end{array}\right]$$

∴ Total work done in raising the potential of uncharged capacitor to V volts is

$$W = \int_0^V C\,v\,dv = C\left[\frac{v^2}{2}\right]_0^V$$

or $$W = \frac{1}{2}C\,V^2 \text{ joules}$$

This work done is stored in the electrostatic field set up in the dielectric.

∴ Energy stored in the capacitor is

$$E = \frac{1}{2}C\,V^2 = \frac{1}{2}{}^{**}Q\,V = \frac{{}^{\dagger}Q^2}{2\,C} \text{ joules}$$

Note that an ideal (or pure) capacitor does not dissipate or consume energy; instead, it *stores* energy.

6.23. Energy Density of Electric Field

The energy stored per unit volume of the electric field is called **energy density** *of the electric field*

∴ Energy density, $$u = \frac{\text{Total energy stored}\,(U)}{\text{Volume of electric field}}$$

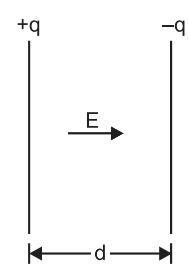


Fig. 6.42

We have seen that energy is stored in the electric field of a capacitor. In fact, wherever electric field exists, there is stored energy. While dealing with electric fields, we are generally interested in energy density (u) *i.e.* energy stored per unit volume. Consider a charged parallel plate capacitor of plate area A and plate separation d as shown in Fig. 6.42.

Energy stored $$= \frac{1}{2}CV^2$$

Volume of space between plates = $A\,d$

∴ Energy density, $$u = \frac{\text{Energy stored}}{\text{Volume}} = \frac{CV^2}{2Ad}$$

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* Electrons are being pushed to the negative plate which tends to repel them. Similarly, electrons are removed from the positive plate which tends to attract them. In either case, forces oppose the transfer of electrons from one plate to the other. This opposition increases as the charge on the plates increases.

** Putting $C = Q/V$ in the exp., $E = \frac{1}{2}QV$

† Putting $V = Q/C$ in the exp., $E = Q^2/2C$

We know that capacitance of a parallel plate capacitor is $C = \varepsilon_0 A/d$.

$$\therefore \quad u = \frac{\varepsilon_0 A}{d} \times \frac{V^2}{2Ad} = \frac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right)^2$$

But V/d is the electric field intensity (E) between the plates.

$$\therefore \quad \text{Energy density, } u = \frac{1}{2}\varepsilon_0 E^2 \quad \text{... in air} \quad ...(i)$$

$$= \frac{1}{2}\varepsilon_0 \varepsilon_r E^2 \quad \text{... in a medium} \quad ...(ii)$$

Obviously, the unit of energy density will be joules/m^3.

Therefore, energy density (i.e., electric field energy stored per unit volume) in any region of space is directly proportional to the square of the electric field intensity in that region.

Note that we derived exps. (*i*) and (*ii*) for the special case of a parallel plate capacitor. But it can be shown to be true for any region of space where electric field exists.

Note. We can also express energy density of electric field in terms of electric flux density D (= $\varepsilon_0 \varepsilon_r E$).

$$u = \frac{1}{2}DE = \frac{D^2}{2\varepsilon_0 \varepsilon_r}$$

Example 6.38. *A 16 μF capacitor is charged to 100 V. After being disconnected, it is immediately connected in parallel with an uncharged capacitor of capacitance 4μF. Determine (i) the p.d. across the combination, (ii) the electrostatic energies before and after the capacitors are connected in parallel and (iii) loss of energy.*

Solution. $C_1 = 16$ μF ; $C_2 = 4$ μF

Before joining

Charge on 16 μF capacitor, $Q = C_1 V_1 = (16 \times 10^{-6}) \times 100 = 1{\cdot}6 \times 10^{-3}$ C

$$\text{Energy stored, } E_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2}(16 \times 10^{-6}) \times 100^2 = \mathbf{0{\cdot}08\ J}$$

After joining. When the capacitors are connected in parallel, the total capacitance $C_T = C_1 + C_2 = 16 + 4 = 20$ μF. The charge $1{\cdot}6 \times 10^{-3}$ C distributes between the two capacitors to have a common p.d. of V volts.

$$\text{P.D. across parallel combination, } V = \frac{Q}{C_T} = \frac{1{\cdot}6 \times 10^{-3}}{20 \times 10^{-6}} = \mathbf{80\ V}$$

$$\text{Energy stored, } E_2 = \frac{1}{2}C_T V^2 = \frac{1}{2}(20 \times 10^{-6}) \times (80)^2 = \mathbf{0.064\ J}$$

$$\text{Loss of energy} = E_1 - E_2 = 0{\cdot}08 - 0{\cdot}064 = \mathbf{0{\cdot}016\ J}$$

It may be noted that there is a loss of energy. This is due to the heat dissipated in the conductor connecting the capacitors.

Example 6.39. *A capacitor-type stored-energy welder is to deliver the same heat to a single weld as a conventional weld that draws 20 kVA at 0·8 p.f. for 0·0625 second/weld. If C = 2000 μF, find the voltage to which it is charged.*

Solution. The energy supplied per weld in a conventional welder is

$$W = VA \times \cos\phi \times \text{time} = (20 \times 10^3) \times (0{\cdot}8) \times 0{\cdot}0625 = 1000 \text{ J}$$

The stored energy in the capacitor should be 1000 J.

$$\therefore \quad 1000 = \frac{1}{2}CV^2$$

$$\text{or} \quad V = \sqrt{\frac{2 \times 1000}{C}} = \sqrt{\frac{2 \times 1000}{2000 \times 10^{-6}}} = \mathbf{1000\ V}$$

Example 6.40. *A parallel plate 100 μF capacitor is charged to 500 V. If the distance between the plates is halved, what will be the new potential difference between the plates and what will be the new stored energy ?*

Solution. $C = 100\ \mu F = 100 \times 10^{-6}\ F = 10^{-4}\ F\ ;\ V = 500$ volts

When plate separation is decreased to half, the new capacitance C' becomes twice *i.e.,* $C' = 2C$. Since the capacitor is not connected to the battery, the charge on the capacitor remains the same. The potential difference between the plates must decrease to maintain the same charge.

$$\therefore \quad Q = CV = C'V' \quad \text{or} \quad V' = \frac{CV}{C'} = \frac{CV}{2C} = \frac{V}{2} = \frac{500}{2} = \mathbf{250\ volts}$$

$$\text{New stored energy} = \frac{1}{2}C'V'^2 = \frac{1}{2}(2C)\left(\frac{V}{2}\right)^2$$

$$= \frac{1}{2}\frac{CV^2}{2} = \frac{1}{2}\left(\frac{1}{2}CV^2\right)$$

$$= \frac{1}{2}\left[\frac{1}{2}\times 10^{-4}\times(500)^2\right] = \mathbf{6{\cdot}25\ J}$$

Example 6.41. *A parallel-plate capacitor is charged with a battery to a charge q_0 as shown in Fig. 6.43 (i). The battery is then removed and the space between the plates is filled with a dielectric of dielectric constant K. Find the energy stored in the capacitor before and after the dielectric is inserted.*

Solution. Energy stored in the capacitor in the absence of dielectric is

$$*E_0 = \frac{1}{2}C_0V_0^2$$

Since $V_0 = q_0/C_0$, this can be expressed as :

$$E_0 = \frac{q_0^2}{2C_0} \qquad \ldots(i)$$

Eq. (*i*) gives the energy stored in the capacitor in the absence of dielectric.

After the battery is removed and the dielectric is inserted between the plates, *charge on the capacitor remains the same.* But the capacitance of the capacitor is increased K times *i.e.,* new capacitance is $C' = K\,C_0$ [See Fig. 6.43 (*ii*)].

$\therefore$ Energy stored in the capacitor after insertion of dielectric is

$$E = \frac{q_0^2}{2C'} = \frac{q_0^2}{2K\,C_0} = \frac{E_0}{K}$$

or
$$E = \frac{E_0}{K} \qquad \ldots(ii)$$

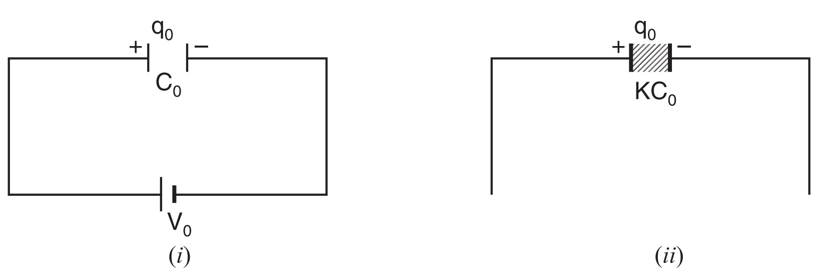


Fig. 6.43

* The subscript 0 indicates the conditions when the medium is air.

Since $K > 1$, we find that final energy is *less* than the initial energy by the factor $1/K$. How will you account for "missing energy" ? When the dielectric is inserted into the capacitor, it gets pulled into the device. The external agent must do negative work to keep the dielectric from accelerating. This work is simply = $E_0 - E$. Alternately, the positive work done by the system = $E_0 - E$.

Example 6.42. *Suppose in the above problem, the capacitor is kept connected with the battery and then dielectric is inserted between the plates. What will be the change in charge, the capacitance, the potential difference, the electric field and the stored energy ?*

Solution. Since the battery remains connected, the potential difference V_0 will **remain unchanged.**

As a result, electric field ($= V_0/d$) will also **remain unchanged.**

The capacitance C_0 will increase to $C = \boldsymbol{K C_0}$.

The charge will also increase to $q = K q_0$ as explained below.

$$q_0 = C_0 V_0 \;;\quad q = CV_0 = KC_0 V_0 = \boldsymbol{K q_0}$$

Initial stored energy, $E_0 = \frac{1}{2} C_0 V_0^2$

Final stored energy, $E = \frac{1}{2} C V_0^2 = \frac{1}{2} K C_0 V_0^2 = K E_0$

$\therefore \quad E = \boldsymbol{KE_0}$

Note that stored energy is increased K times. Will any work be done in inserting the dielectric? The answer is yes. In this case, the work will be done by the battery. The battery not only gives the increased energy to the capacitor but also provides the necessary energy for inserting the dielectric.

Example 6.43. *An air-capacitor of capacitance 0·005 μF connected to a direct voltage of 500 V is disconnected and then immersed in oil with a relative permittivity of 2·5. Find the energy stored in the capacitor before and after immersion.*

Solution. Energy before immersion, $E_1 = \frac{1}{2} CV^2 = \frac{1}{2} \times 0\cdot005 \times 10^{-6} \times (500)^2 = \mathbf{625 \times 10^{-6}\ J}$

When the capacitor is immersed in oil, its capacitance becomes $C' = \varepsilon_r C = 2\cdot5 \times 0\cdot005 = 0\cdot0125$ μF. Since charge remains the same ($V = Q/C$), new voltage is decreased and becomes $V' = V/\varepsilon_r = 500/2\cdot5 = 200$ V.

$\therefore$ Energy after immersion, $E_2 = \frac{1}{2} C'V'^2 = \frac{1}{2} \times 0\cdot0125 \times 10^{-6} \times (200)^2 = \mathbf{250 \times 10^{-6}\ J}$

Example 6.44. *In the circuit shown in Fig. 6.44, the battery e.m.f. is 100 V and the capacitor has a capacitance of 1 μF. The switch is operated 100 times every second. Calculate (i) the average current through the switch between switching operations and (ii) the average power dissipated in the resistor. It may be assumed that the capacitor is ideal and that the capacitor is fully charged or discharged before the subsequent switching.*

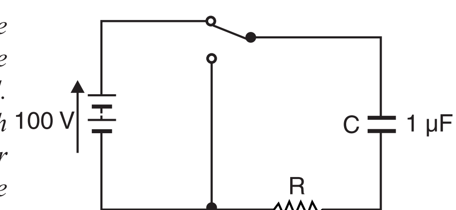


Fig. 6.44

Solution. **(*i*)** Maximum charge on capacitor, $Q = CV = (1 \times 10^{-6}) \times (100) = 10^{-4}$ C

The time taken to acquire this charge (or to lose it) is

$$T = \frac{1}{f} = \frac{1}{100} = 0.01 \text{ s}$$

$\therefore$ Average current, $I_{av} = \frac{\Delta Q}{\Delta T} = \frac{10^{-4}}{0\cdot01} = 0\cdot01 \text{ A} = \mathbf{10\ mA}$

(*ii*) The maximum energy stored during charging is

$$E_m = \frac{1}{2}CV^2 = \frac{1}{2}\times 10^{-6}\times(100)^2 = 0{\cdot}005 \text{ J}$$

During the charging period, a similar quantity of energy must be dissipated in the resistor. In the subsequent discharging period, the stored energy in the capacitor is dissipated in the resistor. Hence for every switching action, 0·005 J is dissipated in the resistor. For 100 switching operations, the energy E dissipated is

$$E = 100 \times 0{\cdot}005 = 0{\cdot}5 \text{ J}$$

$$\text{Average power taken} = \frac{\Delta E}{\Delta T} = \frac{0{\cdot}5}{1} = \mathbf{0{\cdot}5\ W}$$

Note that amount of energy stored in a capacitor is very small because the value of C is very small.

6.24. Force on Charged Plates

Consider two parallel conducting plates x metres apart and carrying constant charges of $+Q$ and $-Q$ coulombs respectively as shown in Fig. 6.45. Let the force of attraction between the two plates be F newtons. If one of the plates is moved away from the other by a small distance dx, then work done is

$$\text{Work done} = F \times dx \text{ joules} \qquad ...(i)$$

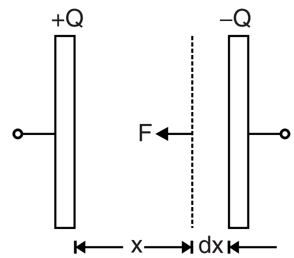


Fig. 6.45

Since the charges on the plates remain constant, no electrical energy can enter or leave the system during the movement dx.

$$\therefore \quad \text{Work done} = \text{Change in stored energy}$$

$$\text{Initial stored energy} = \frac{1}{2}\frac{Q^2}{C} \text{ joules}$$

Since the separation of the plates has increased, the capacitance will decrease by dC. The final capacitance is, therefore, $(C - dC)$.

$$\text{Final stored energy} = \frac{1}{2}\frac{Q^2}{(C - dC)} = \frac{Q^2(C + dC)^*}{2[C^2 - (dC)^2]}$$

Since dC is small compared to C, $(dC)^2$ can be neglected compared to C^2.

$$\therefore \quad \text{Final stored energy} = \frac{Q^2(C + dC)}{2C^2} = \frac{Q^2}{2C} + \frac{Q^2}{2C^2}dC$$

$$\therefore \quad \text{Change in stored energy} = \left(\frac{Q^2}{2\,C} + \frac{Q^2}{2\,C^2}dC\right) - \frac{Q^2}{2\,C} = \frac{Q^2}{2\,C^2}dC \qquad ...(ii)$$

Equating eqs. (*i*) and (*ii*), we get,

$$F \times dx = \frac{Q^2}{2\,C^2}dC$$

or

$$F = \frac{Q^2}{2C^2}\frac{dC}{dx}$$

$$= \frac{1}{2}V^2\frac{dC}{dx} \qquad ...(iii) \quad (\because V = Q/C)$$

Now

$$C = \frac{\varepsilon_0\,\varepsilon_r\,A}{x}$$

* Note this exp. Multiply the numerator and denominator by $(C + dC)$.

$$\therefore \qquad \frac{dC}{dx} = -\frac{\varepsilon_0\,\varepsilon_r A}{x^2}$$

∴ Substituting the value of dC/dx in eq. (*iii*), we get,

$$F = -\frac{1}{2}V^2\frac{\varepsilon_0\,\varepsilon_r\,A}{x^2} = -\frac{1}{2}\varepsilon_0\,\varepsilon_r A\left(\frac{V}{x}\right)^2$$

$$= -\frac{1}{2}\varepsilon_0\,\varepsilon_r\,A\,E^2 \qquad \text{...in a medium}$$

$$= -\frac{1}{2}\varepsilon_0\,A\,E^2 \qquad \text{...in air}$$

This represents the force between the plates of a parallel-plate capacitor charged to a p.d. of V volts. The negative sign shows that it is a force of attraction.

Note. The force of attraction between charged plates may be utilised as a means of measuring potential difference. An instrument of this kind is known as an **electrostatic voltmeter**.

Example 6.45. *A parallel plate capacitor has its plates separated by 0·5 mm of air. The area of plates is 2 m² and they are charged to a p.d. of 100 V. The plates are pulled apart until they are separated by 1 mm of air. Assuming the p.d. to remain unchanged, what is the mechanical force experienced in separating the plates ?*

Solution. Here, $A = 2\text{m}^2$; $d = 0.5$ mm $= 0.5 \times 10^{-3}$ m ; $V = 100$ volts

$$\text{Initial capacitance, } C_1 = \frac{\varepsilon_0\,A}{d} = \frac{8\cdot85\times10^{-12}\times2}{0\cdot5\times10^{-3}} = 35\cdot4\times10^{-9}\,\text{F}$$

$$\text{Initial stored energy, } E_1 = \frac{1}{2}C_1V^2 = \frac{1}{2}\times(35\cdot4\times10^{-9})\times100^2 = 17\cdot7\times10^{-5}\text{ J}$$

$$\text{Final capacitance, } C_2 = \frac{1}{2}C_1 = \frac{1}{2}(35.4\times10^{-9}) = 17.7\times10^{-9}\text{ F}$$

$$\text{Final stored energy, } E_2 = \frac{1}{2}C_2V^2 = \frac{1}{2}(17\cdot7\times10^{-9})\times100^2 = 8\cdot85\times10^{-5}\text{ J}$$

$$\text{Change in stored energy} = (17\cdot7 - 8\cdot85)\times10^{-5} = 8\cdot85\times10^{-5}\text{ J}$$

Suppose F newtons is the average mechanical force between the plates. The plates are separated by a distance $dx = 1 - 0{\cdot}5 = 0{\cdot}5$ mm.

$$\therefore \qquad F\times dx = \text{Change in stored energy}$$

or
$$F = \frac{8\cdot85\times10^{-5}}{0\cdot5\times10^{-3}} = \mathbf{17.7\times10^{-2}\ N}$$

Note that small low-voltage capacitors store microjoules of energy.

6.25. Behaviour of Capacitor in a D.C. Circuit

When d.c. voltage is applied to an uncharged capacitor, there is transfer of electrons from one plate (connected to +ve terminal of source) to the other plate (connected to –ve terminal of source). This is called *charging current* because the capacitor is being charged. The capacitor is *quickly* charged to the applied voltage and charging current becomes zero. Under this condition, the capacitor is said to be fully charged. When a wire is connected across the charged capacitor, the excess electrons on the negative plate move through connecting wire to the positive plate. The energy stored in the capacitor is dissipated in the resistance of the wire. The charge is neutralised when the number of free electrons on both plates are again equal. At this time, the voltage across the capacitor is zero and the capacitor is fully discharged. The behaviour of a capacitor in a d.c. circuit is summed up below :

(*i*) When d.c. voltage is applied to an uncharged capacitor, the capacitor is quickly (*not instantaneously*) charged to the applied voltage.

Charging current, $i = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt}$

When the capacitor is fully charged, capacitor voltage becomes constant and is equal to the applied voltage. Therefore, $dV/dt = 0$ and so is the charging current. Note that dV/dt is the slope of v–t graph of a capacitor.

(*ii*) A capacitor can have voltage across it even when there is no current flowing.

(*iii*) The voltage across a capacitor ($Q = CV$) is proportional to *charge* and not the current.

(*iv*) *There is no current through the dielectric of the capacitor during charging or discharging because the dielectric is an insulating material.* There is merely transfer of electrons from one plate to the other through the connecting wires.

(*v*) When the capacitor is fully charged, there is no circuit current. *Therefore, a fully charged capacitor appears as an open to d.c.*

(*vi*) *An uncharged capacitor is equivalent to a *short circuit as far as d.c. voltage is concerned.* Therefore, a capacitor must be charged or discharged by connecting a resistance in series with it to limit the charging or discharging current.

(*vii*) When the circuit containing capacitor is disconnected from the supply, the capacitor remains charged for a long period. *If the capacitor is charged to a high value, it can be dangerous to someone working on the circuit.*

Example 6.46. *A certain voltage source causes the current to an initially discharged 1000 μF capacitor to increase at a constant rate of 0·06 A/s. Find the voltage across the capacitor after t = 10 s.*

Solution. Charging current, $i_C = 0{\cdot}06t$

∴ Voltage across the capacitor after $t = 10$ s is

$$**v_C = \frac{1}{C}\int_0^{10} i_C\,dt = \frac{1}{1000\times10^{-6}}\int_0^{10} 0\cdot06t$$

$$= 10^3\times0\cdot06\int_0^{10} t\,dt = 60\left[\frac{t^2}{2}\right]_0^{10}$$

$$= 60\times\frac{10^2}{2} = \mathbf{3000\ V}$$

Example 6.47. *A voltage across a 100 μF capacitor varies as follows : (i) uniform increase from 0 V to 700 V in 10 sec (ii) a uniform decrease from 700 V to 400 V in 2 sec (iii) a steady value of 400 V (iv) an instantaneous drop from 400 V to zero. Find the circuit current during each period.*

Solution. $i = C\frac{dv}{dt} = 100\times10^{-6}\frac{dv}{dt} = 10^{-4}\frac{dv}{dt}\,\text{A}$

(*i*) $dv = 700\ \text{V}\ ;\ dt = 10\ \text{sec}$

∴ $i = 10^{-4}\times\frac{700}{10} = 7\times10^{-3}\ \text{A} = \mathbf{7\ mA}$

(*ii*) $dv = 700 - 400 = 300\ \text{V}\ ;\ dt = 2\ \text{sec}$

∴ $i = 10^{-4}\times\frac{300}{2} = 15\times10^{-3}\ \text{A} = \mathbf{15\ mA}$

* When d.c. voltage is applied to an uncharged capacitor, the charging current is limited only by the small resistance of source and any wiring resistance present. The surge current that flows when no resistor is present may be great enough to damage the capacitor, the source or both.

** $i = C\frac{dv}{dt}$ or $\frac{dv}{dt} = \frac{i}{C}$ ∴ Integrating, $v = \frac{1}{C}\int_0^t i\,dt$

(*iii*) $dv/dt = 0$. Therefore, current is **zero**.

(*iv*) $dv = 400 - 0 = 400$ V ; $dt = 0$

$$\therefore \quad i = 10^{-4} \times \frac{400}{0} = \textbf{infinite}$$

Note that in this period, the current is extremely high.

6.26. Charging of a Capacitor

Consider an uncharged capacitor of capacitance C farad connected in series with a resistor R to a d.c. supply of V volts as shown in Fig. 6.46. When the switch is closed, the capacitor starts charging up and charging current flows in the circuit. The charging current is maximum at the instant of switching and decreases gradually as the voltage across the capacitor increases. When the capacitor is charged to applied voltage V, the charging current becomes zero.

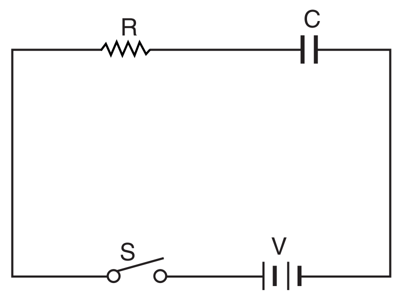


Fig. 6.46

1. **At switching instant.** At the instant the switch is closed, the voltage across capacitor is zero since we started with an uncharged capacitor. The entire voltage V is dropped across resistance R and charging current is maximum (call it I_m).

$\therefore$ Initial charging current, $I_m = V/R$

Voltage across capacitor = 0

Charge on capacitor = 0

2. **At any instant.** After having closed the switch, the charging current starts decreasing and the voltage across capacitor gradually increases. Let at any time t during charging :

i = Charging current

v = P.D. across C

q = Charge on capacitor = $C\,v$

(*i*) Voltage across capacitor

According to Kirchhoff's voltage law, the applied voltage V is equal to the sum of voltage drops across resistor and capacitor.

$$\therefore \quad V = v + iR \qquad \ldots(i)$$

$$\text{or} \quad V = v + CR^{*}\frac{dv}{dt}$$

$$\text{or} \quad -\frac{dv}{V-v} = -\frac{dt}{RC}$$

Integrating both sides, we get,

$$\int -\frac{dv}{V-v} = \int -\frac{dt}{RC}$$

$$\text{or} \quad \log_e (V-v) = -\frac{t}{RC} + K \qquad \ldots(ii)$$

where K is a constant whose value can be determined from the initial conditions. At the instant of closing the switch S, $t = 0$ and $v = 0$.

Substituting these values in eq. (*ii*), we get, $\log_e V = K$.

Putting the value of $K = \log_e V$ in eq. (*ii*), we get,

$$\log_e (V-v) = -\frac{t}{RC} + \log_e V$$

* $i = \frac{dq}{dt} = \frac{d}{dt}(q) = \frac{d}{dt}(Cv) = C\frac{dv}{dt}$

$$\text{or} \qquad \log_e \frac{V - v}{V} = -\frac{t}{RC}$$

$$\text{or} \qquad \frac{V - v}{V} = e^{-t/RC}$$

$$\therefore \qquad v = V[1 - e^{-t/RC}] \qquad \ldots(iii)$$

This is the expression for variation of voltage across the capacitor (v) w.r.t. time (t) and is represented graphically in Fig. 6.47 (*i*). *Note that growth of voltage across the capacitor follows an exponential law*. An inspection of eq. (*iii*) reveals that as t increases, the term $e^{-t/RC}$ gets smaller and voltage v across capacitor gets larger.

(*ii*) Charge on Capacitor

q = Charge at any time t

Q = Final charge

Since $v = q/C$ and $V = Q/C$, the exp. (*iii*) becomes :

$$\frac{q}{C} = \frac{Q}{C}[1 - e^{-t/RC}]$$

$$\text{or} \qquad q = Q(1 - e^{-t/RC}) \qquad \ldots(iv)$$

Again the increase of charge on capacitor plates follows exponential law.

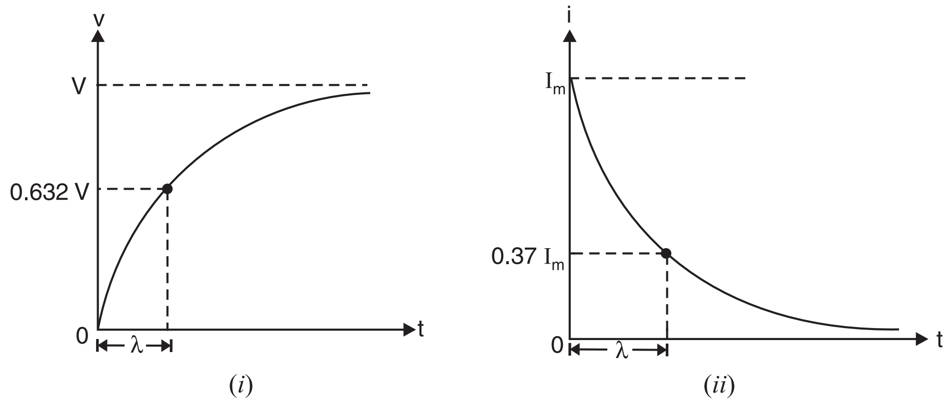


Fig. 6.47

(*iii*) Charging current

From exp. (*i*), $V - v = iR$

From exp. (*iii*), $V - v = V e^{-t/RC}$

$$\therefore \qquad iR = V e^{-t/RC}$$

$$\text{or} \qquad i = \frac{V}{R} e^{-t/RC}$$

$$\therefore \qquad i = I_m e^{-t/RC}$$

where I_m (= V/R) is the initial charging current. *Again the charging current decreases following exponential law.* This is also represented graphically in Fig. 6.47 (*ii*).

(*iv*) Rate of rise of voltage across capacitor

We have seen above that :

$$V = v + CR\frac{dv}{dt}$$

At the instant the switch is closed, $v = 0$.

$$\therefore \qquad V = CR\frac{dv}{dt}$$

or Initial rate of rise of voltage across capacitor is given by ;

$$\frac{dv}{dt} = \frac{V}{CR} \text{ volts/sec} \quad \ldots(iv)$$

Note. *The capacitor is almost fully charged in a time equal to 5 RC i.e., 5 time constants.*

6.27. Time Constant

Consider the eq. (*iii*) above showing the rise of voltage across the capacitor :

$$v = V(1 - e^{-t/RC})$$

The exponent of e is t/RC. The quantity RC has the *dimensions of time so that exponent of e is a number. The quantity RC in called the *time constant* of the circuit and affects the charging (or discharging) time. It is represented by λ (or T or τ).

∴ Time constant, $\lambda = RC$ seconds

Time constant may be defined in one of the following ways :

(*i*) At the instant of closing the switch, p.d. across capacitor is zero. Therefore, putting $v = 0$ in the expression $V = v + CR\frac{dv}{dt}$, we have,

$$V = CR\frac{dv}{dt}$$

or

$$\frac{dv}{dt} = \frac{V}{CR}$$

If this rate of rise of voltage could continue, the capacitor voltage will reach the final value V in time $= V \div V/CR = RC$ seconds = time constant λ.

Hence **time constant** *may be defined as the time required for the capacitor voltage to rise to its final steady value V if it continued rising at its initial rate* (*i.e., V/CR*).

(*ii*) If the time interval $t = \lambda$ (or RC), then,

$$v = V(1 - e^{-t/t}) = V(1 - e^{-1}) = 0{\cdot}632\,V$$

Hence **time constant** *can also be defined as the time required for the capacitor voltage to reach 0·632 of its final steady value V.*

(*iii*) If the time interval $t = \lambda$ (or RC), then,

$$i = I_m e^{-t/t} = I_m e^{-1} = 0{\cdot}37\, I_m$$

Hence **time constant** *can also be defined as the time required for the charging current to fall to 0·37 of its initial maximum value* I_m.

Fig. 6.48 as well as adjoining table shows the percentage of final voltage (V) after each time constant interval during voltage buildup (v) across the capacitor. An uncharged capacitor charges to about 63% of its fully charged voltage (V) in first time constant. A 5 time-constant interval is accepted as the time to fully charge (or discharge) a capacitor and is called the *transient time*.

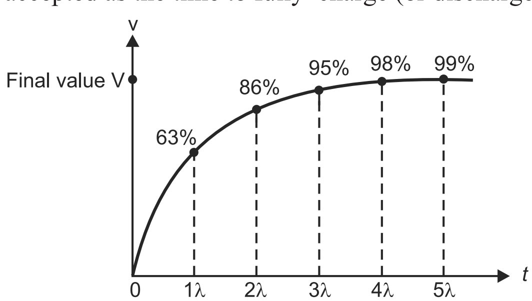


Fig. 6.48

Number of time constants	% of final value
1	63
2	86
3	95
4	98
5	99 considered 100 %

$$* \quad RC = \left(\frac{\text{Volt}}{\text{Ampere}}\right)\times\left(\frac{\text{Coulomb}}{\text{Volt}}\right) = \frac{\text{Volt}}{(\text{Coulomb/sec})}\times\left(\frac{\text{Coulomb}}{\text{Volt}}\right) = \text{seconds}$$

6.28. Discharging of a Capacitor

Consider a capacitor of C farad charged to a p.d. of V volts and connected in series with a resistance R through a switch S as shown in Fig. 6.49 (*i*). When the switch is open, the voltage across the capacitor is V volts. When the switch is closed, the voltage across capacitor starts decreasing. The discharge current rises instantaneously to a value of V/R (= I_m) and then decays gradually to zero.

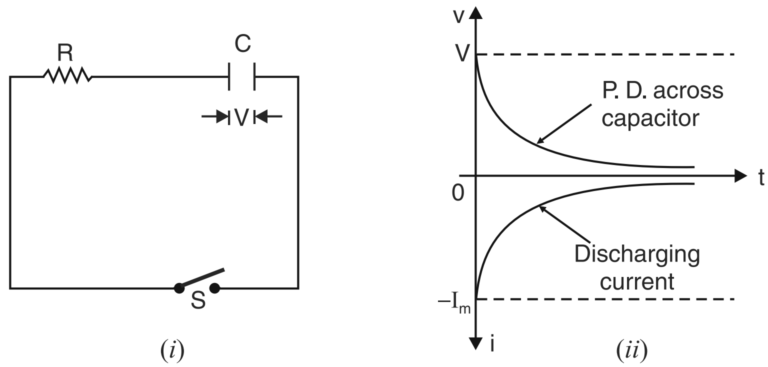


Fig. 6.49

Let at any time t during discharging,

$$v = \text{p.d. across the capacitor}$$
$$i = \text{discharging current}$$
$$q = \text{charge on capacitor}$$

By Kirchhoff's voltage law, we have,

$$0 = v + RC\frac{dv}{dt}$$

or
$$\frac{dv}{v} = -\frac{dt}{RC}$$

Integrating both sides, we get,

$$\int \frac{dv}{v} = -\frac{1}{RC}\int dt$$

$\therefore$
$$\log_e v = -\frac{t}{RC} + K \qquad \ldots(i)$$

At the instant of closing the switch, $t = 0$ and $v = V$. Putting these values in eq. (*i*), we get,

$$\log_e V = K$$

$\therefore$ Equation (*i*) becomes : $\log_e v = (-t/RC) + \log_e V$

or
$$\log_e \frac{v}{V} = -\frac{t}{RC}$$

or
$$\frac{v}{V} = e^{-t/RC}$$

$\therefore$
$$v = V\,e^{-t/\lambda} \qquad \ldots(ii)$$

Again λ (= RC) is the time constant and has the dimensions of time.

Similarly,
$$q = Q\,e^{-t/RC}$$

and
$$i = -I_m\,e^{-t/RC}$$

Note that negative sign is attached to I_m. This is because the discharging current flows in the opposite direction to that in which the charging current flows.

Fig. 6.50 as well as adjoining table shows the percentage of initial voltage (V) after each time constant interval during discharging of capacitor. A fully charged capacitor discharges to about 37% of its initial fully charged value in first time constant. The capacitor is fully discharged in a 5 time-constant interval.

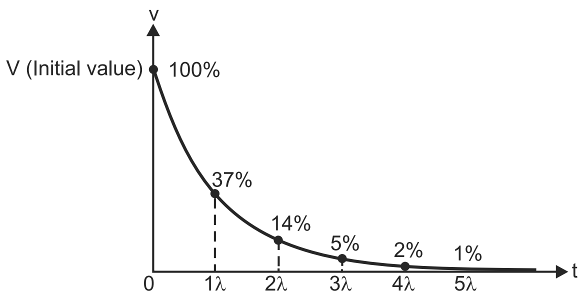


Fig. 6.50

Number of time constants	% of Initial value
1	37
2	14
3	5
4	2
5	1 considered 0

Example 6.48. *A 2 μF capacitor is connected, by closing a switch, to a supply of 100 volts through a 1 MΩ series resistance. Calculate (i) the time constant (ii) initial charging current (iii) the initial rate of rise of p.d. across capacitor (iv) voltage across the capacitor 6 seconds after the switch has been closed and (v) the time taken for the capacitor to be fully charged.*

Solution. (*i*) Time constant, $\lambda = RC = (10^6) \times (2 \times 10^{-6}) = $ **2 seconds**

(*ii*) Initial charging current, $I_m = \dfrac{V}{R} = \dfrac{100}{10^6} \times 10^6 = $ **100 μA**

(*iii*) Initial rate of rise of voltage across capacitor is

$$\frac{dv}{dt} = \frac{V}{CR} = \frac{100}{(2 \times 10^{-6}) \times 10^6} = \textbf{50 V/s}$$

(*iv*) $v = V(1 - e^{-t/RC})$

Here $V = 100$ volts ; $t = 6$ seconds ; $RC = 2$ seconds

∴ $v = 100\,(1 - e^{-6/2}) = 100\,(1 - e^{-3}) = $ **95.1 V**

(*v*) Time taken for the capacitor to be fully charged

$= 5\,RC = 5 \times 2 = $ **10 seconds**

Example 6.49. *A capacitor of 8 μF capacitance is connected to a d.c. source through a resistance of 1 megaohm. Calculate the time taken by the capacitor to receive 95% of its final charge. How long will it take the capacitor to be fully charged ?*

Solution. $q = Q\,(1 - e^{-t/RC})$

Here $RC = (10)^6 \times 8 \times 10^{-6} = 8$ seconds ; $q/Q = 0{\cdot}95$

∴ $0{\cdot}95 = 1 - e^{-t/8}$ or $e^{-t/8} = 0{\cdot}05$

∴ $e^{t/8} = 1/0{\cdot}05 = 20$

or $(t/8)\log_e e = \log_e 20$

∴ $t = 8\log_e 20 = $ **23·96 seconds**

Time taken for the capacitor to be fully charged

$= 5\,RC = 5 \times 8 = $ **40 seconds**

Alternatively. $t = \lambda \log_e \dfrac{V - V_0}{V - v_C}$ ... See Art. 6.30

or $t = \lambda \log_e \dfrac{Q - q_0}{Q - q}$

Here, $\lambda = 8s$; $q_0 = 0$; $q = 95\%$ of $Q = 0.95$ Q

$$\therefore \quad t = 8 \times \log_e \frac{Q - 0}{Q - 0.95Q} = 8 \times \log_e \frac{Q}{0.05Q} = \textbf{23.96 seconds}$$

Example 6.50. *A resistance R and a 4 μF capacitor are connected in series across a 200 V d.c. supply. Across the capacitor is connected a neon lamp that strikes at 120 V. Calculate the value of R to make the lamp strike after 5 seconds.*

Solution. The voltage across the neon lamp has to rise to 120 V in 5 seconds.

Now, $$v = V(1 - e^{-t/\lambda}) \quad \text{or} \quad 120 = 200\,(1 - e^{-5/\lambda})$$

or $$e^{-5/\lambda} = 1 - (120/200) = 0{\cdot}4 \quad \text{or} \quad e^{5/\lambda} = 1/0{\cdot}4 = 2{\cdot}5$$

$$\therefore \quad (5/\lambda)\log_e e = \log_e 2{\cdot}5$$

or $$\lambda = \frac{5}{\log_e 2{\cdot}5} = 5{\cdot}457 \text{ seconds}$$

or $$RC = 5{\cdot}457 \quad \therefore \quad R = \frac{5{\cdot}457}{4 \times 10^{-6}} = 1{\cdot}364 \times 10^6\ \Omega = \mathbf{1{\cdot}364\ M\Omega}$$

Alternatively. $$t = \lambda \log_e \frac{V - V_0}{V - v_C}$$

Here, $t = 5s$; $V = 200$ volts ; $V_0 = 0$; $v_C = 120$ volts

Putting these values in the above expression, we get, $\lambda = 5.457s$.

Now $\lambda = RC$ or $R = \dfrac{\lambda}{C} = \dfrac{5.457}{4 \times 10^{-6}} = 1.364 \times 10^6\Omega =$ **1.364 MΩ**

Example 6.51. *A capacitor of 1 μF and resistance 82 kΩ are connected in series with an e.m.f. of 100 V. Calculate the magnitude of energy and the time in which energy stored in the capacitor will reach half of its equilibrium value.*

Solution. Equilibrium value of energy $= \dfrac{1}{2}CV^2$

$$\therefore \quad \text{Energy stored} \propto V^2$$

Half energy of the equilibrium value will be stored when voltage across capacitor is $v = 100/\sqrt{2}$ $= 70{\cdot}7$ volts.

$$\therefore \quad \text{Energy stored} = \frac{1}{2}Cv^2 = \frac{1}{2}(1 \times 10^{-6}) \times (70{\cdot}7) = \mathbf{0{\cdot}0025\ J}$$

Now, $$v = V(1 - e^{-t/RC})$$

Here, $RC = (82 \times 10^3) \times (1 \times 10^{-6}) = 0{\cdot}082$ s ; $v = 70{\cdot}7$ V ; $V = 100$ V

$$\therefore \quad 70{\cdot}7 = 100\,(1 - e^{-t/0{\cdot}082}) \quad \text{or} \quad e^{-t/0{\cdot}082} = 1 - (70{\cdot}7/100) = 0{\cdot}293$$

$$\therefore \quad e^{t/0{\cdot}082} = 1/0{\cdot}293 = 3{\cdot}413$$

or $$(t/0{\cdot}082)\log_e e = \log_e 3{\cdot}413$$

$$\therefore \quad t = 0{\cdot}082 \times \log_e 3{\cdot}413 = \mathbf{0{\cdot}1\ second}$$

Example 6.52. *When a capacitor C charges through a resistor R from a d.c. source voltage E, determine the energy appearing as heat.*

Solution. When $R - C$ series circuit is switched on to d.c. source of voltage E, the charging current i decreases at exponential rate given by ;

$$i = I\,e^{-t/\lambda}$$

$$\text{where } I = E/R \; ; \; \lambda = RC$$

Energy appearing as heat in small time Δt is

$$\Delta W_R = i^2 R\,\Delta t$$

Total energy appearing as heat in the entire process of charging is

$$W_R = \int_0^\infty i^2 R\,dt = R\int_0^\infty (I e^{-t/\lambda})^2 dt = R\int_0^\infty I^2 e^{-2t/\lambda} dt$$

$$= R \times I^2 \int_0^\infty e^{-2t/\lambda} dt = RI^2 \left[\frac{e^{-2t/\lambda}}{-2/\lambda}\right]_0^\infty$$

$$= R \times (E/R)^2 \left(\frac{-\lambda}{2}\right)[e^{-\infty} - e^0] = \frac{E^2}{R} \times \left(\frac{-RC}{2}\right) \times (-1)$$

$$\therefore \quad W_R = \frac{1}{2} CE^2$$

Although energy stored in a capacitor is very small, it can provide a large current (and hence large power) for a short period of time.

Note. Energy stored in the capacitor at the end of charging process is $CE^2/2$. Also energy appearing as heat in the entire process of charging the capacitor is $CE^2/2$.

$\therefore$ Total energy received from the source $= \frac{1}{2}CE^2 + \frac{1}{2}CE^2 = CE^2$

Thus during charging of capacitor, the total energy received from the source is CE^2 ; half is converted into heat and the rest half stored in the capacitor.

Example 6.53. *Referring to the circuit shown in Fig. 6.51,*

(*i*) *Write the mathematical expression for charging current i and voltage v across capacitor when the switch is placed in position 1.*

(*ii*) *Write the mathematical expression for the discharging current and voltage across capacitor when switch is placed in position 2 after having been in position 1 for 1 s.*

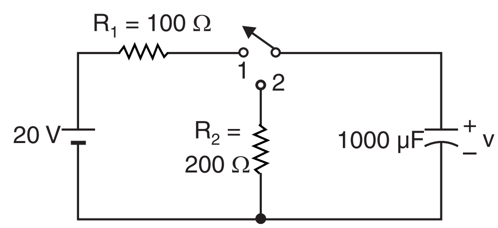


Fig. 6.51

Solution.

(*i*) When the switch is placed in position 1, the capacitor charges through R_1 only. Therefore, time constant during charging is

Time constant, $\lambda = R_1C = (100) \times (1000 \times 10^{-6}) = 0{\cdot}1$ s

Initial charging current, $I_m = V/R_1 = 20/100 = 0{\cdot}2$ A

The charging current at any time *t* is given by ;

$$i = I_m e^{-t/\lambda} \quad \text{or} \quad \boldsymbol{i = 0{\cdot}2\, e^{-t/0{\cdot}1}} \textbf{ A}$$

The voltage *v* across the capacitor at any time *t* is given by ;

$$v = V(1 - e^{-t/\lambda}) \quad \text{or} \quad \boldsymbol{v = 20\,(1 - e^{-t/0{\cdot}1})} \textbf{ volts}$$

(*ii*) Since the switch remains in position 1 for 1 s or 10 time constants, the capacitor charges fully to 20 V. When the switch is placed in position 2, the capacitor discharges through R_2 only. Therefore, time constant during discharge is

Time constant, $\lambda = R_2C = (200) \times (1000 \times 10^{-6}) = 0{\cdot}2$ s

Initial discharging current, $I_m = V/R_2 = 20/200 = 0{\cdot}1$ A

The discharging current at any time *t* is given by ;

$$i = -I_m e^{-t/\lambda} \quad \text{or} \quad \boldsymbol{i = -0{\cdot}1\, e^{-t/0{\cdot}2}} \textbf{ A}$$

The voltage *v* across the capacitor at any time *t* is given by ;

$$v = V e^{-t/\lambda} \quad \text{or} \quad \boldsymbol{v = 20\, e^{-t/0{\cdot}2}} \textbf{ volts}$$

Example 6.54. *A cable 10 km long and of capacitance 2.5μF discharges through its insulation resistance of 50 MΩ. By what percentage the voltage would have fallen 1, 2 and 5 minutes respectively after disconnection from its bus-bars?*

Solution. Capacitance of cable capacitor, $C = 2.5 \times 10^{-6}$ F; Insulation resistance of cable, $R = 50$ MΩ $= 50 \times 10^6$ Ω

Time constant, $\lambda = RC = (50 \times 10^6) \times (2.5 \times 10^{-6}) = 125$ seconds

During discharging, decreasing voltage v across the capacitor is given by ;

$$v = Ve^{-t/\lambda} = Ve^{-t/125}$$

At $t = 1$ min. = 60 seconds, $v_1 = Ve^{-60/125} = 0.618$ V

At $t = 2$ min. = 120 seconds, $v_2 = Ve^{-120/125} = 0.383$ V

At $t = 5$ min. = 300 seconds, $v_3 = Ve^{-300/125} = 0.09$ V

∴ At $t = 1$ min, the % age fall in voltage across capacitor

$$= \frac{V - 0.618V}{V} \times 100 = \mathbf{38.2\%}$$

At $t = 2$ min; the % age fall in voltage across capacitor

$$= \frac{V - 0.383V}{V} \times 100 = \mathbf{61.7\%}$$

At $t = 5$ min; the % age fall in voltage across capacitor

$$= \frac{V - 0.09V}{V} \times 100 = \mathbf{91\%}$$

Tutorial Problems

1. A capacitor is being charged from a d.c. source through a resistance of 2MΩ. If it takes 0·2 second for the charge to reach 75% of its final value, what is the capacitance of the capacitor ? **[18×10^{-4} F]**
2. A 8 μF capacitor is connected is series with 0·5 MΩ resistance across 200 V supply. Calculate (*i*) initial charging current (*ii*) the current and p.d. across capacitor 4 seconds after it is connected to the supply. [(*i*) **400 μA** (*ii*) **147 μA**; **126·4 V**]
3. What resistance connected in series with a capacitance of 4μF will give the circuit a time constant of 2 seconds ? **[500 kΩ]**
4. A series *RC* circuit is to have an initial charging current of 4 mA and a time constant of 3·6 seconds when connected to 120 V d.c. supply. Calculate the values of *R* and *C*. What will be the energy stored in the capacitor ? **[30 kΩ ; 120 μF ; 0·864 J]**
5. A 20μF capacitor initially charged to a p.d. of 500V is discharged through an unknown resistance. After one minute, the p.d. at the terminals of the capacitor is 200 V. What is the value of the resistance ? **[3·274 MΩ]**

6.29. Transients in D.C. Circuits

When a circuit goes from one steady-state condition to another steady-state condition, it passes through a transient state which is of short duration. The word transient means temporary or short-lived. When a d.c. voltage source is first connected to a series *RC* network, the charging current flows only until the capacitor is fully charged. This charging current is called a **transient current**. In connection with d.c. circuits, a transient is a voltage or current that *changes* with time for a short duration of time and remains constant thereafter. As a capacitor charges, its voltage builds up (*i.e.,* changes) until the capacitor is fully charged and its voltage equals the source voltage. After that time, there is no further change in capacitor voltage. Thus the voltage across a capacitor during the time it is being charged is an example of a **transient voltage**.

6.30. Transient Relations During Charging/Discharging of Capacitor

When a capacitor is charging or discharging, it goes from one steady-state condition (called *initial condition*) to another steady-state condition (called *final condition*). During this change, the voltage across and current through the capacitor change continuously. These are called **transient conditions* and exist for a short duration. *It can be shown mathematically that voltage v_C across the capacitor at any time t during charging or discharging* is given by ;

$$v_C = V - (V - V_0)e^{-t/\lambda} \quad ...(i)$$

where v_C = voltage across capacitor at any time t

V = Source voltage during charging

V_0 = Voltage across capacitor at $t = 0$

λ = Time constant (= RC)

Note that for discharging of capacitor, $V = 0$ because there is no source voltage.

1. Transient conditions during charging. When we charge an uncharged capacitor, $V_0 = 0$ so that eq. (*i*) becomes :

$$v_C = V - (V - 0)e^{-t/\lambda} = V - Ve^{-t/\lambda}$$

$$\therefore \quad v_C = V(1 - e^{-t/\lambda}) \quad ...(ii)$$

This is the same equation that we derived in Art. 6.26 for charging of a capacitor.

From eq. (*ii*), $\quad V - v_C = Ve^{-t/\lambda}$

But $V - v_C = iR$, where i is the charging current at time t.

$$\therefore \quad iR = Ve^{-t/\lambda} \text{ or } i = \frac{V}{R}e^{-t/\lambda}$$

$$\therefore \quad i = Ie^{-t/\lambda} \quad ...(iii)$$

where I (= V/R) is the initial charging current.

Note that eq. (*iii*) is the same that we derived in Art. 6.26 for charging of a capacitor. Fig. 6.52 shows capacitor voltage (v_C) and charging current (i) waveforms for a charging capacitor. It may be seen that voltage across the capacitor is building up at an exponential rate while the charging current is decreasing at an exponential rate.

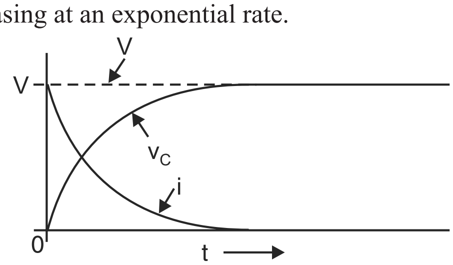


Fig. 6.52

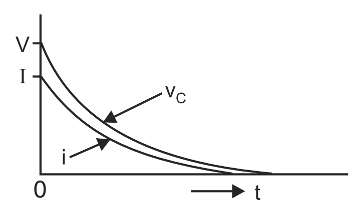


Fig. 6.53

2. Transient conditions during discharging. For discharging of a capacitor, $V = 0$ because there is no source voltage. Therefore, eq. (*i*) becomes :

$$v_C = 0 - (0 - V_0)e^{-t/\lambda}$$

$$\text{or} \quad v_C = V_0 e^{-t/\lambda} \quad ...(iv)$$

Here V_0 is, of course, the voltage to which the capacitor was originally charged. Note that this is the same expression which derived in Art. 6.28 for discharging of a capacitor.

$$\text{Now,} \quad \frac{v_C}{C} = \frac{V_0}{C}e^{-t/\lambda}$$

$$\text{or} \quad q = Q_0 e^{-t/\lambda}$$

* The word transient means temporary or short-lived.

where Q_0 is the initial charge on the capacitor and q is the charge on the capacitor at time t.

$$\text{Similarly, } i = I_0 e^{-t/\lambda}$$

where I is the initial discharging current and i is the discharging current at time t.

Fig. 6.53 shows the capacitor voltage and discharging current waveforms. Both decrease at exponential rate and reach zero value at the same time.

Time for charge or discharge. Sometimes it is desirable to determine how long will it take the capacitor in RC series circuit to charge or discharge to a specified voltage. This can be found as follows : From eq. (i), $v_C = V - (V - V_0)\, e^{-t/\lambda}$

or $$V - v_C = (V - V_0) e^{-t/\lambda}$$

or $$\frac{V - v_C}{V - V_0} = e^{-t/\lambda}$$

or $$\frac{V - V_0}{V - v_C} = e^{t/\lambda}$$

Taking the natural log, we have,

$$\frac{t}{\lambda} \log_e e = \log_e \frac{V - V_0}{V - v_C}$$

$$\therefore \quad t = \lambda \log_e \frac{V - V_0}{V - v_C} \quad ...(v)$$

Exp. (v) is applicable for charging as well as discharging of a capacitor.

For charging. When C is charging from 0V (*i.e.* capacitor is uncharged), $V_0 = 0$. Therefore, putting $V_0 = 0$ in exp. (v), we have,

$$t = \lambda \log_e \frac{V - 0}{V - v_C} = \lambda \log_e \frac{V}{V - v_C}$$

$$\therefore \quad t = \lambda \log_e \frac{V}{V - v_C}$$

If the capacitor has some initial charge instead of zero, then value of V_0 will be corresponding to that charge.

For discharging. In this case, $V = 0$. Therefore, putting $V = 0$ in exp. (v), we have,

$$t = \lambda \log_e \frac{0 - V_0}{0 - v_C} = \lambda \log_e \frac{V_0}{v_C}$$

$$\therefore \quad t = \lambda \log_e \frac{V_0}{v_C}$$

Example 6.55. *The uncharged capacitor in Fig. 6.54 is initially switched to position 1 of the switch for two seconds and then switched to position 2 for the next two seconds. What will be the voltage on the capacitor at the end of this period?*

Solution. When uncharged capacitor is switched to position 1, it will be instantaneously charged to 100 V because there is no resistance in the charging circuit. Therefore, after 2 seconds, the capacitor will be at 100 V. Now when switch is put to position 2, the time of discharge t is given by ;

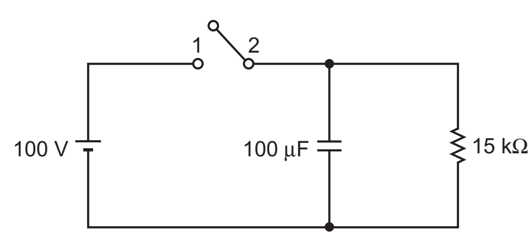


Fig. 6.54

$$t = \lambda \log_e \frac{V - V_0}{V - v_c}$$

Here $t = 2\text{s}$; $\lambda = RC = 15000 \times 100 \times 10^{-6} = 1.5\text{s}$; $V = 0$; $V_0 = 100$ volts

$$\therefore \quad 2 = 1.5 \log_e \frac{0 - 100}{0 - v_c} = 1.5 \log_e \frac{100}{v_c}$$

On solving, v_c = **26.36 V**

Example 6.56. *A 50μF capacitor and a 20 kΩ resistor are connected in series across a battery of 100 V at the instant t = 0. At instant t = 0.5s, the applied voltage is suddenly increased to 150V. Find the charge on the capacitor at t = 0.75 s.*

Solution. Time constant, $\lambda = RC = 20{,}000 \times 50 \times 10^{-6} = 1$ sec.

For first case. $$t = \lambda \log_e \frac{V - V_0}{V - v_c}$$

Here, $t = 0.5\text{s}$; $\lambda = 1\text{s}$; $V = 100$ volts ; $V_0 = 0$; $v_C = ?$

$$\therefore \quad 0.5 = 1 \times \log_e \frac{100 - 0}{100 - v_C} = \log_e \frac{100}{100 - v_C}$$

On solving, v_C = 39.4 volts

For second case. After 0.5 sec., the source voltage is increased to 150 V.

Now $$t = \lambda \log_e \frac{V - V_0}{V - v_C}$$

Here, $t = 0.75 - 0.5 = 0.25\text{s}$; $\lambda = 1\text{s}$; $V = 150$ volts ; $V_0 = 39.4$ volts ; $v'_C = ?$

$$\therefore \quad 0.25 = 1 \times \log_e \frac{150 - 39.4}{150 - v'_C} = \log_e \frac{110.6}{150 - v'_C}$$

On solving, v'_C = 63.6 volts

∴ Charge on capacitor = $C \times v'_C = 50 \times 10^{-6} \times 63.6 =$ **3.18×10^{-3} C**

Example 6.57. *Find how long it takes after the switch S is closed before the total current from the supply reaches 25 mA when V = 10 V, R_1 = 500Ω , R_2 = 700Ω and C = 100μF.*

Solution. When switch S is closed, the current in R_1 = 500Ω is set up instantaneously and its value is $= 10/R_1 = 10/500 = 0.02\text{A} = 20$ mA. In order to draw 25 mA current from the supply, current in capacitor circuit is $= 25 - 20 = 5$mA. Now when switch S is closed, the current in capacitor circuit is maximum and its value is $I = 10/R_2 = 10/700 = 0.0143\text{A} = 14.3$ mA and decreases at exponential rate. Our problem is to find the time t in which charging current in capacitor circuit decreases from 14.3 mA to 5 mA.

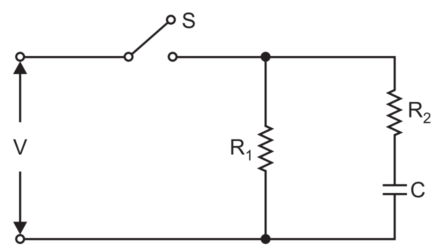


Fig. 6.55

Now, $$i = Ie^{-t/\lambda}$$

Here $i = 5$ mA ; $I = 14.3$ mA ; $\lambda = R_2C = 700 \times 100 \times 10^{-6} = 0.07$ s

$$\therefore \quad 5 = 14.3\, e^{-t/0.07}$$

On solving, t = **0.0735 s**

Example 6.58. *In an RC series circuit, R = 2MΩ, C = 5μF and applied voltage V = 100 volts. Calculate (i) initial rate of change of capacitor voltage (ii) initial rate of change of capacitor current (iii) initial rate of change of voltage across 2MΩ resistor.*

Solution. Time constant, $\lambda = RC = 2 \times 10^6 \times 5 \times 10^{-6} = 10$ seconds

(*i*) $v_C = V(1 - e^{-t/\lambda})$

$$\therefore \quad \frac{dv_C}{dt} = 0 - Ve^{-t/\lambda}\left(-\frac{1}{\lambda}\right) = \frac{V}{\lambda}e^{-t/\lambda}$$

$$\text{At } t = 0, \frac{dv_C}{dt} = \frac{V}{\lambda}e^{-0/\lambda} = \frac{V}{\lambda} = \frac{100}{10} = \mathbf{10\ V/s}$$

(*ii*) $i = I\,e^{-t/\lambda}$

$$\therefore \quad \frac{di}{dt} = Ie^{-t/\lambda}\left(-\frac{1}{\lambda}\right) = -\frac{I}{\lambda}e^{-t/\lambda}$$

$$\text{At } t = 0, \frac{di}{dt} = -\frac{I}{\lambda}e^{-0/\lambda} = -\frac{I}{\lambda} = -\frac{V/R}{\lambda} = -\frac{100/2 \times 10^6}{10} = \mathbf{-5\ \mu A/s}$$

(*iii*) $v_R = iR = (Ie^{-t/\lambda})R = \left(\frac{V}{R}e^{-t/\lambda}\right)R = Ve^{-t/\lambda}$

$$\therefore \quad \frac{dv_R}{dt} = Ve^{-t/\lambda}\left(-\frac{1}{\lambda}\right) = -\frac{V}{\lambda}e^{-t/\lambda}$$

$$\text{At } t = 0, \frac{dv_R}{dt} = -\frac{V}{\lambda}e^{-0/\lambda} = -\frac{V}{\lambda} = -\frac{100}{10} = \mathbf{-10\ V/s}$$

Example 6.59. *Calculate the values of i_2, i_3, v_2, v_3, v_C and v_L in the network shown in Fig. 6.56 at the following times :*

(*i*) *At time, t = 0 immediately after the switch S is closed.*

(*ii*) *At time, $t \to \infty$ i.e. in the steady state. All resistances are in ohms.*

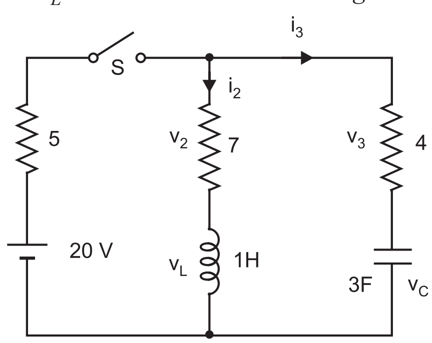


Fig. 6.56

Solution. **(*i*)** At the instant of closing the switch (*i.e.* at $t = 0$), the inductance (= 1 *H*) behaves as an open circuit so that no current flows in the coil.

$\therefore\ i_2 = \mathbf{0\ A}\ ;\ v_2 = \mathbf{0\ V}\ ;\ v_L = \mathbf{20\ V}$

At the instant of closing the switch, the capacitor behaves as a short circuit.

$$\therefore\ i_3 = \frac{20}{5+4} = \mathbf{\frac{20}{9}\ A}\ ;\ v_3 = 4 \times \frac{20}{9} = \mathbf{\frac{80}{9}\ V}\ ;\ v_C = \mathbf{0\ V}$$

(*ii*) Under steady state conditions (*i.e.* when the capacitor is fully charged), the capacitor behaves as an open circuit and the inductance (= 1*H*) as short.

$$\therefore\ i_2 = \frac{20}{5+7} = \mathbf{\frac{5}{3}\ A}\ ;\ v_2 = 7 \times \frac{5}{3} = \mathbf{\frac{35}{3}\ V}\ ;\ v_L = \mathbf{0\ V}\ ;\ i_3 = \mathbf{0 A}\ ;$$

$v_3 = \mathbf{0\ V}\ ;\ v_C = \mathbf{20\ V}$

Example 6.60. *In Fig. 6.57, the capacitor C is uncharged. Determine the final voltage on the capacitor after the switch has been in position 2 for 3s and then in position 3 for 5s.*

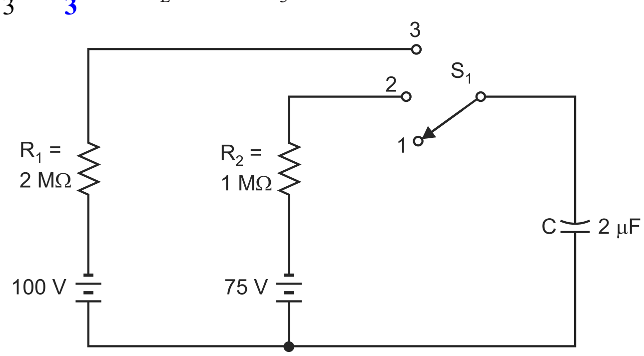


Fig. 6.57

Solution. When the switch is in position 2, the voltage v_C across the capacitor is

$$v_C = V(1 - e^{-t/\lambda})$$

Here, $V = 75$ volts ; $t = 3$s ; $\lambda = R_2C = (1 \times 10^6) \times 2 \times 10^{-6} = 2$s

$$\therefore \quad v_C = 75(1 - e^{-3/2}) = 75\ (1 - 0.223) = 58.3 \text{ V}$$

Therefore, after 2s, voltage across capacitor is 58.3 V.

When switch is in position 3, voltage v'_C across capacitor is

$$v'_C = V - (V - v_C)e^{-t/\lambda}$$

Here, $V = 100$ volts ; $t = 5$s ; $v_C = 58.3$ volts ; $\lambda = R_1C = 2 \times 10^6 \times 2 \times 10^{-6} = 4$s

$$\therefore \quad v'_C = 100 - (100 - 58.3)e^{-5/4}$$

$$= 100 - (100 - 58.3) \times 0.287 = \mathbf{88.0\ V}$$

Therefore, final voltage across the capacitor is 88.0 V.

Tutorial Problems

1. A capacitor of capacitance 12μF is allowed to discharge through its own leakage resistance and a fall of p.d. from 120 V to 100 V is recorded in a time interval of 300 seconds by an electrostatic voltmeter connected in parallel. Calculate the leakage resistance of the capacitor. **[137 MΩ]**

2. When a capacitor charged to a p.d. of 400 V is connected to a voltmeter having a resistance of 25 MΩ, the voltmeter reading is observed to have fallen to 50 V at the end of an interval of 2 minutes. Find the capacitance of the capacitor. **[2.31 μF]**

3. An 8μF capacitor is connected through a 1.5 MΩ resistance to a direct current source. After being on charge for 24 seconds, the capacitor is disconnected and discharged through a resistor. Determine what % age of the energy input from the supply is dissipated in the resistor. **[43.2%]**

4. An 8μF capacitor is connected in series with a 0.5 MΩ resistor across a 200V d.c. supply. Calculate (*i*) the time constant (*ii*) the initial charging current (*iii*) the time taken for the p.d. across the capacitor to grow to 160 V and (*iv*) the current and the p.d. across the capacitor in 4 seconds after it is connected to the supply. **[(*i*) 4s (*ii*) 0.4 mA (*iii*) 6.4s (*iv*) 0.14 mA ; 126.4 V]**

Objective Questions

1. The capacitance of a capacitor is .... relative permittivity.
(*i*) directly proportional to
(*ii*) inversely proportional to
(*iii*) independent of
(*iv*) directly proportional to square of

2. An air capacitor has the same dimensions as that of a mica capacitor. If the capacitance of mica capacitor is 6 times that of air capacitor, then relative permittivity of mica is
(*i*) 36 (*ii*) 12
(*iii*) 3 (*iv*) 6

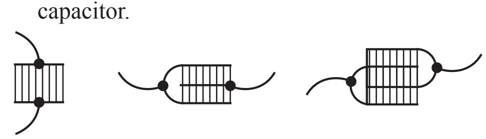
3. The most convenient way of achieving large capacitance is by using
(*i*) multiplate construction
(*ii*) decreased distance between plates
(*iii*) air as dielectric
(*iv*) dielectric of low permittivity

4. Another name for relative permittivity is
(*i*) dielectric strength
(*ii*) breakdown voltage
(*iii*) specific inductive capacity
(*iv*) potential gradient

5. A capacitor opposes
(*i*) change in current
(*ii*) change in voltage
(*iii*) both change in current and voltage
(*iv*) none of the above

6. If a multiplate capacitor has 7 plates each of area 6 cm^2, then,
(*i*) 6 capacitors will be in parallel
(*ii*) 7 capacitors will be in parallel
(*iii*) 7 capacitors will be in series
(*iv*) 6 capacitors will be in series

7. The capacitance of three-plate capacitor [See Fig. 6.58 (*ii*)] is .... that of 2-plate capacitor.



2-plate capacitor (*i*) 3-plate capacitor (*ii*) 4-plate capacitor (*iii*)

Fig. 6.58

(*i*) 3 times (*ii*) 6 times
(*iii*) 4 times (*iv*) 2 times

8. The capacitance of a 4-plate capacitor [See Fig. 6.58 (*iii*)] is .... that of 2-plate capacitor.
(*i*) 2 times (*ii*) 4 times
(*iii*) 3 times (*iv*) 6 times

9. Two capacitors of capacitances 3 μF and 6 μF in series will have a total capacitance of
(*i*) 9 μF (*ii*) 2 μF
(*iii*) 18 μF (*iv*) 24 μF

10. The capacitance of a parallel-plate capacitor does not depend upon
(*i*) area of plates
(*ii*) medium between plates
(*iii*) separation between plates
(*iv*) metal of plates

11. In order to increase the capacitance of a parallel-plate capacitor, one should introduce between the plates a sheet of
(*i*) mica (*ii*) tin
(*iii*) copper (*iv*) stainless steel

12. The capacitance of a parallel-plate capacitor depends upon
(*i*) the type of metals used
(*ii*) separation between plates
(*iii*) thickness of plates
(*iv*) potential difference between plates

13. The force between the plates of a parallel plate capacitor of capacitance C and distance of separation of plates d with a potential difference V between the plates is
(*i*) $\frac{CV^2}{2d}$ (*ii*) $\frac{C^2V^2}{2d^2}$
(*iii*) $\frac{C^2V^2}{d^2}$ (*iv*) $\frac{V^2d}{C}$

14. A parallel-plate air capacitor is immersed in oil of dielectric constant 2. The electric field between the plates is
(*i*) increased 2 times
(*ii*) increased 4 times
(*iii*) decreased 2 times
(*iv*) none of above

15. Two capacitors of capacitances C_1 and C_2 are connected in parallel. A charge Q given to them is shared. The ratio of charges Q_1/Q_2 is
(*i*) C_2 / C_1 (*ii*) C_1 / C_2
(*iii*) $C_1C_2 / 1$ (*iv*) $1 / C_1C_2$

16. The dimensional formula of capacitance is
(*i*) $M^{-1}L^{-2}T^{-4}A^2$ (*ii*) $M^{-1}L^2T^4A^2$
(*iii*) $ML^2T^{-4}A$ (*iv*) $M^{-1}L^{-2}T^4A^2$

17. Four capacitors are connected as shown in Fig. 6.59. What is the equivalent capacitance between A and B ?

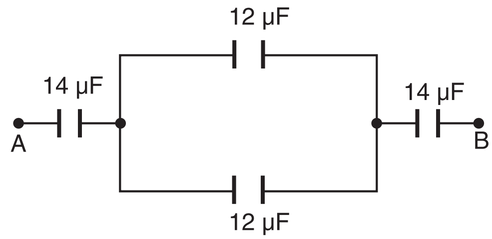


Fig. 6.59

(*i*) 36 μF (*ii*) 5·4 μF
(*iii*) 52 μF (*iv*) 11·5 μF

18. The empty space between the plates of a capacitor is filled with a liquid of dielectric constant K. The capacitance of capacitor
(*i*) increases by a factor K
(*ii*) decreases by a factor K
(*iii*) increases by a factor K^2
(*iv*) decreases by a factor K^2

19. A parallel plate capacitor is made by stacking n equally spaced plates connected alternately. If the capacitance between any two plates is C, then the resulting capacitance is
(*i*) C (*ii*) nC
(*iii*) $(n - 1) C$ (*iv*) $(n + 1) C$

20. 64 drops of radius r combine to form a bigger drop of radius R. The ratio of capacitances of bigger to smaller drop is
(*i*) 1 : 4 (*ii*) 2 : 1
(*iii*) 1 : 2 (*iv*) 4 : 1

21. Two capacitors have capacitances 25 μF when in parallel and 6 μF when in series. Their individual capacitances are
(*i*) 12 μF and 13 μF
(*ii*) 15 μF and 10 μF
(*iii*) 10 μF and 8 μF
(*iv*) none of above

22. A capacitor of 20 μF charged to 500 V is connected in parallel with another capacitor of 10 μF capacitance and charged to 200 V. The common potential is
(*i*) 200 V (*ii*) 250 V
(*iii*) 400 V (*iv*) 300 V

23. Which of the following does not change when a glass slab is introduced between the plates of a charged parallel plate capacitor?

(*i*) electric charge (*ii*) electric energy

(*iii*) capacitance

(*iv*) electric field intensity

24. A capacitor of 1 μF is charged to a potential of 50 V. It is now connected to an uncharged capacitor of capacitance 4 μF. The common potential is

(*i*) 50 V (*ii*) 20 V

(*iii*) 15 V (*iv*) 10 V

25. Three parallel plates each of area A with separation d_1 between first and second and d_2 between second and third are arranged to form a capacitor. If the dielectric constants are K_1 and K_2, the capacitance of this capacitor is

(*i*) $\dfrac{\varepsilon_0 K_1 K_2}{A(d_1 + d_2)}$ (*ii*) $\dfrac{\varepsilon_0}{A\left(\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2}\right)}$

(*iii*) $\dfrac{\varepsilon_0 A K_1 K_2}{d_1 + d_2}$ (*iv*) $\dfrac{\varepsilon_0 A}{\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2}}$

Answers

1. (*i*)	**2.** (*iv*)	**3.** (*i*)	**4.** (*iii*)	**5.** (*ii*)
6. (*i*)	**7.** (*iv*)	**8.** (*iii*)	**9.** (*ii*)	**10.** (*iv*)
11. (*i*)	**12.** (*ii*)	**13.** (*i*)	**14.** (*iii*)	**15.** (*ii*)
16. (*iv*)	**17.** (*ii*)	**18.** (*i*)	**19.** (*iii*)	**20.** (*iv*)
21. (*ii*)	**22.** (*iii*)	**23.** (*i*)	**24.** (*iv*)	**25.** (*iv*)

7

Magnetism and Electromagnetism

Introduction

In the ancient times people believed that the invisible force of magnetism was purely a magical quality and hence they showed little practical interest. However, with steadily increasing scientific knowledge over the passing centuries, magnetism assumed a larger and larger role. Today magnetism has attained a place of pride in electrical engineering. Without the aid of magnetism, it is impossible to operate such devices as electric generators, electric motors, transformers, electrical instruments etc. Without the use of magnetism, we should be deprived of such valuable assets as the radio, television, telephone, telegraph and the ignition systems of our cars, airplanes, trucks etc. In fact, electrical engineering is so much dependent on magnetism that without it a very few of our modern devices would be possible. The purpose of this chapter is to present the salient features of magnetism.

7.1. Poles of a Magnet

If we take a bar magnet and dip it into iron filings, it will be observed that the iron filings cluster about the ends of the bar magnet. The ends of the bar magnet are apparently points of maximum magnetic effect and for convenience we call them the **poles* of the magnet. A magnet has two poles *viz* north pole and south pole. In order to determine the polarity of a magnet, suspend or pivot it at the centre. The magnet will then come to rest in north-south direction. The end of the magnet pointing north is called *north pole* of the magnet while the end pointing south is called the *south pole.* The following points may be noted about the poles of a magnet :

(*i*) The poles of a magnet cannot be separated. If a bar magnet is broken into two parts, each part will be complete magnet with poles at its ends. No matter how many times a magnet is broken, each piece will contain *N*-pole at one end and *S*-pole at the other.

(*ii*) The two poles of a magnet are of equal strength. The pole strength is represented by m.

(*iii*) Like poles repel each other and unlike poles attract each other.

7.2. Laws of Magnetic Force

Charles Coulomb, a French scientist observed that when two **isolated poles are placed near each other, they experience a force. He performed a number of experiments to study the nature and magnitude of force between the magnetic poles. He summed up his conclusions into two laws, known as Coulomb's laws of magnetic force. These laws give us the magnitude and nature of magnetic force between two magnetic poles.

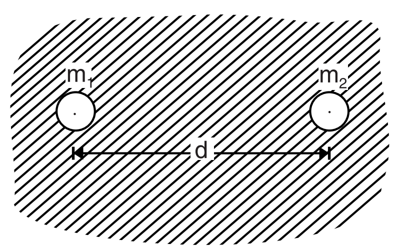


Fig. 7.1

* Magnetic poles have no physical reality, but the concept enables us to appreciate magnetic effects more easily.

** It is not possible to get an isolated pole because magnetic poles exist in pairs. However, if we take thin and long steel rods (about 50 cm long) with a small steel ball on either end and then magnetise them, *N* and *S* poles become concentrated in the steel balls. Such poles may be assumed point poles for all practical purposes.

(*i*) *Like poles repel each other while unlike poles attract each other.*

(*ii*) *The force between two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of distance between their centres.*

Consider two poles of magnetic strength m_1 and m_2 placed at a distance d apart in a medium as shown in Fig. 7.1. According to Coulomb's laws, the force between the two poles is given by ;

$$F \propto \frac{m_1 m_2}{d^2}$$

$$= K \frac{m_1 m_2}{d^2}$$

where K is a constant whose value depends upon the surrounding medium and the system of units employed. In SI units, force is measured in newtons, pole strength in *weber, distance in metres and the value of K is given by ;

$$K = \frac{1}{4\pi \mu_0 \mu_r}$$

where μ_0 = Absolute permeability of vacuum or air

μ_r = Relative permeability of the surrounding medium. For vacuum or air, its value is 1.

The value of $\mu_0 = 4\pi \times 10^{-7}$ H/m and the value of μ_r is different for different media.

$$\therefore \quad F = \frac{m_1 m_2}{4\pi \mu_0 \mu_r d^2} \text{ newtons} \qquad \text{...in a medium}$$

$$= \frac{m_1 m_2}{4\pi\mu_0 d^2} \text{ newtons} \qquad \text{...in air}$$

Unit of pole strength. By unit pole strength we mean 1 weber. It can be defined from Coulomb's laws of magnetic force. Suppose two equal point poles placed 1 m apart in *air* exert a force of 62800 newtons *i.e.*

$$m_1 = m_2 = m \; ; \quad d = 1 \text{ m} \; ; F = 62800 \text{ N}$$

$$\therefore \quad F = \frac{m_1 m_2}{4\pi\mu_0 d^2} \qquad (\because \text{ For air, } \mu_r = 1)$$

$$\text{or} \quad 62800 = \frac{m^2}{4\pi \times 4\pi \times 10^{-7} \times (1)^2}$$

$$\text{or} \quad m^2 = (62800) \times (4\pi \times 4\pi \times 10^{-7} \times 1) = 1$$

$$\therefore \quad m = \pm 1 \text{ Wb}$$

Hence a **pole of unit strength** *(i.e. 1 Wb) is that pole which when placed in air 1 m from an identical pole, repels it with a force of 62800 newtons.*

In vector form :

$$\vec{F} = \frac{m_1 m_2}{4\pi\mu_0 \mu_r d^2} \hat{d}$$

where $\hat{d}$ is a unit vector to indicate the direction of d.

Example 7.1. *Two magnetic S poles are located 5 cm apart in air. If each pole has a strength of 5 mWb, find the force of repulsion between them.*

Solution.

$$F = \frac{m_1 m_2}{4\pi\mu_0 d^2} \qquad (\because \text{ For air, } \mu_r = 1)$$

---

* The unit of magnetic flux is named after Wilhelm Weber (1804–1890), the founder of electrical system of measurements.

Here $m_1 = m_2 = 5$ mWb $= 5 \times 10^{-3}$ Wb ; $d = 5$ cm $= 0{\cdot}05$ m

$$\therefore \qquad F = \frac{(5\times10^{-3})\times(5\times10^{-3})}{4\pi\times4\pi\times10^{-7}\times(0.05)^2} = \mathbf{633\ N}$$

7.3. Magnetic Field

Just as electric field exists near a charged object, similarly magnetic field exists around a magnet. If an isolated magnetic pole is brought near a magnet, it experiences a force according to Coulomb's laws. The region near the magnet where forces act on magnetic poles is called a magnetic field. The magnetic field is strongest near the pole and goes on decreasing in strength as we move away from the magnet.

The space (or field) in which a magnetic pole experiences a force is called a **magnetic field.**

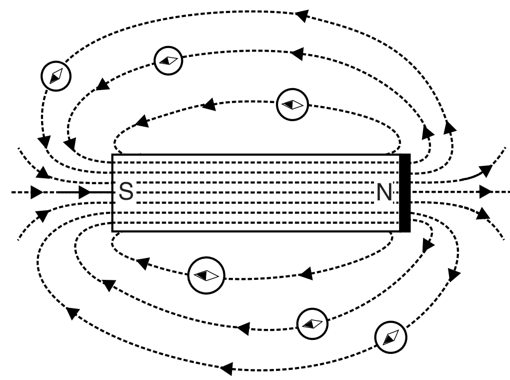


Fig. 7.2

The magnetic field around a magnet is represented by imaginary lines called *magnetic lines of force.* By convention, the direction of these lines of force at any point is the direction along which an *isolated unit *N*-pole (*i.e. N*-pole of 1 Wb) placed at that point would move or tends to move. Following this convention, it is clear that magnetic lines of force would emerge from *N*-pole of the magnet, pass through the surrounding medium and re-enter the *S*-pole. Inside the magnet, each line of force passes from *S*-pole to *N*-pole (See Fig. 7.2), thus forming a closed loop or magnetic circuit. Although magnetic lines of force have no real existence and are purely imaginary, yet they are a useful concept to describe the various magnetic effects.

Properties of magnetic lines of force. The important properties of magnetic lines of force are :

(*i*) *Each magnetic line of force forms a closed loop i.e.* outside the magnet, the direction of a magnetic line of force is from north pole to south pole and it continues through the body of the magnet to form a closed loop (See Fig. 7.2).

(*ii*) *No two magnetic lines of force intersect each other*. If two magnetic lines of force intersect, there would be two directions of magnetic field at that point which is not possible.

(*iii*) *Where the magnetic lines of force are close together, the magnetic field is strong and where they are well spaced out, the field is weak.*

(*iv*) *Magnetic lines of force contract longitudinally and widen laterally.*

(*v*) *Magnetic lines of force are always ready to pass through magnetic materials like iron in preference to pass through non-magnetic materials like air.*

It may be noted that in practice, magnetic fields are produced by (*a*) current carrying conductor or coil or (*b*) a permanent magnet. Both these means of producing magnetic fields are widely used in electrical engineering.

7.4. Magnetic Flux

The number of magnetic lines of force in a magnetic field determines the value of magnetic flux. The more the magnetic lines of force, the greater the magnetic flux and the stronger the magnetic field.

* Theoretically, it is not possible to get an isolated *N*-pole. However, a small compass needle well approximates to an isolated *N*-pole. The marked end (*N*-pole) of the compass needle indicates the direction of magnetic lines of force as shown in Fig. 7.2.

The total number of magnetic lines of force produced by a magnetic source is called **magnetic flux**. It is denoted by Greek letter ϕ (phi).

A unit *N*-pole is supposed to radiate out a flux of one weber. Therefore, the magnetic flux coming out of *N*-pole of *m* weber is

$$\phi = m \text{ Wb}$$

Now $$1\text{Wb} = 10^8 \text{ lines of force}$$

Sometimes we have to use smaller unit of magnetic flux *viz* microweber (μWb).

$$1\ \mu\text{Wb} = 10^{-6}\ \text{Wb} = 10^{-6} \times 10^8 \text{ lines} = 100 \text{ lines}$$

7.5. Magnetic Flux Density

The **magnetic flux density** *is defined as the magnetic flux passing normally per unit area i.e.*

$$\text{Magnetic flux density, } B = \frac{\phi}{A}\ \text{Wb/m}^2$$

where ϕ = flux in Wb

A = area in m^2 normal to flux

The SI unit of magnetic flux density is Wb/m^2 or *tesla. Flux density is a measure of field concentration *i.e.* amount of flux in each square metre of the field. In practice, it is much more important than the total amount of flux. Magnetic flux density is a *vector quantity*.

(*i*) When the plane of the coil is perpendicular to the flux direction [See Fig. 7.3], maximum flux will pass through the coil *i.e.*

$$\text{Maximum flux, } \phi_m = B\,A \text{ Wb}$$

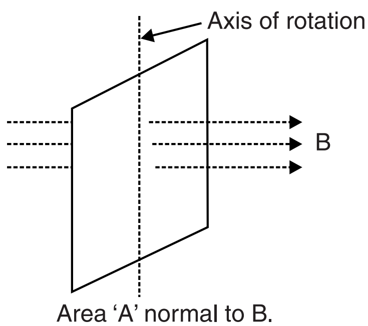


Fig. 7.3

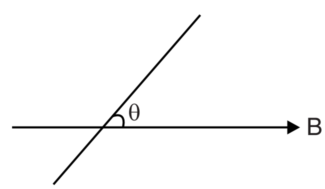


Fig. 7.4

(*ii*) When the plane of the coil is inclined at an angle θ to the flux direction [See Fig. 7.4], then flux ϕ through the coil is

$$\phi = B\,A \sin\theta \text{ Wb}$$

(*iii*) When the plane of the coil is parallel to the flux direction, $\theta = 0°$ so that no flux will pass through the coil ($\phi = BA \sin 0° = 0$).

Example 7.2. *A circular coil of 100 turns and diameter 3·18 cm is mounted on an axle through a diameter and placed in a uniform magnetic field, where the flux density is 0·01 Wb/m², in such a manner that axle is normal to the field direction. Calculate :*

(*i*) *the maximum flux through the coil and the coil position at which it occurs.*

(*ii*) *the minimum flux and the coil position at which it occurs.*

(*iii*) *the flux through the coil when its plane is inclined at 60° to the flux direction.*

* Named in honour of Nikola Tesla (1857–1943), an American electrician and inventor.

Solution. Fig. 7.5 shows the conditions of the problem.

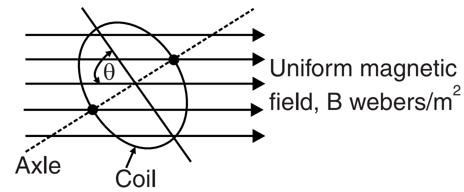


Fig. 7.5

(*i*) The maximum flux will pass through the coil when the plane of the coil is perpendicular to the flux direction.

$\therefore$ Maximum flux, $\phi_m = B \times$ Total coil area

$= (0{\cdot}01) \times \pi\, r^2$

$= 0{\cdot}01 \times \pi \times \left(\dfrac{3{\cdot}18}{2}\right)^2 \times 10^{-4}$ = **$0{\cdot}795 \times 10^{-5}$ Wb**

(*ii*) When the plane of the coil is parallel to the flux direction, no flux will pass through the coil. This is the minimum flux coil position and the minimum flux is **zero.**

(*iii*) When the plane of the coil is inclined at an angle θ to the flux direction, the flux ϕ through the coil is

$$\phi = B\,A \sin\theta = (B\,A)\sin\theta = (0{\cdot}795 \times 10^{-5}) \times \sin 60^\circ$$

$$= \mathbf{0{\cdot}69 \times 10^{-5}\ Wb}$$

Example 7.3. *The total flux emitted from the pole of a bar magnet is 2×10^{-4} Wb (See Fig. 7.6).*

(i) If the magnet has a cross-sectional area of 1 cm^2, determine the flux density within the magnet.

(ii) If the flux spreads out so that a certain distance from the pole, it is distributed over an area of 2 cm by 2 cm, find the flux density at that point.

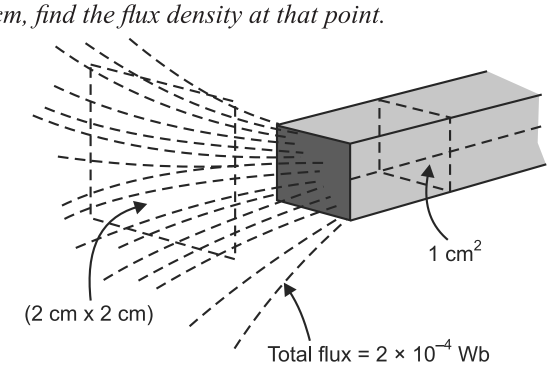


Fig. 7.6

Solution. (*i*) *Flux density within magnet.* $\phi = 2 \times 10^{-4}$ Wb ; $A = 1\ cm^2 = 1 \times 10^{-4}\ m^2$

$\therefore$ Flux density, $B = \dfrac{\phi}{A} = \dfrac{2 \times 10^{-4}}{1 \times 10^{-4}}$ = **2 Wb/m²**

(*ii*) *Flux density away from the pole.*

$$\phi = 2 \times 10^{-4}\ \text{Wb} \ ; \ A = 2 \times 2 = 4\ cm^2 = 4 \times 10^{-4}\ m^2$$

$\therefore$ Flux density, $B = \dfrac{\phi}{A} = \dfrac{2 \times 10^{-4}}{4 \times 10^{-4}}$ = **0.5 Wb/m²**

Example 7.4. *Flux density in the air gap between N and S poles is 2.5 Wb/m². The poles are circular with a diameter of 5.6 cm. Calculate the total flux crossing the air gap.*

Solution. $B = 2.5\ Wb/m^2$; Area of each pole, $A = \pi r^2 = \pi \times (5.6/2)^2 = 24.63\ cm^2 = 24.63 \times 10^{-4}\ m^2$

$\therefore$ Flux crossing the air gap is given by ;

$$\phi = B \times A = 2.5 \times 24.63 \times 10^{-4} = 6.16 \times 10^{-3}\ \text{Wb} = \mathbf{6.16\ mWb}$$

7.6. Magnetic Intensity or Magnetising Force (H)

Magnetic intensity (or field strength) at a point in a magnetic field is the force acting on a unit *N*-pole (*i.e.*, *N*-pole of 1 Wb) placed at that point. Clearly, the unit of *H* will be N/Wb.

Suppose it is desired to find the magnetic intensity at a point *P* situated at a distance *d* metres from a pole of strength *m* webers (See Fig. 7.7). Imagine a unit north pole (*i.e. N*-pole of 1 Wb) is placed at *P*. Then, by definition, magnetic intensity at *P* is the force acting on the unit *N*-pole placed at *P i.e.*

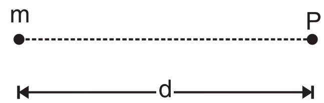


Fig. 7.7

Magnetic intensity at *P*, *H* = Force on unit *N*-pole placed at *P*

or $$H = \frac{m \times 1}{4\pi\mu_0 d^2} \text{ N/Wb} \qquad [\because \mu_r = 1 \text{ for air}]$$

or $$H = \frac{m}{4\pi\mu_0 d^2} \text{ N/Wb}$$

The reader may note the following points carefully :

(*i*) Magnetic intensity is a vector quantity, possessing both magnitude and direction. In vector form, it is given by ;
$$\vec{H} = \frac{m}{4\pi\mu_0 d^2}\hat{d}$$

(*ii*) If a pole of *m* Wb is placed in a uniform magnetic field of strength *H* newtons/Wb, then force acting on the pole, *F* = *m H* newtons.

7.7. Magnetic Potential

The **magnetic potential** *at any point in the magnetic field is measured by the work done in moving a unit N-pole (i.e. 1 Wb strength) from infinity to that point against the magnetic force.*

Consider a magnetic pole of strength *m* webers placed in a medium of relative permeability μ_r. At a point at a distance *x* metres from it, the force on unit *N*-pole is

$$F = \frac{m}{4\pi\mu_0\mu_r x^2}$$

If the unit *N*–pole is moved towards *m* through a small distance *dx*, then work done is

$$dW = \frac{m}{4\pi\mu_0\mu_r x^2} \times (-dx)$$

The negative sign is taken because *dx* is considered in the negative direction of *x*.

Therefore, the total work done (*W*) in bringing a unit *N*-pole from infinity to any point which is *d* metres from *m* is

$$W = \int_{x=\infty}^{x=d} -\frac{m}{4\pi\mu_0\mu_r x^2}dx = \frac{m}{4\pi\mu_0\mu_r d} \text{ J/Wb}$$

By definition, *W* = Magnetic potential *V* at that point.

∴ Magnetic potential, $V = \dfrac{m}{4\pi\mu_0\mu_r d}$ J/Wb

Note that magnetic potential is a scalar quantity.

7.8. Absolute and Relative Permeability

Permeability of a material means its conductivity for magnetic flux. The greater the permeability of a material, the greater is its conductivity for magnetic flux and *vice-versa.* Air or vacuum is the poorest conductor of magnetic flux. The absolute (or actual) permeability *μ_0 (Greek letter "*mu*")

* The absolute (or actual) permeability of all non-magnetic materials is also $4\pi \times 10^{-7}$ H/m.

of air or vacuum is $4\pi \times 10^{-7}$ H/m. The absolute (or actual) permeability μ of magnetic materials is much greater than μ_0. The ratio μ/μ_0 is called the relative permeability of the material and is denoted by μ_r *i.e.*

$$\mu_r = \frac{\mu}{\mu_0}$$

where μ = absolute (or actual) permeability of the material

μ_0 = absolute permeability of air or vacuum

μ_r = relative permeability of the material

Obviously, the relative permeability for air or vacuum would be $\mu_0/\mu_0 = 1$. The value of μ_r for all non-magnetic materials is also 1. However, relative permeability of magnetic materials is very high. For example, soft iron (*i.e.* pure iron) has a relative permeability of 8,000 whereas its value for permalloy (an alloy containing 22% iron and 78% nickel) is as high as 50,000.

Concept of relative permeability. The relative permeability of a material is a measure of the relative ease with which that material conducts magnetic flux compared with the conduction of flux in air. Fig. 7.8 illustrates the concept of relative permeability. In Fig. 7.8 (*i*), the magnetic flux passes between the poles of a magnet in air. Consider a soft iron ring (μ_r = 8,000) placed between the same poles as shown in Fig. 7.8 (*ii*). Since soft iron is a very good conductor of magnetic flux, the flux follows a path entirely within the soft iron itself. The flux density in the soft iron is much greater than it is in air. In fact, flux density in soft iron will be 8,000 times (*i.e.* μ_r times) the flux density in air.

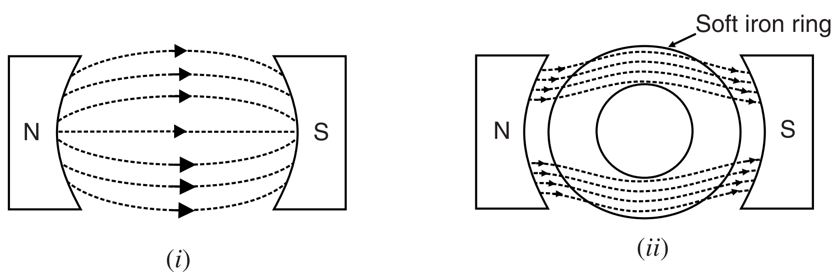


Fig. 7.8

Due to high relative permeability of magnetic materials (*e.g.* iron, steel and other magnetic alloys), they are widely used for the cores of all electromagnetic equipment.

7.9. Relation Between B and H

The flux density *B* produced in a material is directly proportional to the applied magnetising force *H*. In other words, the greater the magnetising force, the greater is the flux density and *vice-versa i.e.*

$$B \propto H$$

or

$$\frac{B}{H} = \text{Constant} = \mu$$

The ratio *B*/*H* in a material is always constant and is equal to the absolute permeability μ ($= \mu_0 \mu_r$) of the material. This relation gives yet another definition of absolute permeability of a material.

Obviously, $B = \mu_0 \mu_r H$ *...in a medium*

$= \mu_0 H$ *...in air*

Suppose a magnetising force *H* produces a flux density B_0 in air. Clearly, $B_0 = \mu_0 H$. If air is replaced by some other material (relative permeability μ_r) and the same magnetising force *H* is applied, then flux density in the material will be $B_{mat} = \mu_0 \mu_r H$.

$$\therefore \quad \frac{B_{mat}}{B_0} = \frac{\mu_0 \mu_r H}{\mu_0 H} = \mu_r$$

Hence **relative permeability** *of a material is equal to the ratio of flux density produced in that material to the flux density produced in air by the same magnetising force.*

Thus when we say that μ_r of soft iron is 8000, it means that for the same magnetising force, flux density in soft iron will be 8000 times its value in air. In other words, for the same cross-sectional area and H, the magnetic lines of force will be 8000 times greater in soft iron than in air.

7.10. Important Terms

(*i*) Intensity of magnetisation (I). When a magnetic material is subjected to a magnetising force, the material is magnetised. Intensity of magnetisation is a measure of the extent to which the material is magnetised and depends upon the nature of the material. It is defined as under :

The intensity of magnetisation of a magnetic material is defined as the magnetic moment developed per unit volume of the material.

$$\therefore \quad \text{Intensity of magnetisation, } I = \frac{M}{V}$$

where M = magnetic moment developed in the material

V = volume of the material

If m is the pole strength developed, a is the area of X-section of the material and $2l$ is the magnetic length, then,

$$I = \frac{m \times 2l}{a \times 2l} = \frac{m}{a}$$

Hence intensity of magnetisation of a material may be defined as the pole strength developed per unit area of cross-section of the material.

$$I = \frac{\text{magnetic moment}}{\text{volume}} = \frac{\text{Amp. (metre)}^2}{(\text{metre})^3} = \text{A m}^{-1}$$

$\therefore$ SI units of I are A m^{-1}.

(*ii*) Magnetic susceptibility (χ_m). The magnetic susceptibility of a material indicates how easily the material can be magnetised. It is defined as under :

The magnetic susceptibility of a material is defined as the ratio of intensity of magnetisation (I) developed in the material to the applied magnetising force (H). It is represented by χ_m (Greek alphabet Chi).

$$\therefore \quad \text{Magnetic susceptibility, } \chi_m = \frac{I}{H}$$

The unit of I is the same as that of H so that χ_m is a number. Since I is magnetic moment per unit volume, χ_m is also called *volume susceptibility* of the material.

7.11. Relation Between μ_r and χ_m

Consider a current carrying toroid having core material of relative permeability μ_r. The total magnetic flux density in the material is given by ;

$$B = B_0 + B_M$$

where B_0 = magnetic flux density due to current in the coils.

B_M = magnetic flux density due to the magnetisation of the material.

Now $B_0 = \mu_0 H$ and $B_M = \mu_0 I$*

* We can imagine that B_M is produced by a fictitious current I_M in the coils.

$$\therefore \quad B_M = \mu_0\, n\, I_M = \mu_0 \frac{N}{l} I_M = \mu_0 \frac{N I_M A}{Al} = \mu_0 I$$

where $N I_M A$ = magnetic dipole moment developed and $A\,l$ is the volume of the specimen.

$\therefore \quad B = \mu_0 H + \mu_0 I = \mu_0 (H + I)$

or $\quad B = \mu_0 (H + I)$

Now $\quad \chi_m = \dfrac{I}{H}$ so that $I = \chi_m H$

$\therefore \quad B = \mu_0 (H + \chi_m H) = \mu_0 H (1 + \chi_m)$

But $\quad B = \mu H = \mu_0 \mu_r H$

$\therefore \quad \mu_0 \mu_r H = \mu_0 H (1 + \chi_m)$

or $\quad \mu_r = 1 + \chi_m$

Example 7.5. *The magnetic moment of a magnet (10 cm × 2 cm × 1 cm) is 1 Am². What is the intensity of magnetisation ?*

Solution. Volume of the magnet, $V = 10 \times 2 \times 1 = 20 \text{ cm}^3 = 20 \times 10^{-6} \text{ m}^3$

Magnetic moment of magnet, $M = 1 \text{ Am}^2$

$\therefore$ Intensity of magnetisation, $I = \dfrac{M}{V} = \dfrac{1}{20 \times 10^{-6}} = \mathbf{5 \times 10^4 \text{ A/m}}$

Example 7.6. *A specimen of iron is uniformly magnetised by a magnetising field of 500 A/m. If the magnetic induction in the specimen is 0·2 Wb/m², find the relative permeability and susceptibility.*

Solution. $\quad B = \mu H = \mu_0 \mu_r H$

$\therefore$ Relative permeability of the specimen is

$$\mu_r = \frac{B}{\mu_0 H} = \frac{0 \cdot 2}{4\pi \times 10^{-7} \times 500} = \mathbf{318 \cdot 5}$$

Now $\quad \mu_r = 1 + \chi_m$

$\therefore$ Susceptibility, $\chi_m = \mu_r - 1 = 318 \cdot 5 - 1 = \mathbf{317 \cdot 5}$

7.12. Refraction of Magnetic Flux

When magnetic flux passes from one medium to another of different permeabilities, the magnetic flux gets refracted at the boundary of the two media [See Fig. 7.9]. Under this condition, the following two conditions exist at the boundary (called **boundary conditions**) :

(*i*) The normal components of magnetic flux density are equal *i.e.*

$$B_{1n} = B_{2n}$$

(*ii*) The tangential components of magnetic field intensities are equal *i.e.*

$$H_{1t} = H_{2t}$$

As proved in Art. 5.25, in a similar way, it can be proved that :

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_1}{\mu_2}$$

This relation is called law of magnetic flux refraction.

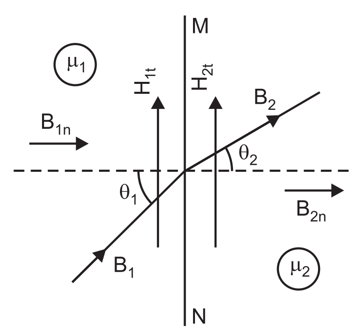


Fig. 7.9

7.13. Molecular Theory of Magnetism

The molecular theory of magnetism was proposed by Weber in 1852 and modified by Ewing in 1890. *According to this theory, every molecule of a magnetic substance (whether magnetised or not) is a complete magnet in itself having a north pole and a south pole of equal strength.*

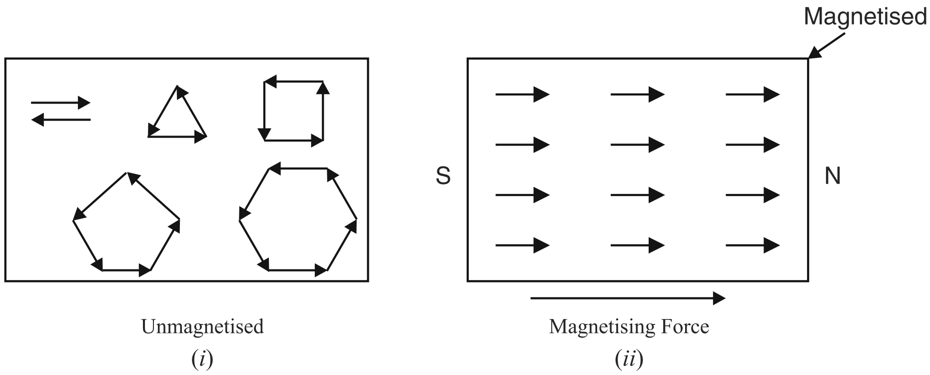


Fig. 7.10

(*i*) In an unmagnetised substance, the molecular magnets are randomly oriented and form closed chains as shown in Fig. 7.10 (*i*). The north pole of one molecular magnet cancels the effect of the south pole of the other so that the substance does not show any net magnetism.

(*ii*) When a magnetising force is applied to the substance (*e.g.* by rubbing a magnet or by passing electric current through a wire wound over it), the molecular magnets are turned and tend to align in the same direction with *N*-pole of one molecular magnet facing the *S*-pole of other as shown in Fig. 7.10 (*ii*). The result is that magnetic fields of the molecular magnets aid each other and two definite *N* and *S* poles are developed near the ends of the specimen ; the strength of the two poles being equal. Hence the substance gets magnetised.

(*iii*) The extent of magnetisation of the substance depends upon the extent of alignment of molecular magnets. When all the molecular magnets are fully aligned, the substance is said to be *saturated* with magnetism.

(*iv*) When a magnetised substance (or a magnet) is heated, the molecular magnets acquire kinetic energy and some of them go back to the closed chain arrangement. For this reason, a magnet loses some magnetism on heating.

Curie temperature. The magnetisation of a magnetised substance decreases with the increase in temperature. It is because when a magnetised substance is heated, random thermal motions tend to destroy the alignment of molecular magnets. As a results, the magnetisation of the substance decreases. At sufficiently high temperature, the magnetic property of the substance suddenly disappears and the substance loses magnetism.

The temperature at which a magnetised substance loses its magnetism is called **Curie temperature** *or* **Curie point** *of the substance*.

For example, the curie temperature of iron is 770°C. Therefore, if the temperature of the magnetised iron piece becomes 770°C, it will loose its magnetism. Similarly, the curie temperatures of nickel and cobalt are 358°C and 1121°C respectively.

7.14. Modern View about Magnetism

According to modern view, the magnetic properties of a substance are attributed to the motions of electrons (orbital and spin) in the atoms. We know that an atom consists of central nucleus with electrons revolving around the nucleus in different orbits. This motion of electrons is called *orbital motion* [See Fig. 7.11 (*i*)]. The electrons also rotate around their own axis. This motion of electrons is called *spin motion* [See Fig. 7.11 (*ii*)]. Due to these two motions, each atom is equivalent to a current loop *i.e.* each atom behaves as a magnetic dipole.

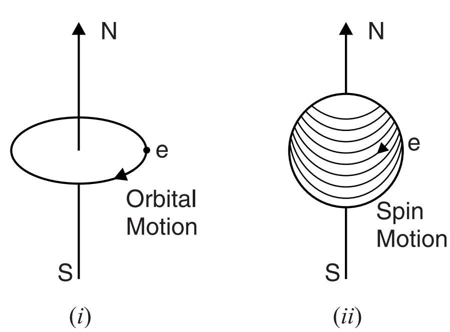


Fig. 7.11

(*i*) In the unmagnetised substances, the magnetic dipoles are randomly oriented so that magnetic fields mutually cancel. When the substance is magnetised, the magnetic dipoles are aligned in the same direction. Hence the substance shows net magnetism.

(*ii*) Since the revolving and spinning electrons in each atom cause magnetism, no substance is non-magnetic.

(*iii*) It is important to note that spinning motion of electrons in particular is responsible for magnetism of a substance.

7.15. Magnetic Materials

We can classify materials into three categories *viz*. **diamagnetic**, **paramagnetic** and **ferromagnetic**. The behaviour of these three classes of substances is different in an external magnetic field.

(*i*) When a diamagnetic substance (*e.g*. copper, zinc, bismuth etc.) is placed in a magnetic field, the substance if *feebly* magnetised in a direction opposite to that of the applied field. Therefore, a diamagnetic substance is feebly repelled by a strong magnet.

(*ii*) When a paramagnetic substance (*e.g*. aluminium, antimony etc.) is placed in a magnetic field, the substance is *feebly* magnetised in the direction of the applied field. Therefore, a paramagnetic substance is feebly attracted by a strong magnet.

(*iii*) When a ferromagnetic substance (*e.g*. iron, nickel, cobalt etc.) is placed in a magnetic field, the substance is *strongly* magnetised in the direction of the applied field. Therefore, a ferromagnetic substance is strongly attracted by a magnet.

Note that diamagnetism and paramagnetism are weak forms of magnetism. However, ferromagnetic substances exhibit very strong magnetic effects.

7.16. Electromagnetism

The first discovery of any connection between electricity and magnetism was made by Hans Christian Oersted, a Danish physicist in 1819. On one occasion at the end of his lecture, he inadvertently placed a wire carrying current parallel to a compass needle. To his surprise, needle was deflected. Upon reversing the current in the wire, the needle deflected in the opposite direction.

Oersted found that the compass deflection was due to a magnetic field established around the current carrying conductor. This accidental discovery was the first evidence of a long suspected link between electricity and magnetism. The production of magnetism from electricity (which we call electromagnetism) has opened a new era. The operation of all electrical machinery is due to the applications of magnetic effects of electric current in one form or the other.

7.17. Magnetic Effect of Electric Current

When an electric current flows through a conductor, magnetic field is set up all along the length of the conductor. Fig. 7.12 shows the magnetic field produced by the current flowing in a straight wire. The magnetic lines of force are in the form of *concentric circles around the conductor.

The direction of lines of force depends upon the direction of current and may be determined by **right-hand rule.** *Hold the conductor in the right-hand with the thumb pointing in the direction of current (See Fig. 7.12). Then the fingers will point in the direction of magnetic field around the*

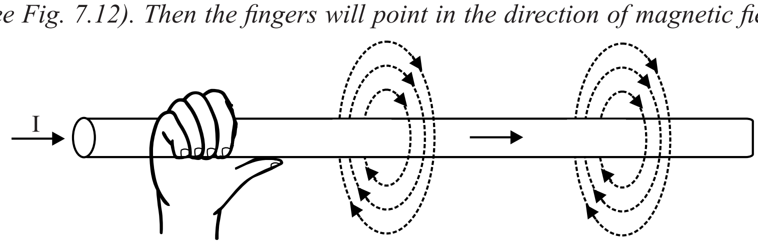


Fig. 7.12

* This can be readily established with a compass needle. If a compass needle is placed near the conductor and it is progressively moved in the direction of its north pole, it will be seen that the paths of magnetic lines of force are concentric circles.

conductor. Applying this rule to Fig. 7.12, it is clear that when viewed from left-hand side, the direction of magnetic lines of force will be clockwise.

The following points may be noted about the magnetic effect of electric current :

(*i*) The greater the current through the conductor, the stronger the magnetic field and *vice-versa.*

(*ii*) The magnetic field neart the conductor is stronger and becomes weaker and weaker as we move away from the conductor.

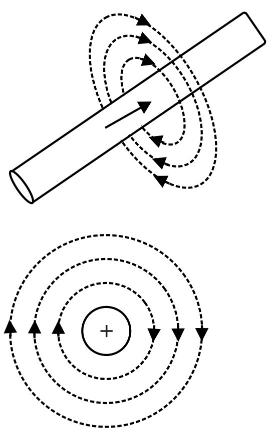
(*iii*) The magnetic lines of force around the conductor will be either clockwise or anticlockwise, depending upon the direction of current. One may use *right-hand rule* to determine the direction of magnetic field around the conductor.

(*iv*) The shape of the magnetic field depends upon the shape of the conductor.

7.18. Typical Electromagnetic Fields

The current carrying conductor may be in the form of a straight wire, a loop of one turn, a coil of several turns. The shape of the magnetic field would eventually depend upon the shape of conductor. By way of illustration, we shall discuss magnetic fields produced by some current carrying conductor arrangements.

(*i*) Long straight conductor. If a straight long conductor is carrying current, the magnetic lines of force will be concentric circles around the conductor as shown in Fig. 7.13. In Fig. 7.13 (*i*), the conductor is carrying current into the plane of paper (usually represented by a cross inside the *X*-section of the conductor). Applying right-hand rule, it is clear that direction of magnetic lines of force will be clockwise. In Fig. 7.13 (*ii*), the conductor is carrying current out of the plane of paper (usually represented by a dot inside the *X*-section of the conductor). Clearly, the direction of magnetic lines of force will be anticlockwise.


(*i*)

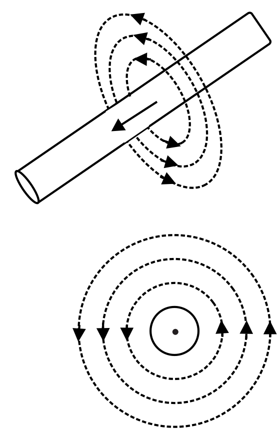

(*ii*)

Fig. 7.13

(*ii*) Parallel conductors. Consider two parallel conductors *A* and *B* placed close together and carrying current into the plane of the paper as shown in Fig. 7.14 (*i*). The magnetic lines of force will be clockwise around each conductor. In the space between *A* and *B*, the lines of force due to the conductors are in the opposite direction and hence they cancel out each other. This results in a field that entirely surrounds the conductors as shown in Fig. 7.14 (*ii*).

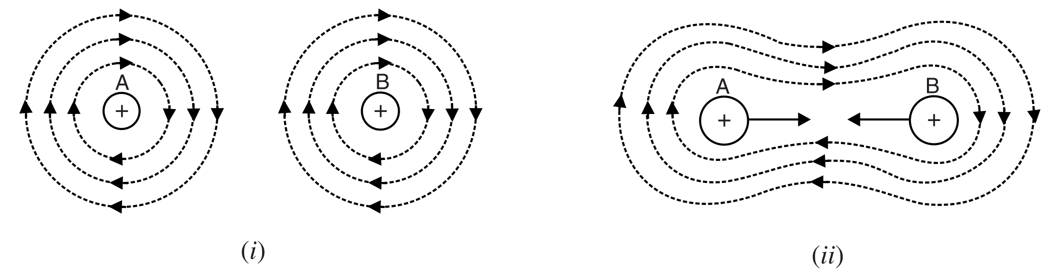


Fig. 7.14

If there are several parallel conductors placed close together and carrying current into the plane of the paper as shown in Fig. 7.15 (*i*), the magnetic field envelops the conductors. If the direction of current is reversed, the direction of field is also reversed as shown in Fig. 7.15 (*ii*).

Fig. 7.15

(*iii*) Coil of several turns. Consider a coil of several truns wound on a hollow tube or iron bar as shown in Fig. 7.16 (*i*). Such an arrangement is called a *solenoid. Suppose current flows through the coil in the direction shown. In the upper part of each turn (at points 1, 2, 3, 4 and 5), the current is flowing into the plane of the paper and in the lower part of each turn (at points 6, 7, 8 and 9), current is flowing out of the plane of paper. This is shown in the cross-sectional view of the coil in Fig. 7.16 (*ii*). It is clear that a clockwise field entirely surrounds the conductors 1, 2, 3, 4 and 5 while anticlockwise field completely envelops the conductors 6, 7, 8 and 9. As a result, the field becomes similar to that of a bar magnet with flux emerging from one end of the coil and entering the other.

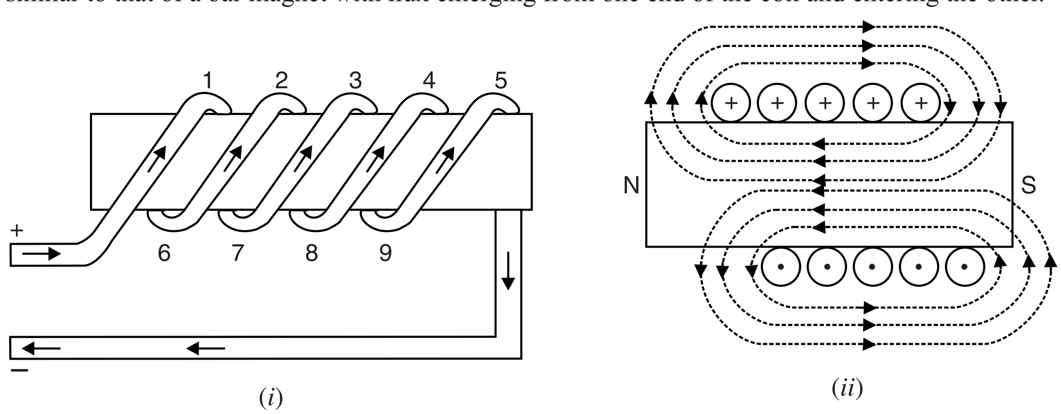


Fig. 7.16

It is clear that left-hand face of the coil [See Fig. 7.16 (*ii*)] becomes a *N*-pole and right-hand face *S*-pole. The magnetic polarity of the coil can also be determined by the **right-hand rule for coil.** *Grasp the whole coil with right-hand so that the fingers are curled in the direction of current. Then thumb stretched parallel to the axis of the coil will point towards the N-pole end of the coil (See*

* Solenoid is Greek word meaning "tube-like."

Fig. 7.17). It may be noted that both right-hand rules (for a conductor and for a coil) discussed so far can be applied in reverse. If we know the direction of magnetic field encircling a conductor or the magnetic polarity of a coil, we can determine the directiron of current by applying appropriate right-hand rule.

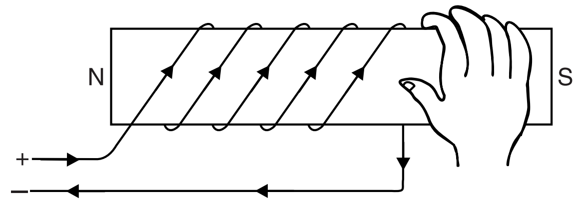


Fig. 7.17

7.19. Magnetising Force (H) Produced by Electric Current

The magnetic flux (ϕ) can be produced by (*i*) current-carrying conductor or coil or (*ii*) a permanent magnet. We generally use current-carrying conductor or coil to produce magnetic flux. Experiments show that magnetic flux (ϕ) produced by a current-carrying coil is directly proportional to the product of number of turns (N) of the coil and electric current (I) which the coil carries. The quantity NI is called *magnetomotive force (m.m.f)* and is measured in *ampere-turns* (AT) or **amperes* (A)

$$\therefore \quad \text{m.m.f.} = NI \text{ Ampere-turns (AT)}$$

Just as e.m.f. (electromotive force) is required to produce electric current in an electric circuit, similarly, m.m.f. is required to produce magnetic flux in a **magnetic circuit. The greater the m.m.f., the greater is the magnetic flux produced in the magnetic circuit and *vice-versa*.

The **magnetising force** (H) *produced by an electric current is defined as the m.m.f. set up per unit length of the magnetic circuit i.e.*

$$\text{Magnetising force, } H = \frac{NI}{l} \text{ AT/m}$$

where NI = m.m.f. (AT)

l = length of magnetic circuit in m

Different current-carrying conductor arrangements produce different magnetising force. Magnetising force (H) is known by different names such as *magnetic field strength, magnetic intensity* and *magnetic potential gradient*.

Example 7.7. *A toroidal coil has a magnetic path length of 33 cm and a magnetic field strength of 650 A/m. The coil current is 250 mA. Determine the number of coil turns.*

Solution. $$H = \frac{NI}{l}$$

Here, H = 650 A/m ; I = 250 mA = 0.25A ; l = 33 cm = 0.33 m

$$\therefore \quad 650 = \frac{N \times 0.25}{0.33} \quad \text{or} \quad N = \frac{650 \times 0.33}{0.25} = \textbf{858 turns}$$

Example 7.8. *Determine the m.m.f. required to generate a total flux of 100µWb in an air gap 0.2 cm long. The cross-sectional area of the air gap is 25 cm^2.*

Solution. $\phi = 100\ \mu\text{Wb} = 100 \times 10^{-6}$ Wb ; $l = 0.2 \times 10^{-2}$ m ; $A = 25 \times 10^{-4}$ m^2

$$\text{Flux density, } B = \frac{\phi}{A} = \frac{100 \times 10^{-6}}{25 \times 10^{-4}} = 4 \times 10^{-2} \text{ Wb/m}^2$$

$$\text{Magnetising force, } H = \frac{B}{\mu_0} = \frac{4 \times 10^{-2}}{4\pi \times 10^{-7}} = 3.18 \times 10^4 \text{ AT/m}$$

* Since number of turns is dimensionless, ampere turns and amperes are the same as for as dimensions are concerned.

** The closed path followed by magnetic flux is called a magnetic circuit; just as the closed path followed by electric current is called an electric circuit.

Now, $$H = \frac{\text{m.m.f.}}{l}$$

∴ $$\text{m.m.f.} = H \times l = 3.18 \times 10^4 \times 0.2 \times 10^{-2} = \mathbf{63.7\ AT}$$

An air gap is a necessity in a rotating machine such as a motor or a generator. It provides mechanical clearance between the fixed and moving parts. Air gaps are also used to prevent saturation in some magnetic devices.

7.20. Force on Current-carrying Conductor Placed in a Magnetic Field

When a current-carrying conductor is placed at right angles to a magnetic field, it is found that the conductor experiences a force which acts in a direction perpendicular to the direction of both the field and the current. Consider a straight current-carrying conductor placed in a uniform magnetic field as shown in Fig. 7.18.

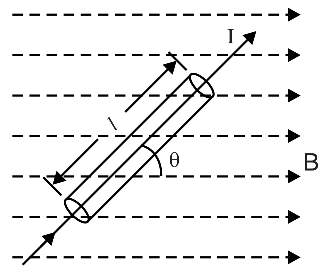


Fig. 7.18

Let B = magnetic flux density in Wb/m^2

I = current through the conductor in amperes

l = effective length of the conductor in metres *i.e.* the length of the conductor lying in the magnetic field

θ = angle which the conductor makes with the direction of the magnetic field

It has been found experimentally that the magnitude of force (F) acting on the conductor is directly proportional to the magnitudes of flux density (B), current (I), length (l) and sin θ *i.e.*

$$F \propto BIl \sin\theta \text{ newtons}$$

or $$F = k\,BIl \sin\theta$$

where k is a constant of proportionality. Now SI unit of B is so defined that value of k becomes unity.

∴ $$F = BIl \sin\theta$$

By experiment, it is found that the direction of the force is always perpendicular to the plane containing the conductor and the magnetic field.

Both magnitude and direction of the force will be given by the following vector equation :

$$\vec{F} = I\left(\vec{l} \times \vec{B}\right)$$

The direction of this force is perpendicular to the plane containing $\vec{l}$ and $\vec{B}$. It can be found by using right-hand rule for cross product.

Special Cases. $$F = BIl \sin\theta$$

(i) When θ = 0° or 180° ; sin θ = 0

∴ $$F = BIl \times 0 = 0$$

Therefore, if a current-carrying conductor is placed parallel to the direction of magnetic field, the conductor will experience no force.

(ii) When θ = 90° ; sin θ = 1

∴ $$F = BIl \quad \text{...maximum value}$$

Therefore, a current-carrying conductor will experience a maximum force when it is placed at right angles to the direction of the magnetic field.

Direction of force. The direction of force $\vec{F}$ is always perpendicular to the plane containing $\vec{l}$ and $\vec{B}$ and can be determined by *right-hand rule for cross product* stated below :

Orient your right hand so that your outstretched fingers point along the direction of the conventional current; the orientation should be such that when you bend your fingers, they must *point along the direction of the magnetic field* $(\vec{B})$. *Then your extended thumb will point in the direction of the force on the conductor.*

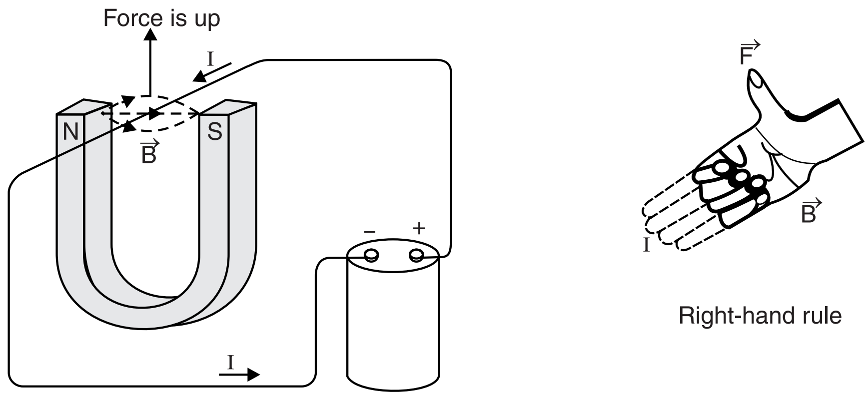


Fig. 7.19

Thus applying right-hand rule for cross product to Fig. 7.19, it is clear that magnetic force on the conductor is vertically upward.

Note. If the current-carrying conductor is at right angles to the magnetic field, the direction of force can also be found by Fleming's Left-hand rule stated below :

Fleming's Left-hand Rule. Stretch out the First finger, seCond finger and thuMb of your left hand so that they are at right angles to one another. If the first finger points in the direction of magnetic field (North to South) and second finger (*i.e.* middle finger) points towards the direction of current, then the thumb will point in the direction of motion of the conductor.

Example 7.9. *A conductor of length 100 cm and carrying 100 A is situated in and at right angles to a uniform magnetic field produced by the pole core of an electrical machine. If the pole core has a circular cross-section of 120 mm diameter and the total flux in the core is 16 mWb, find (i) the mechanical force on the conductor and (ii) power required to move the conductor at a speed of 10 m/s in a plane at right angles to the magnetic field.*

Solution. In this case, mechanical force acts on the conductor.

X-sectional area of pole core $= (\pi/4) \times (0{\cdot}12)^2 = 0{\cdot}0113 \text{ m}^2$

$$\text{Flux density of field, } B = \frac{\text{Flux}}{\text{Pole core area}} = \frac{16\times10^{-3}}{0.0113} = 1{\cdot}416 \text{ Wb/m}^2$$

(*i*) Force on the conductor is given by ;

$$F = B\,I\,l = 1{\cdot}416 \times 100 \times 1 = \mathbf{141{\cdot}6\ N}$$

(*ii*) Power required $=$ Force $\times$ distance/second

$= 141{\cdot}6 \times 10 =$ **1416 watts**

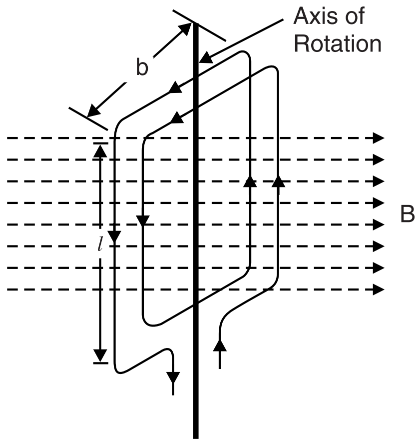
Example 7.10. *The plane of a rectangular coil makes an angle of 60° with the direction of a uniform magnetic field of flux density* 4×10^{-2} *Wb/m*^{2}*. The coil is of 20 turns, measuring 20 cm by 10 cm, and carries a current of 0·5 A. Calculate the torque acting on the coil.*

Solution. Consider a rectangular coil, measuring b by l, of N turns carrying a current of I amperes and placed in a uniform magnetic field of B Wb/m^2. The coil is pivoted about the mid points of the sides b and is free to rotate about an axis in its own plane ; this axis being at right angles to the field density B [See Fig. 7.20 (*i*)]. When current is passed through the coil, forces acting on the coil sides are :

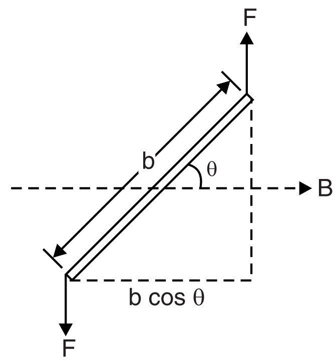
(*i*) The forces developed on each half of coil sides b are equal and produce torques of opposing sense. They, therefore, cancel each other.

(*ii*) The coil sides l always remain at right angles to the field as the coil rotates. The force F acting on each of the coil sides l gives rise to a torque as shown in Fig. 7.20 (*ii*).

Force on each coil side l, $F = B\,I\,l\,N$ newtons



(*i*)



(*ii*)

Fig. 7.20

The perpendicular distance between the lines of action of the two forces is $b \cos\theta$.

$\therefore$ Torque, $T = F \times b\cos\theta = (B\,I\,l\,N)\,b\cos\theta$

or $T = BINA\cos\theta$ newton-metre

where A $(= l \times b)$ is the area of the coil. By an extension of this reasoning, the expression may be proved quite generally for a coil of area A and of any shape.

In the given problem, the data is

$$B = 4 \times 10^{-2}\ \text{Wb/m}^2\ ;\ A = 20 \times 10 = 200\ \text{cm}^2 = 2 \times 10^{-2}\ \text{m}^2\ ;\ I = 0{\cdot}5\ \text{A}\ ;\ \theta = 60°\ ;\ N = 20$$

$\therefore$ Torque, $T = (4 \times 10^{-2}) \times (0{\cdot}5) \times (20) \times (2 \times 10^{-2}) \times \cos 60° =$ **4×10^{-3} Nm**

Tutorial Problems

1. A straight conductor 0·4m long carries a current of 12 A and lies at right angles to a uniform field of 2·5 Wb/m^2. Find the mechanical force on the conductor when (*i*) it lies in the given position (*ii*) it lies in a position such that it is inclined at an angle of 30º to the direction of field. **[(*i*) 12 N (*ii*) 6 N]**

2. A conductor of length 100 cm and carrying 100 A is situated in and at right angles to a uniform magnetic field of strength 1 Wb/m^2. Calculate the force and power required to move the conductor at a speed of 100 m/s in a plane at right angles to the magnetic field. **[100 N ; 1000 watts]**

3. A d.c. motor consists of an armature winding of 400 turns (equivalent to 800 conductors). The effective lengths of conductor in the field is 160 mm and the conductors are situated at a radius of 100 mm from the centre of the motor shaft. The magnetic flux density is 0·6 Wb/m^2 and a current of 25 A flows through the winding. Calculate the torque available at the motor shaft. **[192 Nm]**

4. A d.c. motor is to provide a torque of 540 Nm. The armature winding consists of 600 turns (equivalent to 1200 conductors). The effective length of a conductor in the field is 250 mm and the conductors are situated at a radius of 150 mm from the centre of the motor shaft. Each conductor carries a current of 10 A. Calculate the flux density which must be provided by the radial field in which the conductors lie. **[1·2 Wb/m^2]**

7.21. Ampere's Work Law or Ampere's Circuital Law

The magnetising force (H) at any point in an electromagnetic field is the force experienced by a unit N-pole placed at that point. If the unit N-pole is made to move in a complete path around N current-carrying conductors, then work is done provided the unit N-pole is moved in opposition to the lines of force. Conversely, if the unit N-pole moves in the direction of magnetic field, then work will be done by the magnetic force on whatever force is restraining the movement of the pole. In either case, unit N-pole makes one complete loop around the N conductors. The work done is given by Ampere's work law stated below :

The work done on or by a unit N-pole in moving once around any complete path is equal to the product of current and number of turns enclosed by that path i.e.

$$^{*}\oint \vec{H}_r \cdot \vec{dr} = N I$$

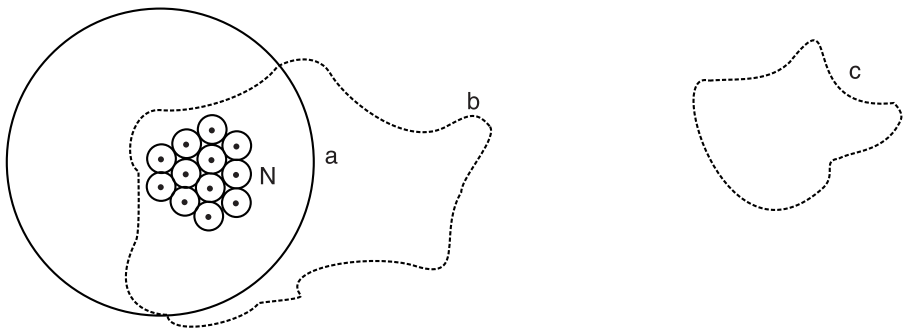


Fig. 7.21

where $\vec{H}_r$ is the magnetising force at a distance r. The circle around the integral sign indicates that the integral is around a complete path.

The work law is applicable regardless of the shape of complete path. Thus in Fig. 7.21, paths '*a*' and '*b*' completely enclose N conductors. If a unit N-pole is moved once around any of these complete paths, the work done in each case will be equal to $N I$. Although path '*c*' is a complete path, it fails to enclose any current carrying conductor. Hence, no work is done in moving a unit N-pole around such a path.

Note. The work law is applicable for all magnetic fields, irrespective of the shape of the field or of the materials which may be present.

7.22. Applications of Ampere's Work Law

Ampere's work law can be used to find magnetising force (H) in simple conductor arrangements. We shall discuss two cases by way of illustration.

1. Magnetising force around a long straight conductor. Consider the case of a long straight conductor carrying a current of I amperes as shown in Fig. 7.22. The conductor will set up magnetic lines of force which encircle it. Consider a circular path of radius r metres. By symmetry, the field intensity H on all the points of this circular path will be the same. If a unit N-pole is moved once around this circular path, then work done is = $2\pi rH$. By work law, this must be equal to the product of current and number of turns enclosed by this circular path.

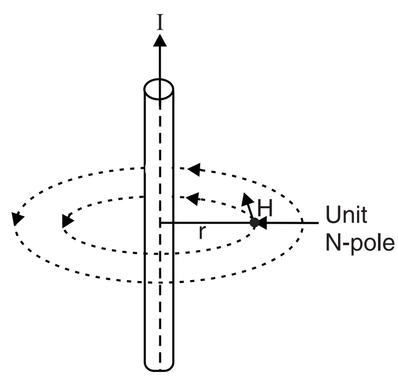


Fig. 7.22

$$\therefore \qquad 2\pi\, r\, H = I \qquad (\because N = 1)$$

$$\text{or} \qquad H = \frac{I}{2\pi r}$$

* This law can also be stated as *the closed line integral of magnetic field intensity (H) is equal to the encloed ampere-turns that produce the magnetic field.*

Note that magnetic lines of force encircle the conductor like concentric circles. The direction of magnetic lines of force can be determined by right-hand rule.

If there had been N turns enclosed by the path, then,

$$H = \frac{NI}{2\pi r}$$

$$\text{Flux density, } B = \mu_0 H = \frac{\mu_0 NI}{2\pi r} \quad \text{... in air}$$

$$= \frac{\mu_0 \mu_r NI}{2\pi r} \quad \text{... in a medium}$$

The following points may be noted carefully :

(*i*) If we choose a complete path for which r is smaller, H on that circle will be large. However, $2\pi r H$ will be still equal to NI.

(*ii*) Inspection of above expression reveals that H can also be expressed in ampere-turns per metre (AT/m).

(*iii*) It is reminded that the quantity NI (*i.e.* product of the number of turns in a winding and the current flowing through it) is called **magnetomotive force** (m.m.f.).

$$\text{m.m.f.} = NI \text{ ampere-turns}$$

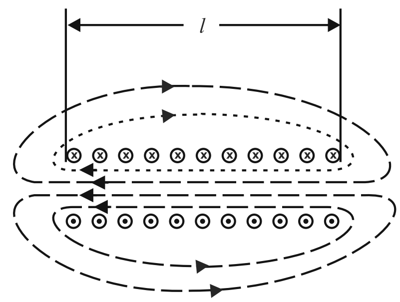


Fig. 7.23

2. Magnetising force due to long solenoid. Consider a long solenoid of length l and wound uniformly with N turns (See Fig. 7.23). The length of the solenoid is much greater than the breadth, say 10 times greater. The following assumptions are permissible :

(*i*) The field strength external to the solenoid is effectively zero.

(*ii*) The field strength inside the solenoid is uniform.

Suppose the current I flowing through the solenoid produces uniform magnetic field strength H within the solenoid. Applying work law to any closed path say dotted one shown in Fig. 7.23,

Total work done around closed path = Ampere turns linked

Since there is negligible field strength (H) outside the solenoid, the only work done will be in travelling length l within the solenoid.

$$\therefore \quad H \times l = NI$$

$$\text{or} \quad H = \frac{NI}{l} \text{ AT/m or A/m}$$

$$\text{Incidentally, } B = \mu_0 H = \frac{\mu_0 NI}{l} \text{ Wb/m}^2 \quad \text{...in air}$$

$$= \mu_0 \mu_r H = \frac{\mu_0 \mu_r NI}{l} \text{ Wb/m}^2 \quad \text{...in a medium}$$

It is reminded that the magnetic field strength (H) is a vector quantity since it has magnitude and direction.

Example 7.11. *An air-cored toroidal coil shown in Fig. 7.24 has 3000 turns and carries a current of 0.1A. The cross-sectional area of the coil is 4 cm² and the length of the magnetic circuit is 15 cm. Determine the magnetic field strength, the flux density and the total flux within the coil.*

Solution. $N = 3000$ turns ; $I = 0.1$ A ; $A = 4 \times 10^{-4}$ m² ; $l = 15 \times 10^{-2}$ m

$$\text{Magnetic field strength, } H = \frac{NI}{l} = \frac{3000 \times 0.1}{15 \times 10^{-2}}$$

$$= \mathbf{2000\ AT/m}$$

Flux density, $B = \mu_0 H = 4\pi \times 10^{-7} \times 2000$

$= \mathbf{2.5 \times 10^{-3}\ Wb/m^2}$

Total flux, $\phi = B \times A = 2.5 \times 10^{-3} \times 4 \times 10^{-4}$

$= 1 \times 10^{-6}$ Wb = **1 μWb**

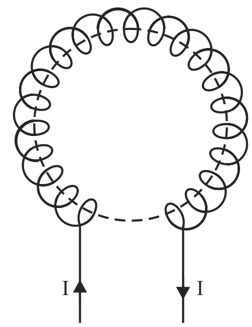


Fig. 7.24

Example 7.12. *An air-cored solenoid has length of 15 cm and inside diameter of 1.5 cm. If the coil has 900 turns, determine the total flux within the solenoid when the coil current is 100 mA.*

Solution. For a solenoid, the length of the magnetic circuit, l = coil length = 15×10^{-2} m.

$$D = 1.5 \times 10^{-2} \text{ m} \;;\; N = 900 \text{ turns} \;;\; I = 100 \times 10^{-3} \text{ A}$$

$$\therefore \quad \text{m.m.f.} = NI = 900 \times 100 \times 10^{-3} = 90 \text{ AT}$$

$$\text{Magnetising force, } H = \frac{\text{m.m.f.}}{l} = \frac{90}{15 \times 10^{-2}} = 600 \text{ AT/m}$$

$$\text{Magnetic flux density, } B = \mu_0 H = 4\pi \times 10^{-7} \times 600 = 24\pi \times 10^{-5} \text{ Wb/m}^2$$

$$\therefore \quad \text{Total flux, } \phi = BA = 24\pi \times 10^{-5} \times \pi \frac{D^2}{4}$$

$$= 24\pi \times 10^{-5} \times \pi \times \frac{(1.5 \times 10^{-2})^2}{4} = \mathbf{1.33 \times 10^{-7}\ Wb}$$

If the solenoid were iron-cored, the magnitude of the magnetic flux within the solenoid would have been much greater than the calculated value because of very high relative permeability of iron.

7.23. Biot-Savart Law

A conductor carrying current I produces a magnetic field around it. We can consider the current carrying conductor to be consisting of infinitesimally small **current elements* $I\,\vec{dl}$; each current element contributing to magnetic field. *Biot-Savart law gives us expression for the magnetic field at a point due to a current element*.

Consider a current element $I\,\vec{dl}$ of a conductor XY carrying current I [See Fig. 7.25]. Let P be the point where the magnetic field $\vec{dB}$ due to the current element is to be found. Suppose $\vec{r}$ is the position vector of point P from the current element $I\,\vec{dl}$ and θ is the angle between $\vec{dl}$ and $\vec{r}$.

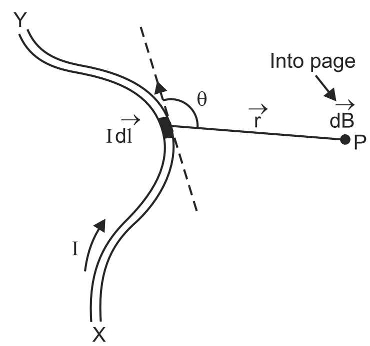


Fig. 7.25

According to Biot-Savart law, the magnitude dB of magnetic field at point P due to the current element depends upon the following factors :

(*i*) $dB \propto I$ (*ii*) $dB \propto dl$ (*iii*) $dB \propto 1/r^2$ (*iv*) $dB \propto \sin\theta$

Combining all these four factors, we get,

$$dB \propto \frac{I\,dl\sin\theta}{r^2}$$

or

$$dB = K\frac{I\,dl\sin\theta}{r^2}$$

* The current element $I\,\vec{dl}$ is a vector. Its direction is tangent to the element and acts in the direction of flow of current in the conductor.

where K is a constant of proportionality. Its value depends on the medium in which the conductor is situated and the system of units adopted.

$$\text{For free space and SI units, K} = \frac{\mu_0}{4\pi} = 10^{-7} \text{ Tm A}^{-1}$$

$$\text{where } \mu_0 = \text{Absolute permeability of free space} = 4\pi \times 10^{-7} \text{ Tm A}^{-1}$$

$$\therefore \quad dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin\theta}{r^2} \quad ...(i)$$

Eq. (i) is known as *Biot-Savart law* and gives the magnitude of the magnetic field at a point due to small current element $I\vec{dl}$. **Note that Biot-Savart law holds strictly for steady currents.**

In vector form.

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I(\vec{dl} \times \vec{r})}{r^3} \quad ...(ii)$$

The Biot-Savart law is analogous to Coulomb's law. Just as the charge q is the source of electrostatic field, similarly, the source of magnetic field is the current element $I\vec{dl}$.

Direction of $\vec{B}$.

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I(\vec{dl} \times \vec{r})}{r^3}$$

The direction of $\vec{dB}$ is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$. By right-hand rule for the cross product, the field is directed *inward*.

Magnetic field due to whole conductor. Eq. (ii) gives the magnetic field at point P due to a small current element $I\vec{dl}$. The total magnetic field at point P is found by summing (integrating) over all current elements.

$$\vec{B} = \int \vec{dB} = \int \frac{\mu_0}{4\pi} \frac{I(\vec{dl} \times \vec{r})}{r^3}$$

where the integration is taken over the entire conductor in which current I flows.

Special cases.

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{r^2}$$

(i) When $\theta = 0°$ *i.e.,* point P lies on the axis of the conductor, then,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin 0°}{r^2} = 0$$

Hence, there is no magnetic field at any point on the thin linear current carrying conductor.

(ii) When $\theta = 90°$ *i.e.,* point P lies at a perpendicular position *w.r.t.* current element, then,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin 90°}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl}{r^2} \quad \textit{... Maximum value}$$

Hence magnetic field due to a current element is maximum in a plane passing through the element and perpendicular to its axis.

(iii) When $\theta = 0°$ or $180°$, $dB = 0$ *...Minimum value*

Important points about Biot-Savart law. This law has the following salient features :

(i) Biot-Savart law is valid for symmetrical current distributions.

(ii) Biot-Savart law cannot be proved experimentally because it is not possible to have a current carrying conductor of length dl.

(iii) Like Coulomb's law in electrostatics, Biot-Savart law also obeys inverse square law.

(iv) The direction of $\vec{dB}$ is perpendicular to the plane containing $I\vec{dl}$ and $\vec{r}$.

7.24. Applications of Biot-Savart Law

Biot-Savart law is very useful in determining magnetic flux density B and hence magnetising force H (= B/μ_0) due to current-carrying conductor arrangements. We shall discuss the following cases by way of illustration.

(*i*) Magnetic flux density at the centre of current-carrying circular coil.

(*ii*) Magnetic flux density due to straight conductor carrying current.

(*iii*) Magnetic flux density on the axis of circular coil carrying current.

7.25. Magnetic Field at the Centre of Current-Carrying Circular Coil

This is a practical case because the operation of many devices depends upon the magnetic field produced by the current-carrying circular coil. Consider a circular coil of radius r and carrying current I in the direction shown in Fig. 7.26. Suppose the loop lies in the plane of paper. It is desired to find the magnetic field at the centre O of the coil. Suppose the entire circular coil is divided into a large number of current elements, each of length dl. According to Biot-Savart law, the magnetic field $\vec{dB}$ at the centre O of the coil due to current element $I\vec{dl}$ is given by ;

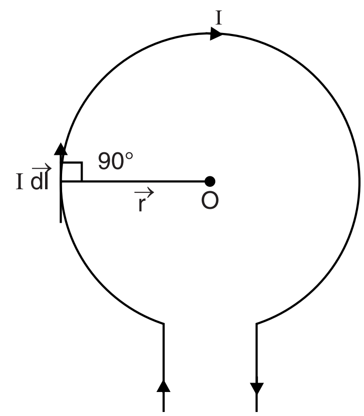


Fig. 7.26

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I(\vec{dl}\times\vec{r})}{r^3}$$

where $\vec{r}$ is the position vector of point O from the current element.

The magnitude of $\vec{dB}$ at the centre O is

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,r\sin\theta}{r^3} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} \qquad ...(i)$$

The direction of $\vec{dB}$ is perpendicular to the plane of the coil and is directed inwards. Since each current element contributes to the magnetic field in the same direction, the total magnetic field B at the centre O can be found by integrating eq. (*i*) around the loop *i.e.*

$$B = \int dB = \int \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

For each current element, angle between $\vec{dl}$ and $\vec{r}$ is 90°. Also distance of each current element from the centre O is r.

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I\sin 90°}{r^2}\int dl$$

Now, $$\int dl = \text{Total length of the coil} = 2\pi r$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I}{r^2}(2\pi r)$$

or $$B = \frac{\mu_0 I}{2r}$$

Also, $$H = \frac{B}{\mu_0} = \frac{1}{\mu_0}\times\frac{\mu_0 I}{2r} = \frac{I}{2r}$$

If the coil has N turns, each carrying current in the same direction, then contributions of all the turns are added up. Therefore, the magnetic field at the centre of the coil is greatly increased and is given by ;

$$B = \frac{\mu_0 N I}{2r}$$

Also, $$H = \frac{B}{\mu_0} = \frac{NI}{2r}$$

Direction of $\vec{B}$. The direction of magnetic field $\vec{B}$ is perpendicular to the plane of the coil and for Fig. 7.27, magnetic field inside the coil is directed inwards. The magnetic lines of force are circular near the wire but practically straight near the centre of the coil. In the middle *M* of the coil, the magnetic field is uniform for a short distance on either side. The direction of magnetic field at the centre of a current-carrying circular coil can be determined by *right-hand palm rule*.

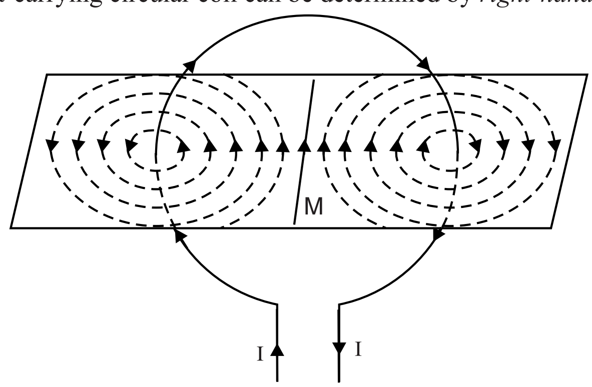


Fig. 7.27

Right-hand palm rule. *Orient the thumb of your right hand perpendicular to the grip of the fingers such that curvature of the fingers points in the direction of current in the circular coil. Then thumb will point in the direction of the magnetic field near the centre of the circular coil.*

7.26. Magnetic Field Due to Straight Conductor Carrying Current

Consider a straight conductor *XY* carrying current *I* in the direction shown in Fig. 7.28. It is desired to find the magnetic field at point *P* located at a perpendicular distance *a* from the conductor (*i.e.* $PQ = a$). Consider a small current element of length *dl*. Let $\vec{r}$ be the position vector of point *P* from the current element and θ be the angle between $\vec{dl}$ and $\vec{r}$ (*i.e.*, $\angle POQ = \theta$). Let us further assume that $QO = l$.

According to Biot-Savart law, the magnitude of magnetic field $\vec{dB}$ at point *P* due to the considered current element is given by ;

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2} \quad ...(i)$$

To get the total magnetic field *B*, we must integrate eq. (*i*) over the whole conductor. As we move along the conductor, the quantities *dl*, θ and *r* change. The integration becomes much easier if we express everything in terms of angle φ shown in Fig. 7.28.

In the right angled triangle *PQO*, $\theta = 90° - \phi$.

∴ $$\sin\theta = \sin(90° - \phi) = \cos\phi$$

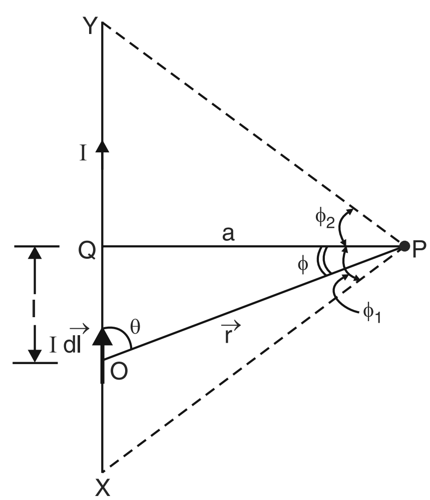


Fig. 7.28

Also, $\cos\phi = \dfrac{a}{r}$ or $r = \dfrac{a}{\cos\phi}$

Further, $\tan\phi = \dfrac{l}{a}$ or $l = a\tan\phi$

or $dl = a\sec^2\phi\, d\phi$

Putting the values of sin θ, *dl* and *r* in eq. (*i*), we have,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,(a\sec^2\phi\, d\phi)\cos\phi}{(a/\cos\phi)^2}$$

or
$$dB = \frac{\mu_0}{4\pi}\frac{I\cos\phi\, d\phi}{a} \quad ...(ii)$$

The direction of $\overrightarrow{dB}$ is perpendicular to the plane of the conductor and is directed inwards (Right-hand grip rule, See section 7.17). Since each current element contributes to the magnetic field in the same direction, the total magnetic field *B* at point *P* can be found by integrating eq. (*ii*) over the length *XY i.e.*

$$B = \int_{-\phi_1}^{\phi_2} dB = \frac{\mu_0}{4\pi}\frac{I}{a}\int_{-\phi_1}^{\phi_2}\cos\phi\, d\phi$$

$$= \frac{\mu_0 I}{4\pi a}[\sin\phi]_{-\phi_1}^{\phi_2} = \frac{\mu_0 I}{4\pi a}(\sin\phi_2 + \sin\phi_1)$$

∴
$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_2 + \sin\phi_1) \quad ...(iii)$$

Also,
$$H = \frac{B}{\mu_0} = \frac{I}{4\pi a}(\sin\phi_2 + \sin\phi_1)$$

Eq. (*iii*) gives the value of *B* at point *P* due to a conductor of *finite length*.

Special cases. We shall discuss a few important cases.

(*i*) *When the conductor XY is of infinite length and point P lies at the centre of the conductor.*

In this case, $\phi_1 = \phi_2 = 90° = \pi/2$.

∴
$$B = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin\pi/2 + \sin\pi/2)$$

or
$$B = \frac{\mu_0}{4\pi}\frac{2I}{a}$$

Also,
$$H = \frac{B}{\mu_0} = \frac{1}{4\pi}\cdot\frac{2I}{a} = \frac{I}{2\pi a}$$

(*ii*) *When conductor XY is of infinite length but point P lies near one end Y* (*or X*). In this case, $\phi_1 = 90°$ and $\phi_2 = 0°$.

∴
$$B = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin 90° + \sin 0°)$$

or
$$B = \frac{\mu_0 I}{4\pi a}$$

Note that it is half of that for case (*i*).

Also,
$$H = \frac{B}{\mu_0} = \frac{I}{4\pi a}$$

(*iii*) *If the length of the conductor is finite (say l) and point P lies on the right bisector of the conductor*. In this case, $\phi_1 = \phi_2 = \phi$.

Now, $$\sin\phi = \frac{l/2}{\sqrt{a^2+(l/2)^2}} = \frac{l}{\sqrt{4a^2+l^2}}$$

∴ $$B = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin\phi+\sin\phi) = \frac{\mu_0}{4\pi}\frac{2I}{a}\sin\phi$$

or $$B = \frac{\mu_0}{4\pi}\frac{2I}{a}\frac{l}{\sqrt{4a^2+l^2}}$$

Also $$H = \frac{B}{\mu_0} = \frac{1}{4\pi}\cdot\frac{2I}{a}\frac{l}{\sqrt{4a^2+l^2}}$$

Direction of $\vec{B}$. For a long straight conductor carrying current, the magnetic lines of force are concentric circles with conductor as the centre ; the direction of magnetic lines of force can be found by *right-hand grip rule*. The direction of $\vec{B}$ at any point is along the tangent to field line at that point as shown in Fig. 7.29.

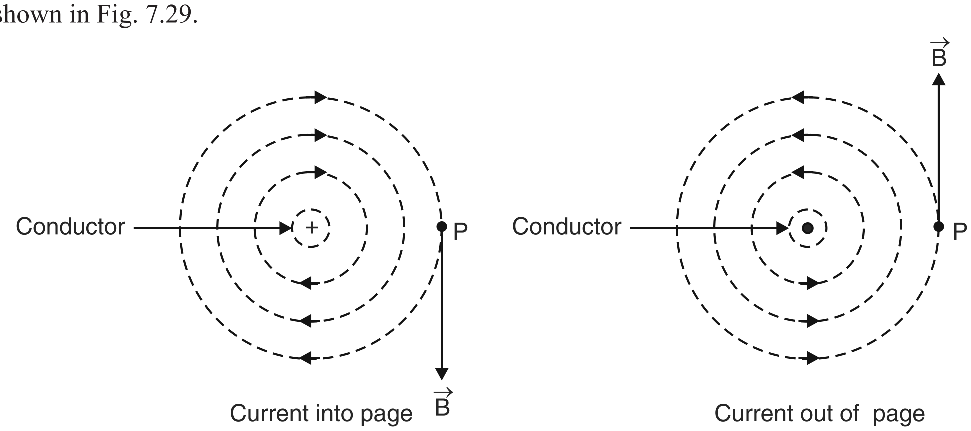


Fig. 7.29

Note. For a given current, $B \propto 1/a$ so that graph between B and a is a hyperbola.

7.27. Magnetic Field on the Axis of Circular Coil Carrying Current

Consider a circular coil of radius a, centre O and carrying a current I in the direction shown in Fig. 7.30. Let the plane of the coil be perpendicular to the plane of the paper. It is desired to find the magnetic field at a point P on the axis of the coil such that $OP = x$.

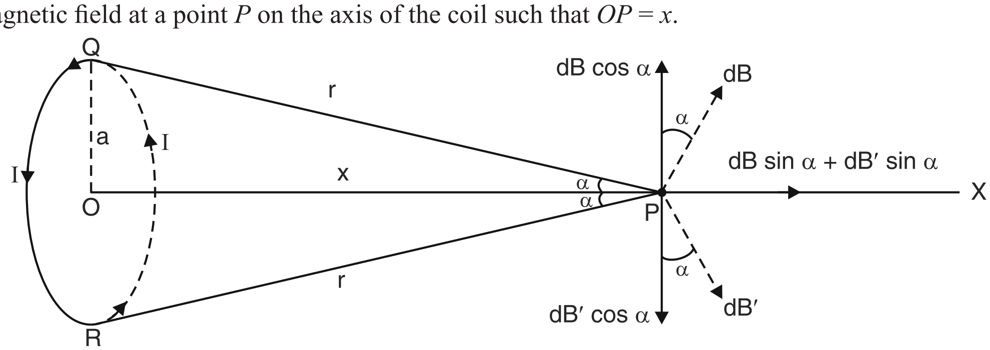


Fig. 7.30

Consider two small current elements, each of length dl, located diametrically opposite to each other at Q and R. Suppose the distance of Q or R from P is r *i.e.* $PQ = PR = r$.

$$\therefore \qquad r = \sqrt{a^2 + x^2}$$

According to Biot-Savart law, the magnitude of magnetic field at P due to current element at Q is given by ;

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl \sin 90°}{r^2} \qquad (\because \theta^* = 90°)$$

or

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{(a^2 + x^2)} \qquad ...(i)$$

The magnetic field at P due to current element at Q is in the plane of paper and at right angles to $\vec{r}$ and in the direction shown.

Similarly, magnitude of magnetic field at point P due to current element at R is given by ;

$$dB' = \frac{\mu_0}{4\pi}\frac{I\,dl}{(a^2 + x^2)} \qquad ...(ii)$$

It also acts in the plane of paper and at right angles to $\vec{r}$ but in opposite direction to dB.

From eqs. (i) and (ii), $dB = dB'$.

It is clear that vertical components ($dB \cos \alpha$ and $dB' \cos \alpha$) will be equal and opposite and thus cancel each other. However, components along the axis of the coil ($dB \sin \alpha$ and $dB' \sin \alpha$) are added and act in the direction PX. This is true for all the diametrically opposite elements of the circular coil. Therefore, when we sum up the contributions of all the current elements of the coil, the perpendicular components will cancel. Hence the resultant magnetic field at point P is the vector sum of all the components $dB \sin \alpha$ over the entire coil.

$$\therefore \qquad B = \int dB \sin\alpha = \int \frac{\mu_0 I\,dl \sin\alpha}{4\pi(a^2 + x^2)} = \frac{\mu_0 I \sin\alpha}{4\pi(a^2 + x^2)}\int dl$$

Now

$$\sin\alpha = \frac{a}{\sqrt{a^2 + x^2}} \quad \text{and} \quad \int dl = 2\pi a$$

$$\therefore \qquad B = \frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}} \quad \text{along } PX \qquad ...(iii)$$

Also,

$$H = \frac{B}{\mu_0} = \frac{Ia^2}{2(a^2 + x^2)^{3/2}}$$

If the circular coil has N turns, then,

$$B = \frac{\mu_0 NI a^2}{2(a^2 + x^2)^{3/2}} \quad \text{along } PX \qquad ...(iv)$$

Also,

$$H = \frac{B}{\mu_0} = \frac{NIa^2}{2(a^2 + x^2)^{3/2}}$$

Different Cases. Let us discuss some special cases.

(*i*) *When point P is at the centre of the coil*. In this case, $x = 0$ and eq. (iv) becomes :

$$B = \frac{\mu_0 NIa^2}{2a^3} = \frac{\mu_0 NI}{2a}$$

This is the expression for the magnetic field at the centre of a current-carrying circular coil already derived in section 7.25. Note that the value of magnetic field is maximum at the centre of the coil.

* The radius vector QP of each current element is perpendicular to it so that $\theta = 90°$ in each case.

Also, $$H = \frac{B}{\mu_0} = \frac{NI}{2a}$$

(*ii*) *When point P is far away from the centre of coil*. In this case, $x >> a$ so that $a^2 + x^2 \simeq x^2$.

$\therefore$ $$B = \frac{\mu_0 \, NI \, a^2}{2x^3}$$

Also, $$H = \frac{B}{\mu_0} = \frac{NIa^2}{2x^3}$$

The magnetic field is directed along the axis of the coil and falls off as the cube of the distance from the coil.

Direction of $\vec{B}$. The magnetic field at the centre of a coil carrying current is along the axis of the coil as shown in Fig. 7.31. The direction of magnetic field can be determined by using **right-hand fist rule.** *Hold the axis of the coil in the right-hand fist in such a way that fingers point in the direction of current in the coil. Then outstretched thumb gives the direction of the magnetic field.* Applying this rule to Fig. 7.31, it is clear that direction of magnetic lines of force is along the axis of the coil as shown.

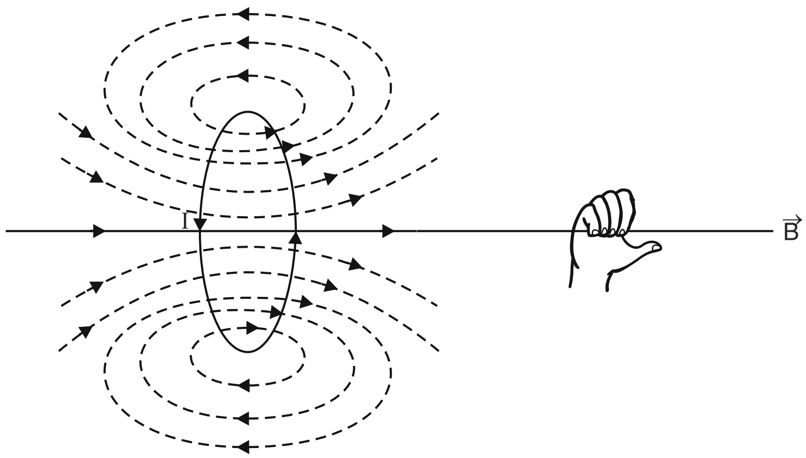


Fig. 7.31

Example 7.13. *How far from a compass should a wire carrying 1 A current be located if its magnetic field at the compass is not to exceed 1 percent of the *earth's magnetic field (3×10^{-5} Wb/m^2) ?*

Solution. Let r metre be the desired distance.

Required flux density at the compass is

$$B = 1\% \text{ of Earth's flux density}$$

$$= 0{\cdot}01 \times 3 \times 10^{-5} = 3 \times 10^{-7} \text{ Wb/m}^2$$

Required magnetising force at the compass is

$$H = \frac{B}{\mu_0} = \frac{3\times10^{-7}}{4\pi\times10^{-7}} = 0{\cdot}239 \text{ AT/m}$$

Now, $$H = \frac{I}{2\pi r} \quad \therefore \quad r = \frac{I}{2\pi H} = \frac{1}{2\pi\times0{\cdot}239} = \mathbf{0{\cdot}67 \ m}$$

Example 7.14. *A horizontal overhead power line carries a current of 50 A in west to east direction. What is the magnitude and direction of the magnetic field 1·5 m below the line ?*

* **Earth's magnetic field.** The earth itself has a weak magnetic field. This is believed to be caused by electric currents circulating within its core. The currents are probably generated by convection in the liquid core maintained by radioactive heating of the earth's interior.

Solution. Figure 7.32 shows the conditions of the problem. The magnitude of magnetic field at point P, 1·5 m below the wire is given by ;

$$B = \frac{\mu_0}{2\pi}\frac{I}{a}$$

Here, $\mu_0 = 4\pi \times 10^{-7}$ H/m ; $I = 50$ A ; $a = 1{\cdot}5$ m

$$\therefore \quad B = \frac{4\pi\times10^{-7}}{2\pi}\times\frac{50}{1\cdot5} = \mathbf{6.7 \times 10^{-6}\ T}$$

According to right-hand grip rule, the direction of magnetic field below the wire is from south to north.

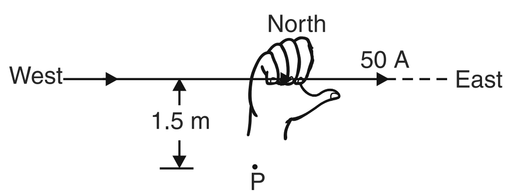


Fig. 7.32

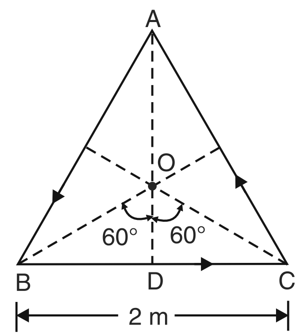


Fig. 7.33

Example 7.15. *A current of 1 A is flowing in the sides of an equilateral triangle of side 2 m. Find the magnetic field at the centroid of the triangle.*

Solution. It is clear that all the three sides of the triangle will produce magnetic field at the centroid O in the same direction. Therefore, total magnetic field at O is = 3 × magnetic field due to one side.

Magnetic field at O due to side BC [See Fig. 7.33] is

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin\phi_1 + \sin\phi_2)$$

Here, $I = 1$ A ; $\phi_1 = \phi_2 = 60°$; $\mu_0 = 4\pi \times 10^{-7}$ H/m

$$a = OD = \frac{BD}{\tan 60°} = \frac{BC/2}{\tan 60°} = \frac{2/2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad B_1 = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{1}{1/\sqrt{3}}(\sin 60° + \sin 60°)$$

$$= 10^{-7}\times\sqrt{3}\left(\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right) = 3 \times 10^{-7}\ \text{T}$$

$\therefore$ Magnetic field at O due to the whole triangle is

$$B = 3B_1 = 3(3 \times 10^{-7}) = \mathbf{9 \times 10^{-7}\ T}$$

Example 7.16. *A square loop of wire of side 2l carries a current I. What is the magnetic field at the centre of the square ? If the square wire is reshaped into a circle, would the magnetic field increase or decrease at the centre ?*

Solution. Square loop. Figure 7.34 (*i*) shows the conditions of the problem. It is clear that each side of the square produces magnetic field at the centre O of the square in the same direction. Therefore, total magnetic field at O = 4 × magnetic field due to one side.

Magnetic field at O due to side AB is given by ;

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin\phi_1 + \sin\phi_2)$$

Here $\mu_0 = 4\pi \times 10^{-7}$ H/m ; $\phi_1 = \phi_2 = 45°$; $a = OM = AB/2 = l$

$$\therefore \quad B_1 = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{I}{l}(\sin 45° + \sin 45°)$$

$$= 10^{-7}\times\frac{I}{l}\left(\frac{2}{\sqrt{2}}\right) = \sqrt{2}\frac{I}{l}\times10^{-7}\,\text{T}$$

Magnetic field at O due to the whole square is

$$B = 4B_1 = \boldsymbol{4\sqrt{2}\frac{I}{l}\times10^{-7}\,T} \quad \ldots(i)$$

Circular loop. The total length of the square loop $= 4 \times 2l = 8l$. When this square loop is shaped into a circular loop of radius r, then [See Fig. 7.34 (*ii*)],

$$2\pi r = 8l \quad \text{or} \quad r = \frac{8l}{2\pi} = \frac{4l}{\pi}$$

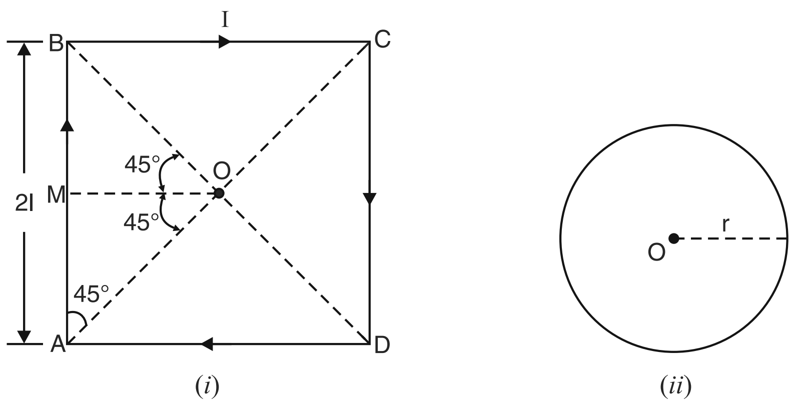


Fig. 7.34

Magnetic field at the centre of the circular loop is

$$B = \frac{\mu_0 I}{2r} = \frac{4\pi\times10^{-7}\times I}{2(4l/\pi)} = \frac{\pi^2}{2}\times\frac{I}{l}\times10^{-7}$$

$$\therefore \quad B = \boldsymbol{4.93\times\frac{I}{l}\times10^{-7}\,T} \quad \ldots(ii)$$

Comments. Inspection of eqs. (*i*) and (*ii*) reveals that magnetic field in case of square loop will be more.

Example 7.17. *A current of 15A is passing along a straight wire. Calculate the force on a unit N-pole placed 0.15 metre from the wire. If the wire is bent to form into a loop, calculate the diameter of the loop so as to produce the same force at the centre of the coil upon a unit N-pole when carrying a current of 15A.*

Solution. By definition, the force on the unit N-pole is the magnetising force H. Therefore, force on a unit N-pole placed at a point 0.15 m (*i.e.* a = 0.15m) from a long straight wire carrying current I(= 15A) is given by ;

$$H = \frac{I}{2\pi a} = \frac{15}{2\pi\times0.15} = \boldsymbol{\frac{50}{\pi}\,\text{AT/m or N/Wb}}$$

Force on a unit N-pole placed at the centre of a loop of radius r when the loop carries a current I(= 15 A) is

$$H' = \frac{I}{2r} = \frac{15}{2r} \text{ AT/m}$$

As per the statement of the problem, $H' = H$.

$$\therefore \quad \frac{15}{2r} = \frac{50}{\pi} \quad \text{or} \quad r = \frac{15\pi}{2 \times 50} = 0.4713 \text{ m}$$

$\therefore$ Diameter of loop, $D = 2r = 2 \times 0.4713 = 0.9426$ m = **94.26 cm**

Tutorial Problems

1. A horizontal overhead power line carries a current of 90 A in east to west direction. What is the magnitude and direction of magnetic field due to the current 1·5 m below the wire ? **[1·2 × 10^{-5} T *towards south*]**
2. A long straight wire is turned into a loop of radius 10 cm as shown in Fig. 7.35. If a current of 8 A is passed, then find the value of magnetic field at the centre O of the loop.
[3·4 × 10^{-5} T *perpendicular to plane of paper pointing upward*]
[**Hint :** The magnetic field at O due to straight wire is perpendicular to the plane of paper and is directed downward. However, field due to circular loop is directed in opposite direction.]

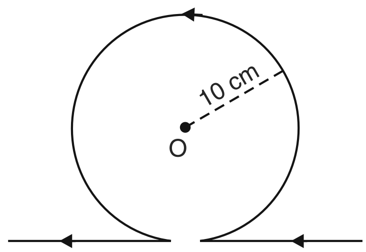


Fig. 7.35

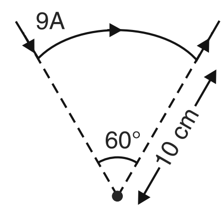


Fig. 7.36

3. A circular segment of radius 10 cm subtends an angle of 60° at its centre. A current of 9 A is flowing through it. Find the magnitude and direction of magnetic field produced at the centre [See Fig. 7.36].
[9·42 × 10^{-6} T *perpendicular to the plane of paper pointing downward*]
[**Hint :** The magnetic field at the centre of a single turn circular coil is

$$B = \frac{\mu_0 I}{2a} \quad \text{... } a \text{ is the radius of coil.}$$

For the given arc, $$B = \frac{60°}{360°}\left(\frac{\mu_0 I}{2a}\right)$$

4. A long wire having a semicircular loop of radius a carries a current I amperes as shown in Fig. 7.37. Find the magnetic field at the centre of the semicircular arc. $\left[\dfrac{\mu_0 I}{4a}\right]$
[**Hint :** The straight portions AB and DE do not contribute to any magnetic field at O. Therefore, magnetic field at O is only due to semicircular loop.]

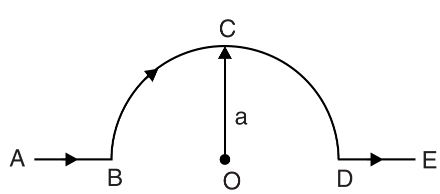


Fig. 7.37

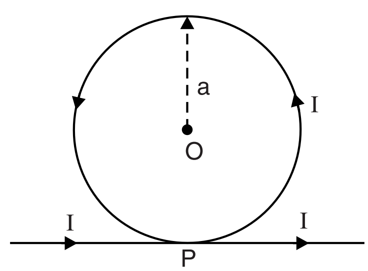


Fig. 7.38

5. The wire shown in Fig. 7.38 carries a current I. What will be the magnitude and direction of magnetic field at the centre O ? Assume that various portions of wire do not touch each other at P.

$$\left[\frac{\mu_0 I}{2a}\left(1+\frac{1}{\pi}\right) \textit{perpendicular to the plane of paper directed upward}\right]$$

[**Hint :** The magnetic field due to straight conductor and that due to circular part aid each other at O.]

7.28. Force Between Current-Carrying Parallel Conductors

When two current-carrying conductors are parallel to each other, a mechanical force acts on each of the conductors. This force is the result of each conductor being acted upon by the magnetic field produced by the other. *If the currents are in the same direction, the forces are attractive ; if currents are in opposite direction, the forces are repulsive.* This can be beautifully illustrated by drawing the magnetic field produced by each conductor.

(*i*) Currents in the same direction. Consider two parallel conductors A and B carrying currents in the same direction (*i.e.* into the plane of paper) as shown in Fig. 7.39 (*i*). Each conductor will set up its own magnetic field as shown. It is clear that in the space between A and B, the two fields are in opposition and hence they tend to cancel each other. However, in the space outside A and B, the two fields assist each other. Hence the resultant field distribution will be as shown in Fig. 7.39 (*ii*).

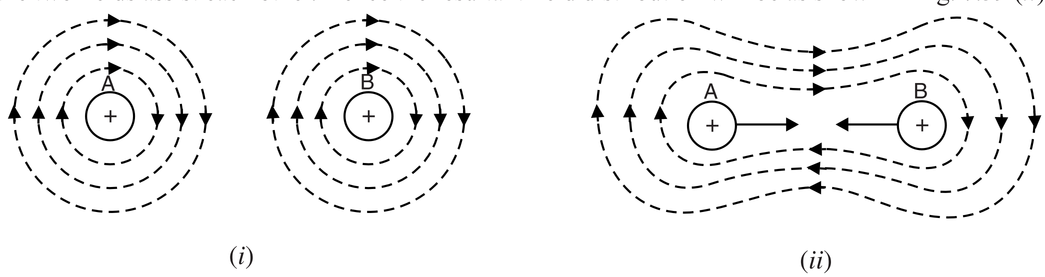


Fig. 7.39

Since magnetic lines of force behave as stretched elastic cords, the two conductors are attracted towards each other. Alternatively, the conductors can be viewed as moving away from the relatively strong field (in the space outside A and B) into the weaker field between the conductors.

(*ii*) Currents in opposite direction. Consider two parallel conductors A and B carrying currents in the opposite direction as shown in Fig. 7.40. Each conductor will set up its own field as shown. It is clear that in the space outside A and B, the two fields are in opposition and hence they tend to cancel each other. However, in the space between A and B, the two fields assist each other. The lateral pressure between lines of force exerts a force on the conductors tending to push them apart. In other words, the conductors experience a repulsive force.

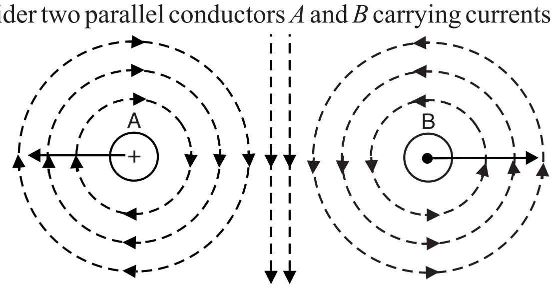


Fig. 7.40

If currents are in the same directions, the conductors attract each other ; if currents are in opposite directions, the conductors repel each other.

7.29. Magnitude of Mutual Force

Fig. 7.41 (*i*) shows two parallel conductors placed in air and carrying currents in the same direction. Here I_1 and I_2 are the currents in conductors 1 and 2 respectively, l is the length of each conductor in metres and d is the distance between conductors in metres. It is clear that each of the two parallel conductors lies in the magnetic field of the other conductor.

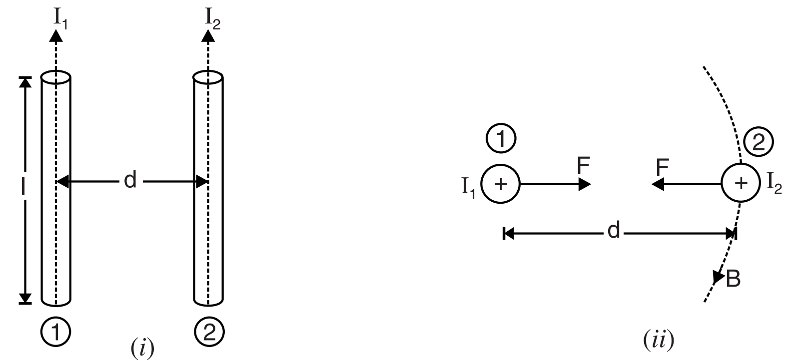


Fig. 7.41

In order to determine the magnitude of force, we can consider conductor 2 placed in the magnetic field produced by conductor 1 as shown in Fig. 7.41 (*ii*). Now field intensity H due to current I_1 in conductor 1 at the centre of conductor 2 is given by ;

$$H = \frac{I_1}{2\pi d}$$

$$\text{But} \quad B = \mu_0 \mu_r H = \mu_0 H = \frac{\mu_0 I_1}{2\pi d} \quad [\text{For air, } \mu_r = 1]$$

Force acting on conductor 2 is given by ;

$$F = B I_2 l = \left(\frac{\mu_0 I_1}{2\pi d}\right) I_2 l$$

$$= \frac{4\pi \times 10^{-7} I_1 I_2 l}{2\pi d} = \frac{2 I_1 I_2 l}{d} \times 10^{-7} \text{ newtons}$$

$$\therefore \quad F = \frac{2 I_1 I_2 l}{d} \times 10^{-7} \text{ N}$$

It can be easily shown that conductor 1 will experience an equal force in the opposite direction [See Fig. 7.41 (*ii*)].

Force per metre run of the conductor is given by ;

$$F' = \frac{2 I_1 I_2}{d} \times 10^{-7} \text{ N/m}$$

According to Fleming's left-hand rule, the two conductors will attract each other.

7.30. Definition of Ampere

The force acting between two parallel conductors has led to the modern definition of an ampere. We have seen above that force between two parallel current-carrying conductors is

$$F = \frac{2 I_1 I_2 l}{d} \times 10^{-7} \text{ newtons}$$

If $I_1 = I_2 = 1\text{A}$; l = 1 m ; d = 1 m, then,

$$F = \frac{2 \times 1 \times 1 \times 1}{1} \times 10^{-7} = 2 \times 10^{-7} \text{ N}$$

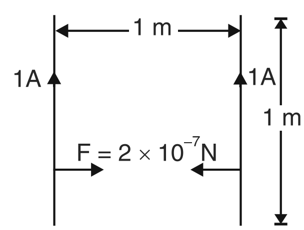


Fig. 7.42

Hence **one ampere** *is that current which, if maintained in two long parallel conductors, and placed 1 m apart in vacuum, would produce between these conductors a force equal to 2 ×* 10^{-7} *newton per metre of length (See Fig. 7.42).*

Historically, the ampere was fixed originally in a very different way. The constant 2 × 10^{-7} that appears in the modern definition was chosen so as to keep the magnitude of ampere the same as formerly.

Example 7.18. *Two long horizontal wires are kept parallel at a distance of 0·2 cm apart in a vertical plane. Both the wires have equal currents in the same direction. The lower wire has a mass of 0·05 kg/m. If the lower wire appears weightless, what is the current in each wire ?*

Solution. Let I amperes be the current in each wire. The lower wire is acted upon by two forces *viz* (*i*) upward magnetic force and (*ii*) downward force due to weight of the wire. Since the lower wire appears weightless, the two forces are equal over 1m length of the wire.

$$\text{Upward force/m length} = \frac{2I_1I_2}{d}\times 10^{-7} = \frac{2\times I\times I\times 10^{-7}}{0.2\times 10^{-2}} = 10^{-4}\,I^2\text{ N}$$

$$\text{Downward force/m length} = mg = 0{\cdot}05\times 9{\cdot}8 = 0{\cdot}49\text{ N}$$

$$\therefore \quad 10^{-4}I^2 = 0{\cdot}49 \quad\text{or}\quad I = \sqrt{0.49\times 10^4} = \mathbf{70\ A}$$

Example 7.19. *A rectangular loop ABCD carrying a current of 16A in clockwise direction is placed with its longer side parallel to a straight conductor 4 cm apart and carrying a current of 20A as shown in Fig. 7.43. The sides of the loop are 15 cm and 6 cm. What is the net force on the loop ? What will be the difference in force if the direction of current in the loop is reversed ?*

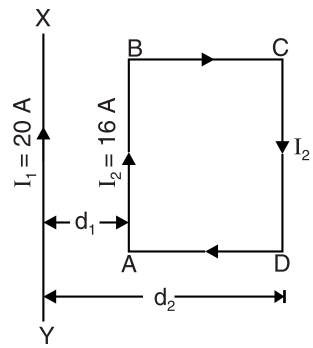


Fig. 7.43

Solution. Fig. 7.43 shows the arrangement. The long straight conductor XY will exert an attractive force on arm AB of the loop while arm CD will experience a repulsive force. The forces on the arms BC and AD will be equal and opposite and hence cancel out. Referring to Fig. 7.43,

$d_1 = 4$ cm $= 0{\cdot}04$ m ; $d_2 = 4 + 6 = 10$ cm $= 0{\cdot}1$ m

$$\text{Force on arm } AB,\ F_1 = \frac{2\,I_1\,I_2}{d_1}\times 10^{-7}\times \text{Length } AB \qquad \text{... towards } XY$$

$$= \frac{2\times 20\times 16}{0\cdot 04}\times 10^{-7}\times 0\cdot 15 = 2\cdot 4\times 10^{-4}\text{ N}$$

$$\text{Force on arm } CD,\ F_2 = \frac{2\,I_1\,I_2}{d_2}\times 10^{-7}\times \text{Length } CD \qquad \text{... away from } XY$$

$$= \frac{2\times 20\times 16}{0\cdot 1}\times 10^{-7}\times 0\cdot 15 = 0\cdot 96\times 10^{-4}\text{ N}$$

Net force on the loop is $F = F_1 - F_2 = 10^{-4}(2{\cdot}4 - 0{\cdot}96) = \mathbf{1{\cdot}44\times 10^{-4}\ N}$

Therefore, the net force on the loop is directed *towards* the current-carrying straight conductor XY. If the direction of current in the loop is reversed, the magnitude of net force on the loop remains the same (*i.e.* $F = 1{\cdot}44\times 10^{-4}$ N) but its direction will be away from the current-carrying straight conductor XY.

Example 7.20. *Two long straight parallel wires, standing in air 2m apart, carry currents I_1 and I_2 in the same direction. The magnetic intensity at a point midway between the wires is 7.95 AT/m. If the force on each wire per unit length is 2.4 × 10^{-4} N, evaluate I_1 and I_2.*

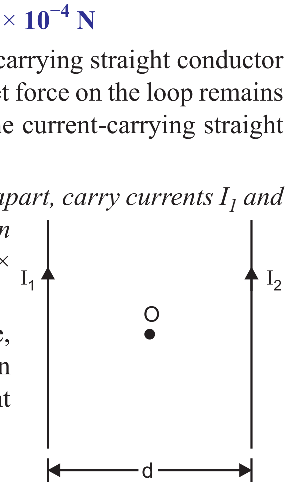


Fig. 7.44

Solution. Fig. 7.44 shows the conditions of the problem. Here, separation between the wires is $d = 2$ m and O is the point midway between the two wires. As proved in Art. 7.26, the magnetic intensity H at a point distant a from a long straight current-carrying wire is

$$H = \frac{I}{2\pi a}$$

Since the two wires are carrying currents in the same direction, the net magnetic intensity H at O is the difference of the magnetic intensities at O due to two currents *i.e.*

$$H = H_1 - H_2$$

or $$7.95 = \frac{I_1}{2\pi \times 1} - \frac{I_2}{2\pi \times 1} \quad (\because \text{ point } O \text{ is 1m from each wire})$$

$\therefore$ $$I_1 - I_2 = 50 \quad ...(i)$$

As proved in Art. 7.29, force per unit length of the conductors is

$$F = \frac{2I_1 I_2}{d} \times 10^{-7}$$

or $$2.4 \times 10^{-4} = \frac{2I_1 I_2}{2} \times 10^{-7}$$

$\therefore$ $$I_1 I_2 = 2400$$

Now, $$(I_1 + I_2)^2 = (I_1 - I_2)^2 + 4I_1 I_2 = (50)^2 + 4 \times 2400 = 12100$$

$\therefore$ $$I_1 + I_2 = 110 \quad ...(ii)$$

From eqs. (*i*) and (*ii*), I_1 = **80A** ; I_2 = **30A**

Example 7.21. *A horizontal straight wire 5 cm long of mass 1·2 g/m is placed perpendicular to a uniform magnetic field of 0·6 T. If resistance of the wire is 3·8 Ω m^{-1}, calculate the p.d. that has to be applied between the ends of the wire to make it just self-supporting.*

Solution. The current (I) in the wire is to be in such a direction that magnetic force acts on it vertically upward. To make the wire self-supporting, its weight should be equal to the upward magnetic force *i.e.*

$$B\,I\,l = m\,g \quad (\because \theta = 90°)$$

or $$I = \frac{mg}{Bl}$$

Here, $m = 1{\cdot}2 \times 10^{-3}\, l$; $B = 0{\cdot}6$ T ; $g = 9{\cdot}8 \text{ ms}^{-2}$

$\therefore$ $$I = \frac{(1{\cdot}2 \times 10^{-3} l) \times 9{\cdot}8}{0{\cdot}6 \times l} = 19{\cdot}6 \times 10^{-3} \text{ A}$$

Resistance of the wire, $R = 0{\cdot}05 \times 3{\cdot}8 = 0{\cdot}19\ \Omega$

$\therefore$ Required P.D., $V = I\,R = (19{\cdot}6 \times 10^{-3})\, 0{\cdot}19 = \mathbf{3{\cdot}7 \times 10^{-3}\ V}$

Tutorial Problems

1. A pair of rising mains has a spacing of 200 mm between centres. If each conductor carries 500 A, determine the force between the conductors for each 10m length of run. **[2·5 N *repulsive*]**
2. Two busbars, each 20 m long, feed a circuit and are spaced at a distance of 80 mm inbetween centres. If a short-circuit current of 20,000 A flows through the conductors, calculate the force per metre between the bars. **[1000 N]**
3. Two long straight parallel conductors carry the same current I in the same direction. The conductors are placed 20 cm apart in air. The magnetic flux density between the conductors 5 cm from one of them is $1{\cdot}33 \times 10^{-5}$ Wb/m^2. If the force on each conductor per metre length is 25×10^{-6} N, find the current in each conductor. **[5 A]**
4. The wires that supply current to a 120 V, 2kW electric heater are 2 mm apart. What is the force per metre between the wires ? **[0·028 N/m]**
5. The busbars 10 cm apart are supported by insulators every metre along their length. The busbars each carry a current of 15 kA. What is the force acting on each insulator ? **[450 N]**

Objective Questions

1. When a magnet is heated,
   (*i*) it gains magnetism
   (*ii*) it loses magnetism
   (*iii*) it neither loses nor gains magnetism
   (*iv*) none of the above
2. The magnetic material used in permanent magnets is
   (*i*) iron (*ii*) soft steel
   (*iii*) nickel (*iv*) hardened steel
3. The magnetic material used in temporary magnets is
   (*i*) hardened steel (*ii*) cobalt steel
   (*iii*) soft iron (*iv*) tungsten steel
4. Magnetic flux density is a
   (*i*) vector quantity (*ii*) scalar quantity
   (*iii*) phasor (*iv*) none of the above
5. The relative permeability of a ferromagnetic material is 1000. Its absolute permeability will be
   (*i*) 10^6 H/m (*ii*) $4\pi \times 10^{-3}$ H/m
   (*iii*) $4\pi \times 10^{-11}$ H/m (*iv*) none of the above
6. The main advantage of temporary magnets is that we can
   (*i*) change the magnetic flux
   (*ii*) use any magnetic material
   (*iii*) decrease the hysteresis loss
   (*iv*) none of the above
7. One weber is equal to
   (*i*) 10^6 lines (*ii*) $4\pi \times 10^{-7}$ lines
   (*iii*) 10^{12} lines (*iv*) 10^8 lines
8. Magnetic field intensity is a
   (*i*) scalar quantity (*ii*) vector quantity
   (*iii*) phasor (*iv*) none of the above
9. The absolute permeability of a material having a flux density of 1 Wb/m^2 is 10^{-3} H/m. The value of magnetising force is
   (*i*) 10^{-3} AT/m (*ii*) $4\pi \times 10^{-3}$ AT/m
   (*iii*) 1000 AT/m (*iv*) $4\pi \times 10^3$ AT/m
10. When the relative permeability of a material is slightly less than 1, it is called a
    (*i*) diamagnetic material
    (*ii*) paramagnetic material
    (*iii*) ferromagnetic material
    (*iv*) none of the above
11. The greater percentage of substances are
    (*i*) diamagnetic (*ii*) paramagnetic
    (*iii*) ferromagnetic (*iv*) none of the above
12. When the relative permeability of material is much greater than 1, it is called
    (*i*) diamagnetic material
    (*ii*) paramagnetic material
    (*iii*) ferromagnetic material
    (*iv*) none of the above
13. The magnetic flux density in an air-cored coil is 10^{-2} Wb/m^2. With a cast iron core of relative permeability 100 inserted, the flux density will become
    (*i*) 10^{-4} Wb/m^2 (*ii*) 10^4 Wb/m^2
    (*iii*) 10^{-2} Wb/m^2 (*iv*) 1 Wb/m^2
14. Which of the following is more suitable for the core of an electromagnet ?
    (*i*) soft iron (*ii*) air
    (*iii*) steel (*iv*) tungsten steel
15. The source of a magnetic field is
    (*i*) an isolated magnetic pole
    (*ii*) static electric charge
    (*iii*) magnetic substances
    (*iv*) current loop
16. A magnetic needle is kept in a uniform magnetic field. It experiences
    (*i*) a force and a torque
    (*ii*) a force but not a torque
    (*iii*) a torque but not a force
    (*iv*) neither a torque nor a force
17. The unit of pole strength is
    (*i*) A/m^2 (*ii*) Am
    (*iii*) Am^2 (*iv*) Wb/m^2
18. When the relative permeability of a material is slightly more than 1, it is called a
    (*i*) diamagnetic material
    (*ii*) paramagnetic material
    (*iii*) ferromagnetic material
    (*iv*) none of the above
19. AT/m is the unit of
    (*i*) m.m.f.
    (*ii*) reluctance
    (*iii*) magnetising force
    (*iv*) magnetic flux density
20. A magnetic needle is kept in a non-uniform magnetic field. It experiences

(*i*) a force and a torque
(*ii*) a force but not a torque
(*iii*) a torque but not a force
(*iv*) neither a force nor a torque

21. Magnetic flux passes more readily through
(*i*) air (*ii*) wood
(*iii*) vacuum (*iv*) iron

22. Iron is ferromagnetic
(*i*) above 770°C
(*ii*) below 770°C
(*iii*) at all temperatures
(*iv*) none of the above

23. The relative permeability of a material is 0·9998. It is
(*i*) diamagnetic (*ii*) paramagnetic
(*iii*) ferromagnetic (*iv*) none of the above

24. Magnetic lines of force
(*i*) intersect at infinity
(*ii*) intersect within the magnet
(*iii*) cannot intersect at all
(*iv*) none of the above

25. Demagnetising of magnets can be done by
(*i*) rough handling (*ii*) heating
(*iii*) magnetising in opposite direction
(*iv*) all of the above

Answers

1. (*ii*)	**2.** (*iv*)	**3.** (*iii*)	**4.** (*i*)	**5.** (*ii*)
6. (*i*)	**7.** (*iv*)	**8.** (*ii*)	**9.** (*iii*)	**10.** (*i*)
11. (*ii*)	**12.** (*iii*)	**13.** (*iv*)	**14.** (*i*)	**15.** (*iv*)
16. (*iii*)	**17.** (*ii*)	**18.** (*ii*)	**19.** (*iii*)	**20.** (*i*)
21. (*iv*)	**22.** (*ii*)	**23.** (*i*)	**24.** (*iii*)	**25.** (*iv*)

8

Magnetic Circuits

Introduction

We have seen that magnetic lines of force form closed loops around and through the magnetic material. The closed path followed by magnetic flux is called a magnetic circuit just as the closed path followed by current is called an electric circuit. Many electrical devices (*e.g.* generator, motor, transformer etc.) depend upon magnetism for their operation. Therefore, such devices have magnetic circuits *i.e.* closed flux paths. In order that these devices function efficiently, their magnetic circuits must be properly designed to obtain the required magnetic conditions. In this chapter, we shall focus our attention on the basic principles of magnetic circuits and methods to obtain their solution.

8.1. Magnetic Circuit

The closed path followed by magnetic flux is called a **magnetic circuit.**

In a magnetic circuit, the magnetic flux leaves the *N*-pole, passes through the entire circuit, and returns to the starting point. A magnetic circuit usually consists of materials having high permeability *e.g.* iron, soft steel *etc*. It is because these materials offer very small opposition to the 'flow' of magnetic flux. The most usual way of producing magnetic flux is by passing electric current through a wire of number of turns wound over a magnetic material. This helps in exercising excellent control over the magnitude, density and direction of magnetic flux.

Consider a coil of *N* turns wound on an iron core as shown in Fig. 8.1. When current *I* is passed through the coil, magnetic flux ϕ is set up in the core. The flux follows the closed path *ABCDA* and hence *ABCDA* is the magnetic circuit. The following points may be noted carefully :

(*i*) The amount of magnetic flux set up in the core depends upon current (*I*) and number of turns (*N*). If we increase the current or number of turns, the amount of magnetic flux also increases and *vice-versa*. The product **N I* is called the **magnetomotive force (m.m.f.)** and determines the amount of flux set up in the magnetic circuit.

$$\text{m.m.f.} = N\,I \text{ ampere-turns}$$

It can just be compared to electromotive force (e.m.f.) which sends current in an electric circuit.

(*ii*) The opposition that the magnetic circuit offers to the magnetic flux is called **reluctance.** It depends upon length of magnetic circuit (*i.e.* length *ABCDA* in this case), area of *X*-section of the circuit and the nature of material that makes up the magnetic circuit.

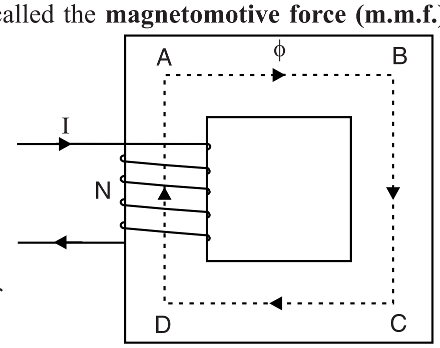


Fig. 8.1

8.2. Analysis of Magnetic Circuit

Consider the magnetic circuit shown in Fig. 8.1. Suppose the mean length of the magnetic circuit (*i.e.* length *ABCDA*) is *l* metres, cross-sectional area of the **core is '*a*' m^2 and relative

* Coiling a conductor into two or more turns has the effect of using the same current for more than once. For example, 5-turn coil carrying a current of 10A produces the same magnetic flux in a given magnetic circuit as a 1-turn coil carrying a current of 50A. Hence m.m.f. is equal to the product of *N* and *I*.

** The arrangement of magnetic materials to form a magnetic circuit is generally called a *core*.

permeability of core material is μ_r. When current I is passed through the coil, it will set up flux ϕ in the material.

Flux density in the material, $B = \dfrac{\phi}{a}\ \text{Wb/m}^2$

Magnetising force in the material, $H = \dfrac{B}{\mu_0\mu_r} = \dfrac{\phi}{a\mu_0\mu_r}\ \text{AT/m}$

According to work law, the work done in moving a unit magnetic pole once around the magnetic circuit (*i.e.* path *ABCDA* in this case) is equal to the ampere-turns enclosed by the magnetic circuit.

$\therefore \quad {}^*H \times l = N\,I$

or $\quad \dfrac{\phi}{a\mu_0\mu_r} \times l = N\,I$

or $\quad \phi = \dfrac{NI}{(l/a\mu_0\mu_r)}$

The quantity *NI* which produces the magnetic flux is called the magnetomotive force (m.m.f.) and is measured in ampere-turns. The quantity $l/a\ \mu_0\ \mu_r$ is called the reluctance of the magnetic circuit. Reluctance is the opposition that the magnetic circuit offers to magnetic flux.

$$\therefore \quad \text{Flux, } \phi = \frac{\text{m.m.f.}}{\text{reluctance}} \qquad ...(i)$$

Note that the relationship expressed in eq. (*i*) has a strong resemblance to Ohm's law of electric circuit ($I = E/R$). The m.m.f. is analogous to e.m.f. in the electric circuit, reluctance is analogous to resistance and flux is analogous to current. Because of this similarity, eq. (*i*) is sometimes referred to as **Ohm's law of magnetic circuit.**

8.3. Important Terms

In the study of magnetic circuits, we generally come across the following terms :

(*i*) Magnetomotive force (m.m.f.). It is a magnetic pressure which sets up or tends to set up flux in a magnetic circuit and may be defined as under :

The work done in moving a unit magnetic pole once around the magnetic circuit is called the magnetomotive force (m.m.f.). It is equal to the product of current and number of turns of the coil *i.e.*

$$\text{m.m.f.} = N\,I \text{ ampere-turns (or AT)}$$

Magnetomotive force in a magnetic circuit corresponds to e.m.f. in an electric circuit. The only change in the definition is the substitution of unit magnetic pole in place of unit charge.

(*ii*) Reluctance. *The opposition that the magnetic circuit offers to magnetic flux is called reluctance.* The reluctance of a magnetic circuit depends upon its length, area of *X*-section and permeability of the material that makes up the magnetic circuit. Its unit is †AT/Wb.

$$\text{Reluctance, } S = \frac{l}{a\,\mu_0\,\mu_r}$$

Reluctance in a magnetic circuit corresponds to resistance ($R = \rho\ l/a$) in an electric circuit. Both of them vary as length ÷ area and are dependent upon the nature of material of the circuit. Magnetic materials (*e.g.* iron, steel *etc.*) have a low reluctance because the value of μ_r is very large in their case. On the other hand, non-magnetic materials (*e.g.* air, wood, copper, brass *etc.*) have a high reluctance because they possess least value of μ_r ; being 1 in case of all non-magnetic materials.

* You may recall that *H* means force acting on a unit magnetic pole. If the unit pole is moved once around the magnetic circuit (*i.e.* distance covered is *l*), then work done = $H \times l$.

† Reluctance $= \dfrac{\text{m.m.f.}}{\text{flux}} = \dfrac{\text{AT}}{\text{Wb}} = \text{AT/Wb}$

The reciprocal of permeability $\mu(=\mu_0\mu_r)$ corresponds to resistivity ρ of the electrical circuit and is called *reluctivity. It may be noted that magnetic permeability (μ) is the analog of electrical conductivity.*

(*iii*) Permeance. *It is the reciprocal of reluctance* and is a measure of the ease with which flux can pass through the material. Its unit is Wb/AT.

$$\text{Permeance} = \frac{1}{\text{Reluctance}} = \frac{a\mu_0\mu_r}{l}$$

Permeance of a magnetic circuit corresponds to conductance (reciprocal of resistance) in an electric circuit.

8.4. Comparison Between Magnetic and Electric Circuits

There are many points of similarity between magnetic and electric circuits. However, the two circuits are not anologous in all respects. A comparison of the two circuits is given below in the tabular form.

Magnetic Circuit

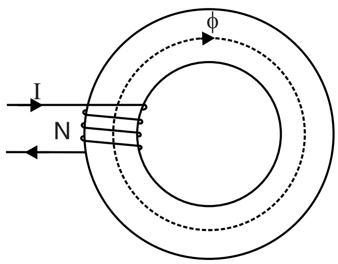


Fig. 8.2

Electric Circuit

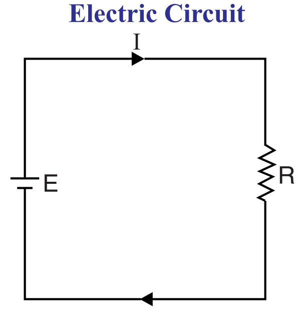


Fig. 8.3

Similarities

1. The closed path for magnetic flux is called a magnetic circuit.	**1.** The closed path for electric current is called an electric circuit.
2. Flux, $\phi = \dfrac{\text{m.m.f.}}{\text{reluctance}}$	**2.** Current, $I = \dfrac{\text{e.m.f.}}{\text{resistance}}$
3. m.m.f. (ampere-turns)	**3.** e.m.f. (volts)
4. Reluctance, $S = \dfrac{l}{a\mu_0\mu_r}$	**4.** Resistance, $R = \rho \dfrac{l}{a}$
5. Flux density, $B = \dfrac{\phi}{a}\,\text{Wb/m}^2$	**5.** Current density, $J = \dfrac{I}{a}\,\text{A/m}^2$
6. m.m.f. drop = $\phi\, S$	**6.** Voltage drop = $I\,R$
7. Magnetic intensity, $H = N\,I/l$	**7.** Electric intensity, $E = V/d$
8. Permeance	**8.** Conductance.
9. Permeability	**9.** Conductivity

Dissimilarities

1. Truly speaking, magnetic flux does not flow.	**1.** The electric current acutally flows in an electric circuit.
2. There is no magnetic insulator. For example, flux can be set up even in air (the best known magnetic insulator) with reasonable m.m.f.	**2.** There are a number of electric insulators. For instance, air is a very good insulator and current cannot pass through it.

3. The value of μ_r is not constant for a given magnetic material. It varies considerably with flux density (B) in the material. This implies that reluctance of a magnetic circuit is not constant rather it depends upon B.	**3.** The value of resistivity (ρ) varies very slightly with temperature. Therefore, the resistance of an electric circuit is practically constant. This salient feature calls for different approach to the solution of magnetic and electric circuits.
4. No energy is expended in a magnetic circuit. In other words, energy is required in creating the flux, and not in maintaining it.	**4.** When current flows through an electric circuit, energy is expended so long as the current flows. The expended energy is dissipated in the form of heat.

8.5. Calculation of Ampere-Turns

In any magnetic circuit, flux produced is given by ;

$$\text{Flux, } \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{AT}{(l/a\mu_0\mu_r)}$$

$$\therefore \quad AT \text{ required} = \phi \times \frac{l}{a\mu_0\mu_r} = \frac{\phi}{a} \times \frac{l}{\mu_0\mu_r}$$

$$= \frac{B}{\mu_0\mu_r} \times l \qquad \left(\because \; B = \frac{\phi}{a}\right)$$

$$= H \times l \qquad (\because \; H = B/\mu_0\,\mu_r)$$

i.e. **AT required for any part = Field strength H in that part × length of that part of magnetic circuit**

8.6. Series Magnetic Circuits

In a series magnetic circuit, the same flux ϕ flows through each part of the circuit. It can just be compared to a series electric circuit which carries the same current throughout.

Consider a ***composite series magnetic circuit** consisting of three different magnetic materials of different relative permeabilities along with an air gap as shown in Fig. 8.4. Each part of this series magnetic circuit will offer reluctance to the magnetic flux ϕ. The reluctance offered by each part will depend upon dimensions and μ_r of that part. Since the different parts of the circuit are in series, the total reluctance is equal to the sum of reluctances of individual parts, *i.e.*

$$\text{Total reluctance} = \frac{l_1}{a_1\,\mu_0\,\mu_{r1}} + \frac{l_2}{a_2\,\mu_0\,\mu_{r2}} + \frac{l_3}{a_3\,\mu_0\,\mu_{r3}} + \frac{l_g}{a_g\,\mu_0}^{**}$$

$$\text{Total m.m.f.} = \text{Flux} \times \text{Total reluctance}$$

$$= \phi\left[\frac{l_1}{a_1\,\mu_0\,\mu_{r1}} + \frac{l_2}{a_2\,\mu_0\,\mu_{r2}} + \frac{l_3}{a_3\,\mu_0\,\mu_{r3}} + \frac{l_g}{a_g\,\mu_0}\right]$$

$$= \frac{\phi}{a_1\,\mu_0\,\mu_{r1}} \times l_1 + \frac{\phi}{a_2\,\mu_0\,\mu_{r2}} \times l_2 + \frac{\phi}{a_3\,\mu_0\,\mu_{r3}} \times l_3 + \frac{\phi}{a_g\,\mu_0} \times l_g$$

$$= \frac{B_1}{\mu_0\,\mu_{r1}} \times l_1 + \frac{B_2}{\mu_0\,\mu_{r2}} \times l_2 + \frac{B_3}{\mu_0\,\mu_{r3}} \times l_3 + \frac{B_g}{\mu_0} \times l_g$$

$$= H_1\,l_1 + H_2\,l_2 + H_3\,l_3 + H_g\,l_g \qquad (\because \; H = B/\mu_0\,\mu_r)$$

* A series magnetic circuit that has parts of different dimensions and materials is called a composite series circuit.

** For air, $\mu_r = 1$.

Hence the total ampere-turns required for a series magnetic circuit can be found as under :

(*i*) Find H for each part of the series magnetic circuit. For air, $H = B/\mu_0$ whereas for magnetic material, $H = B/\mu_0\mu_r$.

(*ii*) Find the mean length (l) of magnetic path for each part of the circuit.

(*iii*) Find AT required for each part of the magnetic circuit using the relation, $AT = H \times l$.

(*iv*) The total AT required for the entire series circuit is equal to the sum of AT for various parts.

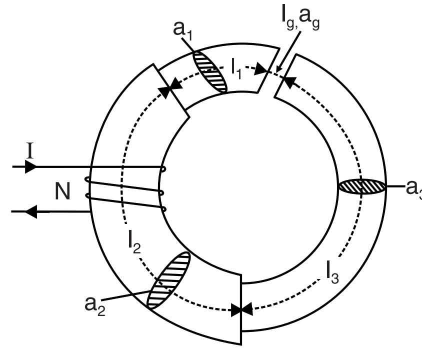


Fig. 8.4

8.7. Air Gaps in Magnetic Circuits

In many practical magnetic circuits, air gap is indispensable. For example, in electromechanical conversion devices like electric motors and generators, the magnetic flux must pass through stator as well as rotor. This necessitates to have a small air gap between the stator and rotor to permit mechanical clearance.

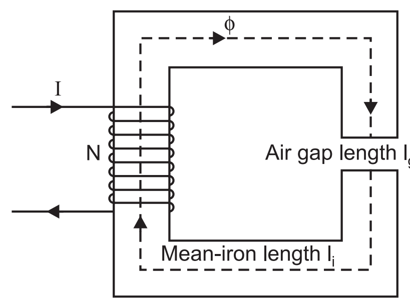


Fig. 8.5

The magnitude of AT required for air gap is much greater than that required for iron part of the magnetic circuit. It is because reluctance of air is very large compared to that offered by iron. Consider a magnetic circuit of uniform cross-sectional area a with an air gap as shown in Fig. 8.5. The length of the air gap is l_g and the mean length of iron part is l_i. The flux density $B(= \phi/a)$ is constant in the magnetic circuit.

$$\therefore \quad \text{Reluctance of air gap} = \frac{l_g}{a\mu_0}$$

$$\text{Reluctance of iron part} = \frac{l_i}{a\mu_0\mu_r}$$

Now relative permeability μ_r of iron is very high (> 6000) so that reluctance of iron part is very small as compared to that of air gap inspite of the fact that $l_i > l_g$. In fact, most of ampere-turns (AT) are required in a magnetic circuit to force the flux through the air gap than through the iron part. In some magnetic circuits, we neglect reluctance of iron part compared to the air gap/gaps. This assumption leads to reasonable accuracy.

8.8. Parallel Magnetic Circuits

A magnetic circuit which has more than one path for flux is called a parallel magnetic circuit. It can just be compared to a parallel electric circuit which has more than one path for electric current.

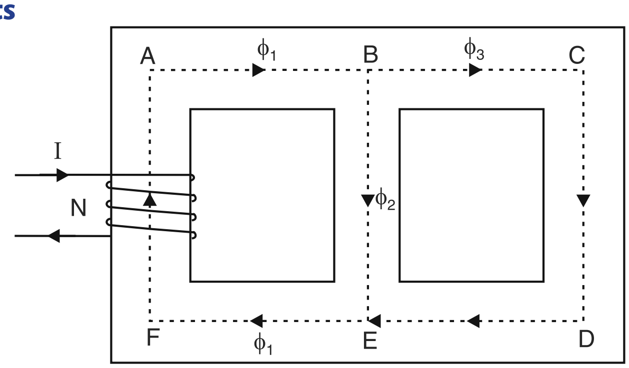


Fig. 8.6

The concept of parallel magnetic circuit is illustrated in Fig. 8.6. Here a coil of N turns wounded on limb AF carries a current of I amperes. The flux ϕ_1 set up by the coil divides at B into two paths, namely ;

(*i*) flux ϕ_2 passes along the path *BE*

(*ii*) flux ϕ_3 follows the path *BCDE*

$$\text{Clearly, } \phi_1 = \phi_2 + \phi_3$$

The magnetic paths *BE* and *BCDE* are in parallel and form a parallel magnetic circuit. The *AT* required for this parallel circuit is equal to *AT* required for any *one of the paths.

Let S_1 = reluctance of path *EFAB*

S_2 = reluctance of path *BE*

S_3 = reluctance of path *BCDE*

$\therefore$ Total m.m.f. required = m.m.f. for path *EFAB* + m.m.f. for path *BE* or path *BCDE*

or $$NI = \phi_1 S_1 + \phi_2 S_2 = \phi_1 S_1 + \phi_3 S_3$$

The reluctances S_1, S_2 and S_3 must be determined from a calculation of $l/a\mu_0\mu_r$ for those paths of the magnetic circuit in which ϕ_1, ϕ_2 and ϕ_3 exist respectively.

8.9. Magnetic Leakage and Fringing

The flux that does not follow the desired path in a magnetic circuit is called a **leakage flux.**

In most of practical magnetic circuits, a large part of flux path is through a magnetic material and the remainder part of flux path is through air. The flux in the air gap is known as *useful flux* because it can be utilised for various useful purposes. Fig. 8.7 shows an iron ring wound with a coil and having a narrow air gap. The total flux produced by the coil does not pass through the air gap as some of it **leaks through the air (path at '*a*') surrounding the iron. These flux lines as at '*a*' are called leakage flux.

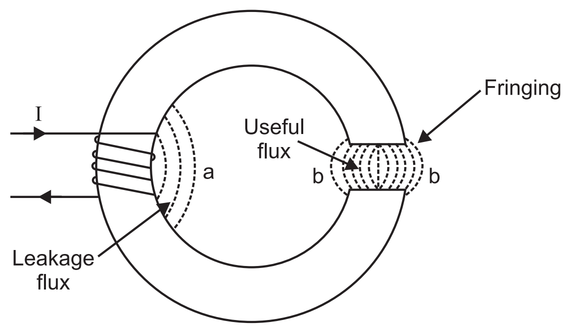


Fig. 8.7

Let ϕ_i = total flux produced *i.e.,* flux in the ***iron ring

ϕ_g = useful flux across the air gap

$\therefore$ Leakage flux, $\phi_{leak} = \phi_i - \phi_g$

$$\text{Leakage coefficient, } \lambda = \frac{\text{Total flux}}{\text{Useful flux}} = \frac{\phi_i}{\phi_g}$$

The value of leakage coefficients for electrical machines is usually about 1.15 to 1.25.

Magnetic leakage is undesirable in electrical machines because it increases the weight as well as cost of the machine. Magnetic leakage can be greatly reduced by placing source of m.m.f. close to the air gap.

Fringing. When crossing an air gap, magnetic lines of force tend to bulge out such as lines of force at *bb* in Fig. 8.7. It is because lines of force repel each other when passing through non-

* This means that we may consider either path, say path *BE*, and calculate *AT* required for it. The same *AT* will also send the flux (ϕ_3 in this case) through the other parallel path *BCDE*. The situation is similar to that of two resistors R_1 and R_2 in parallel in an electric circuit. The voltage V required to send currents (say I_1 and I_2) in the resistors is equal to that appearing across either resistor *i.e.* $V = I_1 R_1 = I_2 R_2$.

** Air is not a good magnetic insulator. Therefore, leakage of flux from iron to air takes place easily.

*** The flux ϕ_i is not constant all around the ring. However, for reasonable accuracy, it is assumed that the iron carries the whole of the flux produced by the coil.

magnetic material such as air. This effect is known as *fringing*. The result of bulging or fringing is to increase the effective area of air gap and thus decrease the flux density in the gap. The longer the air gap, the greater is the fringing and *vice-versa*.

Note. In a short air gap with large cross-sectional area, the fringing may be insignificant. In other situations, 10% is added to the air gap's cross-sectional area to allow for fringing.

8.10. Solenoid

A long coil of wire consisting of closely packed loops is called a **solenoid.**

The word solenoid comes from Greek word meaning 'tube-like'. By a long solenoid we mean that length of the solenoid is very large as compared to its diameter. When current is passed through the coil of air-cored solenoid, magnetic field is set up as shown in Fig. 8.8. The path of the magnetic flux is made up of two components :

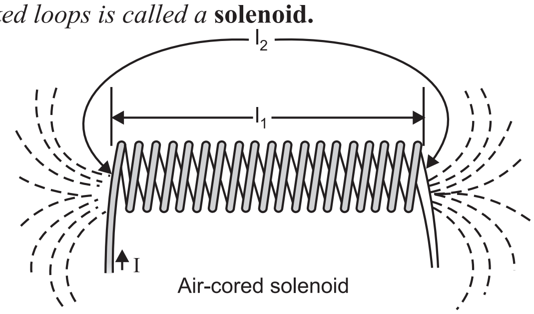


Fig. 8.8

(*i*) length l_1 of the path within the coil

(*ii*) length l_2 of the path outside the coil.

The total m.m.f. required for the solenoid is the sum of m.m.f.s required for these two paths *i.e.*

Total m.m.f. = m.m.f. for path l_1 + m.m.f. for path l_2

But m.m.f. for path l_1 *>> m.m.f. for path l_2

$\therefore$ Total m.m.f. = m.m.f. for path l_1

Hence, for a solenoid (air-cored or iron-cored), the length of the magnetic circuit is the coil length l_1. We can use right-hand rule to determine the direction of magnetic field in the core of the solenoid.

Example 8.1. *A cast steel electromagnet has an air gap length of 3 mm and an iron path of length 40 cm. Find the number of ampere-turns necessary to produce a flux density of 0.7 Wb/m² in the gap. Neglect leakage and fringing. Assume ampere-turns required for air gap to be 70% of the total ampere-turns.*

Solution. Air-gap length, $l_g = 3 \text{ mm} = 3 \times 10^{-3}$ m

Flux density in air gap, $B_g = 0.7 \text{ Wb/m}^2$

$$\therefore \quad \text{Magnetising force, } H_g = \frac{B_g}{\mu_0 \mu_r} = \frac{0.7}{4\pi \times 10^{-7} \times 1} = 5.57 \times 10^5 \text{ AT/m}$$

$$AT \text{ required for air gap, } AT_g = H_g \times l_g = 5.57 \times 10^5 \times 3 \times 10^{-3} = 1671 \text{ AT}$$

It is given that : AT_g = 70% of total AT

$$\therefore \quad \text{Total } AT = \frac{AT_g}{0.7} = \frac{1671}{0.7} = \mathbf{2387\ AT}$$

Example 8.2. *An iron ring has a cross-sectional area of 400 mm² and a mean diameter of 25 cm. It is wound with 500 turns. If the value of relative permeability is 250, find the total flux set up in the ring. The coil resistance is 474 Ω and the supply voltage is 240 V.*

* The lengths l_2 and l_1 do not differ very much. However, the cross-sectional area of path l_2 is very large as compared to that of path l_1. Therefore, reluctance of path l_2 is very small as compared to that of path l_1.

Now, m.m.f. = flux × reluctance

Since reluctance of path l_2 is very small, the m.m.f. required for this path is negligible compared to that for path l_1.

Solution. The conditions of the problem are represented in Fig. 8.9.

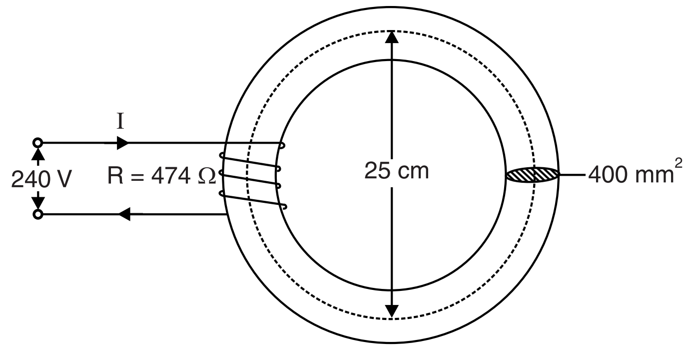


Fig. 8.9

Current through the coil, $I = V/R = 240/474 = 0{\cdot}506$ A

Mean length of magnetic circuit is given by ;

$$l = \pi \times (25 \times 10^{-2}) = 0{\cdot}7854 \text{ m}$$

$$\text{Magnetising force, } H = \frac{Nl}{l} = \frac{500 \times 0.506}{0.7854} = 322.13 \text{ AT/m}$$

$$\text{Flux density, } B = \mu_0 \mu_r H = (4\pi \times 10^{-7}) \times 250 \times 322{\cdot}13 = 0{\cdot}1012 \text{ Wb/m}^2$$

$$\therefore \quad \text{Flux in the ring, } \phi = B \times a = 0{\cdot}1012 \times (400 \times 10^{-6}) = \mathbf{40{\cdot}48 \times 10^{-6}\ Wb}$$

Example 8.3. *An iron ring of cross-sectional area 6 cm² is wound with a wire of 100 turns and has a saw cut of 2 mm. Calculate the magnetising current required to produce a flux of 0·1 mWb if mean length of magnetic path is 30 cm and relative permeability of iron is 470.*

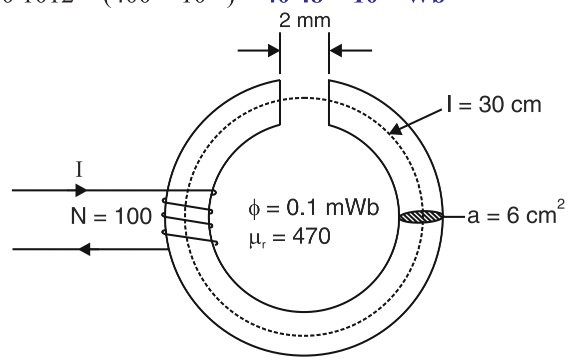


Fig. 8.10

Solution. The conditions of the problem are represented in Fig. 8.10. It will be assumed that flux density in the air gap is equal to the flux density in the core *i.e.* fringing is neglected. This assumption is quite reasonable in this case.

$$\text{Flux density, } B = \frac{\phi}{a} = \frac{0.1 \times 10^{-3}}{6 \times 10^{-4}} = 0.167 \text{ Wb/m}^2$$

Ampere-turns required for iron will be :

$$AT_i = H_i \times l_i$$

$$= \frac{B}{\mu_0 \mu_r} \times l_i = \frac{0.167}{4\pi \times 10^{-7} \times 470} \times 0.3 = 84.83 \text{ AT}$$

Ampere-turns required for air will be :

$$AT_g = \frac{B}{\mu_0} \times l_g = \frac{0{\cdot}167}{4\pi \times 10^{-7}} \times (2 \times 10^{-3}) = 265{\cdot}8 \text{ AT}$$

$$\therefore \quad \text{Total } AT = 265{\cdot}8 + 84{\cdot}83 = 350{\cdot}63 \text{ AT}$$

$$\therefore \quad \text{Magnetising current, } I = 350{\cdot}63/N = 350{\cdot}63/100 = \mathbf{3{\cdot}51\ A}$$

It may be seen that many more ampere-turns are required to produce the magnetic flux through 2 mm of air gap than through the iron part. This is expected because reluctance of air is much more than that of iron.

Example 8.4. *A circular iron ring has a mean circumference of 1·5 m and a cross-sectional area of 0·01 m*^{2}*. A saw-cut of 4 mm wide is made in the ring. Calculate the magnetising current required to produce a flux of 0·8 mWb in the air gap if the ring is wound with a coil of 175 turns. Assume relative permeability of iron as 400 and leakage factor 1·25.*

Solution. $\phi_g = 0{\cdot}8 \times 10^{-3}$ Wb ; $a = 0{\cdot}01$ m^2 ; $l_i = 1{\cdot}5$m ; $l_g = 4 \times 10^{-3}$ m

AT for air gap

$$B_g = \frac{\phi_g}{a} = \frac{0.8\times10^{-3}}{0.01} = 0.08 \text{ Wb/m}^2$$

$$H_g = \frac{B_g}{\mu_0} = \frac{0\cdot08}{4\pi\times10^{-7}} = 63662 \text{ AT/m}$$

$$\therefore \quad AT_g = H_g \times l_g = 63662 \times (4 \times 10^{-3}) = 254{\cdot}6 \text{ AT}$$

AT for iron path

$$\phi_i = \phi_g \times \lambda = 0{\cdot}8 \times 10^{-3} \times 1{\cdot}25 = 10^{-3} \text{ Wb}$$

$$B_i = \phi_i/a = 10^{-3}/0{\cdot}01 = 0{\cdot}1 \text{ Wb/m}^2$$

$$H_i = \frac{B_i}{\mu_0\mu_r} = \frac{0\cdot1}{4\pi\times10^{-7}\times400} = 199 \text{ AT/m}$$

$$\therefore \quad AT_i = H_i \times l_i = 199 \times 1{\cdot}5 = 298{\cdot}5 \text{ AT}$$

$$\therefore \quad \text{Total } AT = 254{\cdot}6 + 298{\cdot}5 = 553{\cdot}1 \text{ AT}$$

$\therefore$ Magnetising current, $I = 553{\cdot}1/N = 553{\cdot}1/175 =$ **3·16 A**

Example 8.5. *A shunt field coil is required to develop 1500 AT with an applied voltage of 60 V. The rectangular coil is having a mean length of 50 cm. Calculate the wire size. Resistivity of copper may be assumed to be 2 × 10*$^{-6}$ *Ω-cm at the operating temperature of the coil. Estimate also the number of turns if the coil is to be worked at a current density of 3 A/mm*^{2}*.*

Solution. Suppose the number of turns of coil is N.

Then the total length of the coil, $l = 50 \times N$ cm

Current in coil, $I = V/R = 60/R$

$$\text{Resistance of coil, } R = \rho\frac{l}{A} = 2\times10^{-6}\times\frac{50\times N}{A} = \frac{N\times10^{-4}}{A} \quad ...(i)$$

Also $NI = 1500$ or $N \times (60/R) = 1500$ $\therefore$ $R = N/25$ ...(*ii*)

From eqs. (*i*) and (*ii*), $\dfrac{N}{25} = \dfrac{N\times10^{-4}}{A}$ or $A = 25 \times 10^{-4}$ cm^2 = 0·25 mm^2

If D is the diameter of the wire, then,

$$\frac{\pi}{4}D^2 = 0{\cdot}25 \quad \text{or} \quad D = \mathbf{0{\cdot}568\ mm}$$

In order to operate the coil at a current density of 3 A/mm^2, the current in the coil is

$$I' = A \times \text{current density} = 0{\cdot}25 \times 3 = 0{\cdot}75 \text{ A}$$

$$\therefore \quad N'I' = 1500 \quad \text{or} \quad N' = 1500/I' = 1500/0{\cdot}75 = \mathbf{2000}$$

Example 8.6. *An iron ring has a mean diameter of 15 cm, a cross-section of 20 cm*^{2} *and a radial gap of 0·5 mm cut in it. It is uniformly wound with 1500 turns of insulated wire and a magnetising current of 1 A produces a flux of 1 mWb. Neglecting the effect of magnetic leakage and fringing, calculate (i) reluctance of the magnetic circuit, (ii) relative permeability of iron and (iii) inductance of the winding.*

Solution. (*ii*) $a = 20 \times 10^{-4}$ m^2 ; $l_i = \pi \times 0{\cdot}15 = 0{\cdot}471$ m ; $l_g = 0{\cdot}5 \times 10^{-3}$ m

$$\text{Flux density in air gap, } B = \frac{\phi}{a} = \frac{1\times10^{-3}}{20\times10^{-4}} = 0{\cdot}5 \text{ Wb/m}^2$$

Magnetising force in air gap, $H_g = B/\mu_0 = 0{\cdot}5/4\pi \times 10^{-7} = 398 \times 10^3$ AT/m

Ampere-turns for air gap, $AT_g = H_g \times l_g = (398 \times 10^3) \times 0{\cdot}5 \times 10^{-3} = 199$ AT

Total AT provided $= NI = 1500 \times 1 = 1500$ AT

$\therefore$ AT available for iron part, $AT_i = 1500 - 199 = 1301$ AT

Magnetising force in iron, $H_i = \dfrac{AT_i}{l_i} = \dfrac{1301}{0.471} = 2762$ AT/m

Now, $$B = \mu_0 \mu_r H_i$$

$$\therefore \quad \mu_r = \frac{B}{\mu_0 H_i} = \frac{0.5}{4\pi \times 10^{-7} \times 2762} = \mathbf{144}$$

(i) Reluctance of air gap $= \dfrac{l_g}{a\mu_0} = \dfrac{0 \cdot 5 \times 10^{-3}}{(20 \times 10^{-4}) \times 4\pi \times 10^{-7}} = 1{\cdot}99 \times 10^5$ AT/Wb

Reluctance of iron part $= \dfrac{l_i}{a\mu_0\mu_r} = \dfrac{0.471}{(20 \times 10^{-4}) \times 4\pi \times 10^{-7} \times 144} = 13{\cdot}01 \times 10^5$ AT/Wb

$\therefore$ Total circuit reluctance $= 10^5\,(1.99 + 13.01) = \mathbf{15 \times 10^5}$ **AT/Wb**

(iii) Inductance of winding $= \dfrac{N\phi}{I} = \dfrac{(1500) \times (1 \times 10^{-3})}{1} = \mathbf{1.5\ H}$

Example 8.7. *A magnetic circuit is constructed as shown in Fig. 8.11. Both sections A and B are of 20 mm by 20 mm square cross-section and the mean dimensions are 100 mm by 80 mm. The relative permeability of section A is 250 and of section B is 500. Find the reluctance of each section and the total circuit reluctance.*

If the joints between sections A and B have an air gap of 0·5 mm at each joint, find the total reluctance of the circuit.

Solution. The conditions of the problem are represented in Fig. 8.11. The area of *X*-section of the core, $a = 20 \times 20 = 400\ \text{mm}^2 = 4 \times 10^{-4}\ \text{m}^2$.

Section A

Length of magnetic path, $l_A = 80 + 10 + 10 = 100$ mm $= 0{\cdot}1$ m

$\therefore$ Reluctance of section $A = \dfrac{l_A}{a\mu_0\mu_r} = \dfrac{0 \cdot 1}{(4 \times 10^{-4}) \times 4\pi \times 10^{-7} \times 250} = \mathbf{0{\cdot}796 \times 10^6}$ **AT/Wb**

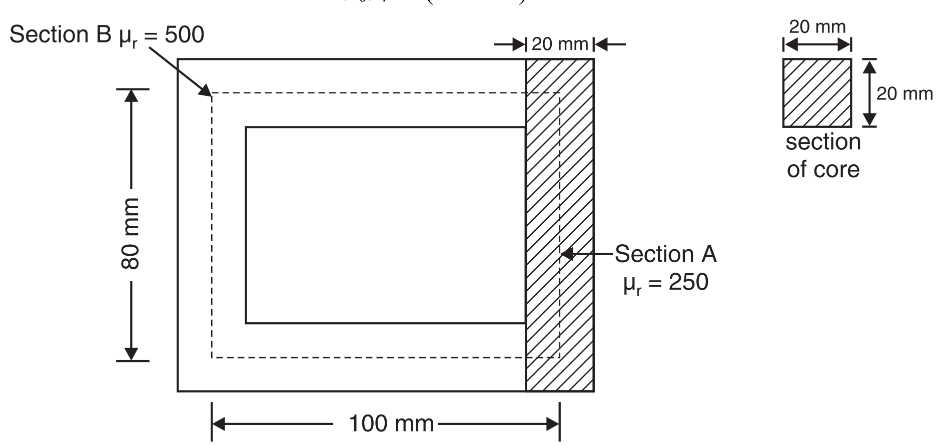


Fig. 8.11

Section B

Length of magnetic path, $l_B = 80 + 90 + 90 = 260$ mm = 0·26 m

$\therefore$ Reluctance of section $B = \dfrac{l_B}{a\mu_0\mu_r} = \dfrac{0\cdot26}{(4\times10^{-4})\times4\pi\times10^{-7}\times500}$ = **1·035 × 10⁶ AT/Wb**

$\therefore$ Total circuit reluctance = 10^6 (0·796 + 1·035) = **$1\cdot831 \times 10^6$ AT/Wb**

Regarding the second part of the problem, the total length of air gaps is $l_g = 2 \times 0\cdot5 = 1$ mm = 0·001 m.

$\therefore$ Reluctance of air gaps = $\dfrac{l_g}{a\mu_0} = \dfrac{0\cdot1}{(4\times10^{-4})\times4\pi\times10^{-7}}$ = $1\cdot99 \times 10^6$ AT/Wb

$\therefore$ Total circuit reluctance = 10^6 (1·831 + 1·99) = **$3\cdot821 \times 10^6$ AT/Wb**

The reader may note that the reluctance of even small air gaps is very large. It is very important, therefore, that the joints of magnetic circuits — for example, the core of a transformer — should be tightly bolted together.

Note. The air gap is very small. Therefore, the magnetic length of iron part is the same in the two cases.

Example 8.8. *A rectangular iron core is shown in Fig. 8.12. It has a mean length of magnetic path of 100 cm, cross-section of 2 cm × 2 cm, relative permeability of 1400 and an air gap of 5 mm cut in the core. The three coils carried by the core have number of turns $N_a = 335$, $N_b = 600$ and $N_c = 600$ and the respective currents are 1·6 A, 4 A and 3 A. The directions of the currents are as shown in Fig. 8.12. Find the flux in the air gap.*

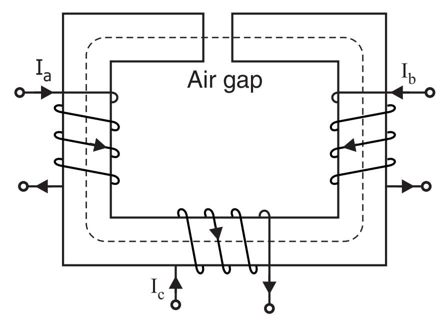


Fig. 8.12

Solution. By applying right-hand rule for the coil, it is easy to see that fluxes produced by currents I_a and I_b are in the clockwise direction through the iron core while the flux produced by current I_c is in the anticlockwise direction through the core.

$\therefore$ Net m.m.f. = $N_aI_a + N_bI_b - N_cI_c = 335 \times 1\cdot6 + 600 \times 4 - 600 \times 3 = 1136$ AT

Reluctance of air gap = $\dfrac{l_g}{\mu_0 a} = \dfrac{5\times10^{-3}}{4\pi\times10^{-7}\times4\times10^{-4}}$ = $9\cdot946 \times 10^6$ AT/Wb

Reluctance of iron path = $\dfrac{l_i}{\mu_0\mu_r a} = \dfrac{(100-0\cdot5)\times10^{-2}}{4\pi\times10^{-7}\times1400\times4\times10^{-4}}$ = $1\cdot414 \times 10^6$ AT/Wb

$\therefore$ Total reluctance = $(9\cdot946 + 1\cdot414) \times 10^6 = 11\cdot36 \times 10^6$ AT/Wb

The statement of the example suggests that there is no leakage flux. Therefore, flux in the air gap is the same as in the iron core.

$\therefore$ Flux in air gap = $\dfrac{\text{Net m.m.f.}}{\text{Total reluctance}} = \dfrac{1136}{11\cdot36\times10^6}$ = 100×10^{-6} Wb = **100 μWb**

Example 8.9. *An angular ring of wood has a cross-sectional area of 4 cm² and a mean diameter of 30 cm. It is uniformly wound with 1200 turns of wire having a resistance of 6 Ω. The core of the second ring, with same dimensions and similarly wound, is made of a magnetic material of relative permeability 50. When the two windings are connected in parallel to a battery, the sum of the two fluxes in the two cores is 0·2 mWb [See Fig. 8.13]. Find the terminal voltage of the battery.*

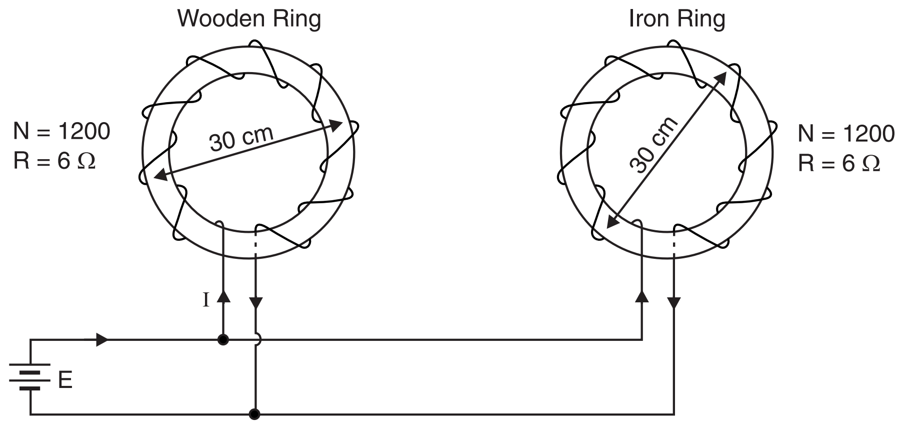


Fig. 8.13

Solution. The windings will carry the same current I as their resistances are equal. Moreover, each ring has the same mean magnetic length $l = \pi \times 0{\cdot}3 = 0{\cdot}942$ m.

Wooden ring. Reluctance $= \dfrac{l}{a\mu_0\mu_r} = \dfrac{0\cdot 942}{(4\times10^{-4})\times4\pi\times10^{-7}\times1} = 18{\cdot}74 \times 10^8$ AT/Wb

Now, m.m.f. = flux × reluctance

$\therefore$ Flux in wooden ring, $\phi_1 = \dfrac{\text{m.m.f.}}{\text{reluctance}} = \dfrac{1200I}{18\cdot74\times10^8} = 6{\cdot}4 \times 10^{-7}\, I$ Wb

Iron ring. Reluctance $= \dfrac{l}{a\mu_0\mu_r} = \dfrac{0\cdot 942}{(4\times10^{-4})\times4\pi\times10^{-7}\times50} = 0{\cdot}375 \times 10^8$ AT/Wb

$\therefore$ Flux in the iron ring, $\phi_2 = \dfrac{1200I}{0\cdot375\times10^8} = 320 \times 10^{-7}\, I$ Wb

$\therefore$ Total flux in the two rings $= (6{\cdot}4 + 320)\, 10^{-7}\, I = 326{\cdot}4 \times 10^{-7}\, I$ Wb

But the sum of two fluxes in the rings is given to be $0{\cdot}2 \times 10^{-3}$ Wb.

$\therefore$ $326{\cdot}4 \times 10^{-7}\, I = 0{\cdot}2 \times 10^{-3}$ or $I = \dfrac{0\cdot2\times10^{-3}}{326\cdot4\times10^{-7}} = 6{\cdot}13$ A

$\therefore$ Battery terminal voltage $= IR = 6{\cdot}13 \times 6 =$ **36·78 V**

Example 8.10. *In the magnetic circuit shown in Fig. 8.14, find (i) the total reluctance of the magnetic circuit and (ii) value of flux linking the coil. Assume that the relative permeability of the magnetic material is 800. The exciting coil has 1000 turns and carries a current of 1·25 A.*

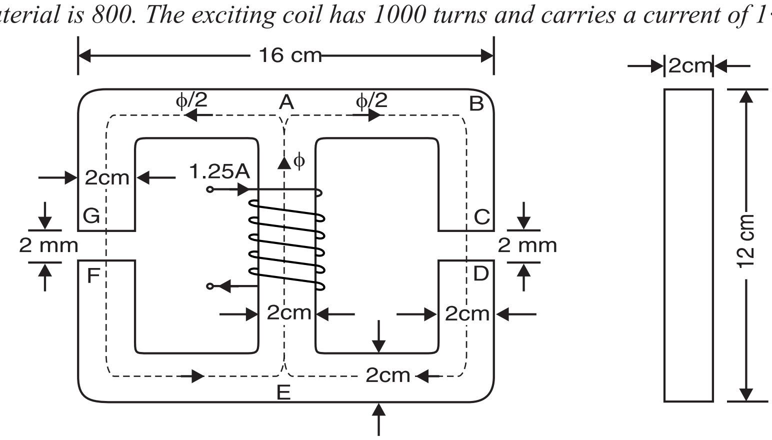


Fig. 8.14

Solution. The total flux ϕ set up by the exciting coil is divided into two parallel paths *viz*. path *AGFE* and path *ABCDE*. Since the two parallel paths are identical, each path will carry a flux = $\phi/2$ and that each parallel path has the same reluctance.

$l_{AE} = 10$ cm ; $l_{AG} = l_{FE} = 12$ cm ; $l_{GF} = 2$ mm ; $a = 2 \times 2 = 4$ cm^2

(*i*) Reluctance of magnetic path *AGFE*

$$= 2^* \text{(Reluct. of path } AG) + \text{Reluct. of air gap } GF$$

$$= 2\left(\frac{l_{AG}}{a\mu_0\mu_r}\right) + \frac{l_{GF}}{a\mu_0}$$

$$= 2\left[\frac{0\cdot 12}{(4\times10^{-4})\times4\pi\times10^{-7}\times800}\right] + \frac{2\times10^{-3}}{4\times10^{-4}\times4\pi\times10^{-7}}$$

$$= 5{\cdot}968 \times 10^5 + 39{\cdot}788 \times 10^5 = 45{\cdot}756 \times 10^5 \text{ AT/Wb}$$

Reluctance of magnetic path *AE* will be

$$= \frac{l_{AE}}{a\mu_0\mu_r} = \frac{0\cdot 1}{(4\times10^{-4})\times4\pi\times10^{-7}\times800} = 2{\cdot}486 \times 10^5 \text{ AT/Wb}$$

Total reluctance of magnetic circuit will be

$$= 45{\cdot}75 \times 10^5 + 2{\cdot}486 \times 10^5 = \mathbf{48{\cdot}242 \times 10^5 \text{ AT/Wb}}$$

(*ii*) m.m.f. = flux × reluctance

or $1000 \times 1{\cdot}25 = \phi \times (48.242 \times 10^5)$

$$\therefore \quad \phi = \frac{1000\times1\cdot25}{48\cdot242\times10^5} = \mathbf{25{\cdot}9 \times 10^{-5} \text{ Wb}}$$

Example 8.11. *A magnetic circuit consists of three parts in series, each of uniform cross-sectional area. They are :*

(*a*) *a length of 80 mm and cross-sectional area 50 mm*^{2}

(*b*) *a length of 60 mm and cross-sectional area 90 mm*^{2}

(*c*) *an air gap of length 0·5 mm and cross-sectional area 150 mm*^{2}.

A coil of 4000 turns is wound on part (b) and the flux density in the air gap is 0·3 Wb/m^2. *Assuming that all the flux passes through the given circuit, and that relative permeability* μ_r *is 1300, estimate the coil current to produce such a flux density.*

Solution. Flux in the circuit, $\phi = B_g \times a_g = 0{\cdot}3 \times 1{\cdot}5 \times 10^{-4} = 0{\cdot}45 \times 10^{-4}$ Wb/m^2

$$\text{m.m.f. required for part } (a) = \phi\, S_a = \phi\times\frac{l_a}{\mu_0\mu_r a_a}$$

$$= 0\cdot45\times10^{-4}\times\frac{80\times10^{-3}}{4\pi\times10^{-7}\times1300\times50\times10^{-6}} = 44{\cdot}07 \text{ AT}$$

$$\text{m.m.f. required for part } (b) = \phi\, S_b = \phi\times\frac{l_b}{\mu_0\mu_r a_b}$$

$$= 0\cdot45\times10^{-4}\times\frac{60\times10^{-3}}{4\pi\times10^{-7}\times1300\times90\times10^{-6}} = 18{\cdot}4 \text{ AT}$$

$$\text{m.m.f. required for part } (c) = \phi\, S_c = \phi\times\frac{l_c}{\mu_0 a_c}$$

$$= 0\cdot45\times10^{-4}\times\frac{0\cdot5\times10^{-3}}{4\pi\times10^{-7}\times150\times10^{-6}} = 119{\cdot}3 \text{ AT}$$

$$\text{Total m.m.f. required} = 44{\cdot}07 + 18{\cdot}4 + 119{\cdot}3 = 181{\cdot}77 \text{ AT}$$

* Reluctance of path *AG* = Reluctance of path *FE*

∴ $NI = 181{\cdot}77$ or $I = 181{\cdot}77/N = 181{\cdot}77/4000 = 45{\cdot}4 \times 10^{-3}$ A = **45·4 mA**

Since the absolute permeability of air (μ_0) is much smaller than that of a ferromagnetic material, the value of reluctance of air gap (= $l_g/a_g\mu_0$) is much greater than the reluctance of adjacent magnetic material (= $l_i/a_i\mu_0\mu_r$). Therefore, the m.m.f. required to force flux through the air gap can be quite large.

Example 8.12. *A laminated soft-iron ring has a mean circumference of 600 mm, cross-sectional area 500 mm² and has a radial air gap of 1 mm cut through it. It is wound with a coil of 1000 turns. Estimate the current in the coil to produce a flux of 0·5 mWb in the air gap assuming :*

(*i*) *the relative permeability of the soft iron is 1000,* (*ii*) *the leakage factor is 1·2,* (*iii*) *fringing is negligible,* (*iv*) *the space factor is 0·9.*

Solution. AT for air-gap

$\phi_g = 0{\cdot}5 \text{ mWb} = 5 \times 10^{-4}$ Wb ; $l_g = 1 \times 10^{-3}$ m ; $a_g = 500 \times 10^{-6}$ m²

$$\text{m.m.f. for air gap} = \phi_g S_g = \phi_g \times \frac{l_g}{\mu_0 a_g}$$

$$= 5\times10^{-4}\times\frac{1\times10^{-3}}{4\pi\times10^{-7}\times500\times10^{-6}} = 795{\cdot}7 \text{ AT}$$

AT for iron part

$\phi_i = \phi_g \times 1{\cdot}2^* = 5 \times 10^{-4} \times 1{\cdot}2$ Wb ; $l_i = 600 \times 10^{-3}$ m ; $a_i = 500 \times 10^{-6} \times 0{\cdot}9^{**}$m²

$$\therefore \quad \text{m.m.f. for iron part} = \phi_i S_i = \phi_i \times \frac{l_i}{\mu_0\mu_r a_i}$$

$$= 5\times10^{-4}\times1{\cdot}2\times\frac{600\times10^{-3}}{4\pi\times10^{-7}\times1000\times500\times10^{-6}\times0{\cdot}9}$$

$$= 636{\cdot}6 \text{ AT}$$

∴ Total m.m.f. required = 795·7 + 636·6 = 1432·3 AT

Now $NI = 1432{\cdot}3$ ∴ $I = 1432{\cdot}3/N = 1432.3/1000 =$ **1·432 A**

Note that *AT* for air-gap are comparable to that for iron part. It is because length of air gap is very small.

Example 8.13. *The ring-shaped core shown in Fig. 8.15 is made of material having relative permeability 1000. The flux density in the thicker section is 1·5 T. If the current through the coil is not to exceed 0·5 A, find the number of turns of the coil.*

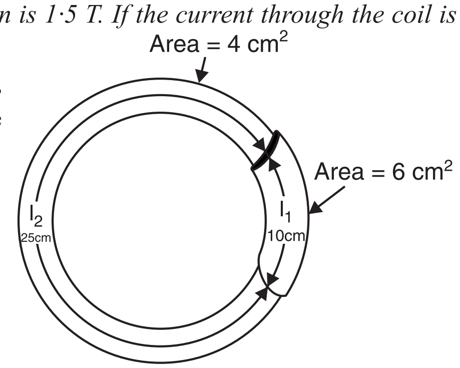


Fig. 8.15

Solution. The statement of the problem suggests that flux in the thicker as well as in thin section is the same *i.e.* it is a series magnetic circuit.

Flux in the magnetic circuit is

$$\phi = 1{\cdot}5 \times 6 \times 10^{-4}$$
$$= 9 \times 10^{-4} \text{ Wb}$$

AT for thick section

$$H_1 = \frac{B_1}{\mu_0\mu_r} = \frac{1{\cdot}5}{4\pi\times10^{-7}\times1000} = 1194 \text{ AT/m}$$

m.m.f. for thick section = $H_1 l_1 = (1194) \times (10 \times 10^{-2})$

* The leakage factor refers to the flux leakage in the iron part of the magnetic circuit.

$$\text{Leakage factor} = \frac{\text{Total flux}}{\text{Useful flux}}$$

** $$\text{Space factor} = \frac{\text{Useful area}}{\text{Total area}}$$

$= 119{\cdot}4$ AT

AT for thin section $B_2 = \dfrac{\phi}{a} = \dfrac{9\times10^{-4}}{4\times10^{-4}} = 2{\cdot}25$ T

$$H_2 = \frac{B_2}{\mu_0\mu_r} = \frac{2\cdot25}{4\pi\times10^{-7}\times1000} = 1790 \text{ AT/m}$$

m.m.f. for thin section $= H_2\, l_2 = (1790)\times(25\times10^{-2}) = 448$ AT

$\therefore$ Total m.m.f. required $= 119{\cdot}4 + 448 = 567{\cdot}4$ AT

Now $NI = 567{\cdot}4$ or $N = 567{\cdot}4/I = 567{\cdot}4/0{\cdot}5 =$ **1135**

Example 8.14. *A steel ring 30 cm mean diameter and of circular section 2 cm in diameter has an air gap 1 mm long. It is wound uniformly with 600 turns of wire carrying current of 2·5 A. Find (i) total m.m.f., (ii) total reluctance and (iii) flux. Neglect magnetic leakage. The iron path takes 40% of the total m.m.f.*

Solution. (*i*) Total m.m.f. $= NI = 600\times2{\cdot}5 =$ **1500 AT**

(*ii*) Let M_1 and M_2 be the m.m.f. for iron part and air gap respectively and S_1 and S_2 their corresponding reluctances.

$$M_1 = 40\% \text{ of } 1500 = (40/100)\times1500 = 600 \text{ AT}$$

$$M_2 = 1500 - 600 = 900 \text{ AT}$$

Now, $M_1 = \phi S_1$ and $M_2 = \phi S_2$

$$\therefore \quad \frac{S_1}{S_2} = \frac{M_1}{M_2} = \frac{600}{900} = 0{\cdot}67$$

$$S_2 = \frac{l_g}{a\mu_0} = \frac{1\times10^{-3}}{\pi(1\times10^{-2})^2\times4\pi\times10^{-7}} = 2{\cdot}5\times10^6 \text{ AT/Wb}$$

$$\therefore \quad S_1 = 0{\cdot}67S_2 = 0{\cdot}67\times(2{\cdot}5\times10^6) = 1{\cdot}675\times10^6 \text{ AT/Wb}$$

Total reluctance $= S_1 + S_2 = (1{\cdot}675 + 2{\cdot}5)\times10^6 =$ **$4{\cdot}175\times10^6$ AT/Wb**

(*iii*)
$$\text{Flux} = \frac{\text{Total m.m.f.}}{\text{Total reluctance}} = \frac{1500}{4\cdot175\times10^6}$$

$= 0{\cdot}36\times10^{-3}$ Wb $=$ **0·36 mWb**

Example 8.15. *A cast steel magnetic structure made of a bar of section 2 cm × 2 cm is shown in Fig. 8.16. Determine the current that the 500 turn magnetising coil on the left limb should carry so that a flux of 2mWb is produced in the right limb. Take $\mu_r = 600$ and neglect leakage.*

Solution. The magnetising coil on the left limb produces flux ϕ which divides into two parallel paths; ϕ_1 in path B and ϕ_2 in path C. Since paths B and C are in parallel, AT required for path B ($= \phi_1 S_B$) are equal to that required for path C ($= \phi_2 S_c$) *i.e.*

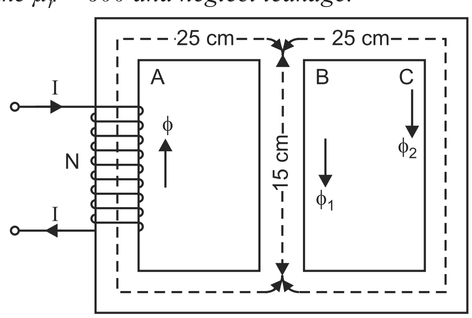


Fig. 8.16

$$\phi_1 S_B = \phi_2 S_c$$

or
$$\phi_1\times\frac{l_B}{\mu_0\mu_r a} = \phi_2\times\frac{l_C}{\mu_0\mu_r a}$$

$$\therefore \quad \phi_1 = \phi_2\times\frac{l_C}{l_B} = 2\times\frac{25}{15} = \frac{10}{3}\text{mWb} \qquad (\because\ \phi_2 = 2\text{mWb})$$

Total flux in path A, $\phi = \phi_1 + \phi_2 = \dfrac{10}{3} + 2 = \dfrac{16}{3}$ mWb

Total AT required for the whole magnetic circuit are equal to the sum of (i) AT required for path A and (ii) AT required for one of the parallel paths B or C.

$$\text{Flux density in path } A, B_A = \frac{\phi}{a} = \frac{(16/3)\times10^{-3}}{4\times10^{-4}} = \frac{40}{3}\ \text{Wb/m}^2$$

$$AT \text{ required for path } A = \frac{B_A}{\mu_0\mu_r}\times l_A \times \frac{(40/3)}{4\pi\times10^{-7}\times600}\times0.25 = 4420 \text{ AT}$$

$$\text{Flux density in path } B, B_B = \frac{\phi_1}{a} = \frac{(10/3)\times10^{-3}}{4\times10^{-4}} = 8.33 \text{ Wb/m}^2$$

$$AT \text{ required for path } B = \frac{B_B}{\mu_0\mu_r}\times l_B = \frac{8.33}{4\pi\times10^{-7}\times600}\times0.15 = 1658 \text{ AT}$$

$\therefore$ Total AT required = 4420 + 1658 = 6078 AT

Now, $NI = 6078 \quad \therefore I = \frac{6078}{N} = \frac{6078}{500} =$ **12.16 A**

Example 8.16. *A magnetic core made of annealed sheet steel has the dimensions as shown in Fig. 8.17. The X-section is 25 cm^2 everywhere. The flux in branches A and B is 3500 µWb but that in the branch C is zero. Find the required ampere-turns for coil A and for coil C. Relative permeability of sheet steel is 1000.*

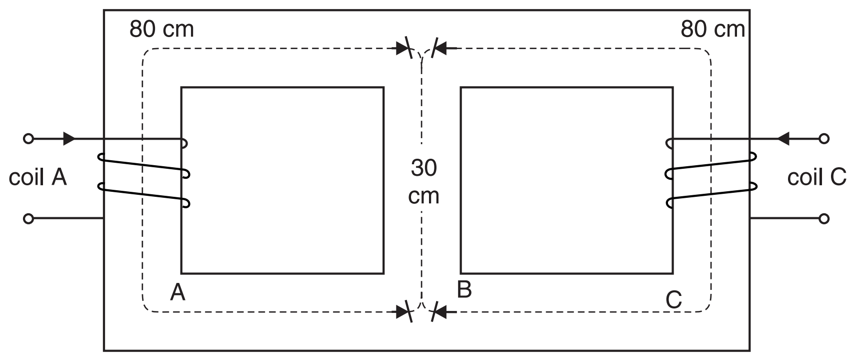


Fig. 8.17

Solution. AT for coil *A*. Flux paths B and C are in parallel. Therefore, AT required for coil A is equal to AT for path A plus AT for path B or path C.

$$\text{AT for path } A = \text{flux} \times \text{reluctance} = (3500\times10^{-6})\times\frac{0\cdot8}{(25\times10^{-4})\times4\pi\times10^{-7}\times1000} = 891\cdot3 \text{ AT}$$

$$\text{AT for path B} = \text{flux} \times \text{reluctance} = (3500\times10^{-6})\times\frac{0\cdot3}{(25\times10^{-4})\times4\pi\times10^{-7}\times1000} = 334\cdot2 \text{ AT}$$

Total AT for coil A = 891·3 + 334·2 = **1225·5 AT**

AT for coil *C*. The coil C produces flux ϕ_C µWb in the opposite direction to that produced by coil A.

$$\text{m.m.f. of path } B = \text{m.m.f. of path } C$$

$$\phi_B S_B = \phi_C S_C$$

$$\text{or} \quad (3500\times10^{-6})\times\frac{l_B}{a\mu_0\mu_r} = \phi_C\times\frac{l_C}{a\mu_0\mu_r}$$

$$\therefore \phi_C = (3500\times10^{-6})\times l_B/l_C = (3500\times10^{-6})\times0\cdot3/0\cdot8 = 1312\cdot5\ \mu\text{Wb}$$

$$\text{Total AT for coil } C = \phi_C \times \text{reluctance}$$

$$= (1312\cdot5\times10^{-6})\times\frac{0\cdot8}{(25\times10^{-4})\times4\pi\times10^{-7}\times1000} = \mathbf{334.22\ AT}$$

Example 8.17. *A magnetic circuit is shown in Fig. 8.18. It is made of cast steel 0.05 m thick. The length of air gap is 0.003 m. Find the m.m.f. to establish a flux of 5 × 10^{-4} Wb in the air gap. The relative permeability for the material is 800.*

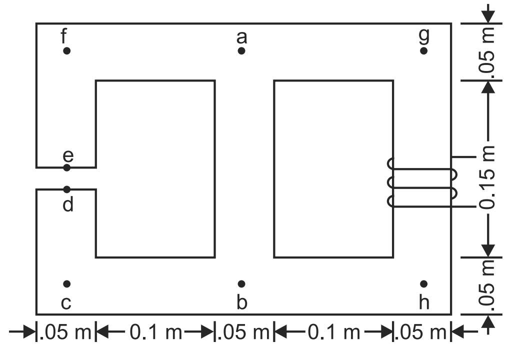


Fig. 8.18

Solution. The flux ϕ set up by the current-carrying coil in the path *bhga* divides into two parallel paths *viz* path *ab* and path *aedb*. Therefore, total m.m.f. required is equal to *AT* required for path *bhga plus AT* required for one of the parallel paths (*i.e.* path *aedb* or path *ab*) *i.e.*

Total m.m.f = *AT* for path *aedb* + *AT* for path *bhga*

1. AT for path aedb. The m.m.f. required for this path is equal to *AT* required for air gap *ed plus AT* required for steel path (*ae* + *db*)

(*i*) *AT for air gap*. $\phi_g = 5 \times 10^{-4}$ Wb ; $a_g = 0.05 \times 0.05 = 0.0025$ m^2 ; $l_g = 0.003$ m

$$\therefore \quad B_g = \frac{\phi_g}{a_g} = \frac{5\times10^{-4}}{0.0025} = 0.2 \text{ Wb/m}^2$$

$$\text{Now,} \quad H_g = \frac{B_g}{\mu_0} = \frac{0.2}{4\pi\times10^{-7}} = 15.92 \times 10^4 \text{ AT/m}$$

$$\therefore \quad AT_g = H_g \times l_g = 15.92 \times 10^4 \times 0.003 = 477.6 \text{ AT}$$

(*ii*) *AT for steel path (ae + db)*. The flux density in this path is also 0.2 Wb/m^2.

$$l_{ae} + l_{bd} = 0.5 - 0.003 = 0.497 \text{ m}$$

$$\text{Magnetising force, } H_s = \frac{0.2}{\mu_0\mu_r} = \frac{0.2}{4\pi\times10^{-7}\times800} = 198.94 \text{ AT/m}$$

$$\therefore \quad AT_s = 198.94 \times 0.497 = 98.87 \text{ AT}$$

$\therefore$ *AT* required for path *aedb* = 477.6 + 98.87 = 576.47 AT = AT_{ab}

2. AT for path *bhga*. We first find flux ϕ in this path. Now, $l_{ab} = 0.2$ m.

Also, AT_{ab} = 576.47 AT ... Calculated above

$$\text{Flux density, } B_{ab} = \frac{AT_{ab}\times\mu_0\mu_r}{l_{ab}} = \frac{576.47\times4\pi\times10^{-7}\times800}{0.2} = 2.898 \text{ Wb/m}^2$$

$$\text{Flux, } \phi_{ab} = B_{ab} \times a = 2.898 \times 0.0025 = 0.007245 \text{ Wb}$$

$$\therefore \quad \phi = \phi_g + \phi_{ab} = 5 \times 10^{-4} + 0.007245 = 0.007745 \text{ Wb}$$

$$\text{Flux density in path } bhga = \frac{\phi}{a} = \frac{0.007745}{0.0025} = 3.098 \text{ Wb/m}^2$$

$$\text{Magnetising force, } H = \frac{3.098}{\mu_0\mu_r} = \frac{3.098}{4\pi\times10^{-7}\times800} = 3081.63 \text{ AT/m}$$

Length of path *bhga*, l = 0.5 m

AT for path *bhga* = $H \times l$ = 3081 .63 × 0.5 = 1540.815 AT

$\therefore$ Total m.m.f. required = 576.47 + 1540.815 = **2117.285 AT**

Example 8.18. *The magnetic core shown in Fig. 8.19 has the following dimensions :*

l_1 = 10 cm ; $l_2 = l_3$ = 18 cm ; cross-sectional area of l_1 path = 6.25 × 10^{-4} m^2 ; cross-sectional areas of l_2 and l_3 paths = 3 × 10^{-4} m^2 ; length of air gap, l_4 = 2mm.

Determine the current that must be passed through the 600-turn coil to produce a total flux of 100 µWb in the air gap. Assume that the metal has relative permeability of 800.

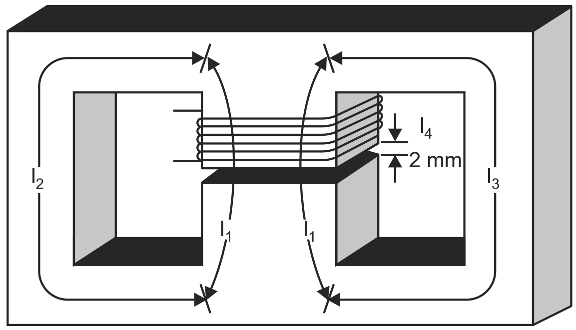


Fig. 8.19

Solution. $\phi_g = 100\ \mu\text{Wb} = 100 \times 10^{-6}$ Wb ; $a_g = 6.25 \times 10^{-4}$ m^2

AT for air gap. $B_g = \dfrac{\phi_g}{a_g} = \dfrac{100\times10^{-6}}{6.25\times10^{-4}} = 0.16$ Wb/m^2

Now, $H_g = \dfrac{B_g}{\mu_0} = \dfrac{0.16}{4\pi\times10^{-7}} = 1.27 \times 10^5$ AT/m

$\therefore$ $AT_g = H_g \times l_g = 1.27 \times 10^5 \times 2 \times 10^{-3} = 254$ AT

AT for path l_1. $B_1 = 0.16$ Wb/m^2 ; $l_1 = 10 \times 10^{-2}$ m

Now, $H_1 = \dfrac{B_1}{\mu_0\mu_r} = \dfrac{0.16}{4\pi\times10^{-7}\times800} = 159$ AT/m

$\therefore$ $AT_1 = H_1 \times l_1 = 159 \times 10 \times 10^{-2} = 15.9$ AT

Here, we neglect l_g, being very small, compared to iron path. Paths l_2 and l_3 are similar so that total flux (= 100×10^{-6} Wb) divides equally between these two paths. Since paths l_2 and l_3 are in parallel, it is necessary to consider m.m.f. for only one of them. Let us find AT for path l_2.

AT for path l_2. $\phi_2 = 50 \times 10^{-6}$ Wb ; $\mu_r = 800$; $l_2 = 18 \times 10^{-2}$ m

$\therefore$ $B_2 = \dfrac{\phi_2}{a} = \dfrac{50\times10^{-6}}{3\times10^{-4}} = 0.167$ Wb/m^2

Now, $H_2 = \dfrac{B_2}{\mu_0\mu_r} = \dfrac{0.167}{4\pi\times10^{-7}\times800} = 166$ AT/m

$\therefore$ $AT_2 = H_2 \times l_2 = 166 \times 18 \times 10^{-2} = 29.9$ AT

$\therefore$ Total $AT = 254 + 15.9 + 29.9 = 300$ AT

Now, $NI = 300$ or $I = \dfrac{300}{N} = \dfrac{300}{600} = 0.5$ A = **500 mA**

Tutorial Problems

1. It is required to produce a flux density of 0·6 Wb/m^2 in an air gap having a length of 8 mm. Calculate the m.m.f. required. **[480 × 10^3 AT/m]**
2. A coil of 200 turns is wound uniformly over a wooden ring having a mean circumference of 60 cm and a uniform cross-sectional area of 5 cm^2. If the current through the coil is 4A, calculate (*i*) the magnetising force (*ii*) the flux density and (*iii*) the total flux. **[(*i*) 1333 AT/m (*ii*) 1675 µWb/m^2 (*iii*) 0·8375 µWb]**
3. A core forms a closed magnetic loop of path length 32 cm. Half of this path has a cross-sectional area of 2 cm^2 and relative permeability 800. The other half has a cross-sectional area of 4 cm^2 and relative

permeability 400. Find the current needed to produce a flux of 0·4 Wb in the core if it is wound with 1000 turns of insulated wire. Ignore leakage and fringing effects. **[636·8 A]**

4. An iron ring has a cross-sectional area of 400 mm^2 and a mean diameter of 250 mm. An air gap of 1 mm has been made by a saw-cut across the section of the ring. If a magnetic flux of 0·3 mWb is required in the air gap, find the current necessary to produce this flux when a coil of 400 turns is wound on the ring. The iron has a relative permeability of 500. **[3·84 A]**

5. An iron ring has a mean circumferential length of 60 cm and a uniform winding of 300 turns. An air gap has been made by a saw-cut across the section of the ring. When a current of 1 A flows through the coil, the flux density in the air gap is found to be 0·126 Wb/m^2. How long is the air gap ? Assume iron has a relative permeability of 300. **[1 mm]**

6. An iron magnetic circuit has a uniform cross-sectional area of 5 cm^2 and a length of 25 cm. A coil of 120 turns is wound uniformly over the magnetic circuit. When the current in the coil is 1·5 A, the total flux is 0·3 Wb. Find the relative permeability of iron. **[663]**

7. The uneven ring-shaped core shown in Fig. 8.20 has $\mu_r = 1000$ and the flux density in the thicker section is to be 0.75 T. If the current through a coil wound on the core is to be 500 mA, determine number of coil turns required. **[567]**

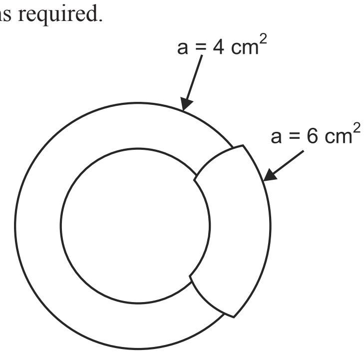


Fig. 8.20

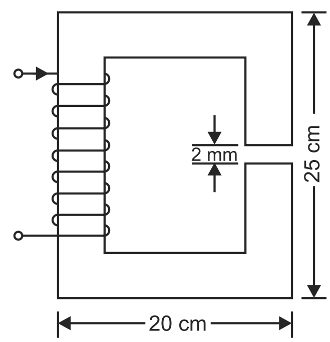


Fig. 8.21

8. A rectangular magnetic core shown in Fig. 8.21 has square cross section of area 16 cm^2. An air gap of 2 mm is cut across one of its limbs. Find the exciting current needed in the coil having 1,000 turns wound on the core to create an air-gap flux of 4 mWb. The relative permeability of the core is 2000. **[4.713 A]**

9. The magnetic circuit of Fig. 8.22 is energised by a current of 3A. If the coil has 1500 turns, find the flux produced in the air gap. The relative permeability of the core material is 3000. **[65.25×10^{-4} Wb]**

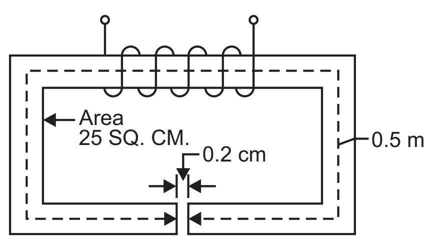


Fig. 8.22

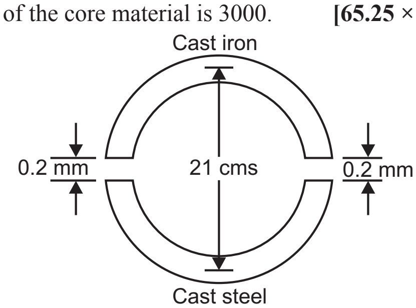


Fig. 8.23

10. A ring [See Fig. 8.23] has a diameter of 21 cm and a cross-sectional area of 10 cm^2. The ring is made up of semicircular sections of cast iron and cast steel with each joint having a reluctance equal to an air gap of 0.2 mm. Find the ampere turns required to produce a flux of 8×10^{-4} Wb. The relative permeabilities of cast steel and cast iron are 800 and 166 respectively. Neglect leakage and fringing effects. **[1783 AT]**

8.11. B-H Curve

The *B-H* curve (or magnetisation curve) indicates the manner in which the flux density (*B*) varies with the magnetising force (*H*).

(*i*) For non-magnetic materials. For non-magnetic materials (*e.g.* air, copper, rubber, wood *etc.*), the relation between B and H is given by ;

$$B = \mu_0 H$$

Since μ_0 (= $4\pi \times 10^{-7}$ H/m) is constant,

$$\therefore \qquad B \propto H$$

Hence, the *B-H* curve of a non-magnetic material is a straight line passing through the origin as shown in Fig. 8.24. Two things are worth noting. First, the curve never saturates no matter how great the flux density may be. Secondly, a large m.m.f. is required to produce a given flux in the non-magnetic material *e.g.* air.

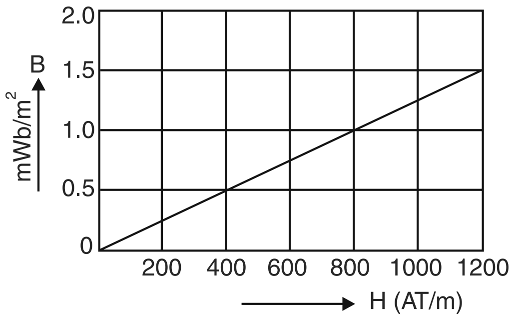


Fig. 8.24

(*ii*) For magnetic materials. For magnetic materials (*e.g.* iron, steel *etc.*), the relation between B and H is given by ;

$$B = \mu_0 \mu_r H$$

Unfortunately, μ_r is not constant but varies with the flux density. Consequently, the *B-H* curve of a magnetic material is not linear. Fig. 8.25 (*i*) shows the general *shape of *B-H* curve of a magnetic material. The non-linearity of the curve indicates that relative permeability μ_r (= $B/\mu_0 H$) of a material is not constant but depends upon the flux density. Fig. 8.25 (*ii*) shows how relative permeability μ_r of a magnetic material (cast steel) varies with flux density.

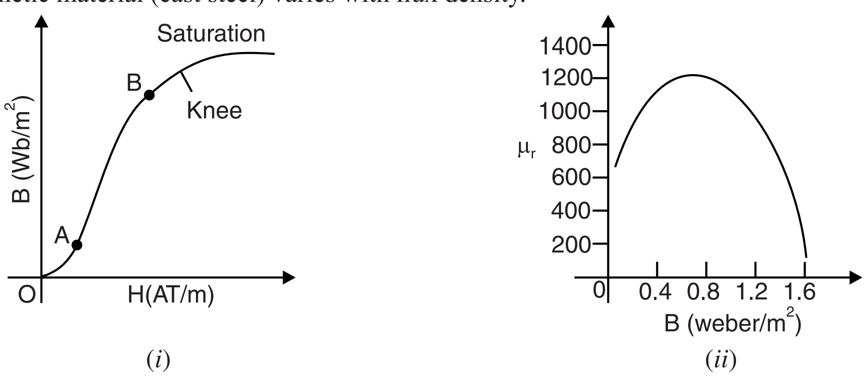


Fig. 8.25

While carrying out magnetic calculations, it should be ensured that the values of μ_r and H are taken at the working flux density. For this purpose, the *B-H* curve of the material in question may be very helpful. In fact, the use of *B-H* curves permits the calculations of magnetic circuits with a fair degree of ease.

8.12. Magnetic Calculations From B-H Curves

The solution of magnetic circuits can be easily obtained by the use of *B-H* curves. The procedure is as under :

(*i*) Corresponding to the flux density B in the material, find the magnetising force H from the *B-H* curve of the material.

* Note the shape of the curve. It is slightly concave up for 'low' flux densities (portion *OA*) and exhibits a straight line character (portion *AB*) for 'medium' flux densities. In the portion *AB* of the curve, the μ_r of the material is almost constant. For higher flux 'densities', the curve concaves down (called the *knee* of the curve). After knee of the curve, any further increase in H does not increase B. From now onwards, the curve is almost flat and the material is said to be *saturated.* In terms of molecular theory, saturation can be explained as the point at which all the molecular magnets are oriented in the direction of applied H.

(ii) Compute the magnetic length l.

(iii) m.m.f. required = $H \times l$

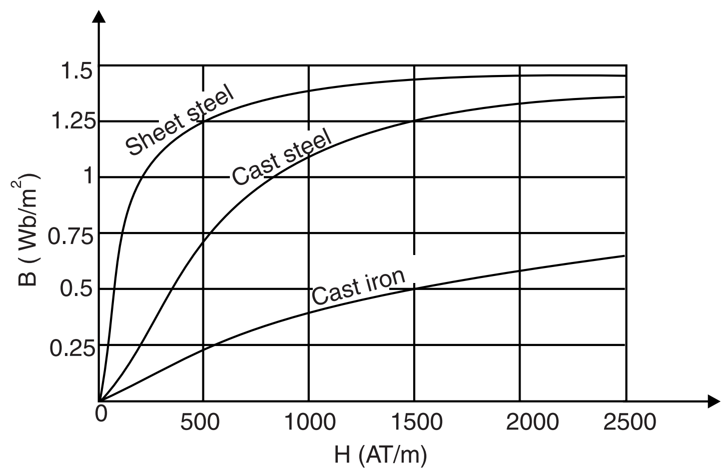


Fig. 8.26

The reader may note that the use of *B-H* curves for magnetic calculations saves a lot of time. Fig. 8.26 shows the *B-H* curves for sheet steel, cast steel and cast iron.

Note. We do not use *B-H* curve to find m.m.f. for air gap. We can find H_g directly from B_g/μ_0 and hence the m.m.f. = $H_g \times l_g$. However, in a magnetic material, *$H_i = B_i/\mu_0 \mu_r$. Since the value of μ_r depends upon the working flux density, this realtion will not yield correct result. Instead, we find H_i corresponding to B_i in the material from the *B-H* curve. Then m.m.f. required for iron path = $H_i \times l_i$.

Example 8.19. *A cast steel ring of mean diameter 30 cm having a circular cross-section of 5 cm^2 is uniformly wound with 500 turns. Determine the magnetising current required to establish a flux of 5 × 10^{-4} Wb (i) with no air gap (ii) with a radial air gap of 1 mm.*

The magnetisation curve for cast steel is given by the following :

B(*Wb*/*m*^{2})	0·2	0·4	0·6	0·8	1	1·2
H(*AT*/*m*)	175	300	400	600	850	1250

Solution. Plot the *B-H* curve from the given data as shown in Fig. 8.27.

***(i)* With no air gap**

$$B_i = \frac{\phi}{a} = \frac{5\times10^{-4}}{5\times10^{-4}} = 1 \text{ Wb/m}^2$$

From the *B-H* curve, we find that for a flux density of 1 Wb/m^2, the value of

$$H_i = 850 \text{ AT/m}$$

Now, $l_i = \pi D = \pi \times 30 \times 10^{-2}$

$= 0{\cdot}942$ m

$\therefore$ Total *AT* required $= H_i \times l_i$

$= 850 \times 0{\cdot}942 = 800{\cdot}7$ AT

$\therefore$ Magnetising current, I = 800·7/500 = **1·6 A**

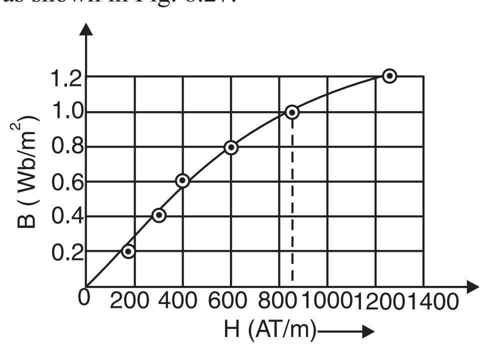


Fig. 8.27

***(ii)* With air gap of 1 mm**

Flux density in air gap, B_g = 1 Wb/m^2 (same as in steel)

* The suffix i denotes iron part while suffix g denotes air gap.

Magnetising force required, $^*H_g = \dfrac{B}{\mu_0} = \dfrac{1}{4\pi\times10^{-7}}$ = 7·96 × 10^5 AT/m

AT required for air gap = $H_g \times l_g$ = (7·96 × 10^5) × (1 × 10^{-3}) = 796 AT

Total AT required = 800·7 + 796 = 1596·7 AT

∴ Magnetising current, I = 1596·7/500 = **3·19 A**

Example 8.20. *A magnetic circuit made of wrought iron is arranged as shown in Fig. 8.28. The central limb has a cross-sectional area of 8 cm^2 and each of the side limbs has a cross-sectional area of 5 cm^2. Calculate the ampere-turns required to produce a flux of 1 mWb in the central limb, assuming the magnetic leakage is negligible. Given that for wrought iron (from B-H curve), H = 500 AT/m at B = 1·25 Wb/m^2 and H = 200 AT/m at B = 1 Wb/m^2.*

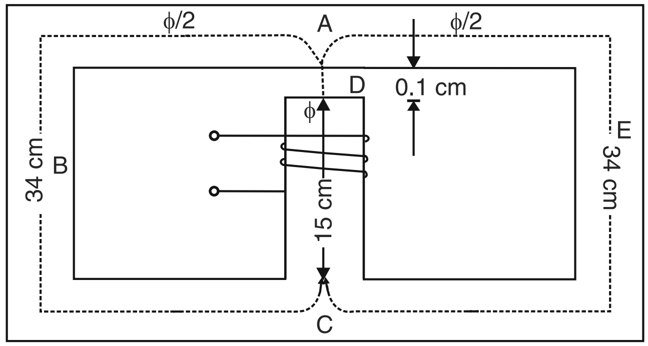


Fig. 8.28

Solution. The flux ϕ set up in the central limb divides equally into two identical parallel paths *viz.* path ABC and path AEC. The toal m.m.f. required for the entire circuit is the sum of the following three m.m.fs' :

(*i*) that required for path CD

(*ii*) that required for air gap DA

(*iii*) that required for either of parallel paths (*i.e.* path ABC or path AEC).

(*i*) AT for path CD

$$B = \frac{\phi}{a} = \frac{1\times10^{-3}}{8\times10^{-4}} = 1.25 \text{ Wb/m}^2$$

Now H at 1·25 Wb/m^2 = 500 AT/m (*given*)

∴ AT required for path CD = 500 × 0·15 = 75 AT

(*ii*) AT for air gap DA

$$H \text{ in air gap} = \frac{B}{\mu_0} = \frac{1\cdot25}{4\pi\times10^{-7}} = 994\cdot7\times10^3 \text{ AT/m}$$

∴ AT required for air gap = (994·7 × 10^3) × (0·1 × 10^{-2}) = 994·7 AT

(*iii*) AT for path ABC

Flux in path ABC = ϕ/2 = 1/2 = 0·5 mWb

$$\text{Flux density in path } ABC = \frac{0.5\times10^{-3}}{5\times10^{-4}} = 1 \text{ Wb/m}^2$$

Now H at 1 Wb/m^2 = 200 AT/m (*given*)

∴ AT required for path ABC = 200 × 0·34 = 68 AT

∴ Total AT required = 75 + 994·7 + 68 = **1137·7 AT**

The reader may note that air gap "grabs" 87 per cent of the applied ampere-turns.

* We do not use *B-H* curve to find AT for air gap. It is because μ_r for air (in fact for all non-magnetic materials) is constant, being equal to 1, and AT can be calculated directly.

Example 8.21. *A series magnetic circuit comprises three sections (i) length of 80 mm with cross-sectional area 60 mm^2, (ii) length of 70 mm with cross-sectional area 80 mm^2 and (iii) air gap of length 0.5 mm with cross-sectional area 60 mm^2. Sections (i) and (ii) are of a material having magnetic characteristics given by the following table.*

H(AT/m)	*100*	*210*	*340*	*500*	*800*	*1500*
B(Tesla)	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*	*1.2*

Determine the current necessary in a coil of 4000 turns wound on section (ii) to produce a flux density of 0.7 T in the air gap. Neglect magnetic leakage.

Solution. Air-gap flux density, $B_g = 0.7$ T ; Air-gap area, $a_g = 60 \times 10^{-6}$ m^2

Air-gap, flux, $\phi_g = B_g \times a_g = 0.7 \times 60 \times 10^{-6} = 42 \times 10^{-6}$ Wb

Since it is a series magnetic circuit, the flux in each of the three sections will be the same $(=\phi_g = 42 \times 10^{-6}$ Wb) but flux density will depend on the area of *X*-section of the section.

AT for section (*i*). $B = 0.7$ T because it has the same cross-sectional area as the air gap. If we plot the $B - H$ curve, it will be found that corresponding to $B = 0.7$ T, $H = 415$ AT/m.

$\therefore$ *AT* required for section (*i*) $= H \times l = 415 \times 80 \times 10^{-3} = 33.2$ AT

AT for section (*ii*). $B = \dfrac{\phi_g}{a} = \dfrac{42\times10^{-6}}{80\times10^{-6}} = 0.525$ T

From *B*–*H* curve, corresponding to $B = 0.525$ T, $H = 285$ AT/m.

$\therefore$ *AT* required for section (*ii*) $= H \times l = 285 \times 70 \times 10^{-3} = 19.95$ AT

AT for section (*iii*). This section is air gap.

$$B_g = 0.7 \text{ T and } H_g = \frac{B_g}{\mu_0} = \frac{0.7}{4\pi\times10^{-7}} = 0.557 \times 10^6 \text{ AT/m}$$

$\therefore$ *AT* required for air gap $= H_g \times l_g = 0.557 \times 10^6 \times 0.5 \times 10^{-3} = 278.5$ AT

Total *AT* required $= 33.2 + 19.95 + 278.5 = 331.6$ AT

Now, $NI = 331.6$ or $I = \dfrac{331.6}{N} = \dfrac{331.6}{4000} =$ **0.083 A**

Example 8.22. *A magnetic circuit is made of mild steel arranged as shown in Fig. 8.29. The central limb is wound with 500 turns and has a cross-sectional area of 8cm^2. Each of the outer limbs has a cross-sectional area of 5 cm^2. The air gap has a length of 1 mm. Calculate the current required to set up a flux of 1·3 mWb in the central limb, assuming no magnetic leakage and fringing. The mean lengths of the various magnetic paths are shown in the diagram. Given that for mild steel (from B-H curve) H = 3800 AT/m at B = 1·625 T and H = 850 AT/m at B = 1·3 T.*

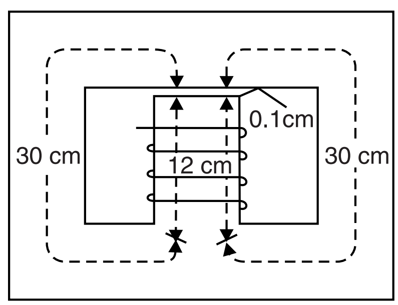


Fig. 8.29

Solution. Flux density in the central limb

$$= \frac{\text{Flux}}{\text{cross-sectional area}} = \frac{1.3\times10^{-3}}{8\times10^{-4}} = 1.625 \text{ T}$$

Given that $H = 3800$ AT/m at $B = 1{\cdot}625$ T

$\therefore$ m.m.f. for central limb $= H_1 l_1 = 3800 \times 0{\cdot}12 = 4{\cdot}56$ AT

Since half the flux returns through one outer limb and half through the other, the two outer limbs are magnetically equivalent to a single limb having a cross-sectional area of 10 cm^2 and a length of 30 cm.

$\therefore$ Flux density in outer limbs $= \dfrac{1.3\times10^{-3}}{10\times10^{-4}} = 1.3$ T

Given that H = 850 AT/m at $B = 1{\cdot}3$ T

∴ m.m.f. for outer limbs = $H_2 l_2 = 850 \times 0{\cdot}3 = 255$ AT

Flux density in airgap, B=1·625 T

Magnetising force for air gap is given by ;

$$H_3 = \frac{B}{\mu_0} = \frac{1\cdot625}{4\pi\times10^{-7}} = 1\cdot294\times10^{6}\ \text{AT/m}$$

m.m.f. for air gap = $H_3 l_3 = (1{\cdot}294 \times 10^6) \times (1 \times 10^{-3}) = 1294$ AT

Total m.m.f. = 456 + 255 + 1294 = 2005 AT

$$\therefore \quad \text{Magnetising current, } I = \frac{\text{Total m.m.f.}}{\text{Turns}} = \frac{2005}{500} \simeq \mathbf{4A}$$

Example 8.23. *Fig. 8.30 shows the cross-section of a simple relay. Calculate the ampere-turns required on the coil for a flux density of 0·1 Wb/m^2 in the air gaps from the following data :*

Cross-sectional area of yoke	=	*2 cm^2*
Magnetic length of yoke	=	*25 cm*
Cross-sectional area of armature	=	*3 cm^2*
Magnetic length of armature	=	*12 cm*
Air gap area	=	*6 cm^2*
Each air gap length	=	*5 mm*
Leakage coefficient	=	*1·33*

The yoke and armature material have the following magnetic characteristics :

H (AT/m)	*100*	*210*	*340*	*500*	*800*	*1500*
B (Wb/m^2)	*0·2*	*0·4*	*0·6*	*0·8*	*1·0*	*1·2*

Solution. Plot the B-H curve from the given data as shown in Fig. 8.31.

Flux in air gap, $\phi_g = 6 \times 10^{-4} \times 0{\cdot}1 = 6 \times 10^{-5}$ Wb = Flux in armature

Flux in yoke, $\phi_y = \lambda\, \phi_g = 1{\cdot}33 \times 6 \times 10^{-6} = 7{\cdot}98 \times 10^{-5}$ Wb

AT for armature

$$\text{Flux density in armature} = \frac{6\times10^{-5}}{3\times10^{-4}} = 0\cdot2\ \text{Wb/m}^2$$

Corresponding to $B = 0{\cdot}2$ Wb/m^2 (See B-H curve), H = 100 AT/m.

∴ AT required for armature = 100 × 0·12 = 12 AT

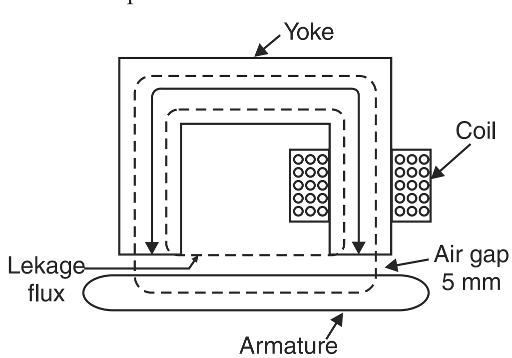


Fig. 8.30

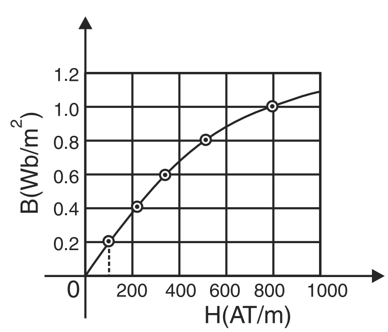


Fig. 8.31

AT for yoke

$$\text{Flux density in the yoke} = \frac{7\cdot98\times10^{-5}}{2\times10^{-4}} = 0\cdot4 \text{ Wb/m}^2$$

Corresponding to B = 0·4 Wb/m^2, H = 210 AT/m.

∴ AT required for yoke = 210 × 0·25 = 52·5 AT

AT for air gaps

$$\text{Magnetising force in air gaps} = \frac{0\cdot1}{\mu_0} = \frac{0\cdot1}{4\pi\times10^{-7}} = 7\cdot96\times10^4 \text{ AT/m}$$

AT for two air gaps = $(7\cdot96\times10^4)\times(10\times10^{-3})$ = 796 AT

Total AT required = 12 + 52·5 + 796 = **860·5 AT**

Example 8.24. *An iron ring of mean diameter 19.1 cm and having cross-sectional area of 4 cm^2 is required to produce a flux of 0.44 mWb. Find the coil m.m.f. required. If a saw-cut 1 mm wide is made in the ring, how many extra ampere-turns are required to maintain the same flux ? $B-\mu_r$ curve is as follows :*

B(Wb/m^2)	*0.8*	*1.0*	*1.2*	*1.4*
μ_r	*2300*	*2000*	*1600*	*1100*

Solution. D_m = 0.191 m ; $a = 4\times10^{-4}$ m^2; $\phi = 0.44\times10^{-3}$ Wb

Length of mean path, $l_m = \pi D_m = \pi\times0.191 = 0.6$ m

$$\text{Flux density in ring, } B_i = \frac{\phi}{a} = \frac{0.44\times10^{-3}}{4\times10^{-4}} = 1.1 \text{ Wb/m}^2$$

By *interpolation, for flux density of 1.1 Wb/m^2, μ_r = 1800.

$$\therefore \quad \text{Magnetising force, } H_i = \frac{B_i}{\mu_0\mu_r} = \frac{1.1}{4\pi\times10^{-7}\times1800} = 486.5 \text{ AT/m}$$

∴ m.m.f. required = $H_i\times l_m$ = 486.5 × 0.6 = **292 AT**

If a saw-cut of 1 mm wide is made in the ring, we require extra *AT* to maintain the same flux (= 0.44 × 10^{-3}Wb).

$$\text{Now } H_g = \frac{B_g}{\mu_0} = \frac{1.1}{4\pi\times10^{-7}} = 875352 \text{ AT/m} ;\ l_g = 1\times10^{-3} \text{ m}$$

∴ Extra m.m.f. required = $H_g\times l_g$ = 875352 × 1 × 10^{-3} = **875 AT**

Example 8.25. *A transformer core made of annealed steel sheet has the form and dimensions shown in Fig. 8.32. A coil of N turns is wound on the central limb. The average length of magnetic circuit (i.e. path ABCDA or path EFGHE) is 30 cm. Determine the ampere-turns of the coil required to produce a flux density of 1 Wb/m^2 in the central leg. What will be the total amount of flux in the central leg and in each outside leg ? Given that for annealed sheet steel (from B-H curve), H = 200 AT/m at 1 Wb/m^2.*

---

* For B = 1.0 Wb/m^2, μ_r = 2000 and for B = 1.2 Wb/m^2, μ_r = 1600. By interpolation, we are to find μ_r for B = 1.1Wb/m^2.

If increase in B is 0.2 Wb/m^2 (= 1.2 – 1.0 = 0.2), then decrease in μ_r is 400 (2000 – 1600 = 400). If increase in B is 0.1/Wb/m^2 (1.1 – 1.0 = 0.1), then decrease in μ_r

$$= \frac{400}{0.2}\times0.1 = 200$$

∴ μ_r at 1.1 Wb/m^2 = 2000 – 200 = 1800

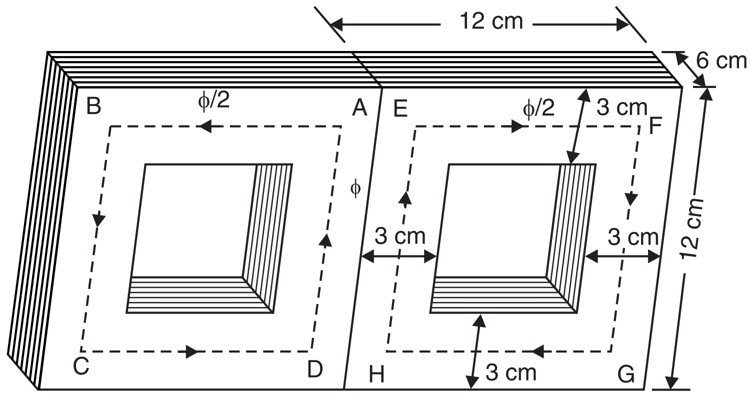


Fig. 8.32

Solution. It is a case of parallel magnetic circuit. It is clear from Fig. 8.32 that central leg has twice the area of an outside leg. The flux ϕ set up in the central limb divides equally into two parallel identical paths *viz*. path *ABCD* and path *EFGH*. It may be noted very carefully that flux density is the *same in the central leg, each outside leg and other parts.

Mean length of magnetic path (*i.e.* path *ABCDA* and *EFGHE*)

$$= 30 \text{ cm} = 0{\cdot}3 \text{ m}$$

$\therefore$ AT required $= 200 \times 0{\cdot}3 =$ **60 AT**

Area of central leg $= 0{\cdot}06 \times 0{\cdot}06 = 0{\cdot}0036 \text{ m}^2$

Flux in central leg $=$ Flux density $\times$ Area $= 1 \times 0{\cdot}0036 =$ **0·0036 Wb**

Area of each outside leg $= 0{\cdot}03 \times 0{\cdot}06 = 0{\cdot}0018 \text{ m}^2$

Flux in each outside leg $= 1 \times 0{\cdot}0018 =$ **0·0018 Wb**

Alternatively, flux in each outside leg will be half that in the central leg *i.e.* 0·0036/2 = **0·0018 Wb.**

Example 8.26. *A ring of cast steel has an external diameter of 24 cm and a square cross-section of 3 cm side. Inside and cross the ring, an ordinary steel bar 18 cm × 3 cm × 0·4 cm is fitted with negligible gap. Calculate the number of ampere-turns required to be applied to one half of the ring to produce a flux density of 1·0 weber per metre^2 in the other half. Neglect leakage. The B-H characteristics are as below :*

	For Cast Steel			*For Ordinary Plate*			
B in Wb/m^2	*1·0*	*1·1*	*1·2*	*B in Wb/m^2*	*1·2*	*1·4*	*1·45*
Amp-turn/m	*900*	*1020*	*1220*	*Amp-turn/m*	*590*	*1200*	*1650*

Solution. The conditions of the problem lead to the magnetic circuit shown in Fig. 8.33. The equivalent electrical circuit is shown in Fig. 8.34. Note that m.m.f. is shown as a battery and reluctances as resistances. Referring to Fig. 8.33, the flux paths *D* and *C* are in parallel. Therefore, total AT required is equal to AT for path *A* plus AT for path *C* or path *D*.

* The area of central leg is 'a' and flux is ϕ so that $B = \phi/a$. The area of each outside and other part of flux path is $a/2$ and flux is $\phi/2$ so that B is again $= \phi/a$.

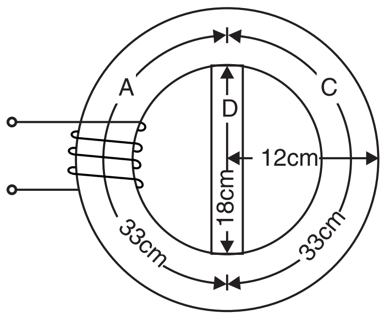


Fig. 8.33

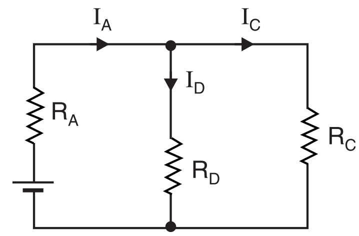


Fig. 8.34

$$\text{Mean diameter of ring} = \frac{24+18}{2} = 21 \text{ cm}$$

$$\text{Mean circumference} = \pi \times 21 = 66 \text{ cm}$$

$$\text{Length of path } A \text{ or } C = 66/2 = 33 \text{ cm} = 0{\cdot}33 \text{ m}$$

AT for path *C*. We shall first determine AT required for path C because flux density in this path is known (1 Wb/m^2). From the B-H characteristic, H corresponding to 1 Wb/m^2 is 900 AT/m.

$$\therefore \quad \text{AT required for path } C = H \times \text{Length of path } C$$

$$= 900 \times 0{\cdot}33 = 297 \text{ AT}$$

AT for path *D*. Since paths C and D are in parallel, AT required for path D = 297 AT and H = 297/0·18 = 1650 AT/m. From the B-H characteristic, B corresponding to 1650 AT/m is 1·45 Wb/m^2.

$$\text{Flux through } C, \phi_C = B \times A = 1 \times 9 \times 10^{-4} = 9 \times 10^{-4} \text{ Wb}$$

$$\text{Flux through } D, \phi_D = (1{\cdot}45) \times (3 \times 0{\cdot}4 \times 10^{-4}) = 1{\cdot}74 \times 10^{-4} \text{ Wb}$$

$$\therefore \quad \text{Flux through } A, \phi_A = \phi_C + \phi_D = (9 + 1{\cdot}74) \times 10^{-4} = 10{\cdot}74 \times 10^{-4} \text{ Wb}$$

$$\text{Flux density in } A = \frac{10 \cdot 74 \times 10^{-4}}{9 \times 10^{-4}} = 1{\cdot}193 \text{ Wb/m}^2$$

From the B-H characteristics, H corresponding to 1·193 Wb/m^2 is 1200 AT/m (approx.).

$$\therefore \quad \text{AT for path } A = 1200 \times 0{\cdot}33 = 396 \text{ AT}$$

$$\therefore \quad \text{Total AT required} = \text{AT for path } C + \text{AT for path } A$$

$$= 297 + 396 = \mathbf{693\ AT}$$

Tutorial Problems

1. A cast iron-cored toroidal coil has 3000 turns and carries a current of 0.1A . The length of the magnetic circuit is 15 cm and cross-sectional area of the coil is 4 cm^2. Find H, B and total flux. Use the following B–H curve for cast iron :

H(AT/m) :	200	400	1000	2000	3000
B(T) :	0.1	0.19	0.375	0.57	0.625

[2000 AT/m ; 0.57 T; 2.28 × 10^{-4} Wb]

2. A series magnetic circuit has an iron path of length 50 cm and an air gap of length 1 mm. The cross-sectional area of the iron is 6 cm^2 and the exciting coil has 400 turns. Determine the current required to produce a flux of 0.9 mWb in the circuit. The following points are taken from the magnetisation characteristic :

B(Wb/m^2) :	1.2	1.35	1.45	1.55
H(AT/m) :	500	1000	2000	4500

[6.35 A]

3. A cast-steel ring of mean circumference 50 cm has a cross-section of 0.52 cm^2. It has a saw-cut of 1 mm at one place. Given the following data :

B(Wb/m^2) :	1.0	1.25	1.46	1.60
μ_r :	714	520	360	247

Calculate how many ampere-turns are required to produce a flux of 0.052 mWb if leakage factor is 1.2. **[1647 AT]**

4. A magnetic circuit with a uniform cross-sectional area of 6 cm^2 consists of a cast steel ring with a mean magnetic length of 80 cm and an air gap of 2 mm. The magnetising winding has 540 ampere-turns. Estimate the magnetic flux produced in the gap. The relevant points on the magnetisation curve of cast steel are :

B(Wb/m^2) :	0.12	0.14	0.16	0.18	0.20
H(AT/m) :	200	230	260	290	320

[0.1128 mWb]

8.13. Determination of B/H or Magnetisation Curve

The variation of permeability μ (= $\mu_0\mu_r$) with flux density creates a design problem. Permeability must be known in order to find the flux density ($B = \mu H$) but permeability changes with flux density. This necessitates a graphical approach to magnetic circuit design. We plot *B-H* curves or magnetisation curves for various magnetic materials. The value of permeability is determined from the *B-H* curve of the material. The *B-H* curve can be determined by the following two methods provided the material is in the form of a ring : (*i*) By means of ballistic galvanometer, (*ii*) By means of fluxmeter.

8.14. B-H Curve by Ballistic Galvanometer

A ballistic galvanometer is similar in principle to the permanent moving coil instrument. It has a moving coil suspended between the poles of a permanent magnet. The coil is wound on a *non-metallic* former so that there is very little damping. The first deflection or 'throw' is proportional to the charge passed through the galvanometer if the duration of the charge passed is short compared with the time of one oscillation.

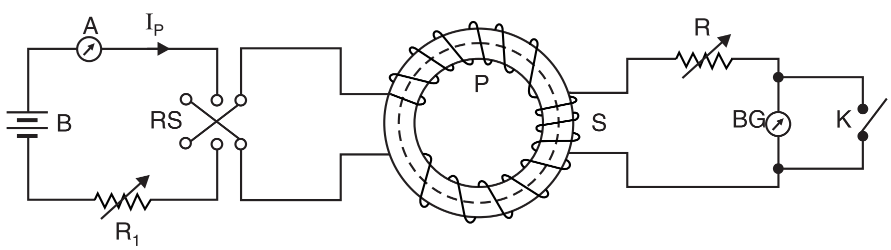


Fig. 8.35

Fig. 8.35 shows the circuit arrangement for the determination of *B-H* curve of a magnetic material by ballistic galvanometer. The specimen ring of uniform cross-section is wound uniformly with a coil *P*, thereby eliminating magnetic leakage. The primary coil *P* is connected to a battery through a reversing switch *RS*, an ammeter *A* and a variable resistor R_1. Another secondary coil *S* (called *search coil*) is wound over a small portion of the ring and is connected through a resistance *R* to the ballistic galvanometer *BG*.

Theory. We shall use subscript *P* for primary and subscript *S* for secondary.

Let θ = first deflection or 'throw' of the galvanometer when primary current I_P is reversed

k = ballistic constant of the galvanometer *i.e.* charge per unit deflection

$\therefore$ Charge passing through $BG = k\,\theta$ coulombs ...(*i*)

If ϕ is the flux produced in the ring by I_P amperes through primary *P* and *t* the time in seconds of *reversal of flux, then,

* The flux changes from ϕ to $-\phi$ by changing reversing switch *RS*. Therefore, change in flux is 2ϕ Wb.

$$\text{Rate of change of flux} = \frac{2\phi}{t} \text{ Wb/s}$$

If N_S is the number of turns in the secondary or search coil, then,

$$\text{Average e.m.f. induced in } S = N_S \times \frac{2\phi}{t} \text{ volts}$$

If R_S is the total resistance in the secondary circuit, then,

$$\text{Current through secondary or } BG, I_S = \frac{2N_S\,\phi}{R_S t} \text{ amperes}$$

$$\therefore \quad \text{Charge through } BG = I_S \times t = \frac{2N_S\,\phi}{R_S\,t} \times t = \frac{2N_S\,\phi}{R_S} \text{ coulombs} \quad ...(ii)$$

From eqs. (*i*) and (*ii*), we get, $k\theta = \dfrac{2N_S\,\phi}{R_S}$ $\quad \therefore \quad \phi = \dfrac{k\,\theta\,R_S}{2N_S}$ Wb

If A is the area of cross-section of the ring in m^2, then,

$$\text{Flux density in the ring, } B = \frac{\phi}{A} = \frac{k\theta\,R_S}{2N_S\,A} \text{ Wb/m}^2$$

If N_P is the number of turns on coil P, l the mean circumference of the ring and I_P is the current through coil P, then,

$$\text{Magnetising force, } H = \frac{N_P I_P}{l}$$

The above experiment is repeated with different values of primary current and from the data obtained, the B-H curve can be plotted.

8.15. B-H Curve by Fluxmeter

In this method, the BG is replaced by the fluxmeter which is a special type of ballistic galvanometer. Its operation is based on the change in flux linkages.

Theory. Let θ = fluxmeter deflection when current through P is reversed

c = fluxmeter constant *i.e.* Wb-turns per unit deflection

$\therefore$ Change of flux linkages with coil $S = c\,\theta$ ...(*i*)

If the flux in the ring changes from ϕ to $-\phi$ when the current through the coil P is reversed and N_S is the number of turns on coil S, then,

Change of flux linkages with coil $S = 2\phi\,N_S$ ...(*ii*)

From eqs. (*i*) and (*ii*), we get, $2\phi\,N_S = c\,\theta$ $\quad \therefore \quad \phi = \dfrac{c\,\theta}{2N_S}$ Wb

If A is the cross-sectional area of the ring in m^2, then,

$$\text{Flux density, } B = \frac{\phi}{A} = \frac{c\,\theta}{2N_S A} \text{ Wb/m}^2$$

Also, $$H = \frac{N_P I_P}{l}$$

where l = mean circumference of the ring in metres

Thus we can plot the B-H curve.

Example 8.27. *A fluxmeter is connected to a search coil having 600 turns and mean area of 4 cm^2. The search coil is placed at the centre of an air-cored solenoid 1 m long and wound with 1000 turns. When a current of 4A is reversed, there is a deflection of 20 scale divisions on the fluxmeter. Calculate the calibration in Wb-turns per scale division.*

Solution. Here, $N_P = 1000$ turns ; $I_P = 4$A ; $l = 1$m ; $N_S = 600$ turns ; $A = 4 \times 10^{-4}$ m^2.

Since the length of the solenoid is large compared to its diameter, the magnetising force inside the solenoid is uniform. Therefore, magnetising force H at the centre of the solenoid is

$$H = \frac{N_P I_P}{l} = \frac{1000 \times 4}{1} = 4000 \text{ AT/m}$$

∴ Flux density, $B = \mu_0 H = 4\pi \times 10^{-7} \times 4000 = 16\pi \times 10^{-4}$ Wb/m^2

Flux linked with search coil, $\phi = BA = 16\pi \times 10^{-4} \times 4 \times 10^{-4} = 64\pi \times 10^{-8}$ Wb

When current in the solenoid is reversed, the change in flux linkages with search coil

$$= 2N_S\phi = 2 \times 600 \times 64\pi \times 10^{-8} = 7.68\pi \times 10^{-4} \text{ Wb-turns}$$

It c is the fluxmeter constant, then, value of c is given by ;

$$c = \frac{\text{Change in flux linkages}}{\text{Deflection produced}}$$

$$= \frac{7.68\pi \times 10^{-4}}{20} = \mathbf{1.206 \times 10^{-4}} \textbf{ Wb-turns/division}$$

Example 8.28. *A solenoid 1·2 m long is uniformly wound with a coil of 800 turns. A short coil of 50 turns, having a mean diameter of 30 mm, is placed at the centre of the solenoid and is connected to a ballistic galvanometer. The total resistance of the galvanometer circuit is 2000 Ω. When a current of 5 A through the solenoid primary winding is reversed, the initial deflection of the ballistic galvanometer is 85 divisions. Determine the ballistic constant.*

Solution. Within the solenoid, we have,

$$H = \frac{N_P I_P}{l}; \quad B = \mu_0 H = \frac{\mu_0 N_P I_P}{l}$$

∴ Flux passing through the secondary or search coil of area A is

$$\phi = B \times A = \frac{\mu_0 N_P I_P A}{l}$$

Here, $N_P = 800$; $I_P = 5$ A ; $A = \pi \times (15)^2 \times 10^{-6}$ m^2 ; $l = 1·2$ m

∴ $$\phi = \frac{4\pi \times 10^{-7} \times 800 \times 5 \times (\pi \times 15^2 \times 10^{-6})}{1·2} = 2·96 \times 10^{-6} \text{ Wb}$$

Ballistic constant, $$k = \frac{2N_S\phi}{R_S\,\theta} = \frac{2 \times 50 \times 2·96 \times 10^{-6}}{2000 \times 85}$$

$$= 1·74 \times 10^{-9} \text{ C/div} = \textbf{1740 pC/div.}$$

Example 8.29. *A steel ring, 400 mm^2 cross-sectional area with a mean length 800 mm, is wound with a magnetising winding of 1000 turns. A secondary coil with 200 turns of wire is connected to a ballistic galvanometer having a constant of 1 μC/div. The total resistance of the secondary circuit is 2 kΩ. On reversing a current of 1 A in the magnetising coil, the galvanometer gives a throw of 100 scale divisions. Calculate :*

(*i*) *The flux density in the specimen.*

(*ii*) *The relative permeability at this flux density.*

Solution.

(*i*) As proved in Art. 8·14, the flux density B within the ring is given by ;

$$B = \frac{k\,\theta\, R_S}{2N_S\, A}$$

Here, $k = 1$ μC/div $= 1 \times 10^{-6}$ C/div ; $\theta = 100$ divisions ;

$R_S = 2$ kΩ $= 2000$ Ω ; $N_S = 200$; $A = 400$ mm^2 $= 400 \times 10^{-6}$ m^2

∴ $$B = \frac{(1 \times 10^{-6}) \times (100) \times (2000)}{2 \times 200 \times 400 \times 10^{-6}} = \mathbf{1·25\ T}$$

(*ii*) $$H = \frac{N_P I_P}{l} = \frac{1000 \times 1}{800 \times 10^{-3}} = 1{\cdot}25 \times 10^3 \text{ AT/m}$$

Now $$B = \mu_0 \mu_r H$$

$\therefore$ Relative permeability, $$\mu_r = \frac{B}{\mu_0 H} = \frac{1.25}{4\pi \times 10^{-7} \times 1.25 \times 10^3} = \mathbf{796}$$

Example 8.30. *An iron ring has a mean diameter of 0·1 m and a cross-section of 33·5 × 10^{-6} m^2. It is wound with a magnetising winding of 320 turns and the secondary winding of 220 turns. On reversing a current of 10 A in the magnetising winding, a ballistic galvanometer gives a throw of 272 scale divisions, while a Hilbert magnetic standard with 10 turns and a flux of 2·5 × 10^{-4} Wb gives a reading of 102 scale divisions, other conditions remaining the same. Find the relative permeability of the specimen.*

Solution. Within the iron ring, we have,

Length of magnetic path, $l = \pi D = 0{\cdot}1\pi$ m

$$H = \frac{N_P I_P}{l} = \frac{320 \times 10}{0.1\,\pi} = 10186 \text{ AT/m}$$

$$B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times \mu_r \times 10186 = 0.0128\,\mu_r \qquad \text{...(}i\text{)}$$

From Hilbert's magnetic standard, we have,

$$2.5 \times 10^{-4} \times 10 = k \times 102 \quad \therefore \quad k = 2.45 \times 10^{-5} \text{ Wb-turn/div.}$$

On reversing a current of 10 A in the primary coil, change in terms of Wb-turn is

$$2\phi N_S = k\theta \quad \text{or} \quad 2 \times \phi \times 220 = 2.45 \times 10^{-5} \times 272$$

$\therefore$ $$\phi = \frac{2.45 \times 10^{-5} \times 272}{2 \times 220} = 1.51 \times 10^{-5} \text{ Wb}$$

$$B = \frac{\phi}{A} = \frac{1.51 \times 10^{-5}}{33.5 \times 10^{-6}} = 0.45 \text{ Wb/m}^2$$

But $B = 0{\cdot}0128\,\mu_r$ as is evident from eq. (*i*).

$\therefore$ $$0.45 = 0.0128\,\mu_r \quad \text{or} \quad \mu_r = 0.45/0.0128 = \mathbf{35.1}$$

Example 8.31. *A coil of 120 turns is wound uniformly over a steel ring having a mean circumference of 1 m and a cross-sectional area of 500 mm^2. A search coil of 15 turns, wound on the ring, is connected to a fluxmeter having a constant of 300 μWbt/div. When a current of 6 A through the 120-turn coil is reversed, the fluxmeter deflection is 64 divisions. Calculate :*

(*i*) *The flux density in the ring.*

(*ii*) *The corresponding value of relative permeability.*

Solution. (*i*) Fluxmeter constant, $c = \dfrac{2N_S\,\phi}{\theta}$

Here $c = 300 \times 10^{-6}$ Wbt/div. ; $N_S = 15$; $\theta = 64$ div.

$\therefore$ $$\phi = \frac{c\theta}{2N_S} = \frac{300 \times 10^{-6} \times 64}{2 \times 15} = 0{\cdot}64 \times 10^{-3} \text{ Wb}$$

Note that ϕ is the flux passing through the search coil.

$\therefore$ Flux density, $$B = \frac{\phi}{A} = \frac{0.64 \times 10^{-3}}{500 \times 10^{-6}} = \mathbf{1{\cdot}28\ Wb/m^2}$$

(*ii*) Within the ring, $$H = \frac{N_P I_P}{l} = \frac{120 \times 6}{1} = 720 \text{ AT/m}$$

Now, $$B = \mu_0 \mu_r H$$

$$\therefore \qquad \mu_r = \frac{B}{\mu_0 H} = \frac{1 \cdot 28}{4\pi \times 10^{-7} \times 720} = \mathbf{1400}$$

Tutorial Problems

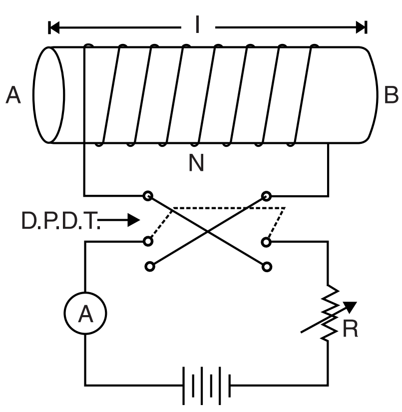
1. A moving coil ballistic galvanometer of 150Ω resistance gives a throw of 75 divisions when the flux through a search coil to which it is connected is reversed. Find the flux density given that the galvanometer constant is 110 μC per scale division and the search coil has 1400 turns, a mean area of 50 cm^2 and a resistance of 20Ω. **[0.1T]**
2. A fluxmeter is connected to a search coil having 500 turns and mean area of 5 cm^2. The search coil is placed at the centre of a solenoid one metre long wound with 800 turns. When a current of 5A is reversed, there is a deflection of 25 scale divisions on the fluxmeter. Calculate the fluxmeter constant. **[10^{-4} Wb-turn/division]**
3. A ballistic galvanometer connected to a search coil for measuring flux density in a core gives a throw of 100 scale divisions on reversal of flux. The galvanometer coil has a resistance of 180Ω. The galvanometer constant is 100μC per scale division. The search coil has an area of 50 cm^2 wound with 1000 turns having a resistance of 20Ω . Calculate the flux density in the core. **[0.2 T]**

8.16. Magnetic Hysteresis

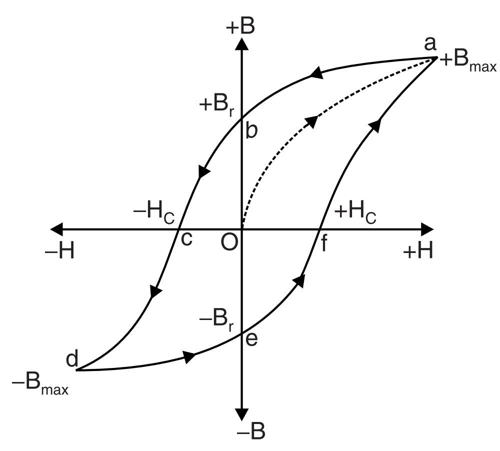
When a magnetic material is subjected to a cycle of magnetisation (*i.e.* it is magnetised first in one direction and then in the other), it is found that flux density B in the material lags behind the applied magnetising force H. This phenomenon is known as hysteresis.

The phenomenon of lagging of flux density (B) behind the magnetising force (H) in a magnetic material subjected to cycles of magnetisation is known as **magnetic hysteresis.**

The term 'hysteresis' is derived from the Greek word *hysterein* meaning to lag behind. If a piece of magnetic material is subjected to *one cycle of magnetisation, the resultant B-H curve is a closed loop *abcdefa* called *hysteresis loop* [See Fig. 8.36 (*ii*)]. Note that B always lags behind H. Thus at point '*b*', H is zero but flux density B has a positive finite value *ob*. Similarly at point '*e*', H is zero, but flux density has a finite negative value *oe*. This tendency of flux density B to lag behind magnetising force H is known as magnetic hysteresis.



(*i*)



(*ii*)

Fig. 8.36

* If we start with unmagnetised iron piece, then magnetise it in one direction and then in the other direction and finally demagnetise it (*i.e.* obtain the original condition we started with), the piece is said to go through one cycle of magnetisation. Compare it with one cycle of alternating current or voltage.

Hysteresis Loop. Consider an unmagnetised iron bar *AB* wound with *N* turns as shown in Fig. 8.36 (*i*). The magnetising force *H* (= *NI*/*l*) produced by this solenoid can be changed by varying the current through the coil. The double-pole, double-throw switch (DPDT) is used to reverse the direction of current through the coil. We shall see that *when the iron piece is subjected to a cycle of magnetisation, the resultant B-H curve traces a loop abcdefa called hysteresis loop.*

(*i*) We start with unmagnetised solenoid *AB*. When the current in the solenoid is zero, *H* = 0 and hence *B* in the iron piece is 0. As *H* is increased (by increasing solenoid current), the flux density (+ *B*) also increases until the point of maximum flux density (+ B_{max}) is reached. The material is saturated and beyond this point, the flux density will not increase regardless of any increase in current or magnetising force. Note that *B*-*H* curve of the iron follows the path *oa*.

(*ii*) If now *H* is gradually reduced (by reducing solenoid current), it is found that the flux density *B* does not decrease along the same line by which it had increased but follows the path *ab*. At point *b*, the magnetising force *H* is zero but flux density in the material has a finite value + B_r (= *ob*) called **residual flux density.** It means that after the removal of *H,* the iron piece still retains some magnetism (*i.e*. + B_r). In other words, *B* lags behind *H*. The greater the lag, the greater is the residual magnetism (*i.e*. ordinate *ob*) retained by the iron piece. The power of retaining residual magnetism is called **retentivity** of the material.

The hysteresis effect (*i.e*. lagging of *B* behind *H*) in a magnetic material is due to the opposition offered by the magnetic domains (or molecular magnets) to the turning effect of magnetising force. Once arranged in an orderly position by the magnetising force, the magnetic domains do not return exactly to the original positions. In other words, the material retains some magnetism even after the removal of magnetising force. This results in the lagging of *B* behind *H*.

(*iii*) To demagnetise the iron piece (*i.e*. to remove the residual magnetism *ob*), the magnetising force *H* is reversed by reversing the current through the coil. When *H* is graudally increased in the reverse direction, the *B*-*H* curve follows the path *bc* so that when *H* = *oc*, the residual magnetism is zero. The value of *H* (= *oc*) required to wipe out residual magnetism is known as **coercive force** (H_c).

(*iv*) If *H* is further increased in the reverse direction, the flux density increases in the reverse direction (− *B*). This process continues (curve *cd*) till the material is saturated in the reverse direction ($-B_{max}$ point) and can hold no more flux.

(*v*) If *H* is now gradually decreased to zero, the flux density also decreases and the curve follows the path *de*. At point *e*, the magnetising force is zero but flux density has a finite value $-B_r$ (= *oe*) — the residual magnetism.

(*vi*) In order to neutralise the residual magnetism *oe*, magnetising force is applied in the positive direction (*i.e*. original direction) so that when *H* = *of* (coercive force H_c), the flux density in the iron piece is zero. Note that the curve follows the path *ef*. If *H* is further increased in the positive direction, the curve follows the path *fa* to complete the loop *abcdefa.*

Thus when a magnetic material is subjected to one cycle of magnetisation, B always lags behind H so that the resultant B-H curve forms a closed loop, called **hysteresis loop.**

For the second cycle of magnetisation, a *similar loop *abcdefa* is formed. If a magnetic material is located within a coil through which alternating current (50 Hz frequency) flows, 50 loops will be formed every second. This hysteresis effect is present in all those electrical machines where the iron parts are subjected to cycles of magnetisation *e.g*. armature of a d.c. machine rotating in a stationary magnetic field, transformer core subjected to alternating flux etc.

* Owing to the nature of magnetic material, a second or even third cycle of *H* would not exactly lie on the tops of the first one. After a relatively few cycles, the successive loops would follow a fixed path.

8.17. Hysteresis Loss

When a magnetic material is subjected to a cycle of magnetisation (*i.e.* it is magnetised first in one direction and then in the other), an energy loss takes place due to the *molecular friction in the material. That is, the domains (or molecular magnets) of the material resist being turned first in one direction and then in the other. Energy is thus expended in the material in overcoming this opposition. This loss is in the form of heat and is called *hysteresis loss*. Hysteresis loss is present in all those electrical machines whose iron parts are subjected to cycles of magnetisation. The obvious effect of hysteresis loss is the rise of temperature of the machine.

(*i*) Transformers and most electric motors operate on alternating current. In such devices, the flux in the iron changes continuously, both in value and direction. Hence hysteresis loss occurs in such machines.

(*ii*) Hysteresis loss also occurs when an iron part rotates in a constant magnetic field *e.g.* d.c. machines.

8.18. Calculation of Hysteresis Loss

We will now show that area of hysteresis loop represents the †energy loss/m^3/cycle.

Let l = length of the iron bar

A = area of X-section of bar

N = No. of turns of wire of solenoid

Suppose at any instant the current in the solenoid is i. Then,

$$H = \frac{Ni}{l} \quad \text{or} \quad i = \frac{Hl}{N}$$

Suppose the current increases by di in a small time dt. This will cause the flux density to increase by dB [See Fig. 8.37] and hence an increase in flux $d\phi$ (= AdB). This causes an e.m.f. e to be induced in the solenoid.

$$\therefore \quad e = N\frac{d\phi}{dt} = NA\frac{dB}{dt}$$

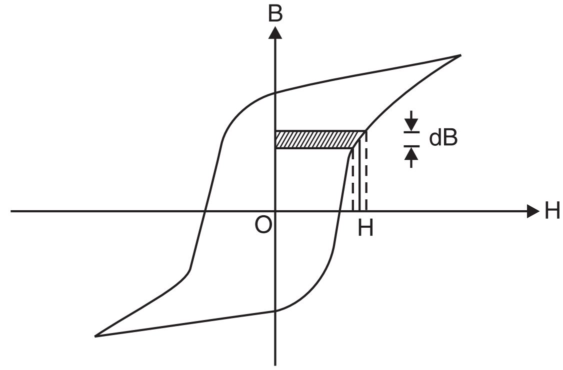


Fig. 8.37

By Lenz's law, this e.m.f. opposes the current i so that energy dW is spent in overcoming this opposing e.m.f.

* The opposition offered by the magnetic domains (or molecular magnets) to the turning effect of magnetising force is sometimes referred to as the molecular friction.

† In order to set up magnetic field, certain amount of energy has to be supplied which is stored in the field. If the field is in free space, the stored energy is returned to the circuit when the field collapses. If the field is in a magnetic material, not all the energy supplied can be returned ; part of it having been converted into heat due to hysteresis effect.

$$\therefore \quad dW = ei\,dt \text{ joules}$$

$$= \left(NA\frac{dB}{dt}\right)\times\left(\frac{Hl}{N}\right)\times dt \text{ joules}$$

$$= Al \times H \times dB \text{ joules}$$

$$= V \times (H \times dB) \text{ joules}$$

where $Al = V$ = volume of iron bar

Now $H \times dB$ is the area of the shaded strip (See Fig. 8.37). For one cycle of magnetisation, the area $H \times dB$ will be equal to the area of hysteresis loop.

$\therefore$ Hysteresis energy loss/cycle, $W_h = V \times$ (area of loop) joules

If f is the frequency of reversal of magnetisation, then,

Hysteresis power loss, $P_h = W_h \times f = V \times$ (area of loop) $\times f$

Note. While calculating the area of hysteresis loop, proper scale factors of B and H must be considered.

For example, if the scales are : 1 cm = x AT/m ...for H

1 cm = y Wb/m^2 ...for B

Then, $W_h = xy \times$ (area of loop in cm^2) $\times V$ joules

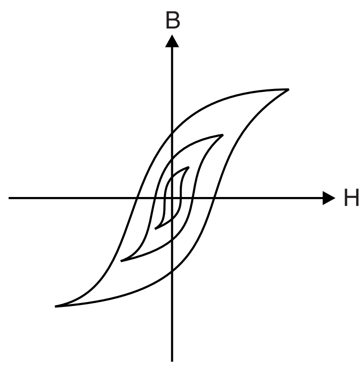
where x and y are the scale factors.

8.19. Factors Affecting the Shape and Size of Hysteresis Loop

There are three factors that affect the shape and size of hysteresis loop.

(*i*) **The material.** The shape and size of the hysteresis loop largely depends upon the nature of the material. If the material is easily magnetised, the loop will be narrow. On the other hand, if the material does not get magnetised easily, the loop will be wide. Further, different materials will saturate at different values of magnetic flux density thus affecting the height of the loop.

(*ii*) **The maximum flux density.** The loop area also depends upon the maximum flux density that is established in the material. This is illustrated in Fig. 8.38. It is clear that the loop area increases as the alternating magnetic field has progressively greater peak values.



Variation of peak flux density

Fig. 8.38

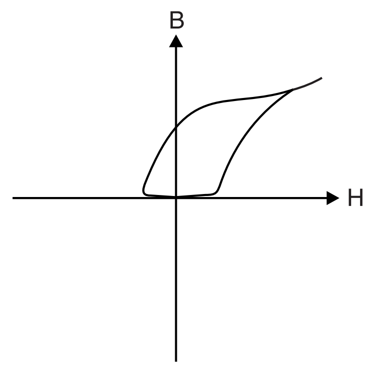


Fig. 8.39

(*iii*) **The initial state of the specimen.** The shape and size of the hysteresis loop also depends upon the initial state of the specimen. To illustrate this point, refer to Fig. 8.39. It is clear that the specimen is already saturated to start with. The magnetic flux density is then reduced to zero and finally the specimen is returned to the saturated condition.

8.20. Importance of Hysteresis Loop

The shape and size of the hysteresis loop *largely depends upon the nature of the material. The choice of a magnetic material for a particular application often depends upon the shape and size of the hysteresis loop. A few cases are discussed below by way of illustration.

(*i*) *The smaller the hysteresis loop area of a magnetic material, the less is the hysteresis loss*. The hysteresis loop for silicon steel has a very small area [See Fig. 8.40 (*i*)]. For this reason, silicon steel is widely used for making transformer cores and rotating machines which are subjected to rapid reversals of magnetisation.

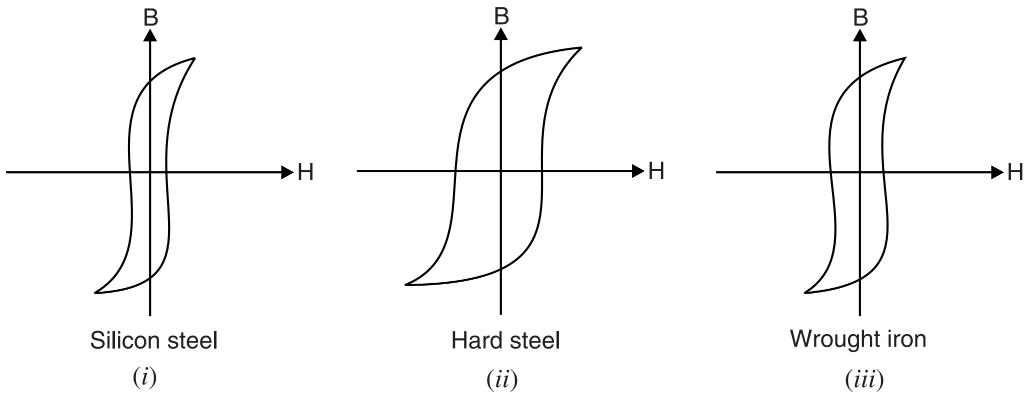


Fig. 8.40

(*ii*) The hysteresis loop for hard steel [See Fig. 8.40 (*ii*)] indicates that this material has high retentivity and coercivity. Therefore, hard steel is quite suitable for making permanent magnets. But due to the large area of the loop, there is greater hysteresis loss. For this reason, hard steel is not suitable for the construction of electrical machines.

(*iii*) The hysteresis loop for wrought iron [See Fig. 8.40 (*iii*)] shows that this material has fairly good residual magnetism and coercivity. Hence, it is suitable for making cores of electromagnets.

8.21. Applications of Ferromagnetic Materials

Ferromagnetic materials (*e.g.* iron, steel, nickel, cobalt etc.) are widely used in a number of applications. The choice of a ferromagnetic material for a particular application depends upon its magnetic properties such as retentivity, coercivity and area of the hysteresis loop. Ferromagnetic materials are classified as being either **soft** (soft iron) and **hard** (steel). Fig. 8.41 shows the hysteresis loop for soft and hard ferromagnetic materials. The table below gives the magnetic properties of hard and soft ferromagnetic materials.

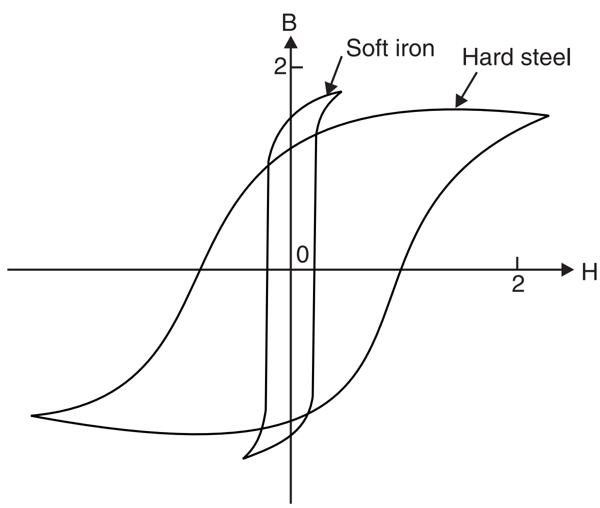


Fig. 8.41

* It also depends upon (*i*) the maximum value of flux density established and (*ii*) the initial magnetic state of the material.

Magnetic property	Soft Iron	Hard Steel
Hysteresis loop	narrow	large area
Retentivity	high	high
Coercivity	low	high
Saturation flux density	high	good

(*i*) The **permanent magnets** are made from hard ferromagnetic materials (steel, cobalt steel, carbon steel etc). Since these materials have high retentivity, the magnet is quite strong. Due to their high coercivity, they are unlikely to be demagnetised by stray magnetic fields.

(*ii*) The **electromagnets** or **temporary magnets** are made from soft ferromagnetic materials (*e.g.* soft iron). Since these materials have low coercivity, they can be easily demagnetised. Due to high saturation flux density, they make strong magnets.

(*iii*) The **transformer cores** are made from soft ferromagnetic materials. When a transformer is in use, its core is taken through many cycles of magnetisation. Energy is dissipated in the core in the form of heat during each cycle. The energy dissipated is known as *hysteresis loss* and is proportional to the area of hysteresis loop. Since the soft ferromagnetic materials have narrow hysteresis loop (*i.e.* smaller hysteresis loop area), they are used for making transformer cores.

Example 8.32. *A magnetic circuit is made of silicon steel and has a volume of* 2×10^{-3} m^3*. The area of hysteresis loop of silicon steel is found to be 7·25* cm^2 *; the scales being 1 cm = 10 AT/m and 1 cm = 4* Wb/m^2*. Calculate the hysteresis power loss when the flux is alternating at 50 Hz.*

Solution. 1 cm = 10 AT/m on *x*-axis and 1 cm = 4 Wb/m^2 on *y*-axis.

$$\text{Area of hysteresis loop in J/m}^3\text{/cycle} = (\text{Area in cm}^2) \times (\text{Scale factors}) = (7{\cdot}25) \times (xy)$$

$$= (7{\cdot}25) \times (10 \times 4) = 290 \text{ J/m}^3\text{/cycle}$$

$$\therefore \quad \text{Hysteresis power loss, } P_h = \text{Volume} \times \text{area of loop} \times \text{frequency}$$

$$= (2 \times 10^{-3}) \times (290) \times (50) \text{ W} = \mathbf{29\ W}$$

Example 8.33. *The area of hysteresis loop obtained with a certain magnetic material was 9·3* cm^2*. The co-ordinates were such that 1 cm = 1000 AT/m and 1 cm = 0·2* Wb/m^2*. If the density of the given material is 7·8* g/cm^3*, calculate the hysteresis loss in watts/kg at 50 Hz.*

Solution. 1 cm = 1000 AT/m on *x*-axis and 1 cm = 0.2 Wb/m^2 on *y*-axis.

$$\text{Volume of 1 kg of material, } V = \frac{10^3}{7 \cdot 8} 10^{-6} = 1{\cdot}282 \times 10^{-4} \text{ m}^3$$

$$\text{Area of hysteresis loop in J/m}^3\text{/cycle} = \text{Area in cm}^2 \times \text{scales factors}$$

$$= (9{\cdot}3) \times (1000 \times 0{\cdot}2) = 1860 \text{ J/m}^3\text{/cycle}$$

$$\text{Hysteresis energy loss, } W_h = V \times (\text{area of loop in J/m}^3\text{/cycle})$$

$$= (1{\cdot}282 \times 10^{-4}) \times 1860 = 0{\cdot}238 \text{ J/cycle}$$

$$\text{Hysteresis power loss, } P_h = W_h \times f = 0{\cdot}238 \times 50 = 11{\cdot}9 \text{ W}$$

Since we have considered 1 kg of material, $\therefore$ Hysteresis power loss, P_h = **11·9 W/kg**

Example 8.34. *Calculate the loss of energy caused by hysteresis in 1 hour in 50 kg of iron when subjected to cyclic magnetic changes. The frequency is 25 Hz, the area of hysteresis loop is equivalent in area to 240* J/m^3*/cycle and the density of iron is 7·8* g/cm^3*.*

Solution. Hysteresis energy loss = 240 J/m^3/cycle

$$\text{Volume of iron} = \frac{\text{mass}}{\text{density}} = \frac{50 \times 10^3}{7 \cdot 8} 10^{-6} = 6{\cdot}41 \times 10^{-3} \text{ m}^3$$

No. of cycles/hour $= 25 \times 60 \times 60 = 9 \times 10^4$

$\therefore$ Energy loss/hour $=$ volume × (area of loop in J/m^3/cycle) × cycles/hour

$= (6{\cdot}41 \times 10^{-3}) \times (240) \times (9 \times 10^4) =$ **138456 J**

Example 8.35. *The armature of a 4-pole d.c. generator has a volume of 12 × 10^{-3} m^3. During rotation, the armature is taken through a hysteresis loop whose area is 20 cm^2 when plotted to a scale of 1 cm = 100 AT/m, 1 cm = 0·1 Wb/m^2. Determine the hysteresis loss in watts when the armature rotates at a speed of 900 r.p.m.*

Solution. 1 cm = 100 AT/m on x–axis and 1 cm = 0.1 Wb/m^2 on y-axis. Since it is a 4-pole machine, two hysteresis loops will be formed in one revolution of the armature.

$\therefore$ No. of loops generated/second, $f = 2 \times 900/60 = 30$

Hysteresis energy loss/cycle $=$ Area of loop in cm^2 × scale factors

$= 20 \times (100 \times 0{\cdot}1) = 200$ J/m^3/cycle

Total hysteresis energy loss/second $=$ volume × (area of loop in J/m^3/cycle) × f

$= (12 \times 10^{-3}) \times 200 \times 30 = 72$ W

i.e. Hysteresis power loss $=$ **72 W**

Example 8.36. *A magnetic circuit core is made of silicon steel and has a volume of 1000000 mm^3. Using the hysteresis loop shown in Fig. 8.42, calculate the hysteresis power loss when the flux is alternating at 50 Hz.*

Solution. Hysteresis power loss, $P_h = V \times f \times$ (area of loop in J/m^3/cycle)

Volume of material, $V = 1000000$ mm^3 $= 1000000 \times 10^{-9}$ m^3

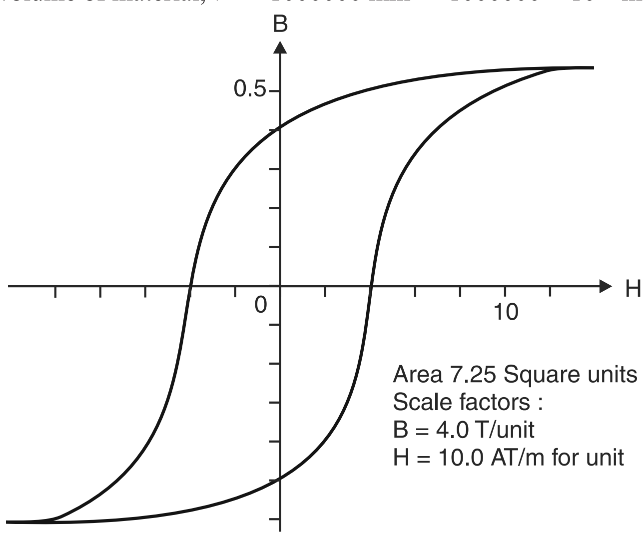


Fig. 8.42

Area of loop in J/m^3/cycle $=$ Area in square units × scale factors

$= 7{\cdot}25 \times 4 \times 10 = 290$ J/m^3/cycle

$\therefore$ $P_h = (1000000 \times 10^{-9}) \times 50 \times 290 =$ **14·5 W**

Example 8.37. *A hysteresis loop is plotted with horizontal axis scale of 1 cm = 1000 AT/m and vertical axis scale of 5 cm = 1T. The area of the loop is 9 cm^2 and overall height is 14 cm. Find (i) hysteresis loss in J/m^3/cycle (ii) B_m and (iii) hysteresis loss in W/kg if density is 7800 kg/m^3. The frequency is 50 Hz.*

Solution. (*i*) 1 cm = 1000 AT/m on x-axis and 1 cm = 0.2T on y-axis.

Area of hysteresis loop in J/m^3/cycle $=$ (Area of loop in cm^2) × scale factors

$= (9) \times (1000 \times 0.2) = 1800$ J/m^3/cycle

i.e. Hysteresis energy loss = **1800 J/m^3/cycle**

(*ii*) In a hysteresis loop, flux density varies from $+B_m$ to $-B_m$. The scale for B is 5 cm = 1T and the overall height of the loop is 14 cm.

$$\therefore \quad 2\,B_m = \frac{14}{5} = 2.8 \text{ T or } B_m = \frac{2.8}{2} = \mathbf{1.4\ T}$$

(*iii*) Volume of 1 kg of material, $V = \dfrac{\text{Mass}}{\text{Density}} = \dfrac{1}{7800}\text{m}^3$

$\therefore$ Hysteresis power loss, P_h = Energy loss/m^3/cycle $\times V \times f$

$$= 1800 \times \frac{1}{7800} \times 50 = 11.538 \text{ W}$$

Since we have considered 1 kg of material, $\therefore P_h$ = **11.538 W/kg**

Tutorial Problems

1. The hysteresis loop for a specimen of mass 12 kg is equivalent to 30 W/mm^3. Find the loss of energy in kWh in one hour at 50 Hz. The density of the specimen is 7·8 g/cm^3. [**0·024 kWh**]
2. A transformer is made of 200 kg of steel plate with a specific gravity of 7·5. It may be assumed that the maximum operating flux density is 1·1 Wb/m^2 for all parts of the steel. When a specimen of the steel was tested, it was found to have a hysteresis loop of area 100 cm^2 for a maximum flux density of 1·1 Wb/m^2. If the scales of the hysteresis loop graph were 1 cm = 50 AT/m and 1 cm = 0·1 Wb/m^2, calculate the hysteresis power loss when the transformer is operated on 50 Hz mains. [**667 W**]
3. A magnetic core is made from sheet steel, the hysteresis loop of which has an area of 2·1 cm^2; the scales being 1 cm = 400 AT/m and 1 cm = 0·4 Wb/m^2. The core measures 100 cm long and has an average cross-sectional area of 10 cm^2. The hysteresis loss is 16·8 W. Calculate the frequency of alternating flux. [**50 Hz**]

8.22. Steinmetz Hysteresis Law

To eliminate the need of finding the area of hysteresis loop for computing the hysteresis loss, Steinmetz devised an empirical law for finding the hysteresis loss. He found that the area of hysteresis loop of a magnetic material is directly proportional to 1·6 the power of the maximum flux density established *i.e.*

$$\text{Area of hysteresis loop} \propto {}^*B_{max}^{1\cdot6}$$

or $$\text{Hysteresis energy loss} \propto B_{max}^{1\cdot6} \text{ joules/m}^3\text{/cycle}$$

or $$\text{Hysteresis energy loss} = \eta\, B_{max}^{1\cdot6} \text{ joules/m}^3\text{/cycle}$$

where η is a constant called **hysteresis coefficient**. Its value depends upon the nature of material. The smaller the value of η of a magnetic material, the lesser is the hysteresis loss. The armatures of electrical machines and transformer cores are made of magnetic materials having low hysteresis coefficient in order to reduce the hysteresis loss. The best transformer steels have η values around 130, for cast steel they are around 2500 and for cast iron about 3750.

If V is the volume of the material in m^3 and f is the frequency of reversal of magnetisation, then,

$$\text{Hysteresis power loss, } P_h = \eta\, f\, B_{max}^{1\cdot6}\, V \text{ J/s or watts}$$

Example 8.38. *The volume of a transformer core built up of sheet steel laminations is 5000 cm^3 and the gross cross-sectional area is 240 cm^2. Because of the insulation between the plates, the net cross-sectional area is 90% of the gross. The maximum value of flux is 22 mWb and the frequency is 50 Hz. Find (i) the hysteresis loss/m^3/cycle and (ii) power loss in watts. Take hysteresis coefficient as 250.*

* The index 1·6 is called **Steinmetz index**. In fact, the value of this index depends upon the nature of material and may vary from 1·6 to 2·5. However, reasonable accuracy is obtained if it is taken as 1·6.

Solution. $a = 0{\cdot}9 \times 240 = 216\ \text{cm}^2$; $B_{max} = \dfrac{22\times10^{-3}}{216\times10^{-4}} = 1{\cdot}019\ \text{Wb/m}^2$

(*i*) Hysteresis energy loss $= \eta\, B^{1{\cdot}6}_{max} = 250 \times (1{\cdot}019)^{1{\cdot}6} =$ **257·6 J/m³/cycle**

(*ii*) Hysteresis power loss, $P_h = \eta\, f\, B^{1{\cdot}6}_{max} \times V = (257{\cdot}6) \times (50) \times (5000 \times 10^{-6}) =$ **64·4 W**

Example 8.39. *The area of hysteresis loop obtained with a certain specimen of iron was 9·3 cm². The co-ordinates were such that 1 cm = 1000 AT/m and 1 cm = 0·2 Wb/m². Calculate (i) the hysteresis loss in J/m³/cycle (ii) hysteresis loss in W/m³ at a frequency of 50 Hz. (iii) If the maximum flux density was 1·5 Wb/m², calculate the hysteresis loss/m³ for a maximum flux density of 1·2 Wb/m², and a frequency of 30 Hz, assuming the loss to be proportional to* $B^{1{\cdot}8}_{max}$.

Solution. 1 cm = 1000 AT/m on x-axis and 1 cm = 0.2 Wb/m² on y-axis.

(*i*) Hysteresis energy loss $= (xy) \times$ (area of loop) J/m³/cycle

$= (1000 \times 0{\cdot}2) \times 9{\cdot}3 =$ **1860 J/m³/cycle**

(*ii*) Hysteresis power loss $= 1860 \times 50 =$ **93,000 W/m³**

(*iii*) Hysteresis power loss/m³ $= \eta\, f\, (B_{max})^{1{\cdot}8}$

or $93000 = \eta \times 50 \times (1{\cdot}5)^{1{\cdot}8}$

$$\therefore \quad \eta = \frac{93000}{50\times(1\cdot5)^{1\cdot8}} = 896{\cdot}5$$

For $B_{max} = 1{\cdot}2\ \text{Wb/m}^2$ and $f = 30$ Hz,

Hysteresis loss/m³ $= \eta\, f\, (B_{max})^{1{\cdot}8}$ W $= 896{\cdot}5 \times 30 \times (1{\cdot}2)^{1{\cdot}8} =$ **37342 W**

Example 8.40. *A cylinder of iron of volume 8 × 10⁻³ m³ revolves for 20 min at a speed of 3000 r.p.m. in a two-pole field of flux density 0.8 Wb/m². If the hysteresis coefficient of iron is 753.6 J/m³, specific heat of iron is 0.11, the loss due to eddy current is equal to that due to hysteresis and 25% of heat produced is lost by radiation, find the temperature rise of iron. Take density of iron as 7.8 × 10³ kg/m³.*

Solution. When an armature revolves in a multipolar field, one magnetic reversal occurs after it passes a pair of poles. If P is the number of poles, the number of magnetic reversals in one revolution is $P/2$. If the speed of the armature is N r.p.m., then number of revolutions/second = $N/60$.

∴ No. of magnetic reversals/second = Reversal in one sec. × No. of revolutions/sec.

or Frequency of magnetic reversals $= \dfrac{P}{2}\times\dfrac{N}{60} = \dfrac{2}{2}\times\dfrac{3000}{60} = 50$ cycles/sec

According to Steinmetz hysteresis law,

Hysteresis power loss, $P_h = \eta f B^{1.6}_{max}\, V$ joules/sec.

$= 753.6 \times 50 \times (0.8)^{1.6} \times 8 \times 10^{-3} = 211$ J/s

∴ Energy loss in 20 min. $= 211 \times (20 \times 60) = 253.2 \times 10^3$ J

Eddy current loss $= 253.2 \times 10^3$ J ... *given*

∴ Total energy loss $= 2 \times 253.2 \times 10^3 = 506.4 \times 10^3$ J

$$\text{Heat produced} = \frac{506.4\times10^3}{J} = \frac{506.4\times10^3}{4200} = 120.57\ \text{kcal}$$

It is given that 25% of heat produced is lost due to radiation.

∴ Heat used to heat iron cylinder = 0.75 × 120.57 = 90.43 kcal

Now, mass of iron cylinder, m = volume × density $= 8 \times 10^{-3} \times 7.8 \times 10^3 = 62.4$ kg; specific heat, $S = 0.11$.

If θ°C is the rise of temperature of iron cylinder, then,

$$mS\theta = 90.43 \quad \text{or} \quad \theta = \frac{90.43}{62.4\times0.11} = \mathbf{13.17°C}$$

Example 8.41. *In a certain transformer, the hysteresis loss was found to be 160 watts when the maximum flux density was 1·1 Wb/m^2 and the frequency 60 Hz. What will be the loss when the maximum flux density is reduced to 0·9 Wb/m^2 and frequency to 50 Hz ?*

Solution. According to Steinmetz hysteresis law,

$$\text{Hysteresis loss, } P_h \propto f\,(B_{max})^{1\cdot 6}$$

$$\text{For the first case, } P_1 \propto 60 \times (1\cdot 1)^{1\cdot 6}$$

$$\text{For the second case, } P_2 \propto 50 \times (0\cdot 9)^{1\cdot 6}$$

$$\therefore \quad \frac{P_2}{P_1} = \frac{50 \times (0\cdot 9)^{1\cdot 6}}{60 \times (1\cdot 1)^{1\cdot 6}} = 0\cdot 604$$

$$\therefore \quad P_2 = 0\cdot 604\, P_1 = 0\cdot 604 \times 160 = \mathbf{96\cdot 64\ W}$$

Example 8.42. *Calculate the loss of energy caused by hysteresis in one hour in 11·25 kg of iron if maximum flux density reached is 1·3 Wb/m^2 and frequency is 50 Hz. Assume Steinmetz coefficient as 500 J/m^3/cycle and density of iron as 7·5 g/cm^3.*

What will be the area of B/H curve (i.e. hysteresis loop) of this specimen if 1 cm = 50 AT/m and 1 cm = 0·1 Wb/m^2 ?

Solution. Volume of iron, $V = \dfrac{11\cdot 25}{7\cdot 5 \times 10^3} = 1\cdot 5 \times 10^{-3}\ \text{m}^3$

$$\text{Hysteresis power loss, } P_h = \eta\, f\,(B_{max})^{1\cdot 6}\, V \text{ watts}$$

$$= 500 \times 50 \times (1\cdot 3)^{1\cdot 6} \times (1\cdot 5 \times 10^{-3}) = 57\cdot 06\ \text{W}$$

$\therefore$ Hysteresis energy loss in 1 hour

$$= 57\cdot 06 \times 3600 = \mathbf{205416\ J}$$

According to Steinmetz hysteresis law,

$$\text{Hysteresis energy loss} = \eta\,(B_{max})^{1\cdot 6}\ \text{J/m}^3\text{/cycle}$$

1 cm = 50 AT/m on x-axis and 1 cm = 0.1 Wb/m^2 on y-axis.

$$\text{Hysteresis energy loss} = xy \times (\text{area of loop})\ \text{J/m}^3\text{/cycle}$$

Equating the two, we get,

$$500 \times (1\cdot 3)^{1\cdot 6} = (50 \times 0\cdot 1) \times \text{Area of loop}$$

$$\therefore \quad \text{Area of loop} = \frac{500 \times (1\cdot 3)^{1\cdot 6}}{50 \times 0\cdot 1} = \mathbf{152\cdot 16\ cm^2}$$

Tutorial Problems

1. The hysteresis loss in an iron specimen is given by the expression; Hysteresis loss is J/m^3/cycle = $\eta B_{max}^{1.7}$ where B_{max} is the maximum flux density. If loss is 5.215 W/kg at a frequency of 50 Hz and a maximum flux density is 1.1 Wb/m^2, find the constant η if density of iron is 7600 kg/m^3. Also find the hysteresis loss at 60 Hz if B_{max} = 1.7 Wb/m^2. **[674.11; 13.117 W/kg]**
2. A sample of silicon steel has a hysteresis coefficient of 100 and a corresponding Steinmetz index of 1.6. Calculate the hysteresis power loss in 10^6 mm^3 when the flux is alternating at 50 Hz, such that the maximum flux density is 2T. **[15.2 W]**
3. The hysteresis loss in an iron specimen is proportional to $(B_{max})^{1.7}$. At B_{max} = 1.1T, the hyteresis loss is 320W at 50 Hz. Find hysteresis loss at 60 Hz if B_{max} = 1.6 T. **[726.05 W]**

8.23. Comparison of Electrostatics and Electromagnetic Terms

It may be worthwhile to compare the terms and symbols used in electrostatics with the corresponding terms and symbols used in electromagnetism. (See table on page 427).

Electrostatics		Electromagnetism	
Term	**Symbol**	**Term**	**Symbol**
Electric flux	ψ	Magnetic flux	ϕ
Electric flux density	D	Magnetic flux density	B
Electric field strength	E	Magnetic field strength	H
Electromotive force	E	Magnetomotive force	—
Electric potential difference	V	Magnetic potential difference	—
Permittivity of free space	ε_0	Permeability of free space	μ_0
Relative permittivity	ε_r	Relative permeability	μ_r
Absolute permittivity $= \dfrac{\text{electric flux density}}{\text{electric field strength}}$		Absolute permeability $= \dfrac{\text{magnetic flux density}}{\text{magnetic field strength}}$	
i.e. $\varepsilon_0\varepsilon_r = \varepsilon = D/E$		*i.e.* $\mu_0\mu_r = \mu = B/H$	

Objective Questions

1. In Fig. 8.43, the magnetic circuit is the path

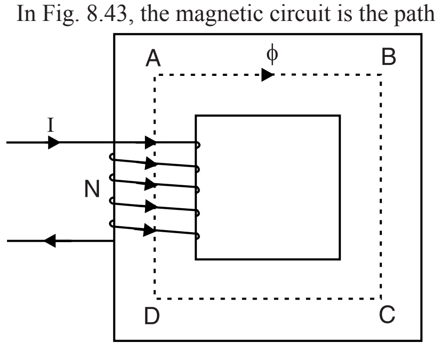


Fig. 8.43

(*i*) *DAB* (*ii*) *ABCDA*
(*iii*) *ABC* (*iv*) *ABCD*

2. If l is the magnetic path in Fig. 8.43, then magnetising force is

(*i*) NI (*ii*) $NI \times l$
(*iii*) l/NI (*iv*) NI/l

3. The reluctance of the magnetic circuit shown in Fig. 8.43 is

(*i*) NI/l (*ii*) ϕ/NI
(*iii*) NI/ϕ (*iv*) ϕ/l

4. The SI unit of reluctance is

(*i*) AT/Wb (*ii*) AT/m
(*iii*) AT (*iv*) N/Wb

5. A magnetic circuit has m.m.f. of 400 AT and reluctance of 2×10^5 AT/Wb. The magnetic flux in the magnetic circuit is

(*i*) 3×10^{-5} Wb (*ii*) 2×10^{-3} Wb
(*iii*) $1{\cdot}5 \times 10^{-2}$ Wb (*iv*) $2{\cdot}5 \times 10^{-4}$ Wb

6. A 2 cm long coil has 10 turns and carries a current of 750 mA. The magnetising force of the coil is

(*i*) 225 AT/m (*ii*) 675 AT/m
(*iii*) 450 AT/m (*iv*) 375 AT/m

7. A magnetic device has a core with cross-section of 1 inch^2. If the flux in the core is 1 mWb, then flux density (1 inch = 2.54 cm) is

(*i*) 2.5 T (*ii*) 1.3 T
(*iii*) 1.55 T (*iv*) 0.25 T

8. The reluctance of a magnetic circuit varies as ....

(*i*) length × area (*ii*) length ÷ area
(*iii*) area ÷ length (*iv*) (length)^2 + area

9. The reluctance of a magnetic circuit is ..... relative permeability of the material comprising the circuit.

(*i*) directly proportional to
(*ii*) inversely proportional to
(*iii*) independent of
(*iv*) none of the above

10. M.M.F. in a magnetic circuit corresponds to ...... in an electric circuit.

(*i*) voltage drop (*ii*) potential difference
(*iii*) electric intensity (*iv*) e.m.f.

11. Permeance of a magnetic circuit is .... area of *x*-section of the circuit.

(*i*) inversely proportional to
(*ii*) directly proportional to

(*iii*) independent of
(*iv*) none of the above.

12. The magnitude of AT required for air gap is much greater than that required for iron part of a magnetic circuit because ....
(*i*) air is a gas
(*ii*) air has the lowest relative permeability
(*iii*) air is a conductor of magnetic flux
(*iv*) none of the above

13. In electro-mechanical conversion devices (*e.g.* motors and generators), a small air gap is left between the rotor and stator in order to .....
(*i*) complete the magnetic path
(*ii*) decrease the reluctance of magnetic path
(*iii*) permit mechanical clearance
(*iv*) increase flux density in air gap

14. A magnetic circuit carries a flux ϕ_i in the iron part and a flux ϕ_g in the air gap. Then leakage coefficient is ......
(*i*) ϕ_i/ϕ_g (*ii*) ϕ_g/ϕ_i
(*iii*) $\phi_g \times \phi_i$ (*iv*) none of the above

15. The value of leakage coefficient for electrical machines is usually about.......
(*i*) 0·5 to 1 (*ii*) 4 to 10
(*iii*) above 10 (*iv*) 1·15 to 1·25

16. The reluctance of a magnetic circuit depends upon ....
(*i*) current in the coil
(*ii*) no. of turns of coil
(*iii*) flux density in the circuit
(*iv*) none of the above

17. The *B-H* curve for .... will be a straight line passing through the origin.
(*i*) air (*ii*) soft iron
(*iii*) hardened steel (*iv*) silicon steel

18. Whatever may be the flux density in ...., the material will never saturate.
(*i*) soft iron (*ii*) cobalt steel
(*iii*) air (*iv*) silicon steel

19. The *B-H* curve of .... will not be a straight line.
(*i*) air (*ii*) copper
(*iii*) wood (*iv*) soft iron

20. The *B-H* curve is used to find the m.m.f. of .... in a magnetic circuit.
(*i*) air gap (*ii*) iron part
(*iii*) both air gap and iron part
(*iv*) none of the above

21. A magnetising force of 800 AT/m will produce a flux density of .... in air.
(*i*) 1 mWb/m^2 (*ii*) 1 Wb/m^2
(*iii*) 10 mWb/m^2 (*iv*) 0·5 Wb/m^2

22. The saturation flux density for most magnetic materials is about ....
(*i*) 0·5 Wb/m^2 (*ii*) 10 Wb/m^2
(*iii*) 2 Wb/m^2 (*iv*) 1 Wb/m^2

23. Hysteresis is the phenomenon of .... in a magnetic circuit.
(*i*) lagging of *B* behind *H*
(*ii*) lagging of *H* behind *B*
(*iii*) setting up constant flux
(*iv*) none of the above

24. In Fig. 8.44, the point .... represents the saturation condition.
(*i*) *b* (*ii*) *c*
(*iii*) *a* (*iv*) *e*

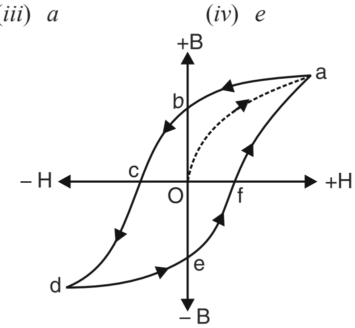


Fig. 8.44

25. In Fig. 8.44, ... represents the residual magnetism.
(*i*) of (*ii*) oc
(*iii*) ob (*iv*) none of the above

26. In Fig. 8.44, oc represents the ....
(*i*) residual magnetism
(*ii*) coercive force
(*iii*) retentivity (*iv*) none of the above

27. If a magnetic material is located within a coil through which alternating current (50 Hz frequency) flows, then .... hysteresis loops will be formed every second.
(*i*) 50 (*ii*) 25
(*iii*) 100 (*iv*) 150

28. Out of the following materials, the area of hysteresis loop will be least for ....
(*i*) wrought iron (*ii*) hard steel
(*iii*) silicon steel (*iv*) soft iron

29. The materials used for the core of a good relay should have .... hysteresis loop.

(*i*) large (*ii*) very large
(*iii*) narrow (*iv*) none of the above

30. The magnetic material used for .... should have a large hysteresis loop.

(*i*) transformers (*ii*) d.c. generators
(*iii*) a.c. motors (*iv*) permanent magnets

Answers

1. (*ii*)	**2.** (*iv*)	**3.** (*iii*)	**4.** (*i*)	**5.** (*ii*)
6. (*iv*)	**7.** (*iii*)	**8.** (*ii*)	**9.** (*ii*)	**10.** (*iv*)
11. (*ii*)	**12.** (*ii*)	**13.** (*iii*)	**14.** (*i*)	**15.** (*iv*)
16. (*iii*)	**17.** (*i*)	**18.** (*iii*)	**19.** (*iv*)	**20.** (*ii*)
21. (*i*)	**22.** (*iii*)	**23.** (*i*)	**24.** (*iii*)	**25.** (*iii*)
26. (*ii*)	**27.** (*i*)	**28.** (*iii*)	**29.** (*iii*)	**30.** (*iv*)

9

Electromagnetic Induction

Introduction

In the beginning of nineteenth century, Oersted discovered that a magnetic field exists around a current-carrying conductor. In other words, magnetism can be created by means of an electric current. Can a magnetic field create an electric current in a conductor ? In 1831, Michael Faraday, the famous English scientist, discovered that this could be done. He demonstrated that when the magnetic flux linking a conductor changes, an e.m.f. is induced in the conductor. This phenomenon is known as *electromagnetic induction*. The great discovery of electromagnetic induction by Faraday through a series of brilliant experiments has brought a revolution in the engineering world. Most of the electrical devices (*e.g.* electric generator, transformer, telephones *etc*.) are based on this principle. In this chapter, we shall confine our attention to the various aspects of eletromagnetic induction.

9.1. Electromagnetic Induction

When the magnetic flux *linking a conductor changes, an e.m.f. is induced in the conductor. If the conductor forms a complete loop or circuit, a current will flow in it. This phenomenon is known as **electromagnetic induction.

The phenomenon of production of e.m.f. and hence current in a conductor or coil when the magnetic flux linking the conductor or coil changes is called **electromagnetic induction.**

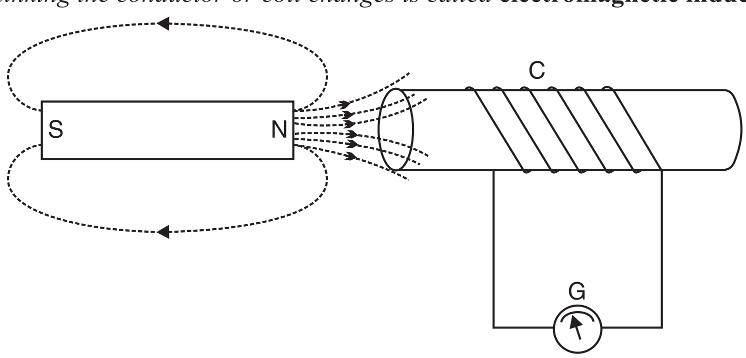


Fig. 9.1

To demonstrate the phenomenon of electromagnetic induction, consider a coil *C* of several turns connected to a centre zero galvanometer *G* as shown in Fig. 9.1. If a permanent magnet is moved towards the coil, it will be observed that the galvanometer shows deflection in one direction. If the magnet is moved away from the coil, the galvanometer again shows deflection but in the opposite direction. In either case, *the deflection will persist so long as the magnet is in motion*. The production of e.m.f. and hence current in the coil *C* is due to the fact that when the magnet is in motion (towards or away from the coil), the amount of flux linking the coil changes—the basic requirement for inducing e.m.f. in the coil. If the movement of the magnet is stopped, though the flux is linking the

* Magnetic lines of force form closed loops. Flux linking the conductor means that the flux embraces it *i.e.* it encircles the conductor.

** So called because electricity is produced from magnetism (*i.e. electromagnetic*) and that there is no physical connection (*induction*) between the magnetic field and the conductor.

coil, there is *no change in flux* and hence no e.m.f. is induced in the coil. Consequently, the deflection of the galvanometer reduces to zero.

The following points may be noted carefully :

(*i*) *The basic requirement for inducing e.m.f. in a coil is not the magnetic flux linking the coil but the change in flux linking the coil. No change in flux, no e.m.f. induced in the coil.*

(*ii*) The change in flux linking the coil can be brought about in two ways. First, the conductors (or coils) are moved through a stationary magnetic field as is the case with d.c. generators. Secondly, the conductors are stationary and the magnetic field is moving as is the case with a.c. generators. In either case, the basic principle is the same *i.e*. the amount of flux linking the conductors (or coils) is changed.

(*iii*) *The e.m.f. and hence current in the conductors (or coils) will persist so long as the magnetic flux linking them is changing.*

Note. We have seen that when magnetic flux linking a conductor changes, an e.m.f. is induced in it. An equivalent statement is like this : *When a conductor cuts magnetic field lines , an e.m.f. is induced in it*. If the conductor moves parallel to the magnetic field lines, no e.m.f. is induced. This terminology is very helpful in visualising the concept of production of e.m.f.

9.2. Flux Linkages

The product of number of turns (N) of the coil and the magnetic flux (ϕ) linking the coil is called **flux linkages** *i.e.*

$$\text{Flux linkages} = N\phi$$

Experiments show that the magnitude of e.m.f. induced in a coil is directly proportional to the rate of change of flux linkages. If N is the number of turns of the coil and the magnetic flux linking the coil changes (say increases) from ϕ_1 to ϕ_2 in t seconds, then,

$$\text{Induced e.m.f., } e \propto \text{Rate of change of flux linkages}$$

or $$e \propto \frac{N\phi_2 - N\phi_1}{t}$$

9.3. Faraday's Laws of Electromagnetic Induction

Faraday performed a series of experiments to demonstrate the phenomenon of electromagnetic induction. He summed up his conclusions into two laws, known as Faraday's laws of electromagnetic induction.

First Law. It tells us about the condition under which an e.m.f. is induced in a conductor or coil and may be stated as under :

When the magnetic flux linking a conductor or coil changes, an e.m.f. is induced in it.

It does not matter how the change in magnetic flux is brought about. The essence of the first law is that the induced e.m.f. appears in a circuit subjected to a changing magnetic field.

Second Law. It gives the magnitude of the induced e.m.f. in a conductor or coil and may be stated as under :

The magnitude of the e.m.f. induced in a conductor or coil is directly proportional to the rate of change of flux linkages i.e.

$$\text{Induced e.m.f., } e \propto \frac{N\phi_2 - N\phi_1}{t}$$

or $$e = k\frac{N\phi_2 - N\phi_1}{t}$$

where the value of k is *unity in SI units.

$$\therefore \quad e = \frac{N\phi_2 - N\phi_1}{t}$$

In differential form, we have, $e = N\dfrac{d\phi}{dt}$

The direction of induced e.m.f. (and hence of induced current if the circuit is closed) is given by **Lenz's law.** The magnitude and direction of induced e.m.f. should be written as :

$$e = -N\frac{d\phi}{dt} \qquad ...(i)$$

The minus sign on the R.H.S. represents Lenz's law mathematically. In SI units, e is measured in volts, ϕ in webers and t in seconds.

9.4. Direction of Induced E.M.F. and Current

The direction of induced e.m.f. and hence current (if the circuit is closed) can be determined by one of the following two methods :

(*i*) Lenz's Law (*ii*) Fleming's right-hand rule

(*i*) Lenz's law. Emil Lenz, a German scientist, gave the following simple rule (known as Lenz's law) to find the direction of the induced current :

The induced current will flow in such a direction so as to oppose the cause that produces it i.e. the induced current will set up magnetic flux to oppose the change in flux.

Note that Lenz's law is reflected mathematically in the minus sign on the R.H.S. of Faraday's second law viz. $e = -N\,d\phi/dt$.

The negative sign simply reminds us that the induced current *opposes* the changing magnetic field that caused the induced current. The negative sign has no other meaning.

Let us apply Lenz's law to Fig. 9.2. Here the N-pole of the magnet is approaching a coil of several turns. As the N-pole of the magnet moves towards the coil, the magnetic flux linking the coil increases. Therefore an e.m.f. and hence current is induced in the coil according to Faraday's laws of electromagnetic induction. According to Lenz's law, the direction of the induced current will be such so as to oppose the cause that produces it. In the present case, the cause of the induced current is the increasing magnetic flux linking the coil. Therefore, the induced current will set up magnetic flux that opposes the increase in flux through the coil. This is possible only if the left hand face of the coil becomes N-pole. Once we know the magnetic polarity of the coil face, the direction of the induced current can be easily determined by applying right-hand rule for the coil. If the magnet is moved away from the coil, then by Lenz's law, the left hand face of the coil will become S-pole. Therefore, by right-hand rule for the coil, the direction of induced current in the coil will be opposite to that in the first case.

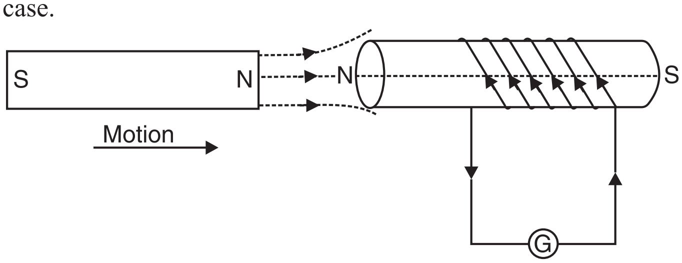


Fig. 9.2

* One volt (SI unit of e.m.f.) has been so defined that the value of k becomes unity. Thus 1V is said to be induced in a coil if the flux linkages change by 1 Wb-turn in 1 second.

Here, $N\phi_2 - N\phi_1 = 1$ Wb-turn, $t = 1$ s and $e = 1$ volt $\therefore \; 1 = k\times\frac{1}{1}$ or $k = 1$.

It may be noted here that Lenz's law directly follows from the law of conservation of energy *i.e.* in order to set up induced current, some energy must be expended. In the above case, for example, when the *N*-pole of the magnet is approaching the coil, the induced current will flow in the coil in such a direction that the left-hand face of the coil becomes *N*-pole. The result is that the motion of the magnet is opposed. The mechanical energy spent in overcoming this opposition is converted into electrical energy which appears in the coil. Thus Lenz's law is consistent with the law of conservation of energy.

(*ii*) Fleming's Right-Hand Rule. This law is particularly suitable to find the direction of the induced e.m.f. and hence current when the conductor moves at right angles to a stationary magnetic field. It may be stated as under :

Stretch out the **forefinger, middle finger** *and* **thumb** *of your right hand so that they are at right angles to one another. If the forefinger points in the direction of magnetic field, thumb in the direction of motion of the conductor, then the middle finger will point in the direction of induced current.*

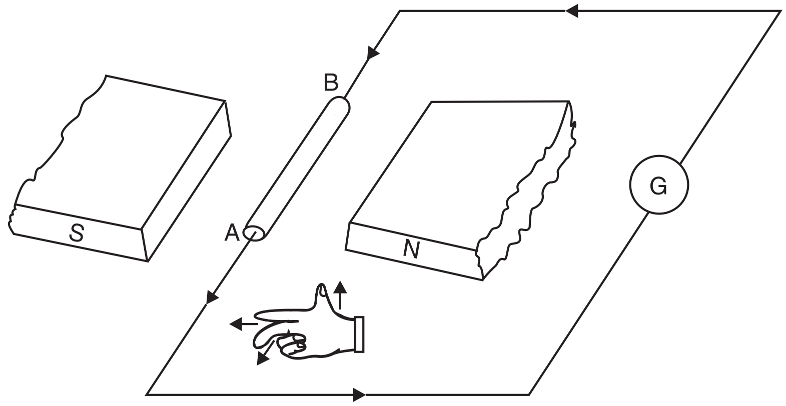


Fig. 9.3

Consider a conductor *AB* moving upwards at right angles to a uniform magnetic field as shown in Fig. 9.3. Applying Fleming's right-hand rule, it is clear that the direction of induced current is from *B* to *A*. If the motion of the conductor is downward, keeping the direction of magnetic field unchanged, then the direction of induced current will be from *A* to *B*.

Example 9.1. *A coil of 200 turns of wire is wound on a magnetic circuit of reluctance 2000 AT/Wb. If a current of 1A flowing in the coil is reversed in 10 ms, find the average e.m.f. induced in the coil.*

Solution. Flux in the coil $= \dfrac{\text{m.m.f.}}{\text{reluctance}} = \dfrac{200 \times 1}{2000} = 0 \cdot 1 \text{ Wb}$

When the current (*i.e.* 1A) in the coil is reversed, flux through the coil is also reversed.

$$e = N\frac{d\phi}{dt}$$

Here, $N = 200$; $d\phi = 0{\cdot}1 - (-0{\cdot}1) = 0{\cdot}2$ mWb ; $dt = 10 \times 10^{-3}$s

$$\therefore \quad e = 200 \times \frac{0 \cdot 2 \times 10^{-3}}{10 \times 10^{-3}} = \mathbf{4\ V}$$

Example 9.2. *The field winding of a 4-pole d.c. generator consists of 4 coils connected in series, each coil being wound with 1200 turns. When the field is excited, there is a magnetic flux of 0·04 Wb/pole. If the field switch is opened at such a speed that the flux falls to the residual value of 0·004 Wb/pole in 0·1 second, calculate the average value of e.m.f. induced across the field winding terminals.*

Solution. Total no. of turns, $N = 1200 \times 4 = 4800$

Total initial flux $= 4 \times 0{\cdot}04 = 0{\cdot}16$ Wb

Total residual flux $= 4 \times 0{\cdot}004 = 0{\cdot}016$ Wb

Change in flux, $d\phi = 0{\cdot}16 - 0{\cdot}016 = 0{\cdot}144$ Wb

Time taken, $dt = 0{\cdot}1$ second

$$\therefore \quad \text{Induced e.m.f., } e = N\frac{d\phi}{dt} = 4800 \times \frac{0.144}{0.1} = \mathbf{6912\ V}$$

Example 9.3. *A fan blade of length 0·5m rotates perpendicular to a magnetic field of 5×10^{-5}T. If the e.m.f. induced between the centre and end of the blade is 10^{-2}V, find the rate of rotation of the blade.*

Solution. Let n be the required number of rotations in one second. The magnitude of induced e.m.f. is given by ;

$$e = N\frac{d\phi}{dt} = N\frac{d}{dt}(BA) = B\frac{dA}{dt} \qquad (\because N = 1)$$

Here dA is the area swept by the blade in one revolution and dt is the time taken to complete one revolution.

Now $e = 10^{-2}$ V ; $B = 5 \times 10^{-5}$T ; $dA = \pi r^2 = \pi \times (0{\cdot}5)^2$ m^2 ; $dt = \frac{1}{n}$ s

$$\therefore \quad 10^{-2} = 5\times10^{-5}\times\frac{\pi\times(0\cdot5)^2}{1/n}$$

or

$$n = \frac{10^{-2}}{(5\times10^{-5})\times\pi\times(0\cdot5)^2} = \mathbf{254.7\ rev\ /\ second}$$

Doubling the speed of rotation of the blade would double the value of dA/dt. Hence, the e.m.f. induced would be doubled.

Example 9.4. *A coil of mean area 500 cm^2 and having 1000 turns is held perpendicular to a uniform field of 0.4 gauss. The coil is turned through 180° in 1/10 second. Calculate the average induced e.m.f.*

Solution. $\phi = NBA \cos\theta$

When the plane of the coil is perpendicular to the field, $\theta = 0°$. When the coil is turned through 180°, $\theta = 180°$. Therefore, initial flux linked with the coil is

$$\phi_1 = NBA \cos 0° = NBA$$

Flux linked with coil when turned through 180° is

$$\phi_2 = NBA \cos 180° = -NBA$$

Change in flux linking the coil is

$$\Delta\phi = \phi_2 - \phi_1 = (-NBA) - (NBA) = -2\,NBA$$

$$\therefore \quad \text{Average induced e.m.f., } e = -\frac{\Delta\phi}{\Delta t} = \frac{2NBA}{\Delta t}$$

Here $N = 1000$; $B = 0.4$ gauss $= 0.4 \times 10^{-4}$ T ; $A = 500 \times 10^{-4}$ m^2 ; $\Delta t = 0.1$ s

$$\therefore \quad e = \frac{2\times1000\times(0.4\times10^{-4})\times500\times10^{-4}}{0.1} = \mathbf{0.04\ V}$$

Example 9.5. *The magnetic flux passing perpendicular to the plane of the coil and directed into the paper (See Fig. 9.4) is varying according to the relation :*

$$\phi_B = 6t^2 + 7t + 1$$

where ϕ_B is in mWb and t in seconds.

(*i*) *What is the magnitude of induced e.m.f. in the loop when t = 2 seconds?*

(*ii*) *What is the direction of current through the resistor R ?*

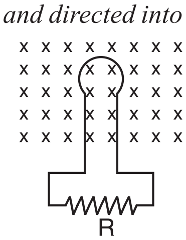


Fig. 9.4

Solution. $\phi_B = (6t^2 + 7t + 1)$ mWb $= (6t^2 + 7t + 1) \times 10^{-3}$ Wb

(*i*) Magnitude of induced e.m.f. is

$$e = \frac{d\phi_B}{dt} = \frac{d}{dt}(6t^2 + 7t + 1) \times 10^{-3} = (12t + 7) \times 10^{-3} \text{ V}$$

$$\text{At } t = 2 \text{ sec}, e = (12 \times 2 + 7) \times 10^{-3} = 31 \times 10^{-3} \text{ V} = \mathbf{31 \text{ mV}}$$

(*ii*) According to Lenz's law, the direction of induced current will be such so as to oppose the change in flux. This means that direction of current in the loop will be such as to produce magnetic field opposite to the given field. For this (*i.e.*, upward field), the current induced in the loop will be anticlockwise. Therefore, **current in resistor *R* will be from left to right.**

Tutorial Problems

1. A square coil of side 5 cm contains 100 loops and is positioned perpendicular to a uniform magnetic field of 0.6 T. It is quickly removed from the field (moving perpendicular to the field) to a region where magnetic field is zero. It takes 0.1 s for the whole coil to reach field-free region. If resistance of the coil is 100 Ω, how much energy is dissipated in the coil ? **[$2{\cdot}3\times10^{-3}$J]**

2. A flat search coil containing 50 turns each of area 2×10^{-4} m^2 is connected to a galvanometer; the total resistance of the circuit is 100Ω. The coil is placed so that its plane is normal to a magnetic field of flux density 0·25 T.

(*i*) What is the change in magnetic flux linking the circuit when the coil is moved to a region of negligible magnetic field ?

(*ii*) What charge passes through the galvanometer ? **[(*i*) $2{\cdot}5\times10^{-3}$ Wb (*ii*) 25 μC]**

3. The magnetic flux passing perpendicular to the plane of a coil and directed into the plane of the paper is varying according to the following equation :

$$\phi = 5\,t^2 + 6\,t + 2$$

where ϕ is in mWb and t in seconds. Find the e.m.f. induced in the coil at t = 1 s. **[16mV]**

4. A coil has an area of 0.04 m^2 and has 1000 turns. It is suspended in a magnetic field of 5×10^{-5} Wb/m^2 perpendicular to the field. The coil is rotated through 90º in 0·2s. Calculate the average e.m.f. induced in the coil due to rotation. **[0·01V]**

5. A gramophone disc of brass of diameter 30 cm rotates horizontally at the rate of 100/3 revolutions per minute. If the vertical component of earth's field is 0·01 T, calculate the e.m.f. induced between the centre and the rim of the disc. **[$3{\cdot}9\times10^{-4}$ V]**

9.5. Induced E.M.F.

When the magnetic flux linking a conductor (or coil) changes, an e.m.f. is induced in it. This change in flux linkages can be brought about in the following two ways :

(*i*) The conductor is moved in a stationary magnetic field in such a way that the flux linking it changes in magnitude. The e.m.f. induced in this way is called **dynamically induced e.m.f**. (as in a d.c. generator). It is so called because e.m.f. is induced in the conductor which is in motion.

(*ii*) The conductor is stationary and the magnetic field is moving or changing. The e.m.f. induced in this way is called **statically induced e.m.f.** (as in a transformer). It is so called because the e.m.f. is induced in a conductor which is stationary.

It may be noted that in either case, the magnitude of induced e.m.f. is given by N$d\phi/dt$ *or derivable from this relation*.

9.6. Dynamically Induced E.M.F.

Consider a single conductor of length l metres moving at *right angles to a uniform magnetic field of B Wb/m^2 with a velocity of v m/s [See Fig. 9.5 (*i*)]. Suppose the conductor moves through a small distance dx in dt seconds. Then area swept by the conductor is = $l \times dx$.

* If the conductor is moved parallel to the magnetic field, there would be no change in flux and hence no e.m.f. would be induced.

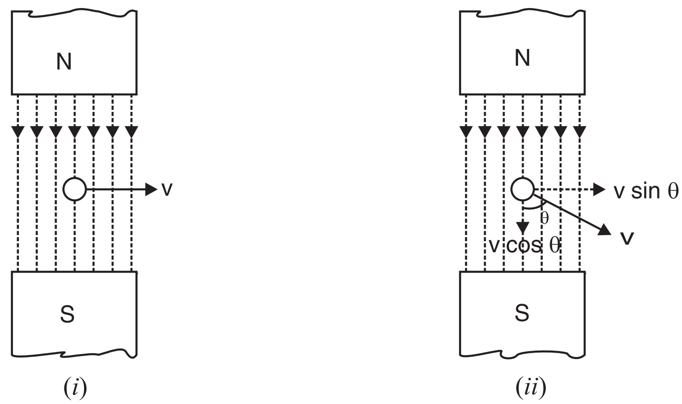


Fig. 9.5

∴ Flux cut, $d\phi$ = Flux density × Area swept = $B\,l\,dx$ Wb

According to Faraday's laws of electromagnetic induction, the magnitude of e.m.f. e induced in the conductor is given by ;

$$e = N\frac{d\phi}{dt} = \frac{B\,l\,dx}{dt} \qquad (\because N = 1)$$

$$\therefore \quad e = B\,l\,v \text{ volts} \qquad (\because dx/dt = v)$$

Special case. If the conductor moves at angle θ to the magnetic field [See Fig. 9.5 (*ii*)], then the velocity at which the conductor moves across the field is *$v \sin\theta$.

$$\therefore \quad e = B\,l\,v\sin\theta$$

The direction of the induced e.m.f. can be determined by Fleming's right-hand rule.

Example 9.6. *An aircraft has a wing span of 56 m. It is flying horizontally at a speed of 810 km/hr and the vertical component of earth's magnetic field is 4×10^{-4} Wb/m^2. Calculate the potential difference between the wing tips of the aircraft.*

Solution. Induced e.m.f. = $B\,lv$

Here $B = 4 \times 10^{-4}$ Wb/m^2 ; $l = 56$ m ; $v = \frac{810 \times 1000}{3600} = 225$ m/s

∴ Induced e.m.f. = $(4 \times 10^{-4}) \times 56 \times (225) = 5{\cdot}04$ V

or Potential difference = **5·04 V**

Example 9.7. *A d.c. generator consists of conductors lying in a radius of 10 cm and the effective length of a conductor in a constant radial field of strength 0·9 Wb/m^2 is 12 cm. The armature rotates at 1400 r.p.m. Given that the generator has 152 conductors in series, calculate the voltage being generated.*

Solution. Since the magnetic field is radial, the conductors cut the magnetic lines of force at right angles.

$$\text{Velocity, } v = \omega \times r = \frac{2\pi N}{60} \times r = \frac{2\pi \times 1400}{60} \times 0{\cdot}1 = 14{\cdot}66 \text{ m/s}$$

Voltage generated in each conductor = $B\,lv = 0{\cdot}9 \times 0{\cdot}12 \times 14{\cdot}66 = 1{\cdot}583$ V

Voltage generated in 152 conductors in series

= 1·583 × 152 = **240·6 V**

Note that effective length (l) is that portion of the conductor which takes part in the actual cutting of magnetic flux lines.

* The component $v \cos\theta$ is parallel to magnetic field and hence no e.m.f. is induced in the conductor due to this component.

Example 9.8. *A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 2\ Wb/m^2$ as shown in Fig. 9.6. The magnetic field lines are perpendicular to the plane of the loop directed into the paper. The loop is connected to a network of resistors each of value 3 Ω. The resistances of lead wires OS and PQ are negligible. What should be the speed v_0 of the loop so as to have a steady current of 1 mA in the loop ? Also indicate the direction of current in the loop.*

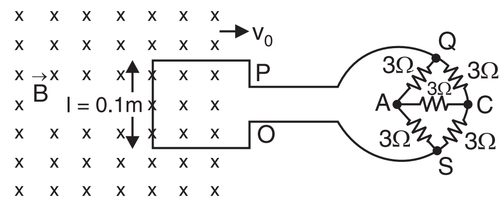


Fig. 9.6

Solution. We shall first find the equivalent resistance of the network. It is clear that network is a balanced Wheatstone bridge. Therefore, the resistance in the branch *AC* is ineffective. The equivalent resistance *R′* of the network is given by ;

$$\frac{1}{R'} = \frac{1}{6}+\frac{1}{6}=\frac{1}{3} \quad \text{or} \quad R' = 3\ \Omega$$

The resistance of the loop is 1 Ω.

∴ Effective resistance of the circuit, $R = R' + 1 = 3 + 1 = 4\ \Omega$

E.M.F. induced in the loop, $e = Bl\,v_0$

$$\text{Current in the loop, } i = \frac{e}{R}=\frac{Blv_0}{R} \quad \therefore \quad \text{Speed of the loop, } v_0 = \frac{iR}{Bl}$$

Here $i = 1\text{ mA} = 10^{-3}$ A ; $R = 4\ \Omega$; $B = 2\ \text{Wb/m}^2$; $l = 0.1$ m

$$\therefore \quad v_0 = \frac{10^{-3}\times 4}{2\times 0.1} = 2\times 10^{-2}\ \text{ms}^{-1} = \textbf{2 cm/second}$$

According to Fleming's right-hand rule, direction of induced current is **clockwise from O to P**.

Example 9.9. *A wheel with 10 metal spokes each 0.5 m long is rotated with a speed of 120 r.p.m. in a plane normal to earth's magnetic field at a place. If the magnitude of the field is 0.4 G, what is the magnitude of induced e.m.f. between the axle and rim of the wheel ?*

Solution. Length of spoke, l = radius $r = 0.5$ m

Frequency of rotation, n = 120 r.p.m. = 2 r.p.s.

Magnetic flux density, $B = 0.4\ G = 0.4 \times 10^{-4}$ T

Angular frequency, $\omega = 2\pi n = 2\pi \times 2 = 4\pi$ rad s^{-1}

As the wheel rotates, the linear velocity of spoke end at the rim = ωr and linear velocity of spoke end at the axle = 0.

$$\therefore \quad \text{Average linear velocity, } v = \frac{0+\omega r}{2} = \frac{1}{2}\omega r$$

Induced e.m.f. across the ends of each spoke is

$$e = Bl\,v = (B)\,(r)\left(\frac{1}{2}\omega r\right) = \frac{1}{2}Br^2\omega$$

or

$$e = \frac{1}{2}Br^2\omega = \frac{1}{2}(0.4\times 10^{-4})\times(0.5)^2\times 4\pi = \mathbf{6.28\times 10^{-5}\ V}$$

One end of all 10 spokes is connected to the rim and the other end to the axle. Therefore, the spokes are connected in parallel. As a result, e.m.f. between rim and axle is equal to the e.m.f. across the ends of each spoke.

Example 9.10. *A conductor 10 cm long and carrying a current of 50 A lies perpendicular to a field of strength 1000 A/m. Calculate :*

(i) the force acting on the conductor.

(ii) the mechanical power to move this conductor against the force with a speed of 1 m/s.

(iii) e.m.f. induced in the conductor.

Solution. **(*i*)** $F = BIl$. Now $H = 1000$ A/m

$$\therefore \quad B = \mu_0 H = 4\pi \times 10^{-7} \times 1000 = 4\pi \times 10^{-4} \text{ Wb/m}^2$$

$$\therefore \quad F = (4\pi \times 10^{-4}) \times 50 \times 0.1 = \mathbf{6.28 \times 10^{-3} \ N}$$

(*ii*) Mechanical power required is given by ;

$$P = F \times v = 6.28 \times 10^{-3} \times 1 = \mathbf{6.28 \times 10^{-3} \ W}$$

(*iii*) E.M.F. induced in the conductor is given by ;

$$e = Blv = (4\pi \times 10^{-4}) \times 0.1 \times 1 = \mathbf{4\pi \times 10^{-5} \ V}$$

Note that electric power developed = $eI = (4\pi \times 10^{-5}) \times 50 = 6.28 \times 10^{-3}$ W. This is equal to the mechanical input power. Therefore, law of conservation of energy is obeyed.

Tutorial Problems

1. A copper disc 40 cm in diameter is rotated at 3000 r.p.m. on a horizontal axis perpendicular to and through the centre of the disc, the axis lying in the magnetic meridian. Two brushes make contact with the disc, one at the edge and the other at the centre. If the horizontal component of earth's field be 0.02 m Wb/m^2, calculate the e.m.f. induced between the brushes. **[0.12 mV]**
2. A meter driving motor consists of a horizontal disc of aluminium 20 cm in diameter, pivoted on a vertical spindle and lying in a permanent magnetic field of density 0.3 Wb/m^2. The current flow is radial from the spindle to the circumference of the disc. The circuit resistance is 0.225 Ω and a p.d. of 2.3 V is required to pass a current of 10 A through the motor. Calculate the rotational speed of the disc and the power lost in friction. **[319 r.p.m. ; 0.5 W]**
3. If the vertical component of earth's magnetic field be 4×10^{-5} Wb/m^2, then what will be the induced potential difference produced between the rails of a metre-gauge when a train is running on them with a speed of 36 km/hr ? **[4×10^{-4} V]**

9.7. Statically Induced E.M.F.

When the conductor is stationary and the field is moving or changing, the e.m.f. induced in the conductor is called statically induced e.m.f. A statically induced e.m.f. can be further sub-divided into :

1. Self-induced e.m.f. 2. Mutually induced e.m.f.

1. Self-induced e.m.f. *The e.m.f. induced in a coil due to the change of its own flux linked with it is called* **self-induced e.m.f.**

When a coil is carrying current (See Fig. 9.7), a magnetic field is established through the coil. If current in the coil changes, then the flux linking the coil also changes. Hence an e.m.f. (= $N \, d\phi/dt$) is induced in the coil. This is known as self-induced e.m.f. The direction of this e.m.f. (by Lenz's law) is such so as to oppose the cause producing it, namely the change of current (and hence field) in the coil. The self-induced e.m.f. will persist so long as the current in the coil is changing. The following points are worth noting :

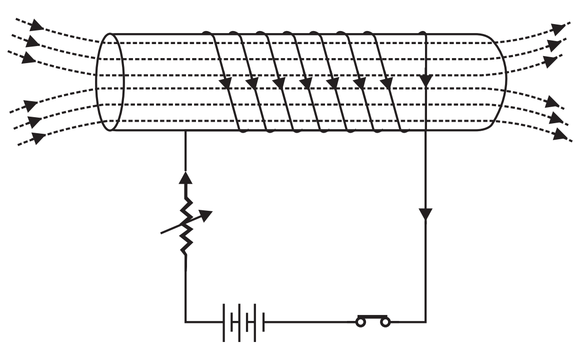


Fig. 9.7

(*i*) When current in a coil changes, the self-induced e.m.f. opposes the change of current in the coil. This property of the coil is known as its *self-inductance* or *inductance.*

(*ii*) *The self-induced e.m.f. (and hence inductance) does not prevent the current from changing ; it serves only to delay the change.* Thus after the switch is closed (See Fig. 9.7), the current will rise from zero ampere to its final steady value in some time (a fraction of a second). This delay is due to the self-induced e.m.f. of the coil.

2. Mutually induced e.m.f. *The e.m.f. induced in a coil due to the changing current in the neighbouring coil is called* **mutually induced e.m.f.**

Consider two coils *A* and *B* placed adjacent to each other as shown in Fig. 9.8. A part of the magnetic flux produced by coil *A* passes through or links with coil *B*. This flux which is common to both the coils *A* and *B* is called *mutual flux* (ϕ_m). If current in coil *A* is varied, the mutual flux also varies and hence e.m.f. is induced in both the coils. The e.m.f. induced in coil *A* is called self-induced e.m.f. as already discussed. The e.m.f. induced in coil *B* is known as *mutually induced e.m.f.*

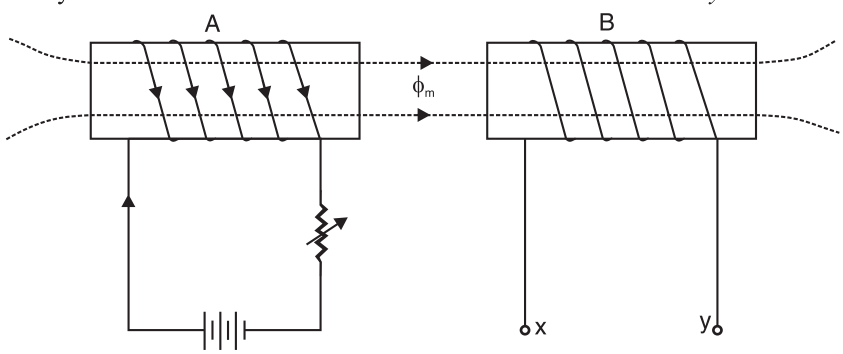


Fig. 9.8

The magnitude of mutually induced e.m.f. is given by Faraday's laws *i.e.* $e_M = N_B\, d\phi_m/dt$ where N_B is the number of turns of coil *B* and $d\phi_m/dt$ is the rate of change of mutual flux *i.e.* flux common to both the coils. The direction of mutually induced e.m.f. (by Lenz's law) is always such so as to oppose the very cause producing it. The cause producing the mutually induced e.m.f. in coil *B* is the changing mutual flux produced by coil *A*. Hence the direction of induced current (when the circuit is completed) in coil *B* will be such that the flux set up by it will oppose the changing mutual flux produced by coil *A*.

The following points may be noted carefully :

(*i*) The mutually induced e.m.f. in coil *B* persists so long as the current in coil *A* is changing. If current in coil *A* becomes steady, the mutual flux also becomes steady and mutually induced e.m.f. drops to zero.

(*ii*) The property of two neighbouring coils to induce voltatge in one coil due to the change of current in the other is called *mutual inductance*.

9.8. Self-inductance (L)

The property of a coil that opposes any change in the amount of current flowing through it is called its **self-inductance** or **inductance.**

This property (*i.e. inductance*) is due to the self-induced e.m.f. in the coil itself by the changing current. If the current in the coil is increasing, the self-induced e.m.f. is set up in such a direction so as to oppose the rise of current *i.e.* direction of self-induced e.m.f. is opposite to that of the applied voltage. Similarly, if the current in the coil is decreasing, self-induced voltage will be such so as to oppose the decrease in current *i.e.* self-induced e.m.f. will be in the same direction as the applied voltage. It may be noted that self-inductance does not prevent the current from changing ; it serves only to delay the change.

Factors affecting inductance. The greater the self-induced voltage, the greater the self-inductance of the coil and hence larger is the opposition to the changing current. According to Faraday's laws of electromagnetic induction, induced voltage in a coil depends upon the number of turns (N) and the rate of change of flux ($d\phi/dt$) linking the coil. Hence, the inductance of a coil depends upon these factors, *viz* :

(*i*) Shape and number of turns.

(*ii*) Relative permeability of the material surrounding the coil.

(*iii*) The speed with which the magnetic field changes.

In fact, anything that affects magnetic field also affects the inductance of the coil. Thus, increasing the number of turns of a coil increases its inductance. Similarly, substituting an iron core for air core increases its inductance.

It may be noted carefully that inductance makes itself felt in a circuit (*or coil*) *only when there is a changing current.* Thus, although a circuit element may have inductance by virtue of its geometrical and magnetic properties, its presence in the circuit is not exhibited unless there is a change of current in the circuit. For example, if a steady direct current (d.c.) is flowing in a circuit, there will be no inductance. However, when alternating current is flowing in the same circuit, the current is constantly changing and hence the circuit exhibits inductance.

Note. The self-inductance of a coil opposes the change of current (increase or decrease) through the coil. This opposition occurs because a changing current produces self-induced e.m.f. (e) which opposes the change of current. For this reason, *self-inductance of a coil is called* **electrical inertia** *of the coil*.

9.9. Magnitude of Self-induced E.M.F.

Consider a coil of N turns carrying a current of I amperes. If current in the coil changes, the flux linkages of the coil will also change. This will set up a self-induced e.m.f. e in the coil given by ;

$$e = N\frac{d\phi}{dt} = \frac{d}{dt}(N\phi)$$

Since flux is due to current in the coil, it follows that flux linkages ($= N\phi$) will be proportional to I.

$$\therefore \quad e = \frac{d}{dt}(N\phi) \propto \frac{dI}{dt}$$

$$\therefore \quad e = \text{Constant} \times \frac{dI}{dt}$$

$$\text{or} \quad e = L\frac{dI}{dt} \quad (\textit{in magnitude}) \quad ...(i)$$

where L is a constant called **self-inductance** or **inductance** of the coil. The unit of inductance is henry (H). If in eq. (i) above, $e = 1$ volt, $dI/dt = 1$ A/second, then $L = 1$ H.

Hence a coil (or circuit) has an inductance of **1 henry** *if an e.m.f. of 1 volt is induced in it when current through it changes at the rate of* 1 *ampere per second.*

Note. The magnitude of self-induced e.m.f. is $e = LdI/dt$. However, the magnitude and direction of self-induced e.m.f. should be written as :

$$e = -L\frac{dI}{dt}$$

The minus sign is because the self-induced e.m.f. tends to send current in the coil in such a direction so as to produce magnetic flux which opposes the change in flux produced by the change in current in the coil. In fact, minus sign represents Lenz's law mathematically.

9.10. Expressions for Self-inductance

The self-inductance (L) of a circuit or coil can be determined by one of the following three ways :

(*i*) First Method. If the magnitude of self-induced e.m.f. (e) and the rate of change of current (dI/dt) are known, then inductance can be determined from the following relation :

$$e = L\frac{dI}{dt}$$

$$\therefore \quad L = \frac{e}{(dI/dt)} \quad ...(i)$$

(*ii*) Second Method. If the flux linkages of the coil and current are known, then inductance can be determined as under :

$$e = L\frac{dI}{dt} = \frac{d}{dt}(LI)$$

$$\text{Also} \quad e = N\frac{d\phi}{dt} = \frac{d}{dt}(N\phi)$$

From the two expressions, we have,

$$LI = N\phi$$

$$\therefore \quad L = \frac{N\phi}{I} \quad ...(ii)$$

Thus, inductance is the flux linkages of the coil per ampere. If $N\phi = 1$ Wb-turn and $I = 1$ A, then $L = 1$H.

Hence a coil has an inductance of **1 henry** *if a current of 1 A in the coil sets up flux linkages of 1 Wb-turn.*

Note. Relation (*ii*) above reveals that inductance depends upon the ratio ϕ/I. Therefore, inductance is constant only when the flux changes uniformly with current. This condition is met only when the flux path is entirely composed of non-magnetic material *e.g.* air. But when the flux path is through a magnetic material (*e.g.* coil wound over iron bar), inductance of the coil will be constant only over the linear portion of the magnetisation curve.

(*iii*) Third Method. The inductance of a magnetic circuit can be found in terms of its physical dimensions. Consider an iron-cored *solenoid of dimensions as shown in Fig. 9.9. Inductance of the solenoid is given by [from exp. (*ii*) above] ;

$$L = N\frac{d\phi}{dI}$$

$$\text{Now} \quad \phi = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{NI}{l/a\mu_0\mu_r}$$

Differentiating ϕ *w.r.t.* I, we get,

$$\frac{d\phi}{dI} = \frac{Na\mu_0\mu_r}{l}$$

$$\therefore \quad L = N\frac{(Na\mu_0\mu_r)}{l}$$

$$\text{or} \quad L = \frac{N^2 a\mu_0\mu_r}{l} \quad ...(iii)$$

$$= \frac{N^2}{l/a\mu_0\mu_r} = \frac{N^2}{\text{Reluctance}(S)} \quad ...(iv)$$

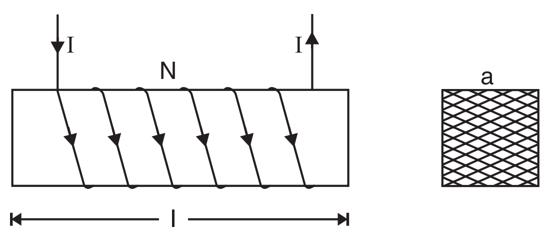


Fig. 9.9

Thus, inductance can be determined by using the relation (*iii*) or (*iv*). *It is important to note [See relation (iv)] that inductance is directly proportional to turns squared and inversely proportional*

* Solenoid is an important winding arrangement, being simple to manufacture, it is found in relays, inductors, small transformers in the form considered.

to the reluctance of the magnetic path. The smaller the reluctance of the magnetic path, the larger the inductance and *vice-versa.* For this reason, an iron-cored coil has more inductance than the equivalent air-cored coil.

Example 9.11. *A coil wound on an iron core of permeability 400 has 150 turns and a cross-sectional area of 5 cm^2. Calculate the inductance of the coil. Given that a steady current of 3 mA produces a magnetic field of 10 lines/cm^2 when air is present as the medium.*

Solution.

$$\mu_i = \frac{\text{Flux density in iron}}{\text{Flux density in air}} = \frac{B_i}{10}$$

$$\therefore \quad B_i = 10 \times \mu_i = 10 \times 400 = 4000 \text{ lines/cm}^2$$

Flux produced by 3 mA current in the iron core is

$$\phi = B_i \times a = 4000 \times 5 = 20{,}000 \text{ lines} = 2 \times 10^{-4} \text{ Wb}$$

$$\therefore \quad L = \frac{N\phi}{I} = \frac{150 \times 2 \times 10^{-4}}{3 \times 10^{-3}} = \mathbf{10\ H}$$

Example 9.12. *A solenoid with 900 turns has a total flux of 1.33 × 10^{-7} Wb through its air core when the coil current is 100 mA. If the flux takes 75 ms to grow from zero to its maximum level, calculate the inductance of the coil. Also, calculate the induced e.m.f. in the coil during the flux growth.*

Solution. The magnitude of induced e.m.f. is given by the following two expressions :

$$e = L\frac{dI}{dt} \ ; \ e = N\frac{d\phi}{dt}$$

$$\therefore \quad L\frac{dI}{dt} = N\frac{d\phi}{dt} \quad \text{or} \quad L = N\frac{d\phi}{dI}$$

Here $N = 900$; $d\phi = 1.33 \times 10^{-7}$ Wb ; $dt = 75$ ms $= 75 \times 10^{-3}$ s ;

$dI = 100$ mA $= 100 \times 10^{-3}$A

$$\therefore \quad L = 900 \times \frac{1.33 \times 10^{-7}}{100 \times 10^{-3}} = 1.2 \times 10^{-3} \text{ H} = \mathbf{1.2\ mH}$$

$$\text{Induced e.m.f., } e = N\frac{d\phi}{dt} = 900 \times \frac{1.33 \times 10^{-7}}{75 \times 10^{-3}} = 1.6 \times 10^{-3} \text{ V} = \mathbf{1.6\ mV}$$

Example 9.13. *An air-cored choke is designed to have an inductance of 20H when operating at a flux density of 1 Wb/m^2 ; the corresponding relative permeability of iron core is 4000. Determine the number of turns in the winding ; given that the flux path has a mean length of 22 cm in the iron core and 1 mm in air gap and that its cross-section is 10 cm^2.*

Solution. $L = N^2/S_T$

where S_T is the total reluctance of the magnetic path.

$$S_T = S_{iron} + S_{air} = \frac{l_{iron}}{a\,\mu_0\,\mu_r} + \frac{l_{air}}{a\,\mu_0\,\mu_r}$$

$$= \frac{0 \cdot 22}{(10 \times 10^{-4}) \times 4\pi \times 10^{-7} \times 4000} + \frac{0 \cdot 001}{(10 \times 10^{-4}) \times 4\pi \times 10^{-7} \times 1}$$

$$= 43767 + 795774 = 839541 \text{ AT/Wb}$$

Now $L = N^2/S_T$

$$\therefore \quad N = \sqrt{L\,S_T} = \sqrt{20 \times 839541} = \mathbf{4097\ turns}$$

Example 9.14. *An iron rod, 1 cm diameter and 50 cm long is formed into a closed ring and uniformly wound with 400 turns of wire. A direct current of 0·5 A is passed through the winding and produces a flux density of 0·75 Wb/m^2. If all the flux links with every turn of the winding, calculate* *(i) the relative permeability of iron (ii) the inductance of the coil (iii) the average value of e.m.f. induced when the interruption of current causes the flux in the iron to decay to 20% of its original value in 0·01 second.*

Solution. (*i*) $$H = \frac{NI}{l} = \frac{400 \times 0\cdot5}{0\cdot5} = 400 \text{ AT/m}$$

$$\mu_r = \frac{B}{\mu_0 H} = \frac{0.75}{4\pi \times 10^{-7} \times 400} = \mathbf{1492}$$

(*ii*) $$\phi = B \times a = 0\cdot75 \times \frac{\pi}{4}\left(1 \times 10^{-2}\right)^2 = 0\cdot589 \times 10^{-4} \text{ Wb}$$

$$\therefore \quad L = \frac{N\phi}{I} = \frac{(400) \times 0\cdot589 \times 10^{-4}}{0\cdot5} = \mathbf{0.0471 \ H}$$

(*iii*) Change in flux, $d\phi$ = 80% of original flux = $0\cdot8 \times 0\cdot589 \times 10^{-4} = 0\cdot47 \times 10^{-4}$ Wb

$$\therefore \quad e = N\frac{d\phi}{dt} = 400 \times \frac{0.47 \times 10^{-4}}{0.01} = \mathbf{1.88 \ V}$$

Example 9.15. *A circuit has 1000 turns enclosing a magnetic circuit 20 cm² in section. With 4A, the flux density is 1 Wb/m² and with 9A, it is 1.4 Wb/m². Find the mean value of the inductance between these current limits and the induced e.m.f. if the current falls from 9A to 4A in 0·05 seconds.*

Solution. $$L = N\frac{d\phi}{dI} = N\frac{d}{dI}(BA) = NA\frac{dB}{dI}$$

Here $N = 1000$; $dB = 1\cdot4 - 1 = 0\cdot4$ Wb/m² ; $dI = 9 - 4 = 5$A

$$\therefore \quad L = (1000) \times (20 \times 10^{-4}) \times \frac{0.4}{5} = \mathbf{0.16 \ H}$$

Also $$e = L\frac{dI}{dt} = 0\cdot16 \times \frac{5}{0\cdot05} = \mathbf{16 \ V}$$

Example 9.16. *A single element has the current and voltage functions graphed in Fig. 9.10 (i) and (ii). Determine the element.*

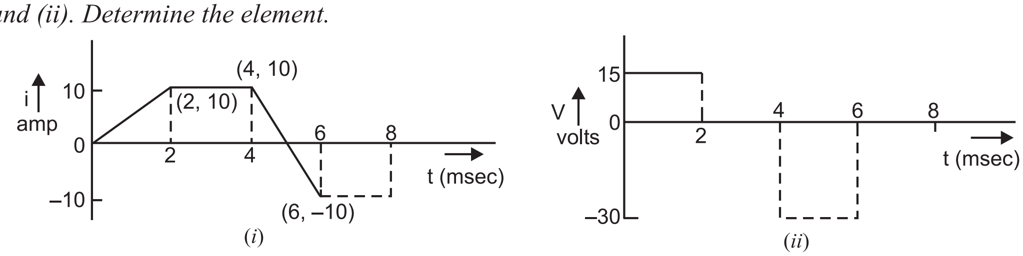


Fig. 9.10

Solution. From $i - t$ and $V - t$ graph of the element, we observe that :

Between 0 – 2 ms ; $di = 10$A ; $dt = 2$ ms ; $V = 15$ volts

$$\therefore \quad \frac{di}{dt} = \frac{10\text{ A}}{2 \times 10^{-3}\text{ s}} = 5000 \text{ A/s. Now, } L = \frac{V}{di/dt} = \frac{15}{5000} = 3 \times 10^{-3} \text{ H} = 3 \text{ mH}$$

Between 4 – 6 ms ; $di = -20$A ; $dt = 2$ ms ; $V = -30$ volts

$$\therefore \quad \frac{di}{dt} = \frac{-20\text{A}}{2 \times 10^{-3}\text{ s}} = -10{,}000 \text{ A/s. Now, } L = \frac{V}{di/dt} = \frac{-30}{-10{,}000} = 3 \times 10^{-3} \text{ H} = 3 \text{ mH}$$

Note that when current is constant, $di/dt = 0$ so that voltage across L is zero. **Hence, the element is 3 mH inductor.**

Example 9.17. *A 300-turn coil has a resistance of 6 Ω and an inductance of 0·5 H. Determine the new resistance and new inductance if one-third of the turns are removed. Assume all the turns have the same circumference.*

Solution. As the resistance of a coil is directly proportional to its length,

$$\therefore \quad R_1/R_2 = N_1/N_2 \quad \text{or} \quad 6/R_2 = 300/200$$

$$\therefore \quad R_2 = 6 \times \frac{200}{300} = \mathbf{4\ \Omega}$$

Also
$$\frac{L_1}{L_2} = \frac{N_1^2/S}{N_2^2/S} \quad \text{or} \quad \frac{0.5}{L_2} = \frac{(300)^2}{(200)^2}$$

$$\therefore \quad L_2 = 0.5 \times \frac{(200)^2}{(300)^2} = \mathbf{0{\cdot}22\ H}$$

Example 9.18. *A battery of 24 V is connected to the primary (coil 1) of a two-winding transformer as shown in Fig. 9.11 and the secondary (coil 2) is open-circuited. The coil parameters are :*

$R_1 = 10\ \Omega$	$R_2 = 30\ \Omega$
$N_1 = 100$ *turns*	$N_2 = 160$ *turns*
$\phi_1 = 0{\cdot}01$ *Wb*	$\phi_2 = 0{\cdot}008$ *Wb*

Calculate (*i*) *the self-inductance of coil 1* (*ii*) *the mutual inductance* (*iii*) *the coefficient of coupling and* (*iv*) *the self-inductance of coil 2.*

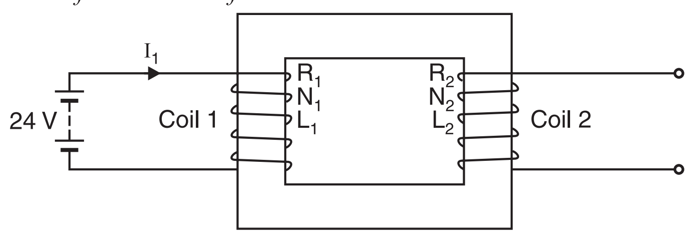


Fig. 9.11

Solution. (*i*) $I_1 = V/R_1 = 24/10 = 2{\cdot}4\text{A}$

$$\therefore \quad L_1 = \frac{N_1\,\phi_1}{I_1} = \frac{100 \times 0.01}{2{\cdot}4} = \mathbf{0.417\ H}$$

(*ii*)
$$M = \frac{N_2\,\phi_2}{I_1} = \frac{160 \times 0.008}{2{\cdot}4} = \mathbf{0.533\ H}$$

(*iii*) $k = 0{\cdot}008/0{\cdot}01 = \mathbf{0{\cdot}8}$

(*iv*) $M = k\sqrt{L_1 L_2}$ or $0{\cdot}533 = 0{\cdot}8\sqrt{0{\cdot}417 \times L_2}$ $\therefore$ $L_2 = \mathbf{1{\cdot}064\ H}$

Example 9.19. *A coil of 1000 turns is wound on a laminated core of steel having a cross-section of 5 cm*^{2}*. The core has an air gap of 2 mm cut at right angle. What value of current is required to have an air gap flux density of 0.5 T? Permeability of steel may be taken as infinity. Determine the coil inductance.*

Solution. $B_g = 0.5$ T ; $a = 5 \times 10^{-4}$ m^2 ; $N = 1000$ turns ; $l_g = 2 \times 10^{-3}$ m ; $\mu_r = \infty$

$$\text{Total } AT \text{ required} = H_i l_i + H_g l_g = \frac{B_g}{\mu_0 \mu_r} l_i + \frac{B_g}{\mu_0} l_g$$

$$= 0 + \frac{B_g}{\mu_0} l_g = 0 + \frac{0.5}{4\pi \times 10^{-7}} \times 2 \times 10^{-3} = 796 \text{ AT} \quad (\because \mu_r = \infty)$$

Now $NI = 796 \quad \therefore I = \frac{796}{N} = \frac{796}{1000} = \mathbf{0.796\ A}$

$$\text{Inductance of coil, } L = \frac{N\phi}{I} = \frac{N \times (B_g \times a)}{I} = \frac{1000 \times (0.5 \times 5 \times 10^{-4})}{0.796} = \mathbf{0.314\ H}$$

Tutorial Problems

1. A current of 2·5 A flows through a 1000-turn coil that is air-cored. The coil inductance is 0·6 H. What magnetic flux is set up ? **[1·5 m Wb]**
2. A 2000-turn coil is uniformly wound on an ebonite ring of mean diameter 320 mm and cross-sectional area 400 mm^2. Calculate the inductance of the toroid so formed. **[2 mH]**
3. A coil has self-inductance of 10 H. If a current of 200 mA is reduced to zero in a time of 1 ms, find the average value of induced e.m.f. across the terminals of the coil. **[2000 V]**
4. A coil consists of 750 turns and a current of 10 A in the coil gives rise to a magnetic flux of 1200 μWb. Calculate the inductance of the coil and determine the average e.m.f. induced in the coil when this current is reversed in 0·01 second. **[0·09 H ; 180 V]**
5. Calculate the inductance of a solenoid of 2000 turns wound uniformly over a length of 50 cm on a cylindrical paper tube 4 cm in diameter. The medium is air. **[12·62 mH]**
6. A circular iron ring of mean diameter 100 mm and cross-sectional area 500 mm^2 has 200 turns of wire uniformly wound around the circumference. If the relative permeability of iron is assumed to be 1200, find the self-inductance of the coil. **[96 mH]**
7. A certain 40-turn coil has an inductance of 6 H. Determine the new inductance if 10 turns are added to the coil. **[9·38 H]**
8. The e.m.f. induced in a coil is 100V when current through it changes from 1A to 10 A in 0·1s. Calculate the inductance of the coil. **[1·11 H]**
9. A 6-pole, 500 V d.c. generator has a flux/pole of 50 mWb produced by a field current of 10 A. Each pole is wound with 600 turns. The resistance of entire field circuit is 50 Ω. If the field circuit is broken in 0·02s, calculate (*i*) the inductance of the field coils (*ii*) the induced e.m.f. and (*iii*) the value of discharge resistance so that the induced e.m.f. should not exceed 1000V. **[(*i*) 18 H (*ii*) 1500 V (*iii*) 50 Ω]**
10. What is the inductance of a single layer 10-turn air-cored coil that is 1 cm long and 0·5 cm in diameter ? **[0·214 μH]**

9.11. Magnitude of Mutually Induced E.M.F.

Consider two coils *A* and *B* placed adjacent to each other as shown in Fig. 9.12. If a current I_1 flows in the coil *A*, a flux is set up and a part ϕ_{12} (*mutual flux*) of this flux links the coil *B*. If current in coil *A* is varied, the mutual flux also varies and hence an e.m.f. is induced in the coil *B*. The e.m.f. induced in coil *B* is termed as mutually induced e.m.f. Note that coil *B* is not electrically connected to coil *A* ; the two coils being magnetically linked.

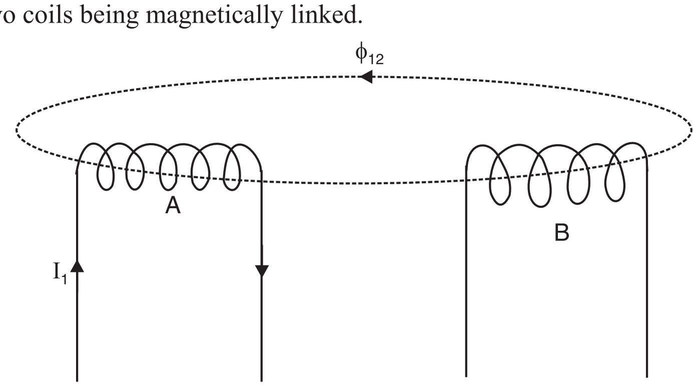


Fig. 9.12

The larger the rate of change of current in coil *A*, the greater is the e.m.f. induced in coil *B*. In other words, mutually induced e.m.f. in coil *B* is directly proportional to the rate of change of current in coil *A i.e.,*

Mutually induced e.m.f. in coil *B* $\propto$ Rate of change of current in coil *A*

or $$e_M \propto \frac{dI_1}{dt}$$

or $$e_M = M\frac{dI_1}{dt} \qquad (\textit{in magnitude}) \qquad ...(i)$$

where *M* is a constant called **mutual inductance** between the two coils. The unit of mutual inductance is henry (H). If in exp. (*i*), e_M = 1 volt, dI_1/dt = 1 A/sec, then, *M* = 1 H.

Hence mutual inductance between two coils is **1 henry** *if current changing at the rate of 1 A/sec in one coil induces an e.m.f. of 1 V in the other coil.*

Mutual inductance comes into picture when two coils are placed close together in such a way that flux produced by one links the other. We say then that the two coils are coupled. Each coil has its own inductance but in addition, there is further inductance due to the induced voltage produced by coupling between the coils. We call this further inductance as mutual inductance. We say the two coils are coupled together by mutual inductance. The terms **magnetic** or **inductive coupling** are sometimes used.

Note. The magnitude of mutually induced e.m.f. in coil *B* (secondary) is $e_M = M\, dI_1/dt$ where dI_1 is the change of current in coil *A* (primary). However, the magnitude and direction of mutually induced e.m.f. in coil *B* should be written as :

$$e_M = -M\frac{dI_1}{dt}$$

The minus sign is because the mutually induced e.m.f. sends current in coil *B* in such a direction so as to produce magnetic flux which opposes the change in flux produced by change in current in coil *A*. In fact, minus sign represents Lenz's law mathematically.

9.12. Expressions for Mutual Inductance

The mutual inductance between two coils can be determined by one of the following three methods :

(*i*) First Method. If the magnitude of mutually induced e.m.f. (e_M) in one coil for the given rate of change of current in the other is known, then *M* between the two coils can be determined from the following relation :

$$e_M = M\frac{dI_1}{dt}$$

or $$M = \frac{e_M}{dI_1/dt} \qquad ...(i)$$

(*ii*) Second Method. Let there be two magnetically coupled coils *A* and *B* having N_1 and N_2 turns respectively (See Fig. 9.13). Suppose a current I_1 flowing in coil *A* produces a mutual flux ϕ_{12}. Note that mutual flux ϕ_{12} is that part of the flux created by coil *A* which links the coil *B*.

$$e_M = M\frac{dI_1}{dt} = \frac{d}{dt}(M\,I_1)$$

Also $$e_M = N_2\frac{d\phi_{12}}{dt} = \frac{d}{dt}(N_2\phi_{12})$$

From these two expressions, we have,

$$MI_1 = N_2\,\phi_{12}$$

or $$M = \frac{N_2\phi_{12}}{I_1} \qquad ...(ii)$$

Thus, mutual inductance between two coils is equal to the flux linkages of one coil ($N_2\phi_{12}$) due to one ampere in the other coil. If $N_2\,\phi_{12} = 1$ Wb-turn and $I_1 = 1$ A, then, $M = 1$ H.

Hence mutual inductance between two coils is **1 henry** *if a current of 1 A flowing in one coil produces flux linkages of 1 Wb-turn in the other.*

(*iii*) Third Method. The mutual inductance between the two coils can be determined in terms of physical dimensions of the magnetic circuit. Fig. 9.13 shows two magnetically coupled coils *A* and *B* having N_1 and N_2 turns respectively. Suppose l and 'a' are the length and area of cross-section of the magnetic circuit respectively. Let μ_r be the relative permeability of the material of which the magnetic circuit is composed.

$$\text{Mutual flux, } \phi_{12} = \frac{\text{m.m.f.}}{\text{reluctance}} = \frac{N_1 I_1}{l / a\,\mu_0\,\mu_r}$$

or
$$\frac{\phi_{12}}{I_1} = \frac{N_1\,a\,\mu_0\,\mu_r}{l}$$

Now
$$M = \frac{N_2\,\phi_{12}}{I_1}$$

$\therefore$
$$M = \frac{N_1 N_2\,a\,\mu_0\,\mu_r}{l} \quad ...(iii)$$

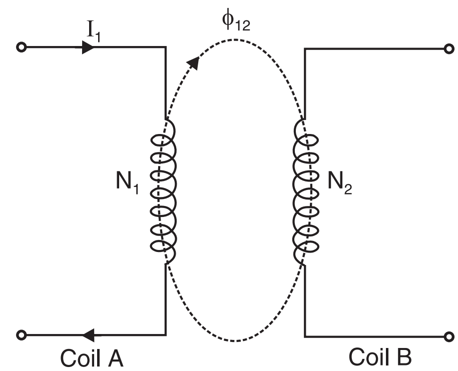


Fig. 9.13

$$= \frac{N_1 N_2}{l / a\,\mu_0\,\mu_r}$$

$$= \frac{N_1\,N_2}{\text{Reluctance}\,(S)} \quad ...(iv)$$

The mutual inductance can be found by using relation (*iii*) or (*iv*). Note that mutual inductance is inversely proportional to the reluctance of the magnetic circuit. The smaller the reluctance of the magnetic circuit, the greater is the mutual inductance and *vice-versa*.

9.13. Coefficient of Coupling

The **coefficient of coupling (*k*)** *between two coils is defined as the fraction of magnetic flux produced by the current in one coil that links the other.*

When the entire flux of one coil links the other, coefficient of coupling is 1 (*i.e.*, 100%). If only half the flux set up in one coil links the other, then coefficient of coupling is 0·5 (or 50%). If two coils have self-inductances L_1 and L_2, then mutual inductance M between them is given by ;

$$M = k\sqrt{L_1\,L_2}$$

where k = coefficient of coupling. Clearly, the mutual inductance M between the coils will be maximum when $k = 1$. If flux of one coil does not at all link with the other coil, then $k = 0$. Under such condition, mutual inductance (M) between the coils will be zero.

Proof. Consider two magnetically coupled coils 1 and 2 having N_1 and N_2 turns respectively (See Fig. 9.14). The current I_1 flowing in coil 1 produces a magnetic flux ϕ_1. Suppose the coefficient of coupling between the two coils is k. It means that flux $k\phi_1$ links with coil 2. Then, by definition,

$$L_1 = \frac{N_1\phi_1}{I_1}$$

and
$$M_{12} = \frac{k\phi_1 N_2}{I_1} \quad ...(i)$$

where M_{12} represents mutual inductance of coil 1 to coil 2.

The current I_2 flowing in coil 2 will produce flux ϕ_2. Since the coefficient of coupling between the coils is k, it means that flux $k\phi_2$ will link with coil 1. Then,

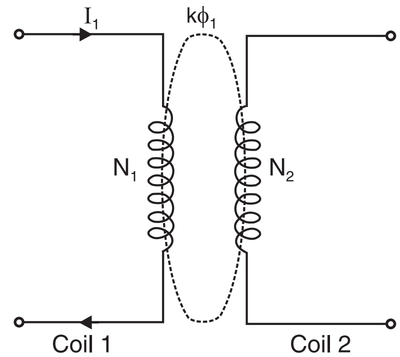


Fig. 9.14

$$L_2 = \frac{\phi_2 N_2}{I_2}$$

and $$M_{21} = \frac{k\phi_2 N_1}{I_2} \quad ...(ii)$$

where M_{21} represents mutual inductance of coil 2 to coil 1.

Mutual inductance between the two coils is exactly the same *i.e.*, $M_{12} = M_{21} = M$.

$$\therefore \quad M_{12} \times M_{21} = \frac{(k\phi_1 N_2)}{I_1} \times \frac{(k\phi_2 N_1)}{I_2}$$

or $$M^2 = k^2 \frac{\phi_1 N_1}{I_1} \times \frac{\phi_2 N_2}{I_2} = k^2 L_1 L_2$$

$$\therefore \quad M = k\sqrt{L_1 L_2} \quad ...(iii)$$

Expression (*iii*) gives the relation between the mutual inductance of the two coils and their self-inductances. The reader may note that mutual inductance between the two coils will be maximum when $k = 1$. Obviously, the maximum value of mutual inductance between the two coils is = $\sqrt{L_1 L_2}$.

$$\therefore \quad k = \frac{M}{\sqrt{L_1 L_2}} = \frac{\text{Actual mutual inductance}}{\text{Max. possible mutual inductance}}$$

Hence, coefficient of coupling can also be defined *as the ratio of the actual mutual inductance* (M) *between the two coils to the maximum possible value* $(\sqrt{L_1 L_2})$.

When two coils are wound on a single ferromagnetic core as shown in Fig. 9.15 (*i*), effectively all of the magnetic flux produced by one coil links with the other. The coils are then said to be **tightly coupled.** Another way to ensure tight coupling is shown in Fig. 9.15 (*ii*) where each turn of the secondary winding is side by side with one turn of primary winding. Coils wound in this fashion are said to be bifilar and it is called **bifilar winding.**

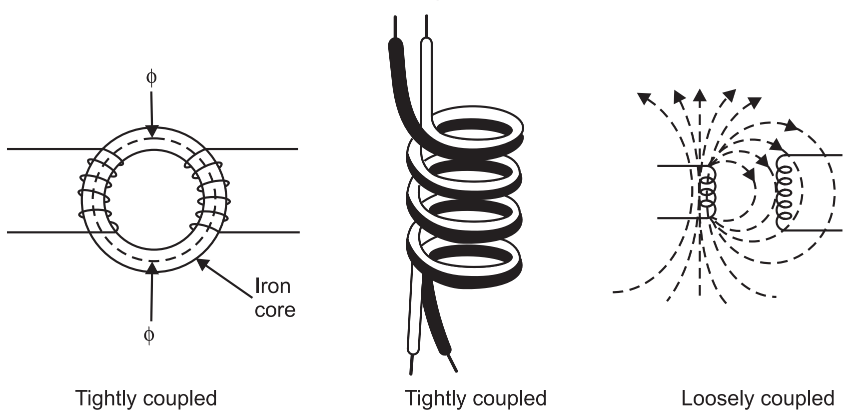


Fig. 9.15

When the two coils are air-cored as shown in Fig. 9.15 (*iii*), then only a fraction of magnetic flux produced by one coil may link with the other coil. The coils are then said to be **loosely coupled.**

Example 9.20. *Two identical coils A and B of 1000 turns each lie in parallel planes such that 80% of flux produced by one coil links with the other. A current of 5 A flowing in coil A produces a flux of 0·05 mWb in it. If the current in coil A changes from + 12A to −12A in 0·02 second, calculate (i) the mutual inductance and (ii) the e.m.f. induced in coil B.*

Solution. (*i*) $$M = \frac{N_2 \phi_{12}}{I_1}$$

Here $N_2 = 1000$; $I_1 = 5$ A ; $^*\phi_{12} = 0{\cdot}8 \times 0{\cdot}05 \times 10^{-3} = 0{\cdot}4 \times 10^{-4}$ Wb

$$\therefore \quad M = \frac{1000 \times 0 \cdot 4 \times 10^{-4}}{5} = \mathbf{0.008\ H}$$

(*ii*) E.M.F. in coil B, $$e_B = M\frac{dI_1}{dt}$$

Here $M = 0{\cdot}008$ H ; $dI_1 = 12 - (-12) = 24$ A ; $dt = 0{\cdot}02$ s

$$\therefore \quad e_B = 0 \cdot 008 \times \frac{24}{0 \cdot 02} = \mathbf{9.6\ V}$$

Example 9.21. *Coils A and B in a magnetic circuit have 600 and 500 turns respectively. A current of 8 A in coil A produces a flux of 0·04 Wb. If the coefficient of coupling is 0·2, calculate* :

(*i*) *Self-inductance of coil A, with B open-circuited.*
(*ii*) *Flux linking with coil B.*
(*iii*) *The average e.m.f. induced in coil B when the flux with it changes from zero to full value in 0·02 second.*
(*iv*) *Mutual inductance.*
(*v*) *Average e.m.f. in B when current in A changes from 0 to 8 A in 0·05 second.*

Solution. (*i*) Inductance of coil A, $$L_A = \frac{N_A\,\phi_A}{I_A} = \frac{600 \times 0 \cdot 04}{8} = \mathbf{3\ H}$$

(*ii*) Flux linking coil B, $\phi_B = k \times \phi_A = 0{\cdot}2 \times 0{\cdot}04 = \mathbf{0{\cdot}008\ Wb}$

(*iii*) e.m.f. in coil B, $$e_B = N_B\frac{\phi_B - 0}{t} = 500\frac{0 \cdot 008}{0 \cdot 02} = \mathbf{200\ V}$$

(*iv*) Mutual inductance, $$M = \frac{k\,\phi_A\,N_B}{I_A} = \frac{0 \cdot 2 \times 0 \cdot 04 \times 500}{8} = \mathbf{0.5\ H}$$

(*v*) e.m.f. in coil B $$= M\frac{dI_A}{dt} = 0.5 \times \frac{8-0}{0.05} = \mathbf{80\ V}$$

Example 9.22. *Two identical coils are wound on a ring-shaped iron core that has a relative permeability of 500. Each coil has 100 turns and the core dimensions are : area, a = 3 cm² and magnetic path length, l = 20 cm. Calculate the inductance of each coil and the mutual inductance between the coils.*

Solution. $N = 100$ turns ; $\mu_r = 500$; $a = 3 \times 10^{-4}$ m^2 ; $l = 20 \times 10^{-2}$ m

The statement of the problem suggests that each coil has the same inductance.

$$\therefore \quad L_1 = L_2 = \mu_0 \mu_r N^2 \frac{a}{l}$$

$$= 4\pi \times 10^{-7} \times 500 \times (100)^2 \times \frac{3 \times 10^{-4}}{20 \times 10^{-2}} = 9.42 \times 10^{-3}\ \text{H} = \mathbf{9.42mH}$$

Since the coils are wound on the same iron core, coefficient of coupling $k = 1$.

$$\therefore \quad M = k\sqrt{L_1 L_2} = 1\sqrt{9.42 \times 9.42} = \mathbf{9.42\ mH}$$

* Note that 80% of flux produced in coil A links with coil B. Therefore, mutual flux (ϕ_{12}) is 80% of 0·05 mWb.

Example 9.23. *Two identical 750-turn coils A and B lie in parallel planes. A current changing at the rate of 1500 A/s in coil A induces an e.m.f. of 11·25 V in coil B. Calculate the mutual inductance of the arrangement. If the self-inductance of each coil is 15 mH, calculate the flux produced in coil A per ampere and the percentage of this flux which links the turns of coil B.*

Solution. Induced e.m.f. in coil B, $e_B = M\dfrac{dI_A}{dt}$

or $11{\cdot}25 = M \times 1500 \quad \therefore \quad M = 7{\cdot}5 \times 10^{-3}$ H = **7·5 mH**

Now $$L_1 = \frac{N_1\phi_1}{I_1} \quad \therefore \quad \frac{\phi_1}{I_1} = \frac{L_1}{N_1} = \frac{15\times10^{-3}}{750} = \mathbf{2 \times 10^{-5}\ Wb/A}$$

Coefficient of coupling, $$k = \frac{M}{\sqrt{L_1L_2}} = \frac{M}{\sqrt{L^2}} = \frac{7{\cdot}5\times10^{-3}}{15\times10^{-3}} = \mathbf{0{\cdot}5} \text{ or } \mathbf{50\%}$$

Example 9.24. *Two coils A and B of 500 and 750 turns respectively are connected in series on the same magnetic circuit of reluctance 1·55 × 10^6 AT/Wb. Assuming that there is no flux leakage, calculate (i) self-inductance of each coil and (ii) mutual inductance between coils.*

Solution. (*i*) $$L_A = \frac{N_A^2}{\text{Reluctance}} = \frac{(500)^2}{1{\cdot}55\times10^6} = \mathbf{0.16\ H}$$

$$L_B = \frac{N_B^2}{\text{Reluctance}} = \frac{(750)^2}{1{\cdot}55\times10^6} = \mathbf{0.36\ H}$$

(*ii*) $$M = \frac{N_AN_B}{\text{Reluctance}} = \frac{500\times750}{1{\cdot}55\times10^6} = \mathbf{0.24\ H}$$

Alternatively. $$M = k\sqrt{L_1L_2} = 1\sqrt{0{\cdot}16\times0{\cdot}36} = \mathbf{0.24\ H}$$

Example 9.25. *Two coils A and B are wound side by side on a paper tube former. An e.m.f. of 0·25 V is induced in coil A when the flux linking it changes at the rate of 10^{-3} Wb/s. A current of 2 A in coil B causes a flux of 10^{-5} Wb to link coil A. What is the mutual inductance between the coils?*

Solution. Induced e.m.f. in coil $A = N_1\dfrac{d\phi}{dt}$ or $0{\cdot}25 = N_1 \times 10^{-3}$

$\therefore \quad N_1 = 0{\cdot}25 / 10^{-3} = 250$ turns

Flux linkages in coil A due to 2 A in coil $B = 250 \times 10^{-5}$ Wb-turns.

$$\therefore \quad M = \frac{\text{Flux linkages in coil } A}{\text{Current in coil } B}$$

$$= 250 \times 10^{-5}/2 = \mathbf{1{\cdot}25 \times 10^{-3}\ H}$$

Example 9.26. *The coefficient of coupling between two coils is 0·6 or 60%. The excited coil produces 0·1 Wb of magnetic flux. How much flux is coupled to the other coil ? What is the value of the leakage flux ?*

Solution. The coefficient of coupling is given by ;

$$k = \frac{\phi_m}{\phi_t}$$

where ϕ_t = flux of the coil receiving current ; ϕ_m = flux that links with the other coil

$\therefore \quad 0{\cdot}6 = \phi_m/\phi_t$

or $\phi_m = 0{\cdot}6 \times \phi_t = 0{\cdot}6 \times 0{\cdot}1 =$ **0·06 Wb**

The difference between ϕ_t and ϕ_m is the leakage flux.

$\therefore$ Leakage flux, $\phi_l = \phi_t - \phi_m = 0{\cdot}1 - 0{\cdot}06 =$ **0·04 Wb**

Example 9.27. *Two coils, A of 12,500 turns and B of 16,000 turns, lie in parallel planes so that 60% of flux produced in A links coil B. It is found that a current of 5A in A produces a flux of 0·6 mWb while the same current in B produces 0·8 mWb. Determine (i) mutual inductance and (ii) coupling coefficient.*

Solution. (*i*) Mutual inductance, $M = \dfrac{k\phi_A N_B}{I_A}$

Here $k = 0{\cdot}6$; $\phi_A = 0{\cdot}6$ mWb $= 0{\cdot}6 \times 10^{-3}$ Wb ; $N_B = 16000$; $I_A = 5$A

$$\therefore \quad M = \frac{0.6 \times 0.6 \times 10^{-3} \times 16000}{5} = \mathbf{1{\cdot}15\ H}$$

(*ii*) Now
$$L_A = \frac{N_A \phi_A}{I_A} = \frac{12500 \times 0.6 \times 10^{-3}}{5} = 1500 \times 10^{-3}\text{ H} = 1500\text{ mH}$$

and
$$L_B = \frac{N_B \phi_B}{I_B} = \frac{16000 \times 0 8 \times 10^{-3}}{5} = 2560 \times 10^{-3}\text{ H} = 2560\text{ mH}$$

$$\therefore \quad \text{Coefficient of coupling, } k = \frac{M}{\sqrt{L_A L_B}} = \frac{115 \times 10^{3}}{\sqrt{1500 \times 2560}} = \mathbf{0{\cdot}586}$$

The coefficient of coupling is a measure of how tightly the two coils are coupled. It is a pure number (no units) that varies from 0 to 1. The only way to closely approach $k = 1$ is to wind both coils on the same high-permeability core. This couples them tightly.

Example 9.28. *The coefficient of coupling between two coils is 0.85. Coil 1 has 250 turns. When the current in coil 1 is 2A, the total flux of this coil is 3×10^{-4} Wb. When I_1 is changed from 2A to zer linearly in 2 ms, the voltage induced in the coil 2 is 63.75 V. Find L_1, L_2, M and N_2.*

Solution. Inductance of coil 1, $L_1 = \dfrac{N_1 \phi_1}{I_1} = \dfrac{250 \times 3 \times 10^{-4}}{2} =$ **37.5×10^{-3} H**

e.m.f. induced in coil 2, $e_2 = M\dfrac{dI_1}{dt}$

Here, $e_2 = 63.75$ V ; $dI_1 = 2 - 0 = 2$A ; $dt = 2$ms $= 2 \times 10^{-3}$ s

$$\therefore \quad 63.75 = M \times \frac{2}{2 \times 10^{-3}} \quad \text{or} \quad M = \mathbf{63.75 \times 10^{-3}\ H}$$

Now, $\quad M = k\sqrt{L_1 L_2}$

Here, $\quad M = 63.75 \times 10^{-3}$ H ; $k = 0.85$; $L_1 = 37.5 \times 10^{-3}$ H

$$\therefore \quad 63.75 \times 10^{-3} = 0.85 \times \sqrt{37.5 \times 10^{-3} \times L_2} \quad \text{or} \quad L_2 = \mathbf{150 \times 10^{-3}\ H}$$

Now
$$\frac{L_1}{L_2} = \frac{N_1^2}{N_2^2} \quad \text{or} \quad \frac{37.5 \times 10^{-3}}{150 \times 10^{-3}} = \frac{(250)^2}{N_2^2}$$

$$\therefore \quad 0.25 = \frac{(250)^2}{N_2^2} \quad \text{or} \quad N_2 = \mathbf{500}$$

Example 9.29. *The dimensions of the magnetic core shown in Fig. 9.16 are :*

Cross-sectional area, a = 3 cm^2 ; magnetic path length, l = 10 cm and the relative permeability is 250.

The primary coil has $N_P = 100$ turns and the secondary coil has $N_S = 75$ turns. If the current is increased from 0 to 5A in 0.1s, determine the e.m.f. induced in the secondary.

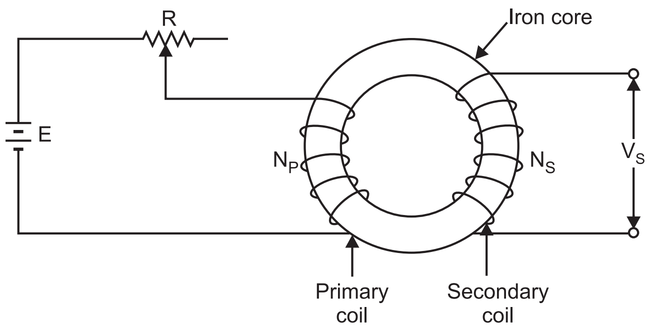


Fig. 9.16

Solution. m.m.f. $= N_P I = 100 \times 5 = 500$ AT

$$\text{Magnetising force, } H = \frac{\text{m.m.f.}}{l} = \frac{500}{10 \times 10^{-2}} = 5000 \text{ AT/m}$$

$$\text{Flux density in core, } B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 250 \times 5000 = 1.57 \text{ Wb/m}^2$$

$$\text{Total flux in core, } \phi = B \times a = 1.57 \times 3 \times 10^{-4} = 471 \times 10^{-6} \text{ Wb}$$

∴ Induced e.m.f. in the secondary is given by ;

$$e_S = N_S \frac{d\phi}{dt} = 75 \times \frac{471 \times 10^{-6}}{0.1} = \mathbf{0.35\ V}$$

Example 9.30. *A long single layer solenoid has an effective diameter of 10 cm and is wound with 2500 T/m. There is a small concenterated coil having its plane lying in the centre cross-sectional plane of the solenoid. Calculate the mutual inductance between the two coils if the concentrated coil has 120 turns on an effective diameter of (i) 8 cm and (ii) 12 cm.*

Solution. Let I_1 be the current flowing through the solenoid.

(*i*) Fig. 9.17 (*i*) shows the conditions of the problem when the effective diameter of concentrated search coil is 8 cm (less than that of the solenoid).

Magnetising force H inside the solenoid is

$$H = \frac{NI_1}{l} = \frac{N}{l} I_1 = 2500\, I_1 \qquad \left(\because \frac{N}{l} = 2500\right)$$

∴ Flux density at the centre of the solenoid is

$$B = \mu_0 H = 2500\, \mu_0 I_1 \text{ Wb/m}^2$$

$$\text{Area of search coil, } a_S = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.08)^2 = 0.005 \text{ m}^2$$

Flux linking with search coil is given by ;

$$\phi_2 = B\, a_S = 2500\, \mu_0 I_1 \times 0.005 = 15.79 \times 10^{-6} I_1 \text{ Wb}$$

$$\therefore \quad M = \frac{N_2 \phi_2}{I_1} = \frac{120 \times 15.79 \times 10^{-6} I_1}{I_1} = \mathbf{1.895 \times 10^{-3}\ H}$$

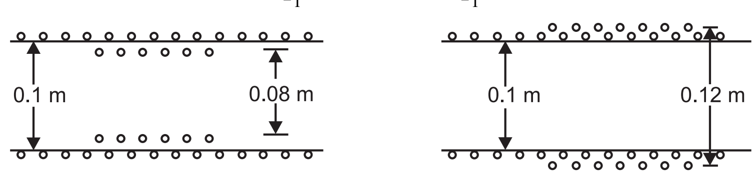


Fig. 9.17

(*ii*) Fig. 9.17 (*ii*) shows the conditions of the problem when the effective diameter of concentrated search coil is 12 cm (*i.e.* more than that of the solenoid). Since the field strength outside the solenoid is negligible, the effective area of search coil will be equal to the area of solenoid *i.e.*

$$a'_S = \frac{\pi}{4}(0.1)^2$$

∴ Flux linking with the search coil is given by ;

$$\phi'_2 = B\,a'_S = 2500\,\mu_0\,I_1 \times \frac{\pi}{4} \times (0.1)^2$$

$$\therefore \quad M = \frac{N_2\phi'_2}{I_1} = 120 \times \frac{2500\mu_0 I_1 \times (\pi/4) \times (0.1)^2}{I_1} = \mathbf{2.962 \times 10^{-3}\ H}$$

Tutorial Problems

1. A solenoid 70 cm in length and of 2100 turns has a radius of 4·5 cm. A second coil of 750 turns is wound upon the middle part of the solenoid. Find the mutual inductance between the two coils. **[18·2 mH]**
2. Two coils having 150 and 200 turns respectively are wound side by side on a closed iron circuit of section 150 cm^2 and mean length of 300 cm. Determine the mutual inductance between the coils and e.m.f. induced in the second coil if current changes from zero to 10A in the first coil in 0·02 second. Relative permeability of iron = 2000. **[0·377 H; 188·5 V]**
3. The self-inductance of a coil of 500 turns is 0·25H. If 60% of the flux is linked with a second coil of 10,000 turns, calculate the mutual inductance between the two coils. **[3 H]**
4. The windings of a transformer has an inductance of L_1 = 6H; L_2 = 0.06 H and a coefficient of coupling k = 0·9. Find the e.m.f. in both the windings when current in primary increases at the rate of 1000 A/s. **[6000 V; 540 V]**
5. An air-cored solenoid with length 30 cm, area of *X*-section 25 cm^2 and number of turns 500 carries a current of 2·5 A. The current is suddenly switched off in a brief time of 10^{-4} second. How much average e.m.f. is induced across the ends of the open switch in the circuit ? Ignore the variation of magnetic field near the ends of the solenoid. **[6·5 V]**

9.14. Inductors in Series

Consider two coils connected in series as shown in Fig. 9.18.

Let L_1 = inductance of first coil

L_2 = inductance of second coil

M = mutual inductance between the two coils

(*i*) Series-aiding. This is the case when the coils are so arranged that their fluxes *aid each other *i.e.* in the same direction as shown in Fig. 9.18 (*i*). Suppose the current is changing at the rate *di/dt*. The total induced e.m.f. in the circuit will be equal to the sum of e.m.f.s induced in L_1 and L_2 *plus* the mutually induced e.m.f.s, *i.e.*

$$e = L_1\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt} \quad \text{... in magnitude}$$

$$= (L_1 + L_2 + 2M)\,di/dt$$

If L_T is the total inductance of the circuit, then,

$$e = L_T\frac{di}{dt}$$

$$\therefore \quad L_T = L_1 + L_2 + 2M \quad \text{... fluxes additive}$$

* **Dot notation.** It is generally not possible to state from the figure whether the fluxes of the two coils are additive or in opposition. Dot notation removes this confusion. The end of the coil through which the current enters is indicated by placing a dot behind it. If the current after leaving the dotted end of coil L_1 enters the dotted end of coil L_2, it means the fluxes of the two coils are additive otherwise in opposition.

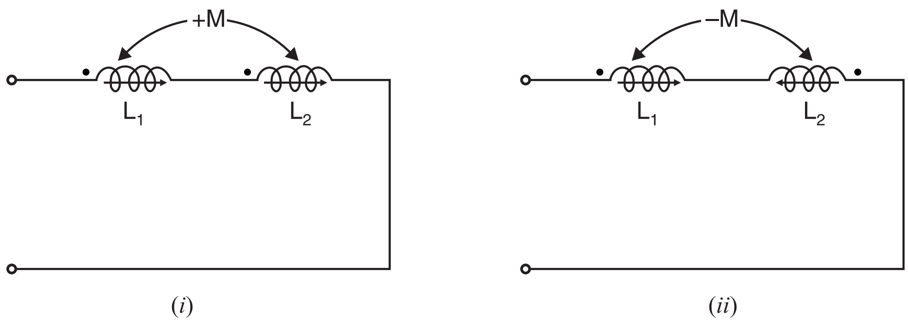


Fig. 9.18

***(ii)* Series-opposing.** Fig. 9.18 (*ii*) shows the series-opposing connection *i.e.* the fluxes of the two coils oppose each other. Suppose the current is changing at the rate *di*/*dt*. The total induced e.m.f. in the circuit will be equal to sum of e.m.f.s induced in L_1 and L_2 *minus* the mutually induced e.m.f.s.

$$e = L_1\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt} - M\frac{di}{dt} = (L_1 + L_2 - 2M)\frac{di}{dt}$$

If L_T is the total inductance of the circuit, then,

$$e = L_T\frac{di}{dt}$$

$$\therefore \quad L_T = L_1 + L_2 - 2M \quad \text{...fluxes subtractive}$$

In general, $\quad L_T = L_1 + L_2 \pm 2M$

Use + sign if fluxes are additive and –ve sign if fluxes are subtractive.

If the two coils are so positioned that *$M = 0$, then, $L_T = L_1 + L_2$.

9.15. Inductors in Parallel with no Mutual Inductance

Consider three inductances L_1, L_2 and L_3 in parallel as shown in Fig. 9.19. Assume that mutual inductance between the coils is zero. Referring to Fig. 9.19, we have,

$$i_T = i_1 + i_2 + i_3$$

or $$\frac{di_T}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt}$$

But $$e = L\frac{di}{dt} \quad or \quad \frac{di}{dt} = \frac{e}{L}$$

$$\therefore \quad \frac{e}{L_T} = \frac{e}{L_1} + \frac{e}{L_2} + \frac{e}{L_3}$$

or $$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \quad ...(i)$$

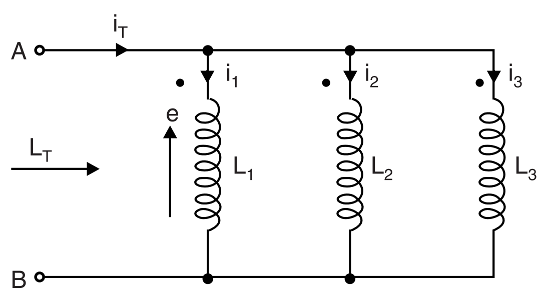


Fig. 9.19

If only two inductors L_1 and L_2 are in parallel, then,

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} = \frac{L_1 + L_2}{L_1 L_2}$$

or $$L_T = \frac{L_1 L_2}{L_1 + L_2} \quad i.e. \quad \frac{\text{Product}}{\text{Sum}}$$

* If the coils are so placed that fluxes produced by them are at right angles to each other, then mutual flux will be zero and hence $M = 0$.

9.16. Inductors in Parallel with Mutual Inductance

Consider two coils A and B of inductances L_1 and L_2 connected in parallel as shown in Fig. 9.20. Let the mutual inductance between the two coils be M. The supply current i divides into two branch currents i_1 and i_2.

By KCL, $i = i_1 + i_2$

$$\therefore \quad \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \qquad ...(i)$$

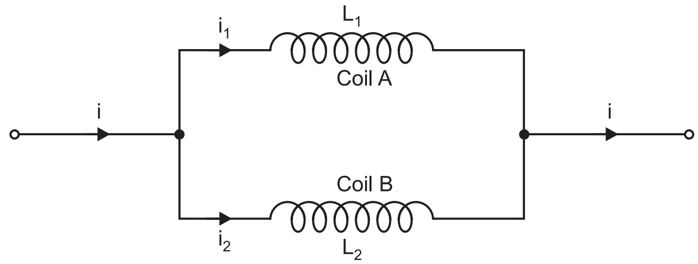


Fig. 9.20

$$\text{Self-induced e.m.f. in coil } A = -L_1 \frac{di_1}{dt}$$

$$\text{Mutually induced e.m.f. in coil } A = -M \frac{di_2}{dt}$$

$$\text{Total e.m.f. induced in coil } A = -\left(L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}\right)$$

$$\text{Similarly, total e.m.f. induced in coil } B = -\left(L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}\right)$$

Since the two coils are in parallel, these e.m.f.s are equal *i.e.*

$$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\text{or} \quad \frac{di_1}{dt}(L_1 - M) = \frac{di_2}{dt}(L_2 - M)$$

$$\therefore \quad \frac{di_1}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right)\frac{di_2}{dt} \qquad ...(ii)$$

Putting this value of di_1/dt in eq. (i), we have,

$$\frac{di}{dt} = \left[\left(\frac{L_2 - M}{L_1 - M}\right) + 1\right]\frac{di_2}{dt} \qquad ...(iii)$$

If L_T is the equivalent inductance of the parallel combination, then,

$$\text{Induced e.m.f.} = -L_T \frac{di}{dt}$$

Since induced e.m.f. in the parallel combination is equal to the e.m.f. induced in any one coil (say coil A),

$$\therefore \quad L_T \frac{di}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\text{or} \quad \frac{di}{dt} = \frac{1}{L_T}\left(L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}\right)$$

Putting the value of di_1/dt from eq. (ii), we have,

$$\frac{di}{dt} = \frac{1}{L_T}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]\frac{di_2}{dt} \qquad ...(iv)$$

From eqs. (*iii*) and (*iv*), we have,

$$\frac{L_2 - M}{L_1 - M} + 1 = \frac{1}{L_T}\left[L_1\left(\frac{L_2 - M}{L_1 - M}\right) + M\right]$$

or
$$\frac{L_1 + L_2 - 2M}{L_1 - M} = \frac{1}{L_T}\left(\frac{L_1L_2 - M^2}{L_1 - M}\right)$$

$\therefore$
$$L_T = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} \quad \text{... when mutual flux aids the individual fluxes}$$

$$= \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} \quad \text{... when mutual flux opposes the individual fluxes}$$

If there is no mutual inductance between the two coils (*i.e.* $M = 0$), then,

$$L_T = \frac{L_1L_2 - (0)^2}{L_1 + L_2 \pm 2(0)} = \frac{L_1L_2}{L_1 + L_2}$$

Example 9.31. *When two coils are connected in series, their effective inductance is found to be 10 H. When the connections of one coil are reversed, the effective inductance is 6 H. If the co-efficient of coupling is 0·6, calculate the self-inductance of each coil and the mutual inductance.*

Solution.
$$10 = L_1 + L_2 + 2M \qquad ...(i)$$
$$6 = L_1 + L_2 - 2M \qquad ...(ii)$$

Subtracting (*ii*) from (*i*), we get, $4 = 4M$ or $M =$ **1 H**

Putting $M = 1$ H in eq. (*i*), we have, $L_1 + L_2 = 8$ ...(*iii*)

Also
$$L_1L_2 = \frac{M^2}{k^2} = \frac{(1)^2}{(0.6)^2} = 2·78 \qquad ...(iv)$$

Now
$$(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1L_2 = (8)^2 - 4 \times 2·78 = 52·88$$

$\therefore$
$$L_1 - L_2 = \sqrt{52.88} = 7·27 \qquad ...(v)$$

Solving eqs. (*iii*) and (*v*), $L_1 =$ **7·635 H** and $L_2 =$ **0·365 H**

Example 9.32. *The total inductance of two coils, A and B, when connected in series, is 0.5 H or 0.2H, depending upon the relative direction of the currents in the coils. Coil A, when isolated from coil B, has a self-inductance of 0.2 H. Calculate (i) the mutual inductance between the two coils, (ii) the self-inductance of coil B, (iii) the coupling factor between the coils, and (iv) the two possible values of the induced e.m.f. in coil A when the current is decreasing at 1000 A/s in the series circuit.*

Solution. ***(i)*** Combined inductance of two coils, $L = L_1 + L_2 \pm 2M$

For series-aiding : $L_1 + L_2 + 2M = 0.5$ ...(*i*)

For series-opposing : $L_1 + L_2 - 2M = 0.2$ ...(*ii*)

Subtracting eq. (*ii*) from eq. (*i*), we have,

$$4M = 0.3 \quad \therefore M = \mathbf{0.075\ H}$$

(ii) Adding eq. (*i*) and eq. (*ii*), we have,

$$2(L_1 + L_2) = 0.7 \quad \text{or} \quad 2(0.2 + L_2) = 0.7 \quad \therefore L_2 = \mathbf{0.15\ H}$$

(iii) Coefficient of coupling is given by ;

$$k = \frac{M}{\sqrt{L_1L_2}} = \frac{0.075}{\sqrt{0.2 \times 0.15}} = \mathbf{0.433\ or\ 43.3\%}$$

(iv)
$$e_1 = L_1\frac{di}{dt} \pm M\frac{di}{dt}$$

$\therefore \qquad e_1 = L_1 \dfrac{di}{dt} + M \dfrac{di}{dt} = 0.2 \times 1000 + 0.075 \times 1000 = \mathbf{275\ V}$

or $\qquad e_1 = L_1 \dfrac{di}{dt} - M \dfrac{di}{dt} = 0.2 \times 1000 - 0.075 \times 1000 = \mathbf{125\ V}$

Example 9.33. *Two mutually coupled coils, A and B, are connected in series to a 360 V d.c. supply. Coil A has a resistance of 6 Ω and inductance 4 H. Coil B has resistance of 11 Ω and inductance 9 H. At a certain instant after the circuit is energised, the current is 10 A and is increasing at the rate of 10 A/s. Calculate (i) the mutual inductance between the coils and (ii) the coefficient of coupling.*

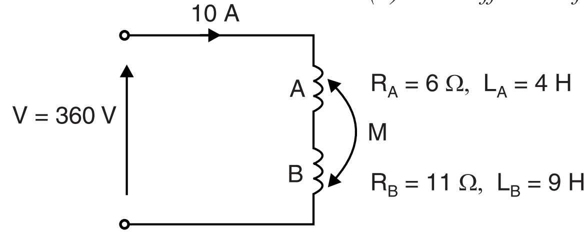


Fig. 9.21

Solution. Fig. 9.21 shows the conditions of the problem.

(*i*) Total circuit resistance, $R_T = R_A + R_B$

$= 6 + 11 = 17\ \Omega$

Total circuit inductance, $L_T = L_A + L_B + 2M$

$= 4 + 9 + 2M = 13 + 2M$

Now $\qquad V = iR_T + L_T \dfrac{di}{dt}$

or $\qquad 360 = 10 \times 17 + (13 + 2M)\ 10 \qquad \therefore \quad M = \mathbf{3\ H}$

(*ii*) Coefficient of coupling, $k = \dfrac{M}{\sqrt{L_A L_B}} = \dfrac{3}{\sqrt{4 \times 9}} = \mathbf{0{\cdot}5}$

Example 9.34. *Two identical coils with terminals, T_1T_2 and T_3T_4 respectively are placed side by side. The inductances measured under different sets of connections are as follows :*

When T_2 is connected to T_3 and inductance measured between T_1 and T_4, it is 4H.

When T_2 is connected to T_4 and inductance measured between T_1 and T_3, it is 0.8 H.

Determine the self inductance of each coil, the mutual inductance between the coils and the coefficient of coupling.

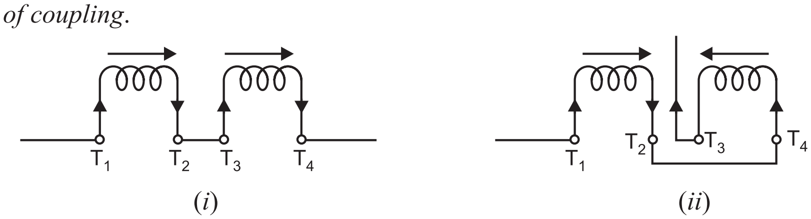


Fig. 9.22

Solution. Since the two coils are identical, each has inductance L (say).

When T_2 is connected to T_3 as shown in Fig. 9.22 (*i*), it is a series-aiding connection so that :

$$L + L + 2M = 4 \quad \text{or} \quad L + M = 2 \qquad ...(i)$$

When T_2 is connected to T_4 as shown in Fig. 9.22 (*ii*), it is a series-opposing connection so that:

$$L + L - 2M = 0.8 \quad \text{or} \quad L - M = 0.4 \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), $L = \mathbf{1.2\ H}$; $M = \mathbf{0.8\ H}$

Coefficient of coupling, $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{0.8}{\sqrt{1.2 \times 1.2}} = \mathbf{0.667}$ or **66.7 %**

Example 9.35. *Find the total inductance of the circuit shown in Fig. 9.23.*

$L_1 = 10\ H$ $M_{12} = 5\ H$

$L_2 = 15\ H$ $M_{23} = 3\ H$

$L_3 = 12\ H$ $M_{13} = 1\ H$

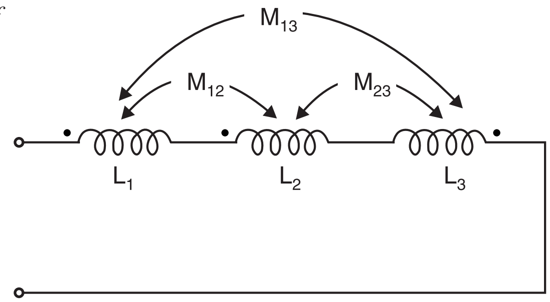


Fig. 9.23

Solution. The fluxes of L_1 and L_2 add to each other and hence M_{12} is positive. The fluxes of L_1 and L_3 are in opposition so M_{13} is negative. Similarly, it can be seen that M_{23} is negative.

$$\therefore \quad L_T = (L_1 + M_{12} - M_{13}) + (L_2 - M_{23} + M_{12}) + (L_3 - M_{23} - M_{13})$$
$$= (10 + 5 - 1) + (15 - 3 + 5) + (12 - 3 - 1)$$
$$= 14 + 17 + 8 = \mathbf{39\ H}$$

Example 9.36. *Fig. 9.24 shows three inductances in series. Find the total inductance of the circuit from the following data :*

$L_1 = 12\ H$ $k_1 = 0{\cdot}33$

$L_2 = 14\ H$ $k_2 = 0{\cdot}37$

$L_3 = 14\ H$ $k_3 = 0{\cdot}65$

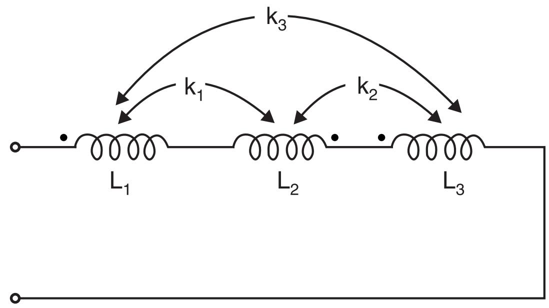


Fig. 9.24

Solution.

$$M_{12} = k_1\sqrt{L_1L_2} = 0.33\sqrt{12\times 14} = 4{\cdot}28\ \text{H}$$

$$M_{23} = k_2\sqrt{L_2L_3} = 0 37\sqrt{14\times 14} = 5{\cdot}18\ \text{H}$$

$$M_{13} = k_3\sqrt{L_1L_3} = 0 65\sqrt{12\times 14} = 8{\cdot}42\ \text{H}$$

$$\therefore \quad L_T = (L_1 - M_{12} + M_{13}) + (L_2 - M_{12} - M_{23}) + (L_3 + M_{13} - M_{23})$$
$$= (12 - 4.28 + 8.42) + (14 - 4.28 - 5.18) + (14 + 8.42 - 5.18)$$
$$= 16{\cdot}14 + 4{\cdot}54 + 17{\cdot}24 = \mathbf{37{\cdot}92\ H}$$

Example 9.37. *Two coils of self-inductances 150 mH and 250 mH and of mutual inductance 120 mH are connected in parallel. Determine the equivalent inductance of the combination if (i) mutual flux helps the individual flux and (ii) mutual flux opposes the individual flux.*

Solution. Here, $L_1 = 0.15$ H ; $L_2 = 0.25$ H ; $M = 0.12$ H

(i) Equivalent inductance L_T of the parallel combination when mutual flux helps the individual flux is

$$L_T = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{0.15\times 0.25 - (0.12)^2}{0.15 + 0.25 - 2\times 0.12} = \mathbf{0.144\ H}$$

(ii) Equivalent inductance L_T of the parallel combination when the mutual flux opposes the individual flux is

$$L_T = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{0.15\times 0.25 - (0.12)^2}{0.15 + 0.25 + 2\times 0.12} = \mathbf{0.036\ H}$$

Example 9.38. *Two coils of inductances 0.3 H and 0.8 H are connected in parallel. If the coefficient of coupling is 0.7, calculate the equivalent inductance of the combination if mutual inductance assists the self-inductance.*

Solution. Here, $L_1 = 0.3$ H ; $L_2 = 0.8$ H ; $k = 0.7$

Mutual inductance M between the two coils is

$$M = k\sqrt{L_1 L_2} = 0.7\sqrt{0.3 \times 0.8} = 0.343 \text{ H}$$

$\therefore$ Equivalent inductance L_T of the combination when mutual inductance assists the self-inductance is

$$L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{0.3 \times 0.8 - (0.343)^2}{0.3 + 0.8 - 2 \times 0.343} = \mathbf{0.2955 \text{ H}}$$

Example 9.39. *Find the equivalent inductance L_{AB} in Fig. 9.25.*

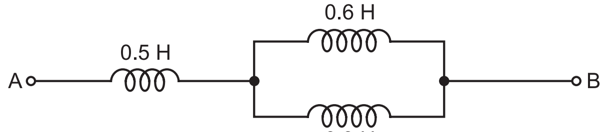


Fig. 9.25

Solution. It is understood that there is no mutual coupling between the coils because it is not given in the problem.

Here, $L_1 = 0.5$ H ; $L_2 = 0.6$H ; $L_3 = 0.3$ H

$$\therefore \qquad L_{AB} = L_1 + \frac{L_2 L_3}{L_2 + L_3} = 0.5 + \frac{0.6 \times 0.3}{0.6 + 0.3} = \mathbf{0.7 \text{ H}}$$

Tutorial Problems

1. The mutual inductance between two coils in a radio receiver is 100 mH. One coil has 100 mH of self-inductance. What is the self-inductance of the other if coefficient of coupling between the coils is 0·5 ? **[400 mH]**
2. The self-inductances of two coils are $L_1 = 150$ mH, $L_2 = 250$ mH. When they are connected in series with their fluxes aiding, their total inductance is 620 mH. When the connection to one of the coils is reversed (they are still in series), their total inductance is 180 mH. How much mutual inductance exists between them ? **[110 mH]**
3. Two coils of self-inductances 5 H and 8 H are connected in series with their fluxes aiding. If the co-efficient of coupling between the coils is 0·45, find the total inductance of the circuit. **[18·06 H]**
4. The self-inductances of three coils are $L_A = 20$ H, $L_B = 30$ H and $L_C = 40$ H. The coils are connected in series in such a way that fluxes of L_A and L_B add, fluxes of L_A and L_C are in opposition and fluxes of L_B and L_C are in opposition. If $M_{AB} = 8$ H, $M_{BC} = 12$ H and $M_{AC} = 10$ H, find the total inductance of the circuit. **[62 H]**

9.17. Energy Stored in a Magnetic Field

In order to establish a magnetic field around a coil, energy is *required, though no energy is needed to ***maintain* it. This energy is stored in the magnetic field and is not used up. When the current is decreased, the flux surrounding the coil is decreased, causing the stored energy to be returned to the circuit. Consider an inductor connected to a d.c. source as shown in Fig. 9.26 (*i*). The inductor is equivalent to inductance L in series with a small resistance R as shown in Fig. 9.26 (*ii*). The energy supplied to the circuit is spent in two ways :

* When the coil is connected to supply, current increases from zero gradually and reaches the final value I (= V/R) after some time. During this change of current, an e.m.f. is induced in L due to the change in flux linkages. This induced e.m.f. opposes the rise of current. Electrical energy must be supplied to meet this opposition. This supplied energy is stored in the magnetic field.

** To impart a kinetic energy of $\frac{1}{2}mv^2$ to a body, energy is required but no energy is required to maintain it at that energy level.

(*i*) A part of supplied energy is spent to meet I^2R losses and cannot be recovered.

(*ii*) The remaining part is spent to create flux around the coil (or inductor) and is stored in the magnetic field. When the field collapses, the stored energy is returned to the circuit.

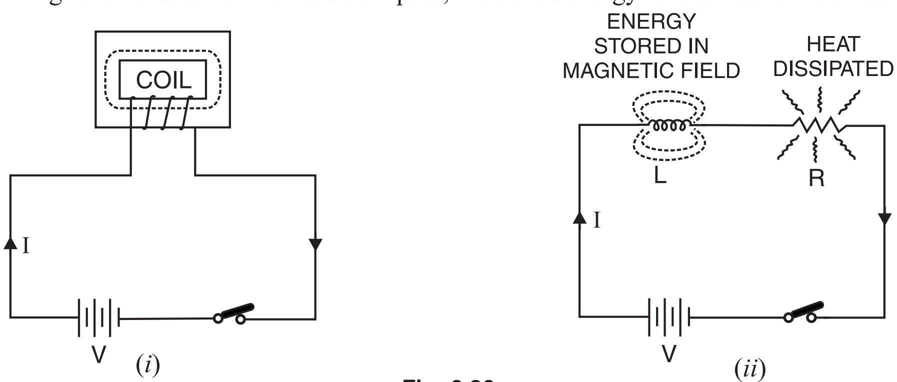


Fig. 9.26

Mathematical Expression. Suppose at any instant the current in the coil is i and is increasing at the rate of di/dt. The e.m.f. e across L is given by ;

$$e = L\frac{di}{dt}$$

$\therefore$ Instantaneous power, $p = ei = Li\frac{di}{dt}$

During a short interval of time dt, the energy dw put into the magnetic field is equal to power multiplied by time *i.e.*

$$dw = p.dt = \left(Li\frac{di}{dt}\right)dt = Li\,di$$

The total energy put into the magnetic field from the time current is zero until it has attained the final steady value I is given by ;

$$W = \int_0^I Li\,di = \frac{1}{2}LI^2$$

$\therefore$ Energy stored in magnetic field, $E = \frac{1}{2}LI^2$ joules

It is clear that energy stored in an inductor depends upon inductance and current through the inductor. For a given inductor, the amount of energy stored is determined by the maximum current through the inductor. Note that energy stored will be in joules if inductance (L) and current (I) are in henry and amperes respectively.

Note. If current in an inductor varies, the stored energy rises and falls in step with the current. Thus, whenever current increases, the coil absorbs energy and whenever current falls, energy is returned to the circuit.

Alternate method. In order to determine the amount of energy an inductor stores, we need to determine inductor's current and voltage during the time it is storing energy. Since the inductor stores energy only during the time the current is increasing, we must determine the average current during the time the current is rising. This can be done by referring to Fig. 9.27 which shows the current in an inductor increasing at a constant rate until it reaches the maximum value I_m. Since the current rises linearly from 0 to I_m, the average value of current is

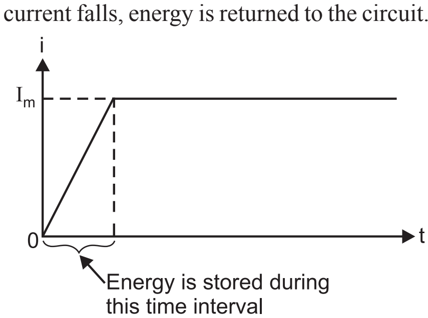


Fig. 9.27

$$I_{av} = \frac{0 + I_m}{2} = 0.5\,I_m$$

The voltage V_L across the inductor during the time it is storing energy is

$$V_L = L\frac{dI}{dt}$$

Since current rises from 0 to I_m linearly, dI/dt remains constant. Therefore, V_L remains constant during the time the current in inductor is increasing. As a result, expression for V_L reduces to :

$$V_L = \frac{LI_m}{t} \qquad (\because dI = I_m \text{ and } dt = t)$$

$\therefore$ Energy stored in the inductor during time t is

$$E = V_L I_{av}\, t = \frac{LI_m}{t} \times 0.5 I_m \times t = 0.5\, I_m^2 L$$

or

$$E = \frac{1}{2} L I_m^2$$

The subscript m is usually dropped so that :

$$E = \frac{1}{2} L I^2$$

Note that I is the final steady current through the inductor. It may be kept in mind that an inductor stores energy in its magnetic field when the current is rising and returns energy to the circuit when the current is falling.

Note. In case of inductors connected in series, the energy stored is given by ;

$$E = \frac{1}{2}(L_1 + L_2 + 2M) I^2 \qquad \text{... series-aiding}$$

$$E = \frac{1}{2}(L_1 + L_2 - 2M) I^2 \qquad \text{... series - opposing}$$

Example 9.40. *A current of 20 mA is passed through a coil of self-inductance 500 mH. Find the magnetic energy stored. If the current is halved, find the new value of energy stored and the energy released back to the electrical circuit.*

Solution. Magnetic energy stored when current is 20 mA is

$$E_1 = \frac{1}{2} L I^2 = \frac{1}{2}(500 \times 10^{-3}) \times (20 \times 10^{-3})^2 = \mathbf{100 \times 10^{-6}\ J}$$

Magnetic energy stored when current becomes 10 mA is

$$E_2 = \frac{1}{2} L I^2 = \frac{1}{2}(500 \times 10^{-3})\, (10 \times 10^{-3})^2 = \mathbf{25 \times 10^{-6}\ J}$$

Magnetic energy released back to the circuit

$$= E_1 - E_2 = (100 - 25) \times 10^{-6} = \mathbf{75 \times 10^{-6}\ J}$$

Example 9.41. *The field winding of a machine consists of 8 coils in series, each containing 1200 turns. When the current is 3A, flux linked with each coil is 20 mWb. Calculate (i) the inductance of the circuit, (ii) the energy stored in the circuit and (iii) the average value of induced e.m.f. if the circuit is broken in 0·1 s.*

Solution.

(*i*) Inductance of each coil, $L = \frac{N\phi}{I} = \frac{1200 \times 20 \times 10^{-3}}{3} = 8\text{H}$

$\therefore$ Total inductance, $L_T = 8\, L = 8 \times 8 = \mathbf{64\ H}$

(*ii*) Magnetic energy stored $= \frac{1}{2} L_T\, I^2 = \frac{1}{2} \times 64 \times 3^2 = \mathbf{288\ J}$

(*iii*) Average e.m.f. induced, $e = L_T \frac{di}{dt} = 64 \times \frac{3 - 0}{0 \cdot 1} = \mathbf{1920\ V}$

Example 9.42. *A coil of inductance 5 H and resistance 100 Ω carries a steady current of 2 A. Calculate the initial rate of fall of current in the coil after a short-circuiting switch connected across its terminals has been suddenly closed. What was the energy stored in the coil and in what form is it dissipated ?*

Solution. The conditions of the problem are represented in Fig. 9.28.

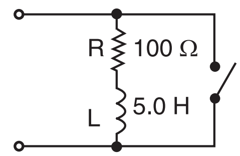


Fig. 9.28

$$V = iR + L\frac{di}{dt}$$

or $$0 = 2 \times 100 + 5\frac{di}{dt}$$

∴ $$\frac{di}{dt} = \frac{-200}{5} = \mathbf{-40\ A/s}$$

Magnetic energy stored in coil $= \frac{1}{2}LI^2 = \frac{1}{2} \times 5 \times (2)^2 = \mathbf{10\ J}$

The stored magnetic energy is dissipated in the form of **heat.**

Example 9.43. *(a) A coil of 100 turns is wound on a toroidal magnetic core having a reluctance of 10^4 AT/Wb. When the coil current is 5A and is increasing at the rate of 200 A/s, determine (i) energy stored in the magnetic circuit and (ii) voltage applied across the coil. Assume coil resistance as zero.*

(b) How are your answers affected if the coil resistance is 2Ω ?

Solution. N = 100 turns ; Reluctance of core, $S = 10^4$ AT/Wb

(*a*) Inductance of coil, $L = \frac{N^2}{S} = \frac{(100)^2}{10^4} = 1$ H

(*i*) Energy stored $= \frac{1}{2}LI^2 = \frac{1}{2} \times 1 \times (5)^2 = \mathbf{12.5\ J}$

(*ii*) Voltage applied across coil = Self-induced e.m.f. in the coil

$$= L\frac{dI}{dt} = 1 \times 200 = \mathbf{200\ V}$$

(*b*) If the coil resistance is 2Ω, the energy stored will remain the same *i.e.,* **12.5 J.**

Voltage across coil $= IR + L\frac{dI}{dt} = 5 \times 2 + 1 \times 200 = \mathbf{210\ V}$

However, there will be a loss of energy $= I^2R = (5)^2 \times 2 = 50$W

Example 9.44. *An iron ring 15 cm in diameter and 10 cm^2 in cross-section is wound with 200 turns of wire. For a flux density of 1 Wb/m^2 and a relative permeability of 500, find the exciting current, the inductance and the stored energy. Find the corresponding quantities when there is a 2 mm air gap.*

Solution. Magnetic flux, $\phi = B \times a = 1 \times (10 \times 10^{-4}) = 10^{-3}$ Wb

Magnetic length, $l = 0{\cdot}15 \times \pi$ m

Now Flux density, $B = \mu_0\, \mu_r\, H$

∴ Magnetising force, $H = \frac{B}{\mu_0 \mu_r} = \frac{1}{(4\pi \times 10^{-7}) \times 500} = 1590$ AT/m

Total ampere-turns $= H \times l = 1590 \times (0{\cdot}15 \times \pi)$ AT

∴ Exciting current, $I = \frac{\text{Total AT}}{N} = \frac{1590 \times (0{\cdot}15 \times \pi)}{200} = \mathbf{3{\cdot}75\ A}$

Inductance, $L = \frac{N\phi}{I} = \frac{200 \times 10^{-3}}{3{\cdot}75} = 53{\cdot}4 \times 10^{-3}$ H = **53·4 mH**

$$\text{Magnetic energy stored} = \frac{1}{2}LI^2 = \frac{1}{2}\times 53{\cdot}4\times 10^{-3}\times(3{\cdot}75)^2 = \mathbf{0{\cdot}375\ J}$$

With 2 mm air gap. The length of air gap, $l_g = 2\text{ mm} = 2 \times 10^{-3}$ m

$$\text{Air gap } AT = H \times l_g = \frac{B}{\mu_0}\times l_g = \frac{1}{4\pi\times 10^{-7}}\times 2\times 10^{-3} = 1590\text{ AT}$$

$$\text{Additional current required} = 1590/200 = 7{\cdot}95\text{ A}$$

∴ Total exciting current, $I_T = 3{\cdot}75 + 7{\cdot}95 = \mathbf{11{\cdot}7\ A}$

$$\text{Inductance, } L = \frac{N\phi}{I_T} = \frac{200\times 10^{-3}}{11{\cdot}7} = 17{\cdot}1\times 10^{-3}\text{ H} = \mathbf{17{\cdot}1\ mH}$$

$$\text{Magnetic energy stored} = \frac{1}{2}LI_T^2 = \frac{1}{2}\times(17{\cdot}1\times 10^{-3})\times(11{\cdot}7)^2 = \mathbf{1{\cdot}17\ J}$$

Example 9.45. *An inductor with 10 Ω resistance and 200 mH inductance is connected across 24 V d.c. source. Calculate (i) energy stored in the inductance, (ii) power dissipated by the resistance and (iii) power dissipated by the inductance.*

Solution. $V = 24$ volts ; $R = 10\ \Omega$; $L = 200$ mH $= 0.2$ H

(i) Final current in inductor, $I = \dfrac{V}{R} = \dfrac{24}{10} = 2.4$ A

Energy stored in inductance $= \dfrac{1}{2}LI^2 = \dfrac{1}{2}\times 0.2\times(2.4)^2 = \mathbf{0.576\ J}$

(ii) Power dissipated by resistor $= I^2R = (2.4)^2 \times 10 = \mathbf{57.6\ W}$

(iii) Power dissipated by inductance = **0 W**

Example 9.46. *A coil of inductance 0.25 H and negligible resistance is connected to a source of supply represented by v = 4 t volts. If the voltage is applied at t = 0 and switched off at t = 5 sec., find (i) the maximum value of current, (ii) r.m.s. value of current and (iii) the energy stored during this period.*

Solution. ***(i)*** $v = 4t$ or $L\dfrac{di}{dt} = 4t$ or $0.25\dfrac{di}{dt} = 4t$

$$\therefore \quad 0.25\int_0^I di = \int_0^5 4t\,dt$$

$$\text{or} \quad 0.25\,I = \left|\frac{4t^2}{2}\right|_0^5 = 50$$

∴ Max. value of current, $I = 50/0{\cdot}25 = \mathbf{200\ A}$

(ii) Suppose i is the current at any time t. Then,

$$0{\cdot}25\,i = \int_0^t 4t\,dt = 2t^2$$

$$\therefore \quad i = 8t^2$$

The sum of squares of current from 0 to 5 sec.

$$= \int_0^5 64t^4 dt = \frac{64\times 5^5}{5} = 64\times 5^4$$

$$\therefore \quad \text{Mean square value} = \frac{64\times 5^4}{5} = 64\times 5^3$$

$\therefore$ R.M.S. value $= \sqrt{64 \times 5^3} =$ **89·5 A**

(iii) Energy stored $= \int_0^5 vi\,dt = \int_0^5 (4t \times 8t^2)\,dt$

$$= \left| \frac{32t^4}{4} \right|_0^5 = \frac{32 \times 5^4}{4} = \textbf{5000 J}$$

Example 9.47. *A direct current of 1 A is passed through a coil of 5000 turns and produces a flux of 0·1 mWb. Assuming that whole of this flux threads all the turns, what is the inductance of the coil ? What would be the voltage developed across the coil if the current were interrupted in 10^{-3} second ? What would be the maximum voltage developed across the coil if a capacitor of 10 µF were connected across the switch breaking the d.c. supply ?*

Solution. Inductance of coil, $L = \frac{N\phi}{I} = \frac{5000 \times 0{\cdot}1 \times 10^{-3}}{1} =$ **0·5 H**

E.M.F. induced in coil, $e = L\frac{dI}{dt} = 0{\cdot}5 \times \frac{1-0}{10^{-3}} =$ **500 V**

When capacitor is connected, the voltage developed will be equal to the p.d. developed across the capacitor plates due to the energy stored in the coil. If V is the value of voltage developed, then,

$$\frac{1}{2}CV^2 = \frac{1}{2}LI^2$$

or $\frac{1}{2} \times (10 \times 10^{-6})V^2 = \frac{1}{2} \times 0{\cdot}5 \times (1)^2$

$\therefore$ $V =$ **2·24 volts**

Tutorial Problems

1. The field winding of a d.c. electromagnet is wound with 960 turns and has resistance of 50 Ω. The exciting voltage is 230 V and the magnetic flux linking the coil is 5 mWb. Find (*i*) self-inductance of the coil and (*ii*) the energy stored in the magnetic field. **[(*i*) 1.043H (*ii*) 11.04 J]**
2. An iron ring of 20 cm mean diameter having a cross-section of 100 cm^2 is wound with 400 turns of wire. Calculate the exciting current required to establish a flux density of 1 Wb/m^2 if the relative permeability of iron is 1000. What is the value of energy stored? **[1.25 A ; 2.5 J]**
3. The inductance of a coil is 0.15H. The coil has 100 turns. Find (*i*) total magnetic flux through the coil when the current is 4A (*ii*) energy stored in the magnetic field (*iii*) voltage induced in the coil when current is reduced to zero in 0.01 second. **[(*i*) 6 mWb (*ii*) 1.2 J (*iii*) 60 V]**
4. An air-cored solenoid has a length of 50 cm and a diameter of 2 cm. Calculate its inductance if it has 1000 turns and also find the energy stored in it if the current rises from zero to 5A. **[0.7 mH ; 8.7 mJ]**

9.18. Magnetic Energy Stored Per Unit Volume

Consider a coil of N turns wound over a magnetic circuit of length l metres and uniform cross-sectional area of 'a' m^2.

Magnetic energy stored $= \frac{1}{2}LI^2 = \frac{1}{2}\left(\frac{N^2 a \mu_0 \mu_r}{l}\right)I^2 = \frac{1}{2}(a\mu_0\mu_r)l\left(\frac{NI}{l}\right)^2$

$$= \frac{1}{2}(\mu_0\mu_r)(a\,l)H^2 \qquad (\because H = NI/l)$$

Now, $al =$ volume of magnetic field in m^3

$\therefore$ Magnetic energy stored/m^3 $= \frac{1}{2}\mu_0\mu_r H^2$

$$= \frac{1}{2}\mu_0\,\mu_r\left(\frac{B}{\mu_0\,\mu_r}\right)^2 \qquad \left[\because H = \frac{B}{\mu_0\mu_r}\right]$$

$$= \frac{B^2}{2\,\mu_0\,\mu_r} \text{ joules} \qquad \text{...in a medium}$$

$$= \frac{B^2}{2\,\mu_0} \text{ joules} \qquad \text{...in air}$$

Note that magnetic energy stored will be in joules if the flux density B is in Wb/m^2.

9.19. Lifting Power of a Magnet

When two opposite polarity magnetic poles are separated by a short distance in air, there is a force of attraction tending to pull the two poles together. The magnitude of this force can be calculated in terms of flux density in the air gap and cross-sectional area of the pole.

Consider two poles, north and south, each of area 'a' square metres separated by a short distance in air as shown in Fig. 9.29. Let P newtons be the force of attraction between the two poles. If one of the poles, say S-pole, is pulled apart through a small distance dx, then work will have to be done against the force of attraction.

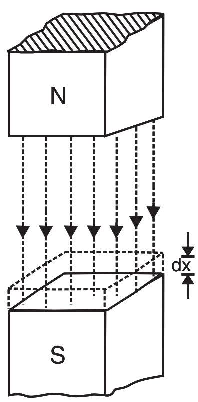


Fig. 9.29

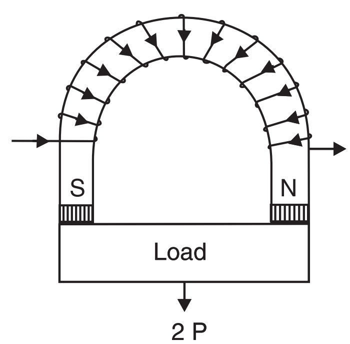


Fig. 9.30

Work done $= P \times dx$ joules

This work done is stored in the additional volume of the magnetic field created.

Additional volume of magnetic field created

$$= a \times dx \text{ m}^3$$

$$\therefore \quad \text{Increase in stored energy} = \frac{B^2}{2\mu_0} \times a\,dx$$

But increase in stored energy = Work done

$$\text{or} \qquad \frac{B^2}{2\mu_0} \times a\,dx = P \times dx$$

$$\therefore \qquad P = \frac{B^2 a}{2\mu_0} \text{ newtons}$$

It may be noted that P will be in newtons if B is in Wb/m^2 and 'a' in m^2.

Note that P is the force of attraction at each pole. In a practical magnet, there are two poles (See Fig. 9.30) so that total force of attraction is $2P$. Electromagnets are widely used for commercial lifting jobs such as loading scrap iron into a truck or raising an armature to a higher position.

Example 9.48. *A lifting magnet of inverted U-shape is formed out of an iron bar 60 cm long and 10 cm² in cross-sectional area [See Fig. 9.31]. Exciting coils of 750 turns each are wound on the two side limbs and are connected in series. Calculate the exciting current necessary for the magnet to lift a load of 60 kg, assuming that the load has negligible reluctance and makes close contact with the magnet. Relative permeability of the material of magnet is 600.*

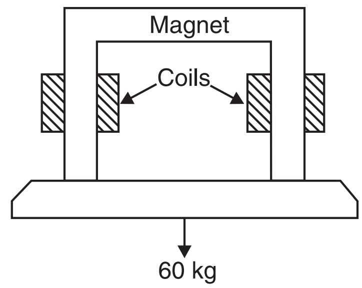


Fig. 9.31

Solution. Attractive force at eact pole is

$$P = \frac{60\times 9.81}{2} = 294.3 \text{ N}$$

Now, $$P = \frac{B^2 a}{2\mu_0}$$

or $$B^2 = \frac{2\mu_0 P}{a}$$

$$= \frac{2\times(4\pi\times 10^{-7})\times 294.3}{10\times 10^{-4}} = 0.74$$

$\therefore$ $$B = \sqrt{0.74} = 0.86 \text{ Wb/m}^2$$

Magnetising force, $$H = \frac{B}{\mu_0\,\mu_r} = \frac{0.86}{4\pi\times 10^{-7}\times 600} = 1141 \text{ AT/m}$$

Length of magnetic path, l = 60 cm = 0·6 m

Total AT required = 0·6 × 1141 = 684·6 AT

Total number of turns = 2 × 750 = 1500

$\therefore$ Exciting current required = 684·6/1500 = **0.456 A**

Example 9.49. *A smooth core armature working in a 4-pole field magnet has a gap (iron to iron) of 0·5 cm. The area of the surface of each pole is 0·1 m². The ampere-turns absorbed by each pole are 3000. Calculate (i) the mechanical force exerted by each pole on the armature and (ii) energy stored in the four air gaps.*

Solution. (*i*) AT per gap = Flux × Reluctance of air gap $= (B\times a)\times\left(\frac{l_g}{a\mu_0}\right) = \frac{B\,l_g}{\mu_0}$

or $$\frac{B}{\mu_0} = \frac{AT \text{ per gap}}{l_g} = \frac{3000}{0\cdot 5\times 10^{-2}} = 6\times 10^5$$

or $$B = \mu_0 \times 6\times 10^5 = (4\pi\times 10^{-7})\times 6\times 10^5 = 0{\cdot}75 \text{ Wb/m}^2$$

Mechanical force exerted by each pole is

$$P = \frac{B^2 a}{2\mu_0} = \frac{(0.75)^2\times 0.1}{2\times 4\pi\times 10^{-7}} = \mathbf{22381\ N}$$

(*ii*) Volume of 4 air gaps $= 4\,a\,l_g = 4\times 0{\cdot}1\times 0{\cdot}5\times 10^{-2} = 0{\cdot}002 \text{ m}^3$

Energy stored in air gaps $= \frac{B^2}{2\mu_0}\times$ Volume of 4 airgaps

$$= \frac{(0.75)^2}{2\times 4\pi\times 10^{-7}}\times 0.002 = \mathbf{448\ J}$$

Example 9.50. *An iron ring having a mean circumference of 30 cm and a cross-sectional area of 1 cm² has two radial saw cuts at diametrically opposite points. A brass plate is inserted in each gap (thickness of each gap being 0·1 mm). If the ring is wound with 200 turns, calculate the magnetising current to exert a pull of 5 kg between the two halves. Assume the magnetic data for the iron to be :*

B (Wb/m²)	*0·79*	*1·0*	*1·3*
H (AT/m)	*250*	*350*	*520*

Solution. The total force of attraction at the two separations is = 5 × 9·80 = 49 N. Therefore, force of attraction at each separation, $P = 49/2 = 24{\cdot}5$ N.

Now, $$P = \frac{B^2 a}{2\mu_0} \quad \therefore \quad B = \sqrt{\frac{2\times 4\pi\times 10^{-7}\times 24{\cdot}5}{1\times 10^{-4}}} = 0{\cdot}79 \text{ Wb/m}^2$$

Corresponding to B = 0·79 Wb/m², we have, H = 250 AT/m.

Length of iron path = 30 cm = 0·3 m

AT for iron path = 250 × 0·3 = 75 AT

H for brass = $B/\mu_0 = 0{\cdot}79/4\pi\times 10^{-7} = 628662$ AT/m

Thickness of brass plates = 0·1 + 0·1 = 0·2 mm = $0{\cdot}2\times 10^{-3}$ m

AT for brass paths = $628662\times 0{\cdot}2\times 10^{-3} = 125{\cdot}73$ AT

Total AT required = 75 + 125·73 = 200·73 AT

∴ Magnetising current required = 200·73/200 = **1 A**

Example 9.51. *The arm of a starter is held in the "ON" position by means of an electromagnet. The torque exerted by the spring is 5 Nm and the effective radius at which the force is exerted is 10 cm. Area of each pole face is 2·5 cm² and each air gap is 0·4 mm. Find the minimum number of AT required to keep the arm in the "ON" position.*

Solution. Fig. 9.32 shows the whole arrangement. Let F newtons be the force exerted by the electromagnet.

Torque = Force × radius

or 5 = $F\times 0.1$ ∴ $F = 5/0.1 = 50$ N

The force exerted at each pole of the magnet, $P = 50/2 = 25$ N

Now $$P = \frac{B^2 a}{2\mu_0}$$

$$\therefore \quad B = \sqrt{\frac{25\times 2\times 4\pi\times 10^{-7}}{2{\cdot}5\times 10^{-4}}} = 0.5 \text{ Wb/m}^2$$

The AT for iron path may be neglected ; being very small.

H in air gap = $B/\mu_0 = 0.5/4\pi\times 10^{-7} = 397887$ AT/m

Total air gap length = $2\times 0.4\times 10^{-3} = 0.8\times 10^{-3}$ m

AT required = $397887\times 0.8\times 10^{-3}$ = **318.3 AT**

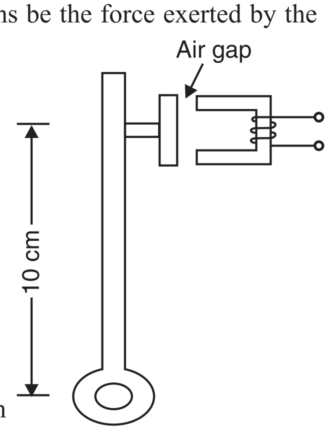


Fig. 9.32

Example 9.52. *The electromagnet shown in Fig. 9.33 has pole pieces each having a cross-sectional area of 25 cm². The total flux crossing each pole is 250 µWb. Determine the maximum weight of iron plate that can be lifted by the magnet. Neglect magnetic leakage and fringing.*

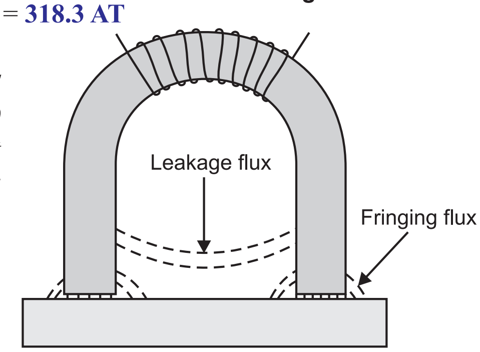


Fig. 9.33

Solution. Flux density in the air gap is

$$B = \frac{\phi}{a} = \frac{250\times 10^{-6}}{25\times 10^{-4}} = 0.1 \text{ Wb/m}^2$$

Attractive force at each pole is

$$P = \frac{B^2 a}{2\mu_0} = \frac{(0.1)^2 \times 25 \times 10^{-4}}{2 \times 4\pi \times 10^{-7}} = 9.95 \text{ N}$$

Total force due to two poles $= 2P = 2 \times 9.95 = 19.9$ N

Let m be the maximum mass of the plate that can be lifted.

$\therefore \quad m \times g = 19.9$

or $\quad m = \frac{19.9}{g} = \frac{19.9}{9.8} = 2.03$ kg

Therefore, maximum weight of the plate that can be lifted is **2.03 kg.**

Tutorial Problems

1. Find the pull exerted on the plunger of an electromagnet when the total flux uniformly distributed is 500 μWb. Diameter of the plunger is 2·54 cm. **[196·3 N]**
2. A horse shoe magnet has two poles, each of area 5 cm^2. Find the pull between the poles and the keeper when the flux density at the contact surface is 1 Wb/m^2. **[398 N]**
3. The core material for use in an electromagnet should not have a flux density more than 1·5 Wb/m^2. How much area each of the two poles should have if the magnet is to lift 200 kg ? **[10·95 cm^2]**
4. A circular crane magnet has an iron cross-section of 200 cm^2 and a mean magnetic path of 80 cm. Assuming the total length of each air gap to be 1·5 mm, calculate (*i*) the *AT* to produce a gap flux of 0·025 Wb (*ii*) the force to separate the contact surface, assuming no leakage or fringing.

B(Wb/m^2)	1·0	1·2	1·4	
H (AT/m)	900	1230	2100	**[(*i*) 4080 AT (*ii*) 2500 kg** (force)**]**

5. In a telephone receiver, the size of each pole of the electromagnet is 1.2 cm × 0.2 cm and flux between each pole and diaphragm is 4 × 10^{-6} Wb. With what force is the diaphragm attracted towards the poles? **[0.532 N]**
6. Magnetic materials having a surface area of 100 cm^2 are in contact with each other. They are in a magnetic circuit of flux 0.01 Wb uniformly distributed across the surface. Calculate the force required to detach the two surfaces. **[3978 N]**
7. Each of the two pole faces of a lifting magnet has an area of 150 cm^2 and this may also be taken as the cross-sectional area of the 40 cm long flux path in the magnet. Determine the *AT* needed on the magnet if it is to lift a 900 kg iron block separated by 0·5 mm from the pole faces. Assume the magnetic leakage factor to be 1·2. Neglect fringing of the gap flux and reluctance of the flux path in the iron block.

H(AT/m)	400	600	800	1200	1600	
B(Wb/m^2)	0·81	0·98	1·1	1·24	1·35	**[955 AT]**

9.20. Closing and Breaking an Inductive Circuit

Consider an inductive circuit shown in Fig. 9.34. When switch S is closed, the current increases gradually and takes some time to reach the final value. The reason the current does not build up *instantly to its final value is that as the current increases, the self-induced e.m.f. in L opposes the change in current (Lenz's Law). Suppose at any instant, the current is i and is increasing at the rate of di/dt.

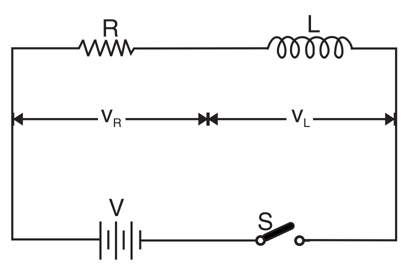


Fig. 9.34

Then, $\quad V = v_R + v_L$

* The current is zero at the instant the switch is closed because it must start from zero.

Now, self-induced e.m.f., $v_L = L\frac{di}{dt}$

If current change (*i.e.di*) is instant, it means $di/dt = 0$. This means that L is infinite which is impossible. So it is not possible for current in inductance to change from one value to the other in zero time.

$$= iR + L\frac{di}{dt}$$

As the current increases, v_R (= iR) increases and v_L decreases since V is constant. The decrease in v_L (= $L\,di/dt$) means that di/dt decreases because L is constant. The result is that after some time, di/dt becomes zero and so does the self- induced e.m.f. v_L (= $L\,di/dt$). At this stage, the current attains the final fixed value I given by ;

$$V = IR + 0 \quad \text{or} \quad I = \frac{V}{R}$$

Thus, when a d.c. circuit containing inductance is switched on, the current takes some time to reach the final value I (= V/R). Note that the role of inductance is to delay the change; it cannot prevent the current from attaining the final value. Similarly, when an inductive circuit is opened, the current does not jump to zero, but falls gradually. In either case, the delay in change depends upon the values of L and R as explained in the next article.

9.21. Rise of Current in an Inductive Circuit

Consider an inductive circuit shown in Fig. 9.34. When switch S is closed, the current rises from zero to the final value I (= V/R) in a small time t. Suppose at any instant, the current is i and is increasing at the rate of di/dt. Then,

$$V = iR + L\frac{di}{dt} \quad \text{or} \quad V - iR = L\frac{di}{dt}$$

or $$\frac{di}{V - iR} = \frac{dt}{L}$$

*Multiplying both sides by $-R$, we get,

$$\frac{-R\,di}{V - iR} = \frac{-R}{L}dt$$

Integrating both sides, we get,

$$\int -\frac{R\,di}{V - iR} = -\frac{R}{L}\int dt$$

or $$\log_e (V - iR) = -\frac{R}{L}t + K \qquad ...(i)$$

where K is a constant whose value can be determined from the initial conditions. At $t = 0$, $i = 0$. Putting these values in exp. (i), we have, $\log_e V = K$.

∴ Equation (i) becomes :

$$\log_e (V - iR) = -\frac{R}{L}t + \log_e V$$

or $$\log_e \frac{V - iR}{V} = -\frac{R}{L}t$$

or $$\frac{V - iR}{V} = e^{-Rt/L}$$

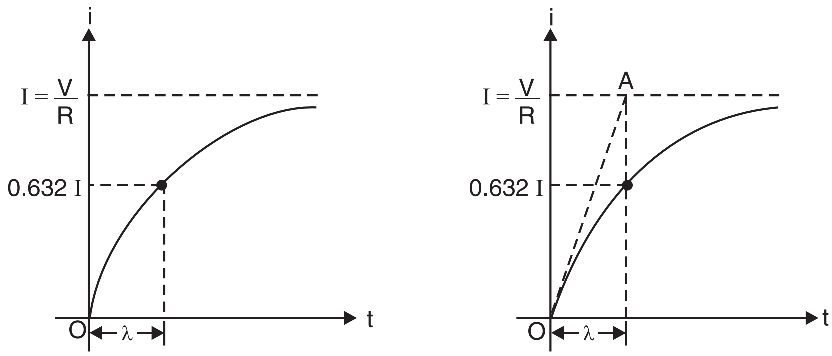
or $$V - iR = V e^{-Rt/L}$$

or $$i = \frac{V}{R}(1 - e^{-Rt/L})$$

But $V/R = I$, the final value of current attained by the circuit.

∴ $$i = I(1 - e^{-Rt/L}) \qquad ...(ii)$$

* This step makes the numerator on the L.H.S. a differential of the denominator.



(*i*) (*ii*)

Fig. 9.35

Eq. (*ii*) shows that rise of current follows an exponential law (See Fig. 9.35). As t increases, the term $e^{-Rt/L}$ gets smaller and current i in the circuit gets larger. Theoretically, the current will reach its final value I (= V/R) in an infinite time. However, practically it reaches this value in a short time.

Note. $V = iR + L\,di/dt$

At the instant the switch is closed, $i = 0$. $\qquad \therefore \quad V = L\,di/dt$

Initial rate of rise of current, $\dfrac{di}{dt} = \dfrac{V}{L}$ A/sec.

The initial rate of rise of current in an inductive circuit helps us in defining the time constant of the circuit.

9.22. Time Constant

Consider the eq. (*ii*) above showing the rise of current w.r.t. time t.

$$i = I\,(1 - e^{-Rt/L})$$

The exponent of e is Rt/L. The quantity L/R has the dimensions of time so that exponent of e (*i.e.* Rt/L) is a number. The quantity L/R is called the *time constant* of the circuit and affects the rise of current in the circuit. It is represented by λ.

$$\therefore \qquad \text{Time constant, } \lambda = L/R \text{ seconds}$$

$$\therefore \qquad i = I(1 - e^{-t/\lambda})$$

Time constant of an inductive circuit can be defined in the following ways :

(*i*) Consider the graph showing the rise of current w.r.t. time t [See Fig. 9.35 (*ii*)]. The initial rate of rise of current (*i.e.* at $t = 0$) in the circuit is

$$\frac{di}{dt} = \frac{V}{L}$$

If this rate of rise of current were maintained, the graph would be linear [*i.e.* OA in Fig. 9.35 (*ii*)] instead of exponential. If this rate of rise could continue, the circuit current will reach the final value I (= V/R) in time

$$= \frac{V}{R} \div \frac{V}{L} = \frac{L}{R} = \text{Time constant } \lambda$$

Hence **time constant** *may be defined as the time required for the current to rise to its final steady value if it continued rising at its initial rate (i.e. V/L).*

(*ii*) If time interval, $t = \lambda$ (or L/R), then,

$$i = I\,(1 - e^{-Rt/L}) = I\,(1 - e^{-1}) = 0{\cdot}632\,I$$

Hence **time constant** *can also be defined as the time required for the current to reach 0·632 of its final steady value while rising.*

Fig 9.36 as well as adjoining table shows the percentage of final current (I) after each time constant interval during current buildup (i) in the inductor. The current will increase to about 63% of its full value (I) in first time constant. A 5 time-constant time interval is accepted as the time for the current to attain its final value I.

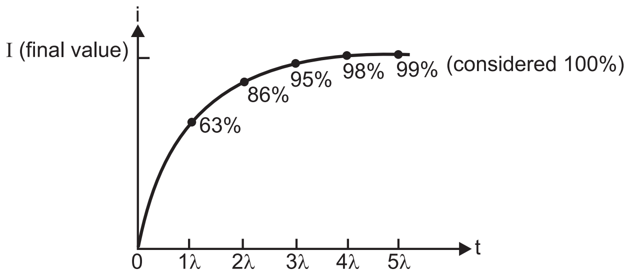


Fig. 9.36

Number of time constants	% of final value
1	63
2	86
3	95
4	98
5	99 (considered 100%)

9.23. Decay of Current in an Inductive Circuit

Consider an inductive circuit shown in Fig. 9.37. When switch S is thrown to position 2, the current in the circuit starts rising and attains the final value I (= V/R) after some time as explained above. If now switch is thrown to position 1, it is found that current in the $R-L$ circuit does not cease immediately but gradually reduces to zero. Suppose at any instant, the current is i and is decreasing at the rate of di/dt. Then,

$$0 = iR + L\frac{di}{dt}$$

or $$\frac{di}{i} = -\frac{R}{L}dt$$

Integrating both sides, we get, $\log_e i = -\frac{R}{L}t + K$ ...(i)

where K is a constant whose value can be determined from the initial conditions. When $t = 0$, then $i = I$ (= V/R).

Putting these values in eq. (i), we have, $\log_e I = 0 + K$ or $K = \log_e I$

∴ Equation (i) becomes : $\log_e i = -\frac{R}{L}t + \log_e I$

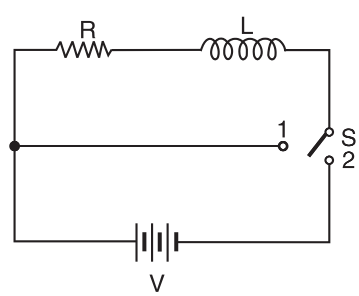


Fig. 9.37

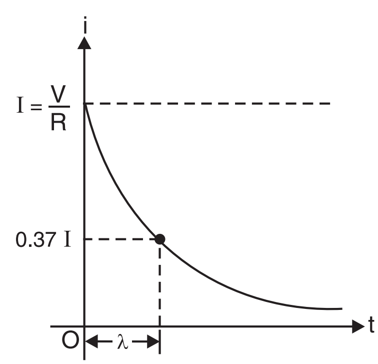


Fig. 9.38

or $$\log_e \frac{i}{I} = -\frac{R}{L}t \quad \text{or} \quad \frac{i}{I} = e^{-Rt/L}$$

∴ $$i = I\,e^{-Rt/L} \quad \text{or} \quad i = I\,e^{-t/\lambda} \qquad ...(ii)$$

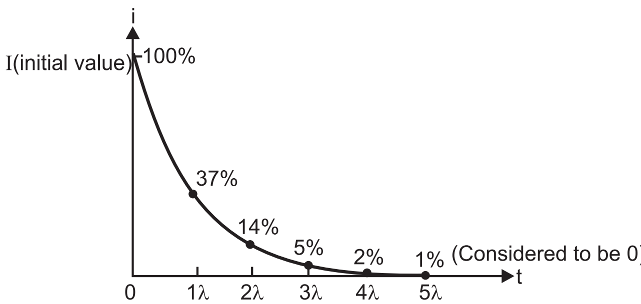
Eq. (*ii*) gives the decay of current in an $R - L$ series circuit with time t and is represented graphically in Fig. 9.38. Note that decay of current follows the exponential law.

Time constant. The quantity L/R in eq. (*ii*) is known as time constant of the circuit. When $t = \lambda\ (= L/R)$,

$$i = I\,e^{-1} = 0{\cdot}37\,I$$

Hence, **time constant** *may also be defined as the time taken by the current to fall to 0·37 of its final steady value I (= V/R) while decaying.*

Fig. 9.39 as well as adjoining table shows the percentage of initial current (*I*) after each time constant interval while the current is decreasing. During the first time constant interval, the current decreases 37% of its initial value. A 5 time-constant interval is accepted as the time for the current to reduce to zero value.



Number of time constants	% of final value
1	37
2	14
3	5
4	2
5	1 (considered 0)

Fig. 9.39

Example 9.53. *The resistance and inductance of a series circuit are 5 Ω and 20 H respectively. At the instant of closing the supply, the current increases at the rate of 4 A/s. Calculate (i) the applied voltage and (ii) the rate of growth of current when the current is 5 A.*

Solution. (*i*) The voltage equation of *R-L* series circuit is

$$V = iR + L\frac{di}{dt}$$

At the instant the switch is closed, $i = 0$.

$$\therefore \quad V = L\frac{di}{dt} = 20 \times 4 = \mathbf{80\ V}$$

(*ii*)

$$V = iR + L\frac{di}{dt}$$

Here $V = 80$ volts ; $i = 5$ A ; $L = 20$ H ; $R = 5\ \Omega$

$$\therefore \quad 80 = 5 \times 5 + 20\frac{di}{dt} \quad \text{or} \quad \frac{di}{dt} = \frac{80 - 25}{20} = \mathbf{2.75\ A/s}$$

An important difference between *RC* and *RL* circuits is the effect of resistance on the duration of the transient. In an *RC* circuit, a large resistance prolongs the transient because it makes the time constant $\lambda\ (= RC)$ large. In an *RL* circuit, a large resistance shortens the transient because it makes time constant $\lambda\ (= L/R)$ small.

Example 9.54. *A constant voltage is applied to a series R – L circuit at t = 0 by closing a switch. The voltage across L is 25 V at t = 0 and drops to 5 V at t = 0.025s. If L = 2H, what must be the value of R ?*

Solution. Applied voltage, $V = iR + L\dfrac{di}{dt}$

At $t = 0$, $i = 0$ and $L\dfrac{di}{dt} = 25$ volts (*given*)

$$\therefore \quad V = 0 + 25 = 25 \text{ volts}$$

At $t = 0.025$ second, $L\dfrac{di}{dt} = 5\text{V}$ so that $iR = 25 - 5 = 20$ V.

Now, $i = I(1 - e^{-t/\lambda}) = \frac{V}{R}(1 - e^{-t/\lambda})$

or $iR = \frac{V}{R} \times R(1 - e^{-t/\lambda})$

$\therefore$ $iR = V(1 - e^{-t/\lambda})$

At $t = 0.025$ second, $iR = 20$ V and $V = 25$ volts.

$\therefore$ $20 = 25(1 - e^{-0.025/\lambda})$

or $1 - e^{-0.025/\lambda} = 0.8$ or $e^{+0.025/\lambda} = 5$

$\therefore$ $\frac{0.025}{\lambda}\log_e e = \log_e 5$ or $\lambda = \frac{0.025}{\log_e 5} = 0.0155$

Now, $\lambda = \frac{L}{R}$ or $R = \frac{L}{\lambda} = \frac{2}{0.0155} = $ **129.03Ω**

Example 9.55. *The steady current flowing in an inductor is 250 mA ; the current flowing 0.1 sec. after connecting the supply voltage is 120 mA. Calculate (i) time constant of the circuit and (ii) the time from closing the circuit at which circuit current has reached 200 mA.*

Solution. (*i*) $i = I(1 - e^{-t/\lambda})$

Here $i = 120$ mA ; $I = 250$ mA ; $t = 0.1$ sec.

$\therefore$ $120 = 250(1 - e^{-0.1/\lambda})$ or $e^{-0.1/\lambda} = 1 - (120/250) = 0{\cdot}52$

$\therefore$ $e^{0.1/\lambda} = 1/0.52 = 1.923$

or $(0.1/\lambda)\log_e e = \log_e 1.923$

$\therefore$ Time constant, $\lambda = \frac{0.1}{\log_e 1.923} = $ **0.153 s**

(*ii*) $i = I(1 - e^{-t/\lambda})$

Here $i = 200$ mA ; $I = 250$ mA ; $\lambda = 0.153$ sec.

$\therefore$ $200 = 250(1 - e^{-t/0.153})$ or $e^{-t/0.153} = 1 - (200/250) = 0.2$

$\therefore$ $e^{t/0.153} = 1/0{\cdot}2 = 5$

or $(t/0{\cdot}153)\log_e e = \log_e 5$

$\therefore$ $t = 0{\cdot}153 \log_e 5 = $ **0·25 s**

Example 9.56. *A coil having $L = 2{\cdot}4$ H and $R = 4\ \Omega$ is connected to a constant 100 V supply source. How long does it take the voltage across the resistance to reach 50 V ?*

Solution. $i = I(1 - e^{-t/\lambda}) = \frac{V}{R}(1 - e^{-t/\lambda})$

or $iR = V(1 - e^{-t/\lambda})$

Here $iR = 50$ volts ; $V = 100$ volts ; $\lambda = L/R = 2{\cdot}4/4 = 0{\cdot}6$ s

$\therefore$ $50 = 100(1 - e^{-t/0.6})$ or $e^{-t/0.6} = 1 - (50/100) = 0{\cdot}5$

$\therefore$ $e^{t/0.6} = 1/0{\cdot}5 = 2$

or $(t/0{\cdot}6)\log_e e = \log_e 2$

$\therefore$ $t = 0{\cdot}6 \log_e 2 = $ **0·416 s**

Example 9.57. *The time constant of a certain inductive coil was found to be 2·5 ms. With a resistance of 80 Ω added in series, a new time constant of 0·5ms was obtained. Find the inductance and resistance of the coil.*

Solution. Time constant, $\lambda = L/R$

For the first case, $L/R = 2{\cdot}5$; For the second case, $L/(R + 80) = 0{\cdot}5$

$\therefore \quad \frac{R+80}{R} = \frac{2\cdot 5}{0\cdot 5} = 5$ or $R = $ **20 Ω**

Now $L/R = 2\cdot 5 \quad \therefore \quad L = 2\cdot 5\, R = 2\cdot 5 \times 20 =$ **50 H**

Example 9.58. *A coil having an effective resistance of 25 Ω and an inductance of 5 H is suddenly connected across a 50 V d.c. supply. What is the rate at which energy is stored in the field of the coil when current is (i) 0·5 A, (ii) 1A and (iii) steady ? Also find the induced EMF in the coil under the above conditions.*

Solution.

(*i*) When current is 0·5 A

Power input = 50 × 0·5 = 25 W ; Power wasted as heat = $i^2R = (0\cdot 5)^2 \times 25 = 6\cdot 25$ W

∴ Rate of energy storage in the field of the coil = 25 – 6·25 = **18·75 W**

(*ii*) When current is 1 A

Power input = 50 × 1 = 50 W ; Power wasted as heat = $(1)^2 \times 25 = 25$ W

∴ Rate of energy stored = 50 – 25 = **25 W**

(*iii*) When current is steady = $V/R = 50/25 = 2$A

Power input = 50 × 2 = 100 W ; Power wasted as heat = $(2)^2 \times 25 = 100$ W

∴ Rate of energy stored = 100 – 100 = **0 W**

Induced e.m.f.

Voltage across coil, $e_L = V - iR$

(*i*) When i = 0·5 A ; $e_L = 50 - 0\cdot 5 \times 25 =$ **37·5 V**

(*ii*) When i = 1 A ; $e_L = 50 - 1 \times 25 =$ **25 V**

(*iii*) When i = 2 A ; $e_L = 50 - 2 \times 25 =$ **0 V**

Example 9.59. *A circuit of resistance R ohms and inductance L henries has a direct voltage of 230 V applied to it. 0.3 second after switching on, the current in the circuit was found to be 5A. After the current had reached its final steady value, the circuit was suddenly short-circuited. The current was again found to be 5A at 0.3 second after short-circuiting the coil. Find the values of R and L.*

Solution. This is a case of growth and decay of current in $R - L$ series circuit. In both cases, i = 5A and t = 0.3 s.

For growth : $i = I(1 - e^{-t/\lambda})$

or $5 = I(1 - e^{-0.3/\lambda})$ ...(*i*)

For decay : $i = I\,e^{-t/\lambda}$

or $5 = I\,e^{-0.3/\lambda}$ ...(*ii*)

From eqs. (*i*) and (*ii*), $I\,e^{-0.3/\lambda} = I(1 - e^{-0.3/\lambda})$

or $2\,e^{-0.3/\lambda} = 1$

$\therefore \quad e^{-0.3/\lambda} = 0.5$ or $\lambda = 0.4328$

Putting $\lambda = 0.4328$ in eq. (*ii*), we get,

$5 = I\,e^{-0.3/0.4328}$ or $I = 5\,e^{+0.3/0.4328} = 5 \times 2 = 10$ A

Now, $I = \frac{V}{R} \quad \therefore R = \frac{V}{I} = \frac{230}{10} =$ **23Ω**

Also, $\lambda = \frac{L}{R}$ or $L = R\lambda = 23 \times 0.4328 =$ **9.95 H**

Example 9.60. *Two mutually coupled coils, A and B, are connected in series to a 400 V d.c. supply. Coil A has a resistance of 14 Ω and inductance of 4 H. Coil B has a resistance of 20 Ω and inductance of 9 H. At a certain instant after the circuit is energised, the current is 5 A and is increasing at the rate of 10 A/s. Calculate (i) the mutual inductance between the coils, and (ii) the coefficient of coupling.*

Solution. Fig. 9.40 shows the conditions of the problem. When not mentioned in the problem, it is understood that the mutual fluxes of the two coils aid each other.

(*i*) $$V = i(R_A + R_B) + L_T \frac{di}{dt}$$

where L_T is the total inductance of the circuit.

or $$400 = 5(14 + 20) + 10\, L_T \quad \therefore \quad L_T = 23 \text{ H}$$

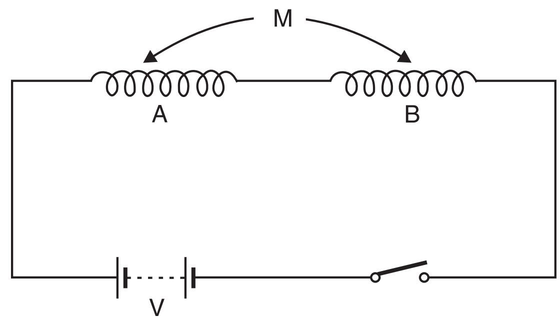
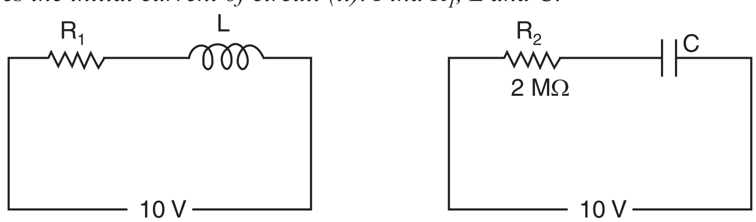


Fig. 9.40

Now, $$L_T = L_A + L_B + 2M \quad \text{or} \quad 23 = 4 + 9 + 2M \quad \therefore \quad M = \mathbf{5\ H}$$

(*ii*) Coefficient of coupling, $$k = \frac{M}{\sqrt{L_A L_B}} = \frac{5}{\sqrt{4 \times 9}} = \mathbf{0{\cdot}83}$$

Example 9.61. *The two circuits of Fig. 9.41 have the same time constant of 0·005 second. With the same d.c. voltage applied to the two circuits, it is found that steady state current of circuit (i) is 2000 times the initial current of circuit (ii). Find R_1, L and C.*



(*i*) (*ii*)

Fig. 9.41

Solution. The time constant for both the circuits is 0·005 s.

$$\therefore \quad R_2 C = 0{\cdot}005 \quad \text{or} \quad C = \frac{0.005}{R_2}$$

$$\therefore \quad C = \frac{0.005}{2 \times 10^6} = 0{\cdot}0025 \times 10^{-6} \text{ F} = \mathbf{0{\cdot}0025\ \mu F}$$

Steady state current in Fig. 9.41 (*i*) = $V/R_1 = 10/R_1$

Initial current in Fig. 9.41 (*ii*) = $V/R_2 = 10/2 \times 10^6 = 5 \times 10^{-6}$ A

As per statement of the problem, we have,

$$10/R_1 = 2000 \times (5 \times 10^{-6}) \quad \therefore \quad R_1 = \mathbf{1000\ \Omega}$$

Now $$L/R_1 = 0{\cdot}005 \quad \therefore \quad L = 1000 \times 0{\cdot}005 = \mathbf{5\ H}$$

Tutorial Problems

1. A 12 V battery is connected in series with 30 Ω resistor and a 220 mH inductor. How long will it take the current to reach half its maximum possible value ? At this instant, at what rate is energy being delivered by the battery ? **[5ms ; 2·4 W]**

2. A p.d. of 100 V is applied to a circuit consisting of a resistance of 50 Ω and an inductance of 5 H. Determine the current in the circuit 0·1 second after the application of the voltage. **[1·264 A]**
3. How many time constants one should wait for the current in an *RL* circuit to grow within 0·1% of its steady value ? **[6·9 time constants]**
4. Calculate the back e.m.f. of a 1 H, 10 Ω coil 0·1 s after 100 V d.c. supply is connected to it. **[36·8 V]**
5. The resistance and inductance of a series circuit are 50 Ω and 20 H respectively. At the instant of closing the supply, the current increases at the rate of 4 A/s. Calculate (*i*) supply voltage (*ii*) the rate of growth of current when current is 5 A. **[(*i*) 80 V (*ii*) 2·75 A/s]**

9.24. Eddy Current Loss

When a magnetic material is subjected to a changing magnetic field, in addition to the hysteresis loss, another loss that occurs in the material is the *eddy current loss*. The changing flux induces voltages in the material according to Faraday's laws of electromagnetic induction. Since the material is conducting, these induced voltages circulate currents within the body of the material. These induced currents do no useful work and are known as eddy currents. These eddy currents develop i^2R loss in the material. Like hysteresis loss, the eddy current loss also results in the rise of temperature of the material. *The hysteresis and eddy current losses in a magnetic material are sometimes called* **core losses** or **iron losses.**

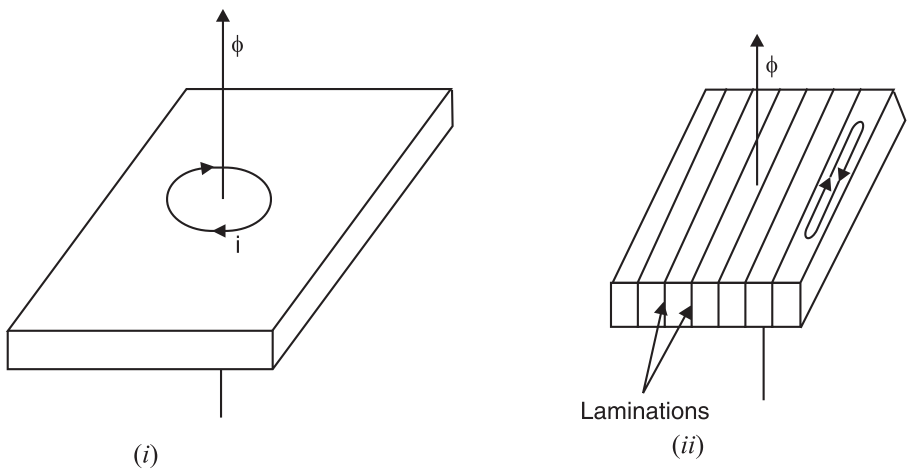


Fig. 9.42

Fig. 9.42 (*i*) shows a solid block of iron subjected to a changing magnetic field. The eddy current power loss in the block will be i^2R where i is the eddy current and R is the resistance to the eddy current path. Since the block is a continuous iron piece of large *X*-section, the magnitude of i will be very *large and hence greater eddy current loss will result. The obvious method of reducing this loss is to reduce the magnitude of eddy current. This can be achieved by splitting the solid block into thin sheets (called **laminations**) in planes parallel to the magnetic flux as shown in Fig.9·42 (*ii*). Each lamination is insulated from the other by a layer of varnish. This arrangement reduces the area of each section and hence the induced e.m.f. It also increases the resistance of eddy current paths since the area through which the currents can pass is smaller. Both effects combine to reduce the eddy current and hence eddy current loss. Further, reduction in this loss can be obtained by using a magnetic material of high resistivity (*e.g.* silicon steel).

The only drawback of laminated core is that the total cross-sectional area of the magnetic material is reduced by the total thickness of the insulation. This is generally taken into account by allowing about 10% reduction in the thickness of core when making the magnetic calculations.

* The large area of the block will have greater e.m.f. induced in it. Larger X-section also means smaller resistance to eddy current path. Both these effects increase the magnitude of eddy current to a great extent.

9.25. Formula for Eddy Current Power Loss

It is difficult to determine the eddy current power loss because the current and resistance values cannot be determined directly. Experiments have shown that eddy current power loss P_e in a magnetic material can be expressed as :

$$P_e = k_e B_m^2 t^2 f^2 V \text{ watts}$$

where k_e = eddy current coefficient and its value depends upon the nature of the material.

B_m = maximum flux density in Wb/m^2

t = thickness of lamination in m

f = frequency of flux in Hz

V = volume of material in m^3

Example 9.62. *The flux in a magnetic core is alternating sinusoidally at 50 Hz. The maximum flux density is 1·5 Wb/m². The eddy current loss then amounts to 140 watts. Find the eddy current loss in the core when the frequency is 75 Hz and the flux density is 1·2 Wb/m².*

Solution. Eddy current power loss, $P_e \propto B_m^2 f^2$

For the first case, $P_{e1} \propto (1{\cdot}5)^2 \times (50)^2$; For the second case, $P_{e2} \propto (1{\cdot}2)^2 \times (75)^2$

$$\therefore \quad \frac{P_{e2}}{P_{e1}} = \left(\frac{1\cdot 2}{1\cdot 5}\right)^2 \times \left(\frac{75}{50}\right)^2 = 1.44$$

$$\therefore \quad P_{e2} = 1{\cdot}44\, P_{e1} = 1{\cdot}44 \times 140 = \mathbf{201{\cdot}6\ W}$$

Example 9.63. *Find the eddy current power loss in a 50 Hz transformer with a maximum flux density of 1 Wb/m². The core is of section 8 cm × 6 cm and total effective length is 50 cm constructed of laminations of thickness 0·4 mm. The eddy current coefficient is 6·58 × 10⁶. Assume a space factor of 0·9.*

Solution. Total core area $= 8 \times 6 = 48\ cm^2 = 48 \times 10^{-4}\ m^2$

*Useful core area $= 0{\cdot}9 \times 48 \times 10^{-4} = 43{\cdot}2 \times 10^{-4}\ m^2$

Volume of iron in core, $V = 43{\cdot}2 \times 10^{-4} \times 0{\cdot}5 = 21{\cdot}6 \times 10^{-4}\ m^3$

Thickness of lamination, $t = 0{\cdot}4\ mm = 0{\cdot}4 \times 10^{-3}\ m$

$$\therefore \quad P_e = k_e B_m^2 t^2 f^2 V \text{ watts}$$

$$= (6{\cdot}58 \times 10^6) \times (1)^2 \times (0{\cdot}4 \times 10^{-3})^2 \times (50)^2 \times 21{\cdot}6 \times 10^{-4} = \mathbf{5{\cdot}68\ W}$$

Example 9.64. *A transformer connected to 25 Hz supply has a core loss of 1500 watts of which 1000 watts are due to hysteresis and 500 watts due to eddy currents. If the flux density is kept constant and frequency is increased to 50 Hz, find the new value of the core loss.*

Solution.

Hysteresis power loss, $P_h \propto B_m^{1\cdot 6} f$

Eddy current power loss, $P_e \propto B_m^2 f^2$

Hysteresis loss

$$\frac{P_{h2}}{P_{h1}} = \left(\frac{B_{m2}}{B_{m1}}\right)^{1\cdot 6} \times \frac{f_2}{f_1} = (1)^{1\cdot 6} \times 50/25 = 2 \qquad (\because B_{m2} = B_{m1})$$

$$\therefore \quad P_{h2} = 2 \times 1000 = 2000\ W$$

* The core of the transformer is laminated to reduce the eddy current loss. The cross-sectional area of iron is now less than the apparent area due to the area taken up by the insulation.

$$\text{Space factor} = \frac{\text{Useful area}}{\text{Total area}}$$

Eddy current loss

$$\frac{P_{e2}}{P_{e1}} = \left(\frac{B_{m2}}{B_{m1}}\right)^2 \times \left(\frac{f_2}{f_1}\right)^2 = (1)^2 \times \left(\frac{50}{25}\right)^2 = 4$$

$\therefore$ $P_{e2} = 4\,P_{e1} = 4 \times 500 = 2000$ W

$\therefore$ New core loss $= P_{h2} + P_{e2} = 2000 + 2000 =$ **4000 W**

Example 9.65. *The core loss in a given specimen is found to be 65 W at a frequency of 30 Hz and a flux density of 1 Wb/m*^{2} *and 190 W at 60 Hz and the same flux density. What are the hysteresis loss and the eddy current loss at each frequency ?*

Solution. Since the flux density, the volume of specimen and the thickness of laminations remain constant, the iron or core loss (= hysteresis loss + eddy current loss) can be written as :

$$\text{Core loss, } P_c = k'_h f + k'_e f^2 \qquad ...(i)$$

where $k'_h = k_h B_m^{1\cdot6} V$ and $k'_e = k_e B_m^2 t^2 V$

Putting the given values in eq. (i), we have,

$$65 = k'_h \times 30 + k'_e \times (30)^2 \qquad ...(ii)$$

$$190 = k'_h \times 60 + k'_e \times (60)^2 \qquad ...(iii)$$

Solving eqs. (ii) and (iii), $k'_h = 1\cdot167$; $k'_e = 0\cdot0333$

At 30 Hz. At 30 Hz, these losses are :

$P_h = k'_h \times 30 = 1\cdot167 \times 30 =$ **35 W**

$P_e = k'_e \times (30)^2 = 0\cdot0333 \times (30)^2 =$ **30 W**

At 60 Hz. At 60 Hz, these losses are :

$P_h = k'_h \times 60 = 1\cdot167 \times 60 =$ **70 W**

$P_e = k'_e \times (60)^2 = 0\cdot0333 \times (60)^2 =$ **120 W**

Objective Questions

1. The basic requirement for inducing e.m.f. in a coil is that ....
   (*i*) flux should link the coil
   (*ii*) there should be change in flux linking the coil
   (*iii*) coil should form a closed loop
   (*iv*) none of the above
2. The e.m.f. induced in a coil is .... the rate of change in flux linkages.
   (*i*) directly proportional to
   (*ii*) inversely proportional to
   (*iii*) independent of
   (*iv*) none of the above
3. The e.m.f. induced in a coil of N turns is given by ....
   (*i*) $d\phi/dt$ (*ii*) $N\,d\phi/dt$
   (*iii*) $-N\,d\phi/dt$ (*iv*) $N\,dt/d\phi$
4. The direction of induced e.m.f. in a conductor (or coil) can be determined by ....
   (*i*) work law
   (*ii*) Ampere's law
   (*iii*) Fleming's right-hand rule
   (*iv*) Fleming's left-hand rule
5. In Fig. 9.43, the conductor is moving upward. The direction of induced e.m.f. is ....

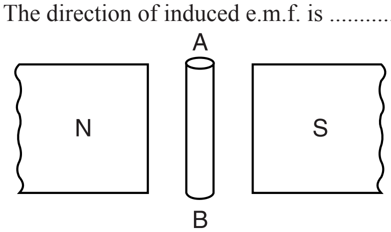


Fig. 9.43

   (*i*) from A to B (*ii*) from B to A
   (*iii*) none of the above
6. In Fig. 9.44, the direction of induced e.m.f. in the conductor A is ....

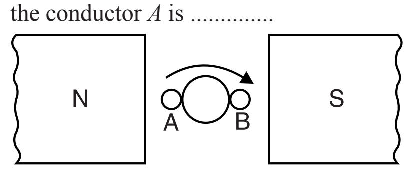


Fig. 9.44

(*i*) into the plane of paper
(*ii*) out of plane of paper
(*iii*) none of the above

7. In Fig. 9.44, the rate of change of flux linkages of conductors *A* and *B* is ....
(*i*) minimum (*ii*) maximum
(*iii*) mid-way between (*a*) and (*b*)
(*iv*) none of the above

8. The e.m.f. induced in a .... is the statically induced e.m.f.
(*i*) d.c. generator (*ii*) transformer
(*iii*) d.c. motor (*iv*) none of the above

9. The e.m.f. induced in a .... is dynamically induced e.m.f.
(*i*) alternator (*ii*) transformer
(*iii*) d.c. generator (*iv*) none of the above

10. In Fig. 9.45, 1 single conductor of length *l* metres moves at right angles to a uniform field of *B* Wb/m^2 with a velocity of *v* m/s. The e.m.f. induced is ....
(*i*) $B\,l/v$ (*ii*) Bv/l
(*iii*) Blv (*iv*) lv/B

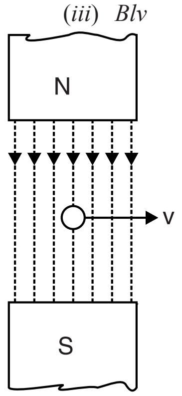


Fig. 9.45

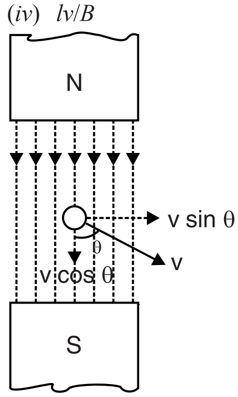


Fig. 9.46

11. In Fig. 9.46, the component of velocity that does not induce any e.m.f. in the conductor is ....
(*i*) $v \sin \theta$ (*ii*) $v \cos \theta$
(*iii*) $v \tan \theta$ (*iv*) none of the above

12. Inductance opposes .... in current in a circuit.
(*i*) only increase (*ii*) only decrease
(*iii*) change (*iv*) none of the above

13. If the number of turns of a coil is increased, its inductance ....
(*i*) remains the same (*ii*) is increased
(*iii*) is decreased (*iv*) none of the above

14. If the relative permeability of the material surrounding the coil is increased, the inductance of the coil ....
(*i*) is increased (*ii*) is decreased
(*iii*) remains unchanged
(*iv*) none of the above

15. Inductance in a circuit ....
(*i*) prevents the current from changing
(*ii*) delays the change in current
(*iii*) causes power loss
(*iv*) causes the current to lead the voltage

16. The inductance of a coil is .... the reluctance of magnetic path.
(*i*) independent of
(*ii*) directly proportional to
(*iii*) inversely proportional to
(*iv*) none of the above

17. If the number of turns of a coil is increased two times, its inductance is ....
(*i*) increased two times
(*ii*) decreased two times
(*iii*) decreased four times
(*iv*) increased four times

18. A circuit has inductance of 2H. If the circuit current changes at the rate of 10 A/second, then self-induced e.m.f. is ....
(*i*) 5 V (*ii*) 0·2 V
(*iii*) 20 V (*iv*) 10 V

19. A current of 2 A through a coil sets up flux linkages of 4 Wb-turn. The inductance of the coil is ....
(*i*) 8 H (*ii*) 0·5 H
(*iii*) 2 H (*iv*) 1 H

20. An air-cored choke is used for .... applications.
(*i*) radio frequency (*ii*) audio frequency
(*iii*) power frequency (*iv*) none of the above

21. If a 10-turn coil has a second layer of 10 turns wound over the first, then total inductance will be about .... the original inductance.
(*i*) two times (*ii*) four times
(*iii*) six times (*iv*) three times

22. An iron-cored coil of 10 turns has reluctance of 100 AT/Wb. The inductance of the coil is ....
(*i*) 1 H (*ii*) 10 H
(*iii*) 0·1 H (*iv*) 5 H

23. An iron-cored coil has an inductance of 2 H. If the reluctance of the magnetic path is 200 AT/Wb, the number of turns on the coil is ....

(*i*) 100 (*ii*) 400
(*iii*) 50 (*iv*) 20

24. The mutual inductance between two coils is .... reluctance of magnetic path.

(*i*) directly proportional to
(*ii*) inversely proportional to
(*iii*) independent of (*iv*) none of the above

25. Mutual inductance between two coils can be decreased by ....

(*i*) increasing the number of turns of either coil
(*ii*) by moving the coils closer
(*iii*) by moving the coils apart
(*iv*) none of the above

26. Mutual inductance between two coils is 4H. If current in one coil changes at the rate of 2 A/second, then e.m.f. induced in the other coil is ....

(*i*) 8 V (*ii*) 2 V
(*iii*) 0·5 V (*iv*) none of the above

27. If in Fig. 9.47, ϕ_{12} = 2 Wb, N_2 = 20 and I_2 = 20 A, then mutual inductance between the coils is ....

(*i*) 200 H (*ii*) 20 H
(*iii*) 4 H (*iv*) 2 H

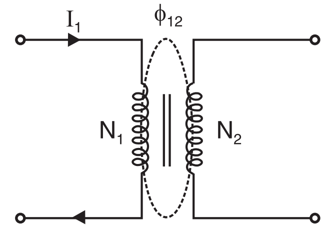


Fig. 9.47

28. If in Fig. 9.47, N_1 = 100, N_2 = 1000 and mutual inductance between the coils is 2H, the reluctance of magnetic circuit is ....

(*i*) 5×10^4 AT/Wb (*ii*) 10^5 AT/Wb
(*iii*) 20 AT/Wb (*iv*) 5 AT/Wb

29. If the coefficient of coupling between two coils is increased, mutual inductance between the coils ....

(*i*) is increased (*ii*) is decreased
(*iii*) remains unchanged
(*iv*) none of the above

30. The maximum mutual inductance between the coils shown in Fig. 9.47 is given by ....

(*i*) $L_A L_B$ (*ii*) L_A / L_B
(*iii*) $\sqrt{L_A L_B}$ (*iv*) $(L_A L_B)^2$

Answers

1. (*ii*)	**2.** (*i*)	**3.** (*iii*)	**4.** (*iv*)	**5.** (*ii*)
6. (*ii*)	**7.** (*ii*)	**8.** (*ii*)	**9.** (*iii*)	**10.** (*iii*)
11. (*ii*)	**12.** (*iii*)	**13.** (*ii*)	**14.** (*i*)	**15.** (*ii*)
16. (*iii*)	**17.** (*iv*)	**18.** (*iii*)	**19.** (*iii*)	**20.** (*i*)
21. (*ii*)	**22.** (*i*)	**23.** (*iv*)	**24.** (*ii*)	**25.** (*iii*)
26. (*i*)	**27.** (*iv*)	**28.** (*i*)	**29.** (*i*)	**30.** (*iii*)

10
Chemical Effects of Electric Current

Introduction

The reader is well acquainted with the passage of electric current through metallic conductors, *e.g.*, copper, aluminium etc. In such conductors, current conduction is due to the movement of free electrons and there is no chemical or physical change except the rise in temperature. However, conduction of current through *certain salt solutions is quite different. Such liquids provide a large number of oppositely charged atoms (called ions) and are known as *electrolytes e.g.*, acids (H_2SO_4, HCl etc.), solutions of inorganic compounds (NaCl, $CuSO_4$, $AgNO_3$ etc.), hydroxides of metals (KOH, NaOH etc). In an electrolyte, conduction is due to the movement of ions (an electrolyte has no free electrons) and chemical changes occur so long as the conduction takes place. Thus passage of electric current through an electrolyte causes chemical changes i.e. electrical energy is converted into chemical energy. The converse of this is also true *i.e.*, we can produce electrical energy from chemical energy. In this chapter, we shall study about the close relationship between electrical energy and chemical energy.

10.1. Electric Behaviour of Liquids

Some liquids conduct current while others do not permit the passage of current through them. On the basis of electrical conductivity, the liquids may be divided into three classes *viz*.

(*i*) Those liquids which do not conduct current are called **insulators** *e.g.,* mineral oils, distilled water etc.

(*ii*) Those liquids which conduct current due to drifting of free electrons are called **conductors** *e.g.,* mercury.

(*iii*) Those liquids which conduct current due to drifting of †ions are known as **electolytes** *e.g.,* solutions of $CuSO_4$, $AgNO_3$ etc. This is the most important class of liquid conductors.

10.2. Electrolytes

A liquid which conducts electric current due to the drifting of ions is called an **electrolyte**.

Salts like silver nitrate ($AgNO_3$), sodium chloride (NaCl), copper sulphate ($CuSO_4$), etc. when dissolved in water dissociate into ions. Their ionic dissociation can be represented as under :

$$AgNO_3 \longrightarrow Ag^+ + NO_3^-$$
$$NaCl \longrightarrow Na^+ + Cl^-$$
$$CuSO_4 \longrightarrow Cu^{++} + SO_4^{--}$$

The atom or group of atoms having positive charge is called a *positive ion*. On the other hand, the atom or group of atoms having negative charge is called a *negative ion*. For example, when NaCl

* Conduction of current is possible only in those liquids which break up into oppositely charged atoms called *ions*. Such liquids are called *electrolytes*. There are, however, many substances (*e.g.,* sugar) which dissolve without splitting into ions. Solutions of these substances do not conduct current and are called non-electrolytes.

† An electrolyte has no free electrons.

is dissolved in water, it splits into positve ions (Na^+) and negative ions (Cl^-). The conduction of current through an electrolyte is due to the drifting of negative and positive ions within the liquid.

10.3. Mechanism of Ionisation

The splitting up of an ionic compound in solution into ions is known as **ionisation** or **ionic dissociation**. Let us take the example of sodium chloride (NaCl). The structure of this solid crystalline salt is made up of Na^+ and Cl^- ions. When in solid state, there is a very strong force of attraction between Na^+ and Cl^- ions which holds them together as a molecule of NaCl. However, when sodium chloride is dissolved in water, the force of attraction between the ions (Na^+ and Cl^-) of sodium chloride molecule is tremendously *reduced due to high permittivity of water (K = 81). In fact, the force of attraction between ions reduces 81 times. The result is that sodium ion (Na^+) and Cl^- ion get separated. This process is called ionisation. It may be noted that as soon as sodium chloride is dissolved in water, ions are formed. In other words, ions are present in an electrolytic solution even before it conducts electric current.

10.4. Electrolysis

The condution of electric current through the solution of an electrolyte together with the resulting chemical changes is called **electrolysis.**

Fig. 10.1 shows the process of electrolysis in a copper voltameter. When copper sulphate ($CuSO_4$) is dissolved in water, it splits up into its components viz. the positive copper ions (Cu^{++}) and negative sulphate ions (SO_4^{--}). This process is called **ionisation. When d.c. voltage is applied across the electrodes, the negative sulphate ions (SO_4^{--}) move towards the anode (+ve electrode) and positive copper ions move towards the cathode (–ve electrode) causing the following chemical changes :

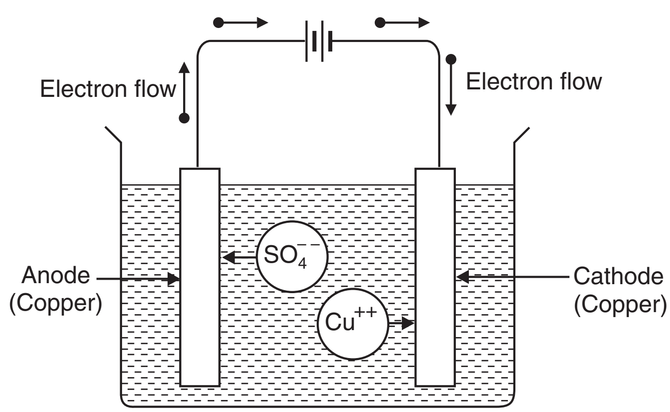


Fig. 10.1

At anode. A sulphate ion (SO_4^{--}) on reaching the anode gives its two extra electrons to it and becomes sulphate radical. These given up electrons continue their journey towards the cathode *via* the external circuit. Now the sulphate radical cannot exist and, therefore, it acts chemically on the anode material to form copper sulphate according to the following reaction :

$$Cu + SO_4 \longrightarrow CuSO_4$$

Thus copper from anode continuously dissolves into the solution so long as this action takes place.

At cathode. At the same time, a copper ion (Cu^{++}) on reaching the cathode takes two electrons from it (these are the same electrons given by the sulphate ion at the anode and have come to cathode

* $F = \dfrac{1}{4\pi\varepsilon_0 K}\dfrac{q_1 q_2}{r^2}$. For air, K = 1 and for water at room temperature, K = 81.

** The reader may recall that copper sulphate is an ionic compound *i.e*,, each molecule of $CuSO_4$ is formed due to the attraction between oppositely charged atoms viz Cu^{++} and SO_4^{--}. When dissolved in water, the force of attraction between them is tremendously reduced due to high relative permittivity of water. The result is that Cu^{++} and SO_4^{--} get separated. These charged atoms (Cu^{++} and SO_4^{--}) are called *ions*.

via the external circuit). The copper ion (Cu^{++}) combines with these two electrons to become copper atom and gets deposited on the cathode.

$$Cu^{++} + 2e \longrightarrow \text{Cu atom}$$

Thus copper from the solution ($CuSO_4$) gets deposited on the cathode.

The following points may be carefully noted :

(*i*) *Electrolysis is possible only if d.c. potential is applied to the electrodes*. It is because we are to attract ions of only one kind to each electrode.

(*ii*) *During electrolysis, either anode material gets deposited over the cathode or gases are liberated at the two electrodes.*

(*iii*) The resulting chemical changes during electrolysis take place so long as the current flows through the electrolyte. When the current through the electrolyte ceases, chemical action also ceases.

10.5. Back e.m.f. or Polarisation Potential

The process of electrolysis is carried out in an apparatus called *voltameter* or *electrolytic cell*. When external d.c. voltage is applied across the electrodes, an e.m.f. is set up between each electrode and the electrolyte which opposes the external d.c. voltage. This opposing e.m.f. is called *back e.m.f.* (E_b) of the electrolyte and is produced due to the coating of electrodes by the products of electrolysis. This effect is called polarisation and for this reason, back e.m.f. is also called *polarisation potential*.

The e.m.f. set up in the voltameter which opposes the external d.c. voltage is called **back e.m.f.** *of the electrolyte*.

The value of back e.m.f. is different for different electrolytes. For acids and alkalies which evolve hydrogen and oxygen, its value is about 1.7 V. For other electrolytes, the value of back e.m.f. depends on the particular salt and generally lies between 0.5 V and 2 V for normal solutions.

Voltage equation for electrolysis. For electrolysis, the applied external d.c. voltage V must overcome the back e.m.f. (E_b) and voltage drop (IR_e) in the electrolyte *i.e.*

$$V = E_b + IR_e \quad \text{... }(i)$$

where V = External d.c. voltage

E_b = Back e.m.f. of electrolyte

I = Circuit current

R_e = Resistance of electrolyte

Therefore, in order to carry out electrolysis at an appreciable rate, the external d.c. voltage V must be atleast equal to $E_b + IR_e$. If the external d.c. voltage is less than this value, electrolysis will not take place.

10.6. Faraday's Laws of Electrolysis

Faraday performed a series of experiments to determine the factors which govern the mass of an element deposited or liberated during electrolysis. He summed up his conclusions into two laws, known as Faraday's laws of electrolysis.

First law. *The mass of an element deposited or liberated at an electrode is directly proportional to the quantity of electricity that passes through the electrolyte*.

If m is the mass of an element deposited or liberated due to the passage of I amperes for t seconds, then according to first law,

$$m \propto Q$$

or $$m \propto It \quad (\because Q = It)$$

or $$m = ZIt \text{ or } m = ZQ$$

where Z is a constant known as *electro-chemical equivalent* (*E.C. E.*) of the element. It has the same value for one element but different for other elements.

If $Q = 1$ coulomb, then, $m = Z$.

Hence **electro-chemical equivalent** (*E.C.E.*) *of an element is equal to the mass of element deposited or liberated by the passage of 1 coulomb of electricity through the electrolyte*. Its unit is gm/C or kg/C.

For example, E.C.E. of copper is 0·000304 gm/C. It means that if 1 coulomb of electricity is passed through a solution of $CuSO_4$, then mass of copper deposited on the cathode will be 0·000304 gm.

The validity of first law is explained by the fact that current inside the electrolyte is carried by the ions themselves. Hence the masses of the chemical substances reaching the anode and cathode are proportional to the quantity of electricity carried by the ions *i.e.*, mass of an ion liberated at any electrode is proportional to the quantity of electricity passed through the electrolyte.

Second law. *The mass of an element deposited or liberated during electrolysis is directly proportional to the chemical equivalent weight of that element i.e.*

$$m \propto \text{Chemical equivalent weight of the element } (E)$$

Faraday's second law is illustrated in Fig. 10.2 where silver and copper voltameters are connected in series. When the same current is passed for the same time through the two voltameters, it will be seen that the masses of silver (Ag) and copper (Cu) deposited on the respective cathodes are in the ratio of 108 : 32. These values of 108 and 32 are respectively the equivalent weights of silver and copper.

$$\frac{\text{Mass of silver deposited}}{\text{Mass of copper deposited}} = \frac{\text{Eq. wt. of Ag}}{\text{Eq. wt. of Cu}} = \frac{108}{32}$$

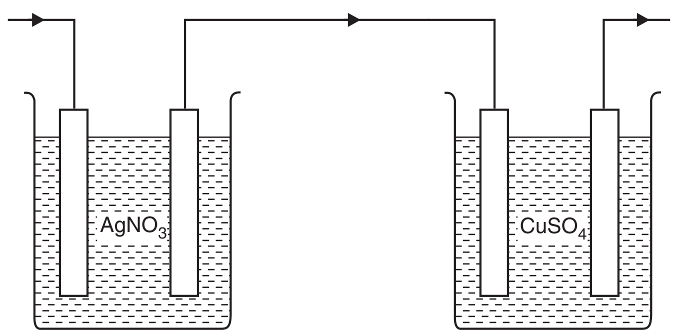


Fig. 10.2

Faraday's second law can be explained as follows. The negative ions (i.e. NO_3^- and SO_4^{--}) from the solutions give up their respective extra electrons to the anodes. These electrons come to cathodes *via* the external circuit and are taken up by the positive ions (Ag^+ and Cu^{++}) to become metallic atoms and get deposited on the respective cathodes. Suppose 10 electrons are flowing in the external circuit. Since silver is monovalent (*i.e.*, its valency is 1), 10 silver ions must be liberated at the cathode of silver voltameter. Again copper is bivalent (*i.e.*, its valency is 2) and hence 5 copper ions must be liberated at the cathode of copper voltameter. This means that mass of an element (silver or copper) liberated is directly proportional to the atomic weight and inversely proportional to the valency of that element *i.e.*

$$\text{Mass liberated, } m \propto \frac{\text{Atomic weight}}{\text{Valency}}$$

$$\propto \text{Chemical equivalent wt. of the element}$$

i.e. $\qquad m \propto E$

10.7. Relation Between E and Z

Suppose the same amount of charge (Q) is passed through the solutions of two electrolytes. If m_1 and m_2 are the masses of the substances liberated/deposited and Z_1 and Z_2 are their electro-chemical equivalents, then,

$$m_1 = Z_1 Q \,;\; m_2 = Z_2 Q$$

$$\therefore \quad \frac{m_1}{m_2} = \frac{Z_1}{Z_2} \quad \text{But} \quad \frac{m_1}{m_2} = \frac{E_1}{E_2} \qquad \text{...Faraday's second law}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{Z_1}{Z_2}$$

$$\text{or} \quad \frac{E}{Z} = \text{Constant}$$

Thus the ratio E/Z is the same for all substances. This constant is called Faraday constant $F(=E/Z)$.

Faraday constant (F). The value of Faraday constant is found to be 96500 C *i.e.* $F = 96500$ C.

Hence **Faraday constant** *is the quantity of charge* (*i.e., 96500 C*) *required to liberate/deposit one gram equivalent (chemical equivalent in gram) of the substance during electrolysis.*

For example, chemical equivalent of silver is 108. When a charge of 96500 C is passed through a silver voltameter, then mass of silver deposited on the cathode will be 108g. Again chemical equivalent of copper is 31·75. If a charge of 96500 C is passed through a copper voltameter, then mass of copper deposited on the cathode will be 31.75 g.

Finding the value of *F*. According to Faraday's first law of electrolysis,

$$m = ZQ$$

Suppose M is the mass of one mole of the substance. If, during electrolysis, the mass of the substance to be deposited is M and p is the valency of the depositing atom, then N_A (= 6.023×10^{23}) atoms will deposit on the electrode.

Now $m = M$ and $Q = N_A pe$

where e = Charge on electron

$$\therefore \quad M = Z N_A pe \qquad \text{...Faraday's first Law}$$

$$\text{or} \quad Z = \frac{1}{N_A e} \cdot \frac{M}{p}$$

$$\text{But} \quad \frac{M}{p} = \frac{\text{Mass of one mole}}{\text{Valency}} = \text{Chemical Equivalent of the substance}$$

$$\therefore \quad Z = \frac{1}{N_A e} E$$

Now, $N_A e$ is a constant, called Faraday constant F.

$$\therefore \quad Z = \frac{E}{F}$$

Now, $1F = N_A e = (6.023 \times 10^{23}) \times 1.602 \times 10^{-19} = 96485 \text{ C} \simeq 96500 \text{ C}$

10.8. Deduction of Faraday's Laws of Electrolysis

Suppose a charge Q is passed through an electrolyte during electrolysis and the mass liberated/deposited on the cathode is m.

$$\therefore \quad m = ZQ$$

$$\text{or} \quad m = \frac{E}{F} Q \qquad \left(\because Z = \frac{E}{F}\right) \qquad \text{...}(i)$$

Equation (*i*) is the **fundamental equation of electrolysis** and contains Faraday's two laws of electrolysis.

(*i*) It is clear from equation (*i*) that :

$$m \propto Q \qquad \text{...Faraday's first law}$$

(*ii*) If the same charge is passed (*i.e.* Q is constant) through different electrolytes during electrolysis, then, $m \propto E$ ...Faraday's second law

Example 10.1. *A current passes through two voltameters in series, one having silver plates and a solution of $AgNO_3$, and the other copper plates and a solution of $CuSO_4$. After the current has ceased to flow, 3·6 gm of silver have been deposited. How much copper will have deposited in the other voltameter ? Take E.C.E. of silver as 0·001118 gm/C and that of copper as $328{\cdot}86 \times 10^{-6}$ gm/C.*

Solution. For silver voltameter, we have,

$$m_1 = Z_1 It$$

or

$$It = \frac{m_1}{Z_1} = \frac{3.6}{0.001118}$$

For copper voltameter, $m_2 = Z_2 It = (328.86 \times 10^{-6}) \times \dfrac{3.6}{0.001118}$ = **1.06 gm**

Example 10.2. *If 16 amperes deposit 12 gm of silver in 9 minutes, how much copper would 10 amperes deposit in 15 minutes? At. wt. of silver = 108 and At. wt. of copper = 63.5.*

Solution. 16A in 9 minutes deposit silver = 12 gm

10A in 15 minutes deposit silver = $12 \times \dfrac{10}{16} \times \dfrac{15}{9}$ = 12.5 gm

Eq. wt. of silver = At. wt./Valency = 108/1 = 108 ; Eq. wt. of copper = 63.5/2 = 31.75

Let m gm be the mass of copper deposited by 10 A in 15 minutes. Then by Faraday's second law of electrolysis,

$$\frac{\text{mass of Cu deposited}}{\text{mass of Ag deposited}} = \frac{\text{Eq. wt. of Cu}}{\text{Eq. wt. of Ag}}$$

or

$$\frac{m}{12.5} = \frac{31.75}{108} \qquad \therefore \quad m = \frac{31.75}{108} \times 12.5 = \mathbf{3.67\ gm}$$

Example 10.3. *A coating of nickel 1 mm thick is to be deposited on a cylinder 2 cm in diameter and 30 cm in length. Calculate the time taken if the current used is 100 A. The following data may be taken. Specific gravity of nickel = 8·9, At. wt. of nickel = 58·7 (divalent), E.C.E. of silver = 1·12 mg/C, At.wt. of silver = 108.*

Solution. Area of curved surface of cylinder = $\pi D \times l = \pi \times 2 \times 30 = 188.5$ cm^2

Volume of Ni to be deposited = Area of curved surface × thickness of Ni

$= 188.5 \times 0.1 = 18.85$ cm^3

Mass of Ni to be deposited, $m = 18.85 \times 8.9 = 167.7$ gm

Eq. wt. of Ni = 58.7/2 = 29.35 ; Eq. wt. of Ag = 108/1 = 108

$$\frac{\text{E.C.E. of Ni}}{\text{E.C.E of Ag}} = \frac{\text{Eq. wt. of Ni}}{\text{Eq. wt. of Ag}} \quad \text{or} \quad \frac{\text{E.C.E of Ni}}{1.12} = \frac{29.35}{108}$$

$$\therefore \quad \text{E.C.E. of Ni} = \frac{29.35}{108} \times 1.12 = 0.304 \text{ mg/C}$$

Now,

$$m = ZIt$$

$$\therefore \quad t = \frac{m}{ZI} = \frac{167.7}{0.304 \times 10^{-3} \times 100} = 5516 \text{ seconds} = \mathbf{91.93\ minutes}$$

Example 10.4. *Find the thickness of copper deposited on a plate area of 0.00025 m^2 during electrolysis if a current of 1 A is passed for 100 minutes. Density of copper = 8900 kg/m^3 and E.C.E. of copper = 32.95 × 10^{-8} kg/C.*

Solution. According to Faraday's law of electrolysis, mass (m) of copper deposited is

$$m = ZIt$$

Here, $Z = 32.95 \times 10^{-8}$ kg/C ; $I = 1$ A ; $t = 100$ min $= 100 \times 60$ sec.

$$\therefore \quad m = 32.95 \times 10^{-8} \times 1 \times 100 \times 60 = 0.001977 \text{ kg}$$

$$\text{Volume of Cu deposited, } v = \frac{\text{Mass}}{\text{Density}} = \frac{0.001977}{8900} = 0.222 \times 10^{-6} \text{ m}^3$$

$$\therefore \text{ Thickness of Cu deposited} = \frac{v}{\text{Plate area}} = \frac{0.222 \times 10^{-6}}{0.00025} = 0.888 \times 10^{-3} \text{ m} = \mathbf{0.888 \text{ mm}}$$

Example 10.5. *An ammeter is being calibrated with the aid of copper voltameter. The ammeter continually reads 2 A when a current is passed through the voltameter for 1 hour. During this time, 2·34 gm of copper was liberated. Taking the electro-chemical equivalent of copper to be 330 × 10^{-9} kg/C, determine the magnitude of error of the ammeter.*

Solution. Let I amperes be the actual current.

Now, $m = ZIt$

Here, $m = 2.34 \times 10^{-3}$ kg ; $Z = 330 \times 10^{-9}$ kg/C ; $t = 1$ hr $= 3600$ s

$$\therefore \quad I = \frac{m}{Zt} = \frac{2.34 \times 10^{-3}}{330 \times 10^{-9} \times 3600} = 1.97\text{A}$$

$\therefore$ Error $= 2 - 1.97 =$ **0.03 A ; *ammeter reads more***

This example shows that the phenomenon of electrolysis can also be used to measure the magnitude of current.

Example 10.6. *Find the mass of zinc which has been dissolved in a simple zinc-copper voltaic cell when 2200 J of energy has been supplied. Assume that electromotive force (e.m.f.) is constant at 1.1 V and that electro-chemical equivalent of zinc is 0·34 × 10^{-6} kg/C.*

Solution. A cell is a device which converts chemical energy into electrical energy. Faraday's law $m = ZQ$ is applicable to cells also but in this case, the mass m refers to the mass dissolved instead of mass liberated. Energy = Volts × Coulombs

or $2200 = 1.1\,Q \quad \therefore \quad Q = 2200/1.1 = 2000$ C

Now, $m = ZQ = (0{\cdot}34 \times 10^{-6}) \times (2000) = 0{\cdot}68 \times 10^{-3}$ kg $= 0{\cdot}68$ g

$\therefore$ Mass of zinc dissolved = **0.68 g**

Example 10.7. *A steady direct current of 100A flows for 5 minutes through fused sodium chloride. How much sodium will be drawn off and how much chlorine will be evolved ? The atomic masses of sodium and chlorine are 23 and 35·5 respectively.*

Solution. $m = E\dfrac{Q}{F}$

Sodium $E_{Na} = \dfrac{23}{1} = 23$; $\dfrac{Q}{F} = \dfrac{100 \times 5 \times 60}{96500} = 0.311$

$\therefore$ $m = 23 \times 0.311 =$ **7.15 g**

Chlorine $E_{Cl} = 35.5/1 = 35.5$; $Q/F = 0.311$ (same as before)

$\therefore$ $m = 35.5 \times 0.311 =$ **11.04 g**

Example 10.8. *A steady current of 10.0 A is passed through a water voltameter for 300 s. Estimate the volume of hydrogen evolved at standard temperature and pressure. Use the known value of Faraday constant. Relative molecular mass of H_2 is 2.016 and molar volume = 22.4 litres (volume of 1 mole of an ideal gas at S.T.P.).*

Solution. $m = Z\,I\,t$

Now, $Z = \dfrac{E}{F} = \dfrac{M}{pF}$ or $m = \dfrac{MIt}{pF}$

Here, $M = 2.016$; $p = 2$; $F = 96500$ C ; $I = 10.0$ A ; $t = 300$ s

$$\therefore \quad m = \frac{2.016 \times 10 \times 300}{2 \times 96500} = 0.0313 \text{ g}$$

$\therefore$ Volume of 0·0313 g of H_2 at STP $= \dfrac{22.4 \times 0.0313}{2.016} =$ **0.35 litres**

Example 10.9. *The potential difference across the terminals of a battery of e.m.f. 12 V and internal resistance 2 Ω drops to 10 V when it is connected to a silver voltameter. Calculate the silver deposited at the cathode in half an hour. Atomic weight of silver is 107.9 g mol^{-1}.*

Solution. E.M.F. of battery, $E = 12$ volts ; Terminal p.d. of battery, $V = 10$ volts ; Internal resistance of battery, $r = 2\ \Omega$; Resistance of voltameter = R.

$$\therefore \quad r = \frac{E - V}{V} \times R \quad \text{or} \quad 2 = \frac{12 - 10}{10} \times R \quad \therefore \; R = 10\ \Omega$$

$$\therefore \quad \text{Circuit current, } I = \frac{E}{R + r} = \frac{12}{10 + 2} = 1\text{A}$$

Now, $E_{Ag} = \dfrac{\text{Atomic weight}}{\text{Valency}} = \dfrac{107.9}{1} = 107.9 \text{ g mole}^{-1}$

Electrochemical equivalent, $Z = \dfrac{E_{Ag}}{F} = \dfrac{107.9}{96500} \text{ g C}^{-1}$

$\therefore$ Mass of silver deposited in half hour ($t = 30 \times 60$ s) is

$$m = ZIt = \frac{107.9}{96500} \times 1 \times 30 \times 60 = \mathbf{2.01\ g}$$

Example 10.10. *A silver and copper voltameters are connected across a 6 V battery of negligible resistance. In half hour, 1 g of copper and 2 g of silver are deposited. Calculate the rate at which energy is supplied by the battery. Given that E.C.E. of Cu is 3294 × 10^{-7} g/C and that of silver is 1118 × 10^{-6} g/C.*

Solution. We know that : $I = \dfrac{m}{Zt}$

For copper voltameter, $I_1 = \dfrac{m_1}{Z_1 t} = \dfrac{1}{3294 \times 10^{-7} \times 1800} = 1{\cdot}687$ A

For silver voltameter, $I_2 = \dfrac{m_2}{Z_2 t} = \dfrac{2}{1118 \times 10^{-6} \times 1800} = 0{\cdot}994$ A

Total current I drawn from the battery is

$$I = I_1 + I_2 = 1{\cdot}687 + 0{\cdot}994 = 2{\cdot}681 \text{ A}$$

Rate at which energy is supplied by the battery is

$$P = VI = 6 \times 2.681 = \mathbf{16.1\ W}$$

Example 10.11. *A refining plant employs 1000 electrolytic cells for copper refining. A current of 5000 A is used and the voltage per cell is 0.25 V. If the plant works for 100 hours/week, determine the annual output of refined copper and the energy consumed in kWh/tonne. The E.C.E. of copper = 1.1844 kg/1000 Ah.*

Solution. Since the voltage drop across each electrolytic cell is less than 1V, a number of cells are connected in series so that the generator can supply current at reasonable voltage.

$$\text{Supply voltage, } V = 0.25 \times 1000 = 250 \text{ volts}$$

$$\text{Circuit current, } I = 5000 \text{ A}$$

$$\text{Plant working time/year, } t = 100 \times 52 = 5200 \text{ hours/year}$$

$$\text{E.C.E. of copper, } Z = 1.1844 \text{ kg/1000 Ah}$$

$$= \frac{1.1844}{1000 \times 3600 \text{As}} = 0.329 \times 10^{-6} \text{ kg/C} \qquad (\because As = C)$$

The amount (m) of refined copper per year is

$$m = ZIt = 0.329 \times 10^{-6} \times 5000 \times 5200 \times 3600 = 30794 \text{ kg}$$

$$= \frac{30794}{1000} \text{tonne} = 30.794 \text{ tonne}$$

$$\text{Energy consumption/year} = VIt = 250 \times 5000 \times 5200 \text{ Wh} = 6500 \times 10^6 \text{ Wh}$$

$$= \frac{6500 \times 10^6}{1000} \text{kWh} = 6500 \times 10^3 \text{ kWh}$$

Since this energy consumption is for refining 30.794 tonne of copper,

$$\therefore \text{ Energy consumption/tonne} = \frac{6500 \times 10^3}{30.794} = \mathbf{211.08 \times 10^3 \text{ kWh/tonne}}$$

Tutorial Problems

1. A current of 5 A flows for 40 minutes through an electrolyte which is a solution of a salt of chromium in water. Calculate the mass of chromium liberated. The electro-chemical equivalent of chromium is 90×10^{-9} kg/C. **[1.08 gm]**
2. How long will it take to deposit, from a copper sulphate solution, a coating of copper 0.05 mm thick on an area of 118 cm^2 if the supply p.d. is 4.5 volts and the total resistance of the circuit is 2.3 Ω. Specific gravity of copper is 8·93 and E.C.E. of copper = 0.329 mg/C. **[2.269 hr]**
3. A metal plate having a surface area of 115 cm^2 is to be silver plated. If a current of 1.5 A is passed for 1 hour and 30 minutes, what thickness of copper will be deposited ? Specific gravity of silver = 10.5 and E.C.E. of silver = 1.118 mg/C. **[0.075 mm]**
4. A worn shaft is to be reconditioned by depositing chromium on its curved surface to a thickness of 0·1 mm. The shaft has a diameter of 3.5 cm and a length of 80 cm. If a current of 4.4 A is passed, calculate how long the plate will take. Density of chromium = 6600 kg/m^3 and E.C.E. of chromium = 90×10^{-9} kg/C. **[41 hours, 44 minutes]**
5. Due to an error, a car battery is overcharged with a current of 5 A for 10 hours. Given that the electro-chemical equivalents of hydrogen and oxygen are 10.4×10^{-9} kg/C and 83.2×10^{-9} kg/C respectively, calculate the volume of distilled water which must be added to compensate for the loss. **[168 c.c.]**

10.9. Practical Applications of Electrolysis

The phenomenon of electrolysis has many industrial and commercial applications. A few of them are discussed below by way of illustration.

(*i*) Electroplating. The process of depositing a thin layer of superior metal (*e.g.*, gold, silver, nickel, etc.) over an inferior metal (*e.g.,* iron) is known as *electroplating*. The aim of electroplating